Analysis of Power Network for Line Reactance Variation to Improve Total Transmission Capacity

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Abstract: The increasing growth in power demand and the penetration of renewable distributed generations in competitive electricity market demands large and flexible capacity from the transmission grid to reduce transmission bottlenecks. The bottlenecks cause transmission congestion, reliability problems, restrict competition, and limit the maximum dispatch of low cost generations in the network. The electricity system requires efficient utilization of the current transmission capability to improve the Available Transfer Capability (ATC). To improve the ATC, power flow among the lines can be managed by using Flexible AC Transmission System (FACTS) devices as power flow controllers, which alter the parameters of power lines. It is important to place FACTS devices on suitable lines to vary the reactance for improving Total Transmission Capacity (TTC) of the network and provide flexibility in the power flow. In this paper a transmission network is analyzed based on line parameters variation to improve TTC of the interconnected system. Lines are selected for placing FACTS devices based on real power flow Performance Index (PI) sensitivity factors. TTC is computed using the Repeated Power Flow (RPF) method using the constraints of lines thermal limits, bus voltage limits and generator limits. The reactance of suitable lines, selected on the basis of PI sensitivity factors are changed to divert the power flow to other lines with enough transfer capacity available. The improvement of TTC using line reactance variation is demonstrated with three IEEE test systems with multi-area networks. The results show the variation of the selected lines’ reactance in improving TTC for all the test networks with defined contingency cases.

Keywords: Available Transfer Capability (ATC); PTDF; FACTS; real power flow Performance Index sensitivity

1. Introduction

In the open electricity market, the transmission networks and especially the European Transmission networks are facing large power flows due to the additional energy transactions for increasing shares of renewable power production. Each participant may try to procure the electrical energy from the cheapest source in order to increase their own profit. This along with some other factors may cause transmission congestion in some parts of the network. The transactions are only possible if there is no bottleneck, which affects the reliability of the power system operation, considering contingency cases. In the ruling of the Federal Energy Regulatory Commission FERC [1], utilities are required to ensure the system reliability by determining their Available Transfer
Capability (ATC) at any instant of time in a competitive market. ATC should be posted on the web at a regular intervals to make it publicly available on Open Access Same-time Information System (OASIS) well before the bid. ATC is defined by the North American Electric Reliability Council (NERC) as the measure of the transfer capability available in the transmission network for other transactions, over and above already committed transactions. Mathematically, ATC is defined as

\[
ATC = TTC - TRM - CBM - ETC
\]

where, the Total Transfer Capability (TTC) is the total amount of electric power that can be transferred over the interconnected transmission network in a reliable manner without violation of specified constraints. Transmission Reliability Margin (TRM) is the amount of transmission transfer capability necessary to ensure that interconnected transmission network is secure under a reasonable range of uncertainties in system conditions. Capacity Benefit Margin (CBM) is the amount of transmission capability reserved by load serving entities to ensure access to generation from interconnected systems to meet generation reliability requirements and Existing Transmission Commitments (ETC) including retail customer service [2].

Determination of TTC is the key component in ATC computation. It is defined as the amount of electric power that can be transferred between two areas of the transmission network without violating the constraints with satisfying a specific set of defined pre- and post-contingency system conditions. TTC is the largest value of transfer power that causes no limit violations, with or without contingency. There are various mathematical based deterministic methods like Continuation Power Flow (CPF) [3], Optimal Power Flow (OPF) [4] and Repeated Power Flow (RPF) method [5], based on the AC power flow method. Sensitivity based methods such as Power Transfer Distribution Factors (PTDFs) [6] or Line Outage Distribution Factors (LODFs) are based on the DC load flow approach, therefore these methods are fast but having less accuracy due to assumptions. Recently in [7] a bi-level optimization framework for the ATC evaluation is proposed in which ATC results can be obtained simultaneously with the ED and ETC in the deregulated electricity market.

In the transmission network, power flow among the lines is not distributed in proportion to their ratings, and also the voltage profile is not smooth in most of the cases. Due to the physical constraints of circuit impedances and phase angles of nodal voltages, most of the lines in high-voltage transmission network are having a line loading far below their thermal rating [8], but violation of one or more lines limit TTC. To improve the TTC, extensive power flow control is required over the lines in interconnected system. Flexible AC Transmission Systems (FACTS) devices are use to dynamically control line reactance, bus voltage magnitude and phase angle, thereby enabling the lines to operate under their thermal ratings [9] and regulate nodal voltages.

The Electric Power Research Institute (EPRI) proposed in 1980s that FACTS not only control line loading of the designated lines, but also could increase the power-transfer capability of transmission network. Therefore, these are the effective alternatives to conventional TTC enhancement methods. FACTS can increase the capacity of individual corridors by up to 80% and, in widespread use, can also increase the overall capacity of a large transmission network by 20% or more [10].

In ATC enhancement, various FACTS devices for controlling power flow, is mostly used by researchers in the last decade. Various methods are devised to investigate their types and locations in the network to improve ATC. Heuristic techniques like Hybrid Particle Swarm Optimization (PSO) used to optimally place multi-type FACTS devices for enhancing power transfer capability [11,12], a dynamic model of Unified Power Flow Controller (UPFC) is developed to improve the power transfer capability in [13], Real-code Genetic Algorithm used as optimization tool to determine the location and control parameters of Thyristor Controlled Series Compensator (TCSC) and Static VAR Compensator (SVC) for ATC enhancement in [14], Multi-type FACTS devices, are optimally sized and located simultaneously for TTC enhancement and improving line congestion through the harmony search algorithm in (HSA) [15], DC load flow based exhaustive analysis of maximum load increase is proposed for Static Synchronous Series Compensator (SSSC) placement to increase ATC to its
maximum in [16], sensitivity analysis based Static Synchronous Compensator (STATCOM) placement for ATC improvement [17], PTDF based locations are selected for the FACTS devices viz. STATCOM, SSSC, and UPFC, which are formulated in optimal power flow problem with an objective function of loss minimization to increase the transmission capability in [18]. Similarly there are many other methods used for various FACTS devices to improve Transmission capacity of the network [19].

The line reactance is one of the most utilized system parameters by FACTS devices, especially the series FACTS devices, it is being proposed to analyze the power network for reactance variation and determine the transmission capability of the system. As all the lines in the network are well below their thermal limits and few lines gets overloading when the power flow is being increased. These overloading lines are the main bottlenecks in transmission networks to facilitate the transaction between the participants in the open access market. This will also limit the transaction of low cost energy from renewable resources [20]. Real power flow Performance Index (PI) [21] is a standard method of measuring line overloading. In this paper the lines are selected based on PI sensitivity factors for FACTS placement. The model of a transmission line with FACTS is simplified by fixed and variable reactance. The reactance of the selected lines, which include FACTS devices are investigated in improving total transfer capability. The proposed algorithm has been demonstrated for three IEEE (24, 30, and 39 bus) test systems.

The paper is structured as follows: Section 2 describes the formulation of Total Transfer Capability computation using the RPF method with a simplified FACTS model. Section 3 explains the method of selecting multiple lines for FACTS placement and real power flow Performance Index (PI) sensitivity factors. Section 4 presents the steps of the procedure to improve Total Transmission Capability of the test networks by varying the lines reactance. The details of the test systems and cases are given in Section 5. Section 6 concludes the paper.

2. Formulation of Total Transfer Capability

TTC is the main component for ATC determination. TTC is the largest power transfer value which causes no line thermal limit or voltage stability limit violation, with and without contingency. For TTC computation between any two areas, It is supposed that there will no change in all other connected areas, because it will affect the TTC value. Several methods for TTC computation have been suggested in the literature [22]. The RPF method is used to calculate TTC based on line thermal limits and voltage stability limits. A few advantages of RPF methods as compared to other methods of TTC computation methods [3], are given as follows:

- The $P – V$ and $V – Q$ curves can be provided by RPF for voltage stability.
- The method for adjusting the control variables is relatively easy in RPF compared to OPF methods.
- The implementation of RPF is much easier than CPF and the convergence time is relatively shorter than for CPF.

2.1. Repeated Power Flow

In the RPF method, conventional power flow equations are solved in each iteration along the specified power transfer directions. The mathematical formulation to calculate TTC using RPF method is expressed as follows:

Maximize $\lambda$

Subject to:

\[
\begin{align*}
    P_Gi - P_{Di} - \sum_{j=1}^{n} |V_i| |V_j| (G_{ij}\cos \delta_{ij} + B_{ij}\sin \delta_{ij}) &= 0 \\
    Q_{Gi} - Q_{Di} - \sum_{j=1}^{n} |V_i| |V_j| (G_{ij}\sin \delta_{ij} - B_{ij}\cos \delta_{ij}) &= 0 \\
    |V_i|_{\min} \leq |V_i| \leq |V_i|_{\max} \\
    S_{ij} \leq S_{ij-\max}.
\end{align*}
\]
where, 
\( \lambda \): Scalar parameter representing the increase in load or generation of the buses 
\( P_{Gi}, Q_{Gi} \): Real and reactive power generation at bus \( i \), 
\( P_{Di}, Q_{Di} \): Real and reactive loads at bus \( i \) 
\( |V_i|, |V_j| \): Voltage magnitude at bus \( i \) and bus \( j \) 
\( \delta_{ij} = \delta_i - \delta_j \): The voltage phase angle difference between bus \( i \) and bus \( j \) 
\( G_{ij} = \frac{r_{ij}}{(r_{ij}^2 + x_{ij}^2)} \) and \( B_{ij} = \frac{-x_{ij}}{(r_{ij}^2 + x_{ij}^2)} \): The real and imaginary parts of the \( ij \)th element of the bus admittance matrix 
\( n \): Total number of buses. 
\( |V_i|_{\text{min}}, |V_i|_{\text{max}} \): Lower and upper limit of the voltage magnitude at bus \( i \) 
\( S_{ij} \): Apparent power flow in line \( ij \) 
\( S_{ij}^{\text{max}} \): Thermal limit of line \( ij \) 

In the power flow equations, the generation and demand are increased by using the following equations:

\[
\begin{align*}
P_{Gi} &= P_{oGi}(1 + \lambda K_{Gi}) \\
P_{Di} &= P_{oDi}(1 + \lambda K_{Di}) \\
Q_{Di} &= Q_{oDi}(1 + \lambda K_{Di})
\end{align*}
\]  

(4)

where, 
\( P_{oGi}, Q_{oDi} \): Initial active and reactive power demand at bus \( i \) in the sink area. 
\( K_{Gi}, K_{Di} \): Constants used to indicate the change rate in the generation and load as \( \lambda \) alters. 

TTC is calculated as follows:

\[
\text{TTC} = \sum_{i=\text{Demands}} P_{Di}(\lambda_{\text{max}}) - \sum_{i=\text{Demands}} P_{Di}^{o}
\]

(5)

where,

\( \sum_{i=\text{Demands}} P_{Di}(\lambda) \) is the total load for \( \lambda = \lambda_{\text{max}} \) 
\( \sum_{i=\text{Demands}} P_{Di}^{o} \) is the total load for \( \lambda = 0 \).

To improve the TTC of the network which is limited by line flow violation or bus voltage violation, FACTS devices control the power flow of the lines by varying line impedance, bus voltage magnitude or phase angle. In the meshed network the power flow is distributed among the lines is due to their physical characteristics. So, by varying the impedance of lines, power flow can be diverted from heavy loaded lines to less loaded lines and thus transmission capability can be improved. The FACTS devices like TCSC, SSSC affect the line impedance and control the power flow through that line. Similarly Distributed FACTS (D-FACTS), proposed by Divan, et al. [23] are small modular types of low cost FACTS devices that provide similar control as TCSC and SSSC.

It is investigated in this paper to analyze the network for varying reactance of multiple lines to improve the transfer capability using FACTS devices. So regardless the type of series FACTS devices, a generalized model of variable reactance is used for FACTS device. In Figure 1 a simplified transmission line is described with fixed line reactance and the FACTS device as variable reactance. So the total reactance of the line having a series FACTS device is modeled as in Equation (6), which describe that the effective reactance of the line become flexible, using FACTS device. The effective reactance of the line can be increased or decreased due the reactance injected by FACTS device.
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Figure 1. Transmission lines with FACTS model.

\[ x_{ij} = x_{ij-\text{line}} + x_k \]
\[ X_{ij-\text{min}} \leq x_k \leq X_{ij-\text{max}} \] (6)

where,
- \( x_{ij} \): Net reactance of line \( ij \)
- \( x_{ij-\text{line}} \): the original reactance of line \( ij \)
- \( x_k \): reactance of Series FACTS
- \( X_{ij-\text{min}} \): lower limit (capacitive reactance)
- \( X_{ij-\text{max}} \): upper limit (inductive reactance).

The power flow equation in Equation (2) is changed by including the FACTS model in the system, given as follows:

\[ \begin{align*}
\text{Maximize } & \lambda \\
\text{Subject to: } & \\
& \{ P_{Gi} - P_{Di} - \sum_{j \neq i} V_j [\Delta G_{ij}\cos\delta_{ij} + \Delta B_{ij}\sin\delta_{ij}] = 0 \\
& Q_{Gi} - Q_{Di} - \sum_{j \neq i} V_j [\Delta G_{ij}\sin\delta_{ij} - \Delta B_{ij}\cos\delta_{ij}] = 0 \}
\end{align*} \] (7)

where,
\[ \Delta G_{ij} = \frac{-r_{ij}x_k(2x_{ij}+x_k)}{(r_{ij}^2+x_{ij}^2)(r_{ij}^2+x_{ij}^2+x_{ij}^2)} \] and \( \Delta B_{ij} = \frac{-x_k(r_{ij}^2-x_{ij}^2+x_{ij}x_k)}{(r_{ij}^2+x_{ij}^2)(r_{ij}^2+x_{ij}^2+x_{ij}^2)} \), \( -X_{ij-\text{min}} \leq x_k \leq X_{ij-\text{max}} \)

Inductive reactance of FACTS are required to those lines where power flows are exceeding their thermal limits, in order to reduce the power flow by increasing their impedances. Similarly capacitive reactance of FACTS are required to increase the power flow over those lines where power flow are well below their thermal limits. To check the maximum variation in reactance of the selected lines for improving the transmission capacity, the upper and lower limits for FACTS are supposed. In this work the upper and lower limits for FACTS is supposed to be the reactance of that line. It means the reactance of the lines with FACTS will vary between 0 to 2\( X_l \). Now the power network is analyzed to choose the lines for FACTS placement which can also impact on the power flow of other lines. Sensitivity factors of real power flow PI are used to select multiple lines for FACTS placement.

3. Method for Multiple Locations Selection of FACTS Devices

In order to improve the total transfer capability various methods are proposed in literature [19] for FACTS placement. In this paper sensitivity of real power flow Performance Index (PI) factors are used to select multiple lines for FACTS placement. The power flow could be diverted from the heavily loaded lines to other parallel lines having less loadings by varying their reactances.

3.1. System Performance Index for Real Power Flow Analysis

The severity of the system loading in normal as well as contingency cases can be measure using real power Performance Index [24]. It can be defined as

\[ PI = \sum_{m=1}^{N_l} \frac{w_m}{2\pi} \left( \frac{P_{lm}}{P_{\text{max}_m}} \right)^2 \] (8)

where,
- \( P_{lm} \): real power flow on line \( m \),
$P_{lm}^{\text{max}}$: $m$th line rated capacity
$z$: specified exponent ($z = 2$ preferred)
$N_L$: total number of lines
$w_m$: non negative real weighting coefficient used to reflect the importance of lines ($w_m = 1$).

The real power flow Performance Index $PI$ contains all the line flows, normalized by their thermal limits. The value of the $PI$ is small when all the lines are under their limits and reaches a high value, when any line is overloaded. So it can measure the line overloading of the system for the given state of a power system, but different cases of thermal violation could not be discriminated based on it. For example it can’t discriminate between one large violation and many small violation cases. Which can be avoided to some extent by using high order performance indices, i.e., $z > 1$. In this paper the exponent value is taken to be 2, as proposed in [25].

3.1.1. PI Sensitivity Factors

The real power flow $PI$ sensitivity factor is defined for series FACTS parameter as,

$$b_k = \left. \frac{\partial P_{lm}}{\partial x_k} \right|_{x_k = 0} = \text{PI Sensitivity w.r.t. series FACTS parameter } x_k \text{ placed at line } k$$

(9)

$$b_k = \sum_{m=1}^{N_L} w_m P_{lm}^3 \left( \frac{1}{P_{lm}^{\text{max}}} \right)^4 \frac{\partial P_{lm}}{\partial x_k}$$

(10)

The real power flow $P_{lm}$ on $m^{th}$ line can be described in terms of real power injection using DC power flow equations.

$$P_{lm} = \begin{cases} \sum_{n=1,n\neq s}^{N_B} S_{mn} P_n & \text{for } m \neq k \\ \sum_{n=1,n\neq s}^{N_B} S_{mn} P_n + P_j & \text{for } m = k \end{cases}$$

(11)

where,

$s$ is index of the slack bus, $S_{mn}$ is the $mn$th element of $[S]$ matrix (given in Appendix A) which relates line flow with bus injections, $N_B$ is the number of buses, $k$ is the line containing the FACTS device from bus $i$ to bus $j$ and $P_j$ is additional flow to bus $j$ due to the FACTS on the line.

$$\frac{\partial P_{lm}}{\partial x_k} = \begin{cases} \left( S_{mi} \frac{\partial P_i}{\partial x_k} + S_{mj} \frac{\partial P_j}{\partial x_k} \right) & \text{for } m \neq k \\ \left( S_{mi} \frac{\partial P_i}{\partial x_k} + S_{mj} \frac{\partial P_j}{\partial x_k} \right) + \frac{\partial P_j}{\partial x_k} & \text{for } m = k \end{cases}$$

(12)

Active power injection at bus $i$ and $j$ due to FACTS device

$$P_i = V_i^2 \Delta G_{ij} - V_i \sum_{j \neq i}^{N_B} V_j \left[ \Delta G_{ij} \cos \delta_{ij} + \Delta B_{ij} \sin \delta_{ij} \right]$$

$$P_j = V_j^2 \Delta G_{ij} - V_j \sum_{i \neq j}^{N_B} V_i \left[ \Delta G_{ij} \cos \delta_{ij} - \Delta B_{ij} \sin \delta_{ij} \right]$$

(13)

Now differentiate Equation (13) with respect to $x_k$

Suppose,

$$\frac{\partial \Delta G_{ij}}{\partial x_k} \bigg|_{x_k = 0} = \frac{-2V_i V_j \delta_{ij}}{(r_i^2 + x_i^2)} = 2G_{ij} B_{ij} = a$$

$$\frac{\partial \Delta B_{ij}}{\partial x_k} \bigg|_{x_k = 0} = \frac{V_i^2 - V_j^2}{(r_i^2 + x_i^2)} = B_{ij}^2 - G_{ij}^2 = b$$

$$\frac{\partial P_i}{\partial x_k} = V_i^2 \left( \frac{\partial \Delta G_{ij}}{\partial x_k} \right) - V_i V_j \left( \frac{\partial \Delta G_{ij}}{\partial x_k} \right) \cos \delta_{ij} - V_i V_j \left( \frac{\partial \Delta B_{ij}}{\partial x_k} \right) \sin \delta_{ij}$$

$$= 2G_{ij} B_{ij} V_i^2 - V_i V_j \left( 2G_{ij} B_{ij} \cos \delta_{ij} + (B_{ij}^2 - G_{ij}^2) \sin \delta_{ij} \right)$$

$$= a V_i^2 - V_i V_j (a \cos \delta_{ij} + b \sin \delta_{ij})$$

(14)
Decrease the power transfer until no constraint is violated, calculate the TTC for the system without FACTS. If any constraint is violated, the RPF stops and TTC is calculated for the system without FACTS. The reactances of the overloaded lines as well as the reactances of selected lines are varied, until the power flow of all the lines are within their thermal limits or the power flow of overloaded lines couldn’t be reduced by reactance variation. Similarly, this process is followed in each RPF iteration until there is no line overload. The power transfer is reduced so that the constraints are within their defined limits and TTC is calculated for the system with FACTS. The selected number of lines is reduced and TTC is calculated for all the normal and contingency cases. The minimum number of selected lines are selected which give large TTC values for most of the contingency cases.

\[
\frac{\partial P_i}{\partial x_k} = V_i^2 \left( \frac{\partial G_{ij}}{\partial x_k} \right) - V_i V_j \left( \frac{\partial G_{ij}}{\partial x_k} \right) \cos \delta_{ij} + V_i V_j \left( \frac{\partial B_{ij}}{\partial x_k} \right) \sin \delta_{ij} \\
= 2G_{ij}B_{ij}V_i^2 - V_i V_j \left( 2G_{ij}B_{ij} - \left( B_{ij}^2 - G_{ij}^2 \right) \sin \delta_{ij} \right) \\
= aV_i^2 - V_i V_j (\cos \delta_{ij} - \sin \delta_{ij})
\]

(15)

Submitting Equations (14) and (15) in Equation (12), the sensitivity factor \( b_k \) for each line can be found and based on these factors the lines are selected for reactance change.

\[
b_k = \frac{\partial P_i}{\partial x_k} = \begin{cases} 
S_m(aV_i^2 - V_i V_j (\cos \delta_{ij} + \sin \delta_{ij})) + S_m(aV_i^2 - V_i V_j (\cos \delta_{ij} - \sin \delta_{ij})) & \text{for } m \neq k \\
S_m(aV_i^2 - V_i V_j (\cos \delta_{ij} + \sin \delta_{ij})) + (S_m + 1)(aV_i^2 - V_i V_j (\cos \delta_{ij} - \sin \delta_{ij})) & \text{for } m = k 
\end{cases}
\]

(16)

4. Determination of TTC with Lines Reactance Variation

Total transfer capability of a system is limited due to the overloading of at least one line in the network or voltage limits violation. In order to reduce the power flow of the overloaded lines without reducing the power transfer from the source to the sink, power should be diverted from the overloaded lines to other lines having enough capacity. In this paper TTC is determined, using the RPF method based on the constraints of line thermal capacity and bus voltage limits. Line reactance is utilized to redistribute the power flow in the network so that the power is diverted from overloaded lines. PI sensitivity factors are used to select lines for reactance variation to extend the TTC value. The procedure of TTC improvement based on lines’ reactance variation is given in the following steps:

1. Select any network from three IEEE(24, 30 & 39 bus) test systems.
2. Solve the power flow for (Normal or any contingency case) with \( \lambda = 0 \).
3. Using Equation (16), calculate PI sensitivity factors \( b_k \) for each line.
4. Select lines (Selected Lines \( L_s \)) based on negative \( b_k \) values.
5. Start RPF with step increase in power transfer and identify source and sink.
6. Solve the power flow problem with the modified power transfer increase in step 5.
7. Identify the lines having line utilization more than 80% of line thermal capacity (Overloaded Lines \( O L_{\text{lines}} \)).
8. Change the reactance of \( O L_{\text{lines}} \) lines and \( L_s \) lines in step 4 using Equation (6), solve the power flow equation for updated line reactance.
9. Check the solution if any constraint is violated, if there is any violation go to next step otherwise go to step 5.
10. Decrease the power transfer until no constraint is violated, Calculate the TTC for source/sink transfer.

The proposed method can also be described in a flow chart as in Figure 2. The first four steps are for base case calculations considering normal or any contingency case. PI sensitivity factors are determined for each line and lines of negative PI sensitivity factors are selected. The RPF is started for the specified source and sink, until there is no line overload or any other constraint violation. If any constraint is violated RPF stopped and TTC is calculated for the system without FACTS. The reactances of the overloaded lines as well as the reactances of selected lines are varied, until the power flow of all the lines are within their thermal limits or the power flow of overloaded lines couldn’t be reduced by reactance variation. Similarly this process is followed in each RPF iteration until there is any line overload. The power transfer is reduced so that the constraints are within their defined limits and TTC is calculated for the system with FACTS. The selected number of lines is reduced and TTC is calculated for all the normal and contingency cases. The minimum number of selected lines are selected which give large TTC values for most of the contingency cases.
5. Test Systems and Case Studies

Three IEEE test networks of 24, 30 and 39 buses are used to investigate the effects of the proposed reactance variation for improving the total transmission capacity, and simulated in MATLAB environment using matpower 5.0. Two case studies are carried out for each of the three IEEE test systems. One case is to determine the maximum contribution of buses and lines by increasing the power transfer in RPF for the overall system, considering all generation buses as source and load buses as sink. The corresponding generations and loads of each area, lines’ loading and bus voltages are compared for the systems with and without FACTS. The overall TTC of each area is also compared to show the improvement based on the proposed method. In the second case study, the inter-area TTC value is determined from area 1 to area 2 for normal and contingency cases. For simplicity of calculation two contingency cases are included i.e., inter-tie lines outages and generators outages from the contingency list. The system with FACTS is considered as variable reactance for the selected lines. The reactance of the selected lines are varied and consequently the power flow on the lines is changed. The power flow of overloaded lines is required to be reduced down to their thermal limits without reducing power transfer from source to the sink. Yan Ou et al. [26] proposed that the line of most negative value of PI sensitivity factor is suitable for placement of series FACTS devices like TCSC. In this paper each test network is investigated for selecting multiple lines based on negative values of PI sensitivity factors to increase the total transfer capacity for normal as well contingency cases.

5.1. IEEE 24-Bus System

The data of IEEE 24 bus system is taken from IEEE reliability test system [27]. The network consists of four areas with 24 buses, 11 generation and 13 load buses interconnected through 38 lines. The following defined cases are studied for this network and results are displayed in figures and tables.
5.1.1. Bus Contribution in Load and Generation

The system is simulated for considering all generation buses as source and all load buses as sink to increase the power transfer among the lines for the constraints of line thermal limit and voltage stability limit. It can be seen in Figure 3a,b that the line flows and bus voltages of the system without FACTS is similar to the base case and there is no increase in load and generation as the power flow on line 6−10 is already exceeded its thermal limit. The load and generation are increased using FACTS devices given in Table 1, which increased the power flow in most of the lines and reduced bus voltage magnitudes of the load buses. A total load is increased by 107.15 MW and generation is increased by 122.40 MW. It can be seen that using FACTS the over all power transfer of the system is increased but all the individual lines’ loading are under their proposed 80% of thermal limits. The line losses are also comparatively increased as line loading are higher in most of the lines.

![Power Flow Diagram](image1)

**Figure 3.** Line power flow and bus voltage of IEEE 24 bus system.
Table 1. Load and generation data of IEEE 24 bus system.

<table>
<thead>
<tr>
<th>Cases</th>
<th>Basecase</th>
<th>Max. without FACTS</th>
<th>Max. with FACTS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MW</td>
<td>Mvar</td>
<td>MW</td>
</tr>
<tr>
<td><strong>Generation</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
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<td>344.00</td>
<td>37.13</td>
<td>344.00</td>
</tr>
<tr>
<td>Area2</td>
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<td>Area4</td>
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<td>256.54</td>
<td>1470.10</td>
</tr>
<tr>
<td><strong>Generation increase</strong></td>
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<td>139.99</td>
<td></td>
</tr>
<tr>
<td><strong>Load</strong></td>
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<td></td>
<td></td>
</tr>
<tr>
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</tr>
<tr>
<td>Area2</td>
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<td>627.00</td>
</tr>
<tr>
<td>Area3</td>
<td>768.00</td>
<td>156.00</td>
<td>768.00</td>
</tr>
<tr>
<td>Area4</td>
<td>750.00</td>
<td>152.00</td>
<td>750.00</td>
</tr>
<tr>
<td><strong>Load increase</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>107.15</td>
<td>21.90</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Losses</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>51.25</td>
<td>454.77</td>
<td>51.25</td>
<td>454.77</td>
</tr>
<tr>
<td>21.90</td>
<td>566.74</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In Figure 4a the variation of line reactance are shown. The original lines’ reactance are shown in black color while red color shows the overloaded lines’ reactance varied in RPF steps for increasing power transfer. Three lines line6–10, line7–8 and line14–16 get overloaded and their reactance are increased by 30.57%, 53.75% and 46.27% respectively. Similarly reactance variation of the selected lines are shown by green color in Figure 4b. The reactances of two of four selected lines line9–11 and line10–11 are increased by 7.15% and 14.30% respectively while reactances of other two selected lines line11–14 and line13–23 are reduced by 102.87% and 32.37% respectively. The TTC values of overall system are given in Table 2. It can be seen TTC values of the system using FACTS are more for all areas except area 4 which has fixed loads.

![Figure 4a](image1.png)
(a) Reactance of Overloaded lines

![Figure 4b](image2.png)
(b) Reactance of Selected lines

Figure 4. Reactance variation of lines in IEEE 24 bus system.
Table 2. TTC values of overall system for IEEE 24 bus system.

<table>
<thead>
<tr>
<th>Areas</th>
<th>TTC without FACTS (MW)</th>
<th>TTC with FACTS (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TTC</td>
<td>0 0 0 0</td>
<td>40.87 41.03 25.25 0</td>
</tr>
</tbody>
</table>

5.1.2. Inter-Area TTC

In this case the TTC value is computed from area 1 to area 2 for normal and contingency cases. The value of TTC for IEEE 24 bus system is listed in Table 3 for normal and contingency cases. For simplicity, the data of three lines and three generators outages are given. It can be seen that there is no increase in TTC values for the system without FACTS although for the contingency case of interlines outage, the TTC values are higher for the respective interline outages. Similarly using FACTS in the system, TTC values are increased for all the selected contingency cases.

Table 3. TTC values from area 1 to area 2 for IEEE 24 bus system.

<table>
<thead>
<tr>
<th>Case</th>
<th>Normal</th>
<th>$L_{10-11}$ outage</th>
<th>$G_{1}$ outage</th>
<th>$L_{10-12}$ outage</th>
<th>$G_{13}$ outage</th>
<th>$L_{14-16}$ outage</th>
<th>$G_{15}$ outage</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.00</td>
<td>126.49</td>
<td>0.00</td>
<td>135.06</td>
<td>0.00</td>
<td>48.91</td>
<td>0.00</td>
</tr>
<tr>
<td>without FACTS</td>
<td></td>
<td>$L_{10-11}$ limiting</td>
<td></td>
<td>$L_{10-12}$ limiting</td>
<td>$L_{3-24}$ limiting</td>
<td>$L_{6-10}$ limiting</td>
<td></td>
</tr>
<tr>
<td>with FACTS</td>
<td>316.39</td>
<td>201.21</td>
<td>332.79</td>
<td>316.39</td>
<td>218.13</td>
<td>316.39</td>
<td></td>
</tr>
<tr>
<td>$L_{5-10}$, $B_{6}$ &amp; $B_{8}$ limiting</td>
<td>206.21</td>
<td>206.21</td>
<td></td>
<td>218.13</td>
<td></td>
<td>256.92</td>
<td></td>
</tr>
<tr>
<td>$L_{5-10}$, $B_{6}$ &amp; $B_{8}$ limiting</td>
<td>322.79</td>
<td>322.79</td>
<td></td>
<td>316.39</td>
<td></td>
<td>316.39</td>
<td></td>
</tr>
</tbody>
</table>

5.2. IEEE 30-Bus System

The data of IEEE 30 bus system is taken from [28]. This network is of three areas with 30 buses, 6 buses consist of generators while loads are interconnected to 20 buses. The buses are interconnected through 41 lines. The base value of 100 MVA is taken for power, the bus voltage magnitude range is assumed to be 0.94 p.u to 1.09 p.u. and line parameters are also shown in p.u.

5.2.1. Bus Contribution in Load and Generation

The system is simulated for considering all the buses with generators are sources and the buses with loads are sinks. The data are given in Table 4 to compare the power generation and demand of each area in the network.

Table 4. Load and generation of IEEE 30 bus system.

<table>
<thead>
<tr>
<th>Cases</th>
<th>Area</th>
<th>Base Case</th>
<th>Max. without FACTS</th>
<th>Max. with FACTS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>P (MW) Q (Mvar)</td>
<td>P (MW) Q (Mvar)</td>
<td>P (MW) Q (Mvar)</td>
</tr>
<tr>
<td>Generation</td>
<td>Area1</td>
<td>86.94 31.00</td>
<td>111.02 44.55</td>
<td>113.50 49.30</td>
</tr>
<tr>
<td></td>
<td>Area2</td>
<td>56.20 19.30</td>
<td>78.03 27.26</td>
<td>78.65 28.41</td>
</tr>
<tr>
<td></td>
<td>Area3</td>
<td>48.50 50.11</td>
<td>66.25 71.11</td>
<td>67.88 74.26</td>
</tr>
<tr>
<td></td>
<td>Area1</td>
<td>84.50 56.40</td>
<td>107.49 72.40</td>
<td>109.59 73.86</td>
</tr>
<tr>
<td>Generation increase</td>
<td></td>
<td>62.40 42.47</td>
<td>68.39 53.55</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Area2</td>
<td>56.20 25.80</td>
<td>75.60 34.66</td>
<td>77.37 35.47</td>
</tr>
<tr>
<td></td>
<td>Area3</td>
<td>48.50 25.00</td>
<td>66.25 34.15</td>
<td>67.88 34.99</td>
</tr>
<tr>
<td>Load</td>
<td>Area1</td>
<td>84.50 56.40</td>
<td>107.49 72.40</td>
<td>109.59 73.86</td>
</tr>
<tr>
<td></td>
<td>Area2</td>
<td>56.20 25.80</td>
<td>75.60 34.66</td>
<td>77.37 35.47</td>
</tr>
<tr>
<td></td>
<td>Area3</td>
<td>48.50 25.00</td>
<td>66.25 34.15</td>
<td>67.88 34.99</td>
</tr>
<tr>
<td>Load increase</td>
<td></td>
<td>60.14 34.01</td>
<td>65.64 37.12</td>
<td></td>
</tr>
<tr>
<td>Losses</td>
<td>2.444</td>
<td>8.99 4.705</td>
<td>17.25 5.19</td>
<td>25.18</td>
</tr>
</tbody>
</table>
It can be seen that loads and generations are increased for all three areas of the system without FACTS. So a net increase of 60.14 MW in loads and 62.40 MW in generations, to increase the overall system power transfer, compared to base case. FACTS increased loads and generations of the system a bit more to improve total power transfer. So a total increase of 65.64 MW in generations and 68.39 MW in loads compared to base case. Thus 9.15% load is increased and 9.6% generation is increased, compared to the system without FACTS.

As the system is limited by line thermal constraint violation of line 12–13 in increasing power transfer. By using FACTS the power flow is reduced on overloaded lines by diverting power to other lines. It can be seen that varying line parameters by FACTS, the line flows in most of the lines are comparatively more than the system without FACTS, as shown in Figure 5a. It means the proposed method efficiently specify the locations and sizes of FACTS for increasing transfer capability without violating the lines flows from the supposed 80% of their thermal capacity.

The bus voltage magnitudes for this test network are also affected in increasing transfer capacity. Which can be seen in terms of reduction in bus voltage magnitudes of sink buses as shown in
Figure 5b. As loads are increased comparatively more than base case. The reactance variation of the overloaded and selected lines are shown in Figure 6a,b. The reactance of two overloading lines $line_{12-13}$ and $line_{15-18}$ are changed to reduced the power flow under their thermal capacity.

![Reactance of Overloaded lines](image)

(a) Reactance of Overloaded lines

![Reactance of Selected lines](image)

(b) Reactance of Selected lines

**Figure 6.** Reactance variation of lines in IEEE 30 bus system.

The reactance of $line_{12-13}$ is increased by 161.43% and $line_{15-18}$ is increased by 6.82%. The power from the overloaded lines are reduced by diverting to other lines rather to reduce power transfer. To lead the access power from overloading lines three other lines $line_{6-7}$, $line_{19-20}$ and $line_{25-27}$ are selected, the reactances of which are increased by 1.86%, 4.29% and 2.14%. The overall system TTC values from source to sink are given in Table 5 which shows that the system with FACTS has more values for all three areas.

<table>
<thead>
<tr>
<th>Cases</th>
<th>TTC without FACTS</th>
<th>TTC with FACTS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A1</td>
<td>A2</td>
</tr>
<tr>
<td>Area</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TTC</td>
<td>22.99</td>
<td>19.40</td>
</tr>
</tbody>
</table>

**Table 5.** TTC values of overall system for IEEE 30 bus system.

5.2.2. Inter-Area TTC

The inter-area total transfer capability from area 1 to area 2 is given in Table 6 for normal and contingency cases. Three inter-tie lines and three generator outages data from contingency list are shown for simplicity. It can be seen that for each of the case the FACTS in the system improve TTC. In normal case TTC is improved by 20.63%, for the contingency cases of $Line_{6-10}$ and $G_1$ outages the TTC is improved by 33.07%, for $Line_{9-10}$ and $G_2$ outages TTC is improved by 49.6% and no improvement in TTC for $Line_{4-12}$ and $G_{13}$. 
Table 6. TTC values from area1 to area 2 for IEEE 30 bus system.

<table>
<thead>
<tr>
<th>Case</th>
<th>Normal</th>
<th>without FACTS</th>
<th>with FACTS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>47.0981</td>
<td>limiting</td>
<td>56.8185</td>
</tr>
<tr>
<td>$L_{6-10}$ outage</td>
<td>40.761</td>
<td>54.2404</td>
<td></td>
</tr>
<tr>
<td>$G_1$ outage</td>
<td>47.0981</td>
<td>56.8185</td>
<td></td>
</tr>
<tr>
<td>$L_{9-10}$ outage</td>
<td>28.6024</td>
<td>51.7638</td>
<td></td>
</tr>
<tr>
<td>$G_2$ outage</td>
<td>33.4037</td>
<td>42.79</td>
<td></td>
</tr>
<tr>
<td>$L_{10-12}$ outage</td>
<td>47.0981</td>
<td>53.8119</td>
<td></td>
</tr>
<tr>
<td>$G_{TS}$ outage</td>
<td>30.1398</td>
<td>30.1398</td>
<td></td>
</tr>
</tbody>
</table>

5.3. IEEE 39-Bus System

The data of IEEE 39 Bus system is taken from [29]. There are 39 buses in which 10 are generation buses while 21 are load buses, divided in three areas. These buses are interconnected via 46 branches. The base power is 100 MVA and bus voltage magnitude and line parameters are in p.u.

5.3.1. Bus Contribution in Load and Generation

The overall power transfer for the network is required to maximized, and the respective data of generation and load of each area are determined and shown in Table 7. It can be seen that the generations and loads are increased in both systems with and without FACTS, as compared to base case because there is enough capacity available in the network.

Table 7. Load and generation of IEEE 39 bus system.

<table>
<thead>
<tr>
<th>Cases</th>
<th>Area</th>
<th>Base Case</th>
<th>without FACTS</th>
<th>with FACTS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>P (MW)</td>
<td>Q (Mvar)</td>
<td>P (MW)</td>
</tr>
<tr>
<td>Generation</td>
<td>Area1</td>
<td>2327.87</td>
<td>507.01</td>
<td>2375.87</td>
</tr>
<tr>
<td></td>
<td>Area2</td>
<td>790.00</td>
<td>160.39</td>
<td>819.66</td>
</tr>
<tr>
<td></td>
<td>Area3</td>
<td>3180.00</td>
<td>607.54</td>
<td>3299.38</td>
</tr>
<tr>
<td>Generation increase</td>
<td></td>
<td>197.03</td>
<td>484.94</td>
<td>343.53</td>
</tr>
</tbody>
</table>

| Load      | Area1 | 2384.03   | 720.60        | 2431.74    | 876.29   | 2466.60 | 892.73 |
|           | Area2 | 1221.60   | 216.30        | 1267.46    | 224.42   | 1301.00 | 230.35 |
|           | Area3 | 2648.60   | 450.20        | 2748.03    | 658.42   | 2820.70 | 675.83 |
| Load increase |       | 193.00    | 372.04        | 334.07     | 411.81   |

So in the system without FACTS generations are increased by 197.10 MW and loads are increased by 193.00 MW for all three areas. Similarly using FACTS in the system, the generations are increased by 343.53 MW and load are increased by 334.07 MW. So the FACTS provide the possibility of increasing the system capacity of adding 74.35% of more generated power and 73.1% of more demand. The active losses are also not increased for the system with FACTS.

The line loadings for this network is given in Figure 7a. Which clearly shows that using FACTS, the power transfer capacity of the network is improved, individual line flows are increased without exceeding the supposed 80% lines’ thermal capacity. The bus voltage magnitudes are lowered in load buses, but are within the voltage stability limits as shown in Figure 7b.
The reactance variation of the lines selected for this network to improve TTC for overall system are shown in Figure 8a,b. The reactances of four lines line2–3, line10–32, line16–19 and line22–35 are increased by 49.67%, 15%, 315.38% and 122.38% respectively to reduced the power from these overloading lines under their thermal capacity. Four other lines line1–39, line2–30, line17–18 and line26–27 are selected, which reactances are increased by 12%, 33.15%, 109.76% and 81.63%, to improve the Power transfer from source to sink. The TTC values of each area are given in Table 8. Which shows that FACTS can improve TTC by varying reactance of lines more than 73%.

Table 8. TTC values of overall system for IEEE 39 bus system.

<table>
<thead>
<tr>
<th>Cases</th>
<th>TTC without FACTS</th>
<th>TTC with FACTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Areas</td>
<td>A1</td>
<td>A2</td>
</tr>
<tr>
<td>TTC (MW)</td>
<td>47.71</td>
<td>45.86</td>
</tr>
</tbody>
</table>
5.3.2. Inter-Area TTC

The inter-area TTC are computed from area1 to area 2 for normal and contingency cases. The detailed results of normal and two contingency cases are given in Table 9. For simplicity three inter-tie lines and generator outages FACTS has improved the TTC values for normal and contingency cases of inter-tie lines and generator outages.

Table 9. TTC values from area1 to area 2 for IEEE 39 bus system.

<table>
<thead>
<tr>
<th>Case</th>
<th>TTC (MW) without FACTS</th>
<th>TTC (MW) with FACTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal</td>
<td>119.02</td>
<td>limiting</td>
</tr>
<tr>
<td>$L_{1-39}$</td>
<td>107.80</td>
<td>107.80</td>
</tr>
<tr>
<td>$G_{30}$ out</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$L_{3-4}$</td>
<td>107.80</td>
<td>0.538</td>
</tr>
<tr>
<td>$G_{32}$ out</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$L_{4-15}$</td>
<td>119.02</td>
<td>7.59</td>
</tr>
<tr>
<td>$G_{37}$ out</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

6. Conclusions

This research is mainly focused on to analyze the power network for the system parameters especially the line reactance which is utilized by series FACTS devices for increasing TTC. It is investigated that the line reactance has strong effect on the transmission capability of the network. To increase the capability of the network for increasing demand as well as making the transmission network more flexible for the open access market, the line reactance has a very important role. The transmission network is the only source of interconnecting different areas of loads and generations. If it is flexible, there is a better chance of participation in energy market by various
power competitors and ultimately easy excess to low cost energy by the consumers. The identification of suitable lines and their reactances can provide the flexibility to the network.

The locations of FACTS are important as there are many lines which has very less or even no effect on power flow of other lines, so it is necessary to analyze the lines of the network before placing FACTS devices. Similarly these locations are also important for diverting power from overloaded lines to other neighboring lines. Lines are selected based on PI sensitivity factors and line capacity utilization for FACTS placement. The most negative values of PI sensitivity factors are used for lines selection and then the number of selected lines are reduced to maximized the TTC value of normal as well as contingency cases. 80% line utilization is selected to ensure the system security and voltage stability. The reactances of the lines are varied for the effective improvement in TTC. The sizes of FACTS devices in terms of total reactance changed depend on thermal lines capacity violation. The lines reactance variation is based on line capacity utilization, to transfer power from overloaded lines to less loaded lines. It is shown in the results that the variation of line reactance regardless of the FACTS device used, but those which change the reactance of the line has improve the TTC value.

The repeated AC power flow method is used to calculate the TTC. Simulations were performed on three IEEE test networks 24, 30 and 39 bus systems. The results of two study cases for each test network are shown. The contribution of each bus in increasing load and generations are much better than other two systems. Inter-area TTC between area 1 and area 2 of three contingency cases of inter-area line outage and generator outage are also compared with other two systems. The results shows the effectiveness of the method in improving the TTC. The power is effectively shifted from heavy loaded lines to other lines with enough available capacity.

The scope of this work is limited to the series compensation by varying line reactance, therefore the bus voltage magnitudes are reduced in some load bus for increasing loads. Which can be improve by using shunt compensation like SVC or distributed generation etc., on suitable buses. This also open opportunities for other non conventional generations like renewable resources as well. In current study the loads and generations are increased only, but further location for loads and generations are yet to be explore in the given network. Similarly the control system is also required for the FACTS devices to provide flexibility to the network. This study demonstrates the selection of the lines in the power network and analyze the network for enhancing TTC by varying the line reactance of selected lines.

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Conflicts of Interest: The authors declare no conflict of interest.

Appendix A. The Relation of Line Power Flow and Bus Power Injection

The Power Transfer Distribution Factor PTDF [30] is defined as the ratio of the fraction of power flow on line \( l \) for a unit MW of power transaction between sending bus \( s \) and receiving bus \( r \). Mathematically it can be describes as follows:

\[
PTDF_{s,r,l} = \frac{\Delta f_l}{\Delta P_{s \to r}}
\]

The DC power flow is one of the simple and fast method to calculate the PTDF. To model the effect on bus phase angles for a transfer of \( P \) MW power from bus \( s \) to bus \( r \) can be done using the linear power flow equation as:
\[ \Delta \theta = [X] \Delta P \]

where

\[ \Delta P = \begin{bmatrix} 0 \\ \vdots \\ +P_s \\ -P_r \\ \vdots \\ 0 \end{bmatrix} \]

The phase angle changes are

\[ \begin{bmatrix} \Delta \theta_1 \\ \Delta \theta_2 \\ \vdots \\ \Delta \theta_n \end{bmatrix} = \begin{bmatrix} X_{11} & X_{12} & \cdots & X_{1n} \\ X_{21} & X_{22} & & \\ \vdots & \vdots & \ddots & \vdots \\ X_{n1} & & \cdots & X_{nn} \end{bmatrix} \begin{bmatrix} 0 \\ \vdots \\ +P_s \\ -P_r \\ \vdots \\ 0 \end{bmatrix} \]

The phase angles change on bus \( i \) and \( j \) are given as

\[ \Delta \theta_i = X_{is} P_s - X_{ir} P_r \]
\[ \Delta \theta_j = X_{js} P_s - X_{jr} P_r \]

The change in flow on line \( l \) between bus \( i \) and bus \( j \), is given as

\[ \Delta f_l = \frac{1}{x_l} (\Delta \theta_i - \Delta \theta_j) \]

then

\[ \Delta f_l = \frac{1}{x_l} \left( (X_{is} P_s - X_{ir} P_r) - (X_{js} P_s - X_{jr} P_r) \right) \]
\[ \Delta f_l = \frac{1}{x_l} (X_{is} P_s - X_{ir} P_r - X_{js} P_s + X_{jr} P_r) \]
\[ \Delta f_l = \frac{1}{x_l} (X_{is} - X_{js}) P_s - (X_{ir} - X_{jr}) P_r \]

\[ \Delta f_l = S_{l,s} P_s - S_{l,r} P_r \]

where

\[ S_{m,n} = \frac{1}{x_l} (X_{mi,n} - X_{mi,n}) \]

The matrix which shows the relation of the power flow over the line \( m \) with the power injection on the bus \( n \).

References


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