Optimal Operation of a Network of Multi-Purpose Reservoir: A Review

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Abstract

Due to the effects of climate change and population growth, reservoirs play a more and more important role in water resources management. The management of a multi-reservoir system is complex due to the curse of dimensionalities, nonlinearities and conflicts between different objectives. The optimal operation of a multi-reservoir system operation typically involves optimization and simulation models, which can provide the quantitative information to improve operational water management. The objectives of this paper are to extend previous state-of-the-art reviews in the operational management of a network of multi-purpose reservoirs with recent developments and to focus on the application of Model Predictive Control for real time control of a reservoir system.

Introduction

Nowadays, effective water management becomes more vital all over the world [1]. Due to the effects of climate change and population growth, reservoirs play a more and more important role in water resources management. Reservoirs can be used for multiple-purposes such as irrigation, municipal and industrial water supply, hydropower generation, flood protection, water quality management, recreation, low flow augmentation and so on. Therefore, the management of a multipurpose reservoir is complex due to conflict interest between these objectives. Thus, in optimal
operation of a network of multipurpose reservoirs it is important to address trade-offs between multiple objectives to achieve the water management goals.

For large scale water systems with more than one reservoir in a river basin, the optimal operation of a network of reservoirs can improve hydropower generation and flood prevention by considering the coordination among the reservoirs [2]. Depending on the spatial distribution of rainfall intensity, some reservoirs are still able to store the water, while other reservoirs are already spilling over into the downstream of a river. The analysis of multi-reservoir system operation typically involves optimization and simulation models which can provide the quantitative information to improve operational water management. The optimization model is used to minimize or maximize an objective function under given constraints and the simulation model is used to examine how a water system behaves under a given set of control actions. In the past, optimization problems have been solved by using Linear Programming (LP), Dynamic Programming (DP), Quadratic Programming (QP) and Non-Linear Programming (NLP) (see [3, 4, 5]). Reservoir system simulation can be done by using hydrological models and/or hydraulic models such as HEC-5, HEC-ResSim, MIKE 11 and SOBEK (see [6, 7, 8, 9]).

Recent developments in optimization and simulation methodology for a reservoir system analysis have been extensively explored in previous literature reviews (see [5, 6, 9, 10]). The current development is using Model Predictive Control (MPC) in a reservoir system analysis for real time management (see [11, 12, 13]). Therefore, the objectives of this review are to extend the previous state-of-the-art reviews for reservoir system management with most recent developments, and to focus on the application of MPC for the control of a reservoir system. MPC can have high performance to control a water system. The main advantage of MPC is that future events are taken into account in every control time-step by using the receding horizon principle. Based on this approach MPC optimizes the control problem over a prediction horizon, but only the first optimal control action is implemented, after getting a new measurement; the optimization is then repeated for every time-step. In this paper, section 2 presents the optimization and simulation model of reservoir system analysis; section 3 discusses recent developments and the application of MPC in reservoir system management. Finally, in the conclusion, we present the development gap and future directions for research.

Models of a reservoir system analysis

The formulation of a reservoir system analysis typically involves three main parts; inflow prediction with a rainfall-runoff model, controlling the inflow from the sub-catchment with the reservoir model, and the river flow simulation with the hydrological or hydrodynamic model. Rainfall runoff modelling is beyond the scope of present article. We mainly focus on the reservoir model and the river flow simulation model.

Reservoir model, control objectives and optimization models

In a reservoir system; the changes of storage over time in each reservoir can be modelled by using the following mass balance equation;

\[ S_i(t + 1) = S_i(t) + [I_i(t) - O_i(t)].\Delta t \]  

where \( S_i(t) \) = storage volume of the reservoir \( i \) at time \( t \) (m\(^3\)), \( I_i(t) \) = inflow volume to the reservoir \( i \) at time \( t \) (m\(^3\)), \( O_i(t) \) = outflow volume from the reservoir \( i \) at time \( t \) (m\(^3\)), \( \Delta t \) = time step (hr). The losses from a reservoir (e.g. evaporation, seepage) and the precipitation over a reservoir surface area are being neglected for simplification in equation (1). The global model of a reservoir system is described in [14] as:

\[ x_{t+1} = f_t(x_t, u_t, \varepsilon_{t+1}) \]  

where \( x_t \in \mathbb{R}^{n_x} \) is the state vector, \( u_t \in \mathbb{R}^{n_u} \) is the input vector and \( \varepsilon_{t+1} \in \mathbb{R}^{n_\varepsilon} \) is the disturbance vector. The input vector \( u_t \) can be manipulated as a control variable to get a desired state in the system. The optimal operation of a reservoir system depends on control objectives. Examples of control objectives for multipurpose considerations include:
- Minimize flood risk at the downstream area
- Minimize erosion and sedimentation
- Minimize water supply shortages for irrigation, municipal and industrial
- Maximize the hydro power generation
- Maximize the environmental flow supply
- Maximize the length of navigation period and others

Therefore, multipurpose reservoir operation needs to address various interactions and trade-offs between the objectives, which are sometimes competitive or conflicting. For example, releases may be required for hydropower generation, at the same time as releases need to be restricted for the prevention of downstream flooding. In order to solve the conflicts between the objectives, an optimization model can be applied to determine the optimal release decisions of a system. An objective function is required to set up in optimization to achieve the water management goals in which the different sub-objectives are considered. The optimal solution can be calculated by giving relative penalty to each of these sub-objectives to indicate the relative importance of the sub-objectives. The constraints can be taken into account in optimization and can be classified as hard and soft constraints [15]. The hard constraints are, for example, the physical limitations in a system, such as storage capacity of a reservoir, maximum release rate of a sluice gate, spillway and pump capacity. The soft constraints are less rigid than the hard constraints and may be required to be implemented into optimization problem, by adding a cost to the objective function whenever the constraints are violated [16]. Linear programming, dynamic programming and quadratic programming are commonly used for reservoir system optimization.

**Linear programming (LP)**

Linear programming is widely used for maximizing or minimizing a linear objective function subject to a given set of linear constraints. Considering a reservoir system, the constant release rate from \( i^{th} \) reservoir is maximized subject to the linear inequality constraints.

\[
\begin{align*}
\text{maximize} & \quad \sum R^t_i \\
\text{subject to} & \quad S^t_{i+1} = S^t_i + I^t_i - R^t_i - O^t_i \\
& \quad S^t_i \leq F \\
& \quad S^t_i, R^t_i, O^t_i \geq 0
\end{align*}
\]  

where \( R \) = constant release rate, \( t = \) month, \( S^t_i \) = storage capacity at the end of month \( t \), \( I^t_i \) = inflow to the reservoir during month \( t \), \( O^t_i \) = all releases from reservoir other than \( R \) during month \( t \), \( F \) = reservoir storage capacity.

Needham et al. [3] presented an operational management of a reservoir system to reduce the flood risk at downstream control points. The optimal releases from three reservoirs are optimized by using the Mixed-Integer Linear Programing Model (MILP). The Muskingum linear channel routing method is applied in simulation model to compute reservoir storage and downstream flow at control point. This analysis is based on reservoir storage balancing approach and reservoir water level balancing approach can be found in [17]. Wei and Hsu proposed a real-time simulation-optimization procedure to control the releases from two reservoirs during a flood event. The Balanced Water Level Index (BWLI) method is applied in the reservoir operation and the optimization models are also developed with MILP. The BWLI method is based on balancing water levels among reservoirs. Priority for releases is chosen by the water level index which means that a reservoir reaches the highest level at the end of the current time period. The Muskingum linear channel routing is applied to simulate the streamflow along a reach between two reservoirs in [17]. The main limitation inherent to LP model is that the objective function and every constraint need to be linear. However, the problems in water systems are non-linear in nature. Moreover, linear programing cannot be applied to problems where the coefficients of decision variables are probabilistic.
Dynamic programming (DP)

The complex optimization problems can be solved by using the dynamic programming algorithms. By applying this approach, the given problem is decomposed into a series of sub-problems which are solved recursively, and then the solutions of the sub-problems are combined to reach an overall solution. DP is widely used in reservoir system analysis because of its ability to deal with the non-linearity and stochastic features of a water system [4]. Stochastic Dynamic Programming (SDP) models are similar in DP style, but try to take advantage of the fact that the disturbance vectors can be described in terms of probability distribution functions in a model. SDP is the most suitable approach for solving optimization problems which involve uncertain or stochastic features [14].

Chen et al. [2] presented the applicability of the Dynamic Control of Flood Limit Water Level (DC-FLWL) method for an effective trade-off between flood control and hydropower generation. The Progressive Optimality Algorithm (POA) is used to find and to update the optimal storage allocation strategy in order to maximize the benefits of cascade reservoirs based on operation rules. The POA divides a multi-stage problem into several two-stage problems then it is run iteratively to solve the optimization of a two-stage problem, while the other stage variables remain fixed. The model can generate more hydropower from cascade reservoirs without affecting original flood prevention standards. However, the problem will become increasingly complex in practice, when the number of reservoirs within the system is increased. Braga et al. [18] used SDP to maximize the monthly hydropower production of a reservoir system in Brazil. Online SDP optimization is used to find the optimal set of reservoir releases by using probability transition matrices. The storages in three reservoirs for a particular month is determined by the off-line deterministic DP. This study presents that a combination of off-line and on-line procedures which can reduce the computational requirements inherent in a multidimensional stochastic DP. The above mentioned studies show how to solve control problems in small scale reservoir systems (involved two or three reservoirs) using DP or SDP. In case of large scale reservoir systems, the control problems become more complex because of increasing number of reservoirs and uncontrolled catchments, nonlinearities and the presence of stochastic variables. Cervellera et al. [19] presented a numerical solution to a 30-dimensional water reservoir network optimization problem, based on Stochastic Dynamic Programming (SDP). To solve such a high-dimensional problem an approach is utilized based on efficient discretization of the state-space and on the approximation of the value functions over the continuous state-space by means of a flexible feedforward neural network. SDP is the most commonly used technique in the reservoir system analysis research and literature. The main limitation of SDP is the curse of dimensionality caused by exponential growth on state and control dimensions.

Quadratic programming (QP)

In a control problem, if the objective function $f$ is quadratic and the constraints are linear in input variables $x \in \mathbb{R}^n$, then this type of problem can be solved by using quadratic programing. Its general form is:

$$
\begin{align*}
\text{minimize} \quad & f(x) = \frac{1}{2}x^TQx + c^Tx \\
\text{subject to} \quad & Ax = b, \\
& Cx \geq d
\end{align*}
$$

where $Q \in \mathbb{R}^{n \times n}$ is a symmetric weight matrix, $c \in \mathbb{R}^n$, $A \in \mathbb{R}^{m \times n}$, $C \in \mathbb{R}^{p \times n}$, $b \in \mathbb{R}^m$, $d \in \mathbb{R}^p$, $x^T$ and $c^T$ are transport of $x$ and $c$. The constraints $Ax = b$ are referred to as equality constraints, while $Cx \geq d$ is known as inequality constraints. The application of the quadratic programing in control of water systems is shown in [15, 20]. The advantage of this approach is the easy computation of the derivative of the objective function given as a quadratic scheme, the minimum of the objective function can then be found by making the derivative equal to zero.

River flow simulation models

In the reservoir system analysis, optimization models are used to find optimal decisions for system operation. On the other hand, simulation models provide the response of the system under a given set of control actions. The outflow
from reservoirs and uncontrolled catchments can be simulated along river reaches by using a hydrological model and/or a hydraulic model. The hydrological simulation model is usually used for conservation analysis and the hydraulic simulation model is mostly used for solving a flood control problem. In general, water movement along a channel reach can be simply described by using one dimensional De Saint-Venant equations, which are mass balance and momentum balance equation (Eq. (5) and (6)).

$$\frac{\partial Q}{\partial x} + \frac{\partial A_f}{\partial t} = q$$  \hspace{1cm} (5)

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x} \left( \frac{Q^2}{A_f} \right) + g_A \frac{\partial h}{\partial x} + \frac{g}{C^2} R_f A_f = 0$$  \hspace{1cm} (6)

where $A_f$ = wetted area of the flow ($m^2$); $q$ = lateral inflow ($m^3/s$); $C$ = Chezy friction coefficient ($m^{1/2}/s$); $g$ = gravitational acceleration ($m^2/s$); $R_f$ = hydraulic radius (m). These partial differential equations can be applied in a numerical model of an open channel network by using the finite element method with discretization in space ($\Delta x$) and time ($\Delta t$) [21]. A recent development for robust shallow water simulation is using a staggered grid with an implicit numerical integration scheme which can also deal with super critical flow and sudden transitions [22].

A simulation model is typically used to compute the reservoir storage, the discharge and the water level. Various simulation models have been reported in literature. The U.S. Army Corps of Engineers developed the reservoir system simulation software like HEC-3, HEC-5 and HEC-ResSim for various purposes (e.g. conservation, flood control and environmental flow). Several studies have used the mass-balance accounting procedure and the Muskingum channel routing procedure for reservoir system simulation (see [3, 17, 18, 23]). For a detailed simulation of flood control operations, Mike 11, HEC-RAS, SOBEK and other hydrodynamic models are available to study the effects of a flood wave moving through a stream (or) channel. These models are able to represent the dynamic of open channel flow by applying St. Venant equations. Ngo et al. [7] proposed control strategies for the trade-off between flood control and hydropower generation of the Hoa Binh reservoir operation in the flood season by applying a combination of simulation and optimisation models. The MIKE 11 model is used to simulate the releases of the reservoir system based on the current storage level, on the hydro-meteorological conditions, and on the time of the year. Recently, Seibert et al. [8] presented the potential of coordinated reservoir operation for flood mitigation in a large river basin (45,000 km$^2$) involving nine reservoirs. The 2D hydrodynamic model is used to improve the model performance instead of using hydrologic channel routing in this model.

Application of Model Predictive Control

A technique which has recently been developed for real time control of dynamic systems is the Model Predictive Control (MPC). MPC is an advanced control method that has been used in industries since 1970s. The main components of MPC are optimization and the internal model. The optimal control actions are calculated by optimizing the objective function based on given constraints. Then, the internal model uses the present and future disturbances and the present and future control actions to predict the future states of the water system. The other advantage of MPC is using the receding horizon principle. In recent years, MPC has been utilized in water resources management such as for the control of irrigation and drainage systems [15, 24]. It has also been applied for single or multi-reservoir management for minimizing downstream flood risk; maximizing hydropower generation and efficient irrigation water supply (see [11, 12, 25]). Blanco et al. [12] presented real time flood regulation of the Demer River Basin, Belgium by using MPC strategy. In this study, the authors assume that disturbances are known, however, the problem can result the uncertainties in the internal model when the disturbances are predicted. Van Overloop et al. [26] proposed Multiple Model Predictive Control (MMPC) which can incorporate uncertainties in the internal models and the optimal control actions is determined based on the risk-approach in the objective function. Delgoda et al. [25] presented an alternative approach to real time flood control based on optimal operation of a reservoir system under uncertain inflows. The alternative is called Adaptive Multi Model Predictive Control (AMMPC), in which multiple models are derived based on multiple disturbance scenarios. Raso [27] also proposed a method to handle the uncertainties in the control problem, called Tree-Based MPC, in which ensemble forecast data are used to generate a tree structure, the tree is then used in the optimization algorithm to find different optimal control strategies. To control a large scale water system, several
MPC controller design for a reservoir system

An example of a flood control problem is presented to discuss the MPC formulation for a reservoir system. A water system is composed of a main river and its tributaries. The reservoirs have been constructed on tributaries of a main river for flood control and other purposes. Each reservoir has two outlet structures for operational management, a spillway and an under-shoot gate. Among these structures, the spillway is a free overflow structure with no gates, thus, it cannot be controlled. The release from an under-shoot gate is to be controlled to maintain the desired water level at the control point. Therefore, the changes of storage in each reservoir over time can be modelled by using the following mass balance equation;

\[ \frac{dV}{dt} (t) = Q_{in}(t) - Q_s(t) - Q_g(t) \]  

where \( V(t) \) = storage change at time \( t \) (m\(^3\)/s); \( Q_{in}(t) \) = inflow to reservoir at time \( t \) (m\(^3\)/s); \( Q_s(t) \) = outflow from spillway at time \( t \) (m\(^3\)/s); \( Q_g(t) \) = outflow from under-shoot gate at time \( t \) (m\(^3\)/s). The losses from a reservoir (e.g. evaporation, seepage) and precipitation over reservoir surface are being neglected for simplification. For the \( i^{th} \) reservoir, equation (7) can be express with discrete time step as follows:

\[ V_i(k + 1) = V_i(k) + [Q_{in}(k) - Q_s(k) - Q_g(k)].\Delta t \]  

The releases from the \( i^{th} \) reservoir can be modelled by using the following linearized structure equations [15]. For spillway (or) over-shoot gate in equation (9) and under-shoot gate (free flow) in equation (10);

\[ Q_s(k + 1) = Q_s(k) + C_g \cdot W_g \cdot \sqrt{\frac{2}{g}} \cdot g \cdot (h_1(k) - h_{cr}(k)). \Delta h_1(k) - C_g \cdot W_g \cdot \sqrt{\frac{2}{g}} \cdot g \cdot (h_1(k) - h_{cr}(k)). \Delta h_{cr}(k) \]  

\[ Q_g(k + 1) = Q_g(k) + \frac{g \cdot C_g \cdot W_g \cdot \mu_g \cdot (h_g - h_{cr})}{\sqrt{2 \cdot g \cdot h_1(k) - (h_{cr} + \mu_g \cdot (h_g(k) - h_{cr}))}} \cdot \Delta h_1(k) + \left[ C_g \cdot W_g \cdot \mu_g \cdot \sqrt{2 \cdot g \cdot [h_1(k) - (h_{cr} + \mu_g \cdot (h_g(k) - h_{cr}))]} \right] - \frac{g \cdot C_g \cdot W_g \cdot \mu_g \cdot (h_g - h_{cr})}{\sqrt{2 \cdot g \cdot h_1(k) - (h_{cr} + \mu_g \cdot (h_g(k) - h_{cr}))}} \cdot \Delta h_g(k) \]  

where \( Q(k) \) = flow through structure at discrete time step \( (k) \) (m\(^3\)/s); \( C_g \) = calibration coefficient; \( W_g \) = width of the gate (m); \( \mu_g \) = contraction coefficient; \( h_1(k) \) = upstream water level at time step \( (k) \) (m); \( h_{cr}(k) \) = crest level at time step \( (k) \) (m); \( h_g(k) \) = gate height at time step \( (k) \) (m). For a river system, the water movement along a channel can be described by using one dimensional De Saint-Venant equations which are shown in (Eq. (5) and Eq. (6)). In this paper, a centralized distance downstream controller is implemented by using the MPC strategy. The gate position of the under-shoot gate is chosen as an input variable because this variable can be manipulated to directly control the water level. As the objective is to maintain the water level at a desired value, the water level is chosen to be the output of the system. The water level in single river reach can be calculated by using;

\[ h(k + 1) = h(k) + \frac{\Delta h}{A} \cdot [Q_d(k) + Q_g(k)] \]  

\[ Q_d(k) = Q_{in}(k) + Q_s(k) - Q_g(k) \]

where \( Q_d \) = disturbance flow (m\(^3\)/s); \( Q_g \) = outflow from river reach (m\(^3\)/s); \( k \) = discrete time step; \( h \) = water level (m); \( \Delta t \) = control time step (s); \( A \) = storage area of river reach (m\(^2\)). The change of water level and the change of
control flow can be defined as:
\[ e(k) = h(k) - h_{ref} \]  \hspace{1cm} (13)
\[ e(k + 1) = e(k) + \frac{\Delta t}{A} [Q_d(k) + Q_g(k)] \]  \hspace{1cm} (14)
\[ \Delta Q_g(k) = Q_g(k + 1) - Q_g(k) \]  \hspace{1cm} (15)

where \( e = \text{deviation of water level at control point (m)} \) and \( h_{ref} = \text{set point (m)}. \) Then, the general state space representation of the controlled water system:

\[
\begin{bmatrix}
  e(k + 1) \\
  e^*(k + 1) \\
  Q_g(k + 1)
\end{bmatrix} =
\begin{bmatrix}
  1 & 0 & \frac{\Delta t}{A} \\
  0 & 1 & \frac{\Delta t}{A} \\
  0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
  e(k) \\
  e^*(k) \\
  Q_g(k)
\end{bmatrix} +
\begin{bmatrix}
  0 & 0 & \frac{\Delta t}{A} \\
  0 & -1 & \frac{\Delta t}{A} \\
  1 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
  \Delta Q_g(k) \\
  u^*(k) \\
  Q_d(k)
\end{bmatrix}
\]  \hspace{1cm} (16)

A soft constraint is used in this internal model to avoid the non-feasibility problem in optimization processes where \( e^* = \text{the water level outside of the allowed range around set point} \) and \( u^* = \text{the virtual signal that is subtracted from the water level deviation to make } e^* \text{ either zero or a value equal to the exceeding of the range around set point} \) [15, 16].

**Constraints on the system**

- A minimum control flow \( Q_{g, min} \) equal to zero when no water is released into the system during flood control;
- A maximum control flow \( Q_{g, max} \) equal to the maximum design flow of the under-shoot gate;
- No flow from spillway, when the water level is under the spillway crest level;
- A maximum outflow from a spillway equal to the maximum design discharge of the spillway;
- A minimum water level equal to the minimum allowed water level;
- A maximum water level equal to the maximum allowed water level;

**The objective function**

The following objective function is used to minimize the deviation of the water level from set point and changes of gate setting along the prediction horizon. Considering the prediction horizon \( n \), number of controlled river reaches \( m \) and number of controlled structures \( l \):

\[ J_{min} = \sum_{i=1}^{n} \sum_{j=1}^{m} [e_i(k + i|k)^T Q_{e,j} e_j(k + i|k)] + \sum_{i=1}^{n} \sum_{j=1}^{m} [e_i^*(k + i|k)^T Q_{e^*,j} e_j^*(k + i|k)] + \sum_{i=1}^{n} \sum_{c=1}^{l} \sum_{c=1}^{l} [\Delta Q_{g,c}(k + i|k)^T R_{u,c} \Delta Q_{g,c}(k + i|k)] \]  \hspace{1cm} (17)

subject to
\[
\begin{align*}
  e_{min} &\leq e \leq e_{max} \\
  Q_{g,min} &\leq Q_s \leq Q_{g,max} \\
  Q_{s,min} &\leq Q_s \leq Q_{s,max}
\end{align*}
\]

where \( Q_e \) & \( Q_{e^*} \) represent the relative penalties on the states and \( R_{u} \) & \( R_{Q_{g,c}} \) represent the relative penalties on the inputs.
Conclusions and direction for future research

As mentioned above, several techniques have been used to analyze reservoir systems for various purposes. Numerous studies have been conducted in reservoir system analysis; the largest example involving 10 reservoirs with 30 dimensional problems. Therefore, it can be concluded that a gap still exists for the control of a large scale reservoir system (more than 10 reservoirs) and the application of such system approach in practice. To control such large scale water systems, it is important to select a suitable model which should be able to handle the complex system.

For optimization models, Linear Programming models mostly present trial and error solutions and it is difficult to find out real optimal solutions to the various complexities. Dynamic Programming techniques are more complex, but can overcome certain limitations of LP. Nonlinear properties of a problem can be readily reflected in a Dynamic Programming formulation. DP is applicable to multi-objective problems, which can be formulated as optimizing a multiple-stage decision process. If the control problems are formulated as the quadratic cost functions with linear constraints then they can be solved by using quadratic programming. Regarding simulation models, HEC-3, HEC-5 and HEC-ResSim are freely available to be used for reservoir system simulation. These models have capabilities for hydrological simulation of reservoir operations involving water supply, hydro-power generation and flood control. However, for detailed simulation of flood control operations, hydrodynamic models such as Mike 11, HEC-RAS, and SOBEK are more suitable to predict the dynamic changes of water levels at the downstream river reaches. In past literature, several methods are used to control a multi-reservoir system in which optimization and simulation models need to be effectively combined for better system performance. MPC can have such ability by using an optimization model and a simulation model as its components. As mentioned in section 3, MPC is successfully used to control a small scale reservoir system. In future research, it will be interesting to implement the MPC on a large scale reservoir system and find a way that how to balance between the model complexity and uncertainty.

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