THz Imaging Using Uncooled Wideband Direct Detection Focal Plane Arrays

Sven van Berkel, Student Member, IEEE, Ozan Yurduseven, Student Member, IEEE, Angelo Freni, Senior Member, IEEE, Andrea Neto, Fellow, IEEE, and Nuria Llombart, Senior Member, IEEE

(Invited Paper)

Abstract—In millimeter and submillimeter-wave radiometric imaging systems, a persistent goal is the increase in the speed of acquisition of the image while maintaining a high sensitivity. Typically, the highest sensitivity is achieved by cryogenically cooling the detectors, specifically in astronomical applications. However, for the purpose of low-cost imaging applications, it is desirable to operate at room temperature. Without cryogenically cooling, the electronic noise introduced by the detectors becomes dominant, making the detectors less sensitive. Resorting to detection architectures containing amplification circuitry might be impractical for implementation in large focal plane arrays (FPAs) fabricated in integrated technologies. This contribution derives the focal plane architecture that maximizes the imaging speed of radiometers operating at room temperature without using any amplification circuitry. It is shown that in such a scenario a practical image acquisition speed can still be achieved when a very broad portion of the THz-band is exploited. Ultimately, the imaging speed is maximized when the FPA is undersampled, implying a tradeoff in the size of the optics. The analysis is substantiated by a case study with recently developed wideband leaky lens antenna feeds operating from 200 to 600 GHz.

Index Terms—Direct detection, focal plane arrays (FPAs), leaky lens antennas, passive imaging, radiometry, wideband antennas.

I. INTRODUCTION

IN MICROWAVE and millimeter wave radiometry, much effort has been made in the development of efficient amplification circuitry and tunable coherent sources for large arrays. However, in the submillimeter regime of the electromagnetic spectrum, i.e., from 300 GHz and above, the design of active circuitry is still considered to be a challenging task [1]. Detection at submillimeter-wave frequencies would then preferably be performed incoherently. In this contribution, we focus on incoherent detection (i.e., direct detection) architectures in the absence of amplifiers or mixers. Such architectures require a quasi-optical system with lenses or reflectors in combination with a focal plane array (FPA) in order to generate the image. Incoherent detectors have the advantage of an easier read-out and a low power consumption. This enables the fabrication of a large number of detectors in the FPA when using integrated solutions such as CMOS [2] or superconducting technologies [3], [4]. In [5], direct detectors are roughly classified into three categories: thermal detection, cryogenically cooled detectors based on superconducting technologies, and rectifying diodes. The first (passive mm-wave) imaging systems, consisting of diodes operating around 90 GHz [6] and bolometers [7], only had few detectors, resulting in a large image acquisition time and/or a small field-of-view (FoV). Since then, in order to achieve a sufficient signal-to-noise ratio (SNR) for fast frame rates and extreme sensitivities, FPAs have been cryogenically cooled and fabricated in integrated technologies to decrease the electronic/thermal noise contributions and increase the FoV [3], [4], [8]–[10]. Ultimately, the SNR is limited by the (thermal) fluctuations of the signal itself, i.e., background-noise limited.

A cornerstone in the dimensioning and characterization of direct detection systems that are based on absorbers or background-noise limited detectors is presented in [11], where different FPA configurations are compared in terms of imaging speed and SNR. However, [11] is limited in its scope since it focuses only on cooled detectors and, more importantly, the effect of bandwidth is not included. Cryogenically cooling the detectors is expensive and results in a bulky system; for the purpose of low-cost imaging applications, it is desired to have a large integrated FPA operating at room temperature. In this scenario, the electronic noise introduced by the detectors is much higher such that none of the existing uncooled direct detectors operate near the background-noise limit [12]; the current state-of-the-art uncooled passive submillimeter-wave imagers [2], [13]–[19] are insufficiently sensitive to operate without actively illuminating the source of interest.

This contribution derives a focal plane architecture, for antenna-coupled direct detectors, that is able to provide a practical sensitivity and imaging speed (e.g., 1 K [20] at near real-time...
refresh rates for security screening) without cooling, amplification, or active illumination. As will be derived in Section II-A, the temperature sensitivity, also referred to as the noise equivalent temperature difference (NETD), of a direct detector after \(\tau_{\text{int}}\) seconds of integration and without any preamplification can be expressed as

\[
\text{NETD} = \frac{\text{NEP}}{k_B \Delta f_{\text{RF}} \sqrt{2\tau_{\text{int}}}}
\]

with \(k_B = 1.38 \times 10^{-23} \text{ m}^2 \text{ kg s}^{-2} \text{ K}^{-1}\) being Boltzmann’s constant, \(\Delta f_{\text{RF}}\) is the effective bandwidth\(^1\)\(^2\) and \(\text{NEP}\) is the noise equivalent power for a 1 Hz postdetection bandwidth, \(\Delta f_{\text{RF}}\).\(^2\) As is clear from (1), the crucial aspect in achieving a practical NETD is the utilization of detectors characterized by a low \(\text{NEP}\) that are matched over a broad portion of the THz-spectrum to an antenna. To illustrate this, Fig. 1 shows the imaging speed as \(1/\tau_{\text{int}}\) versus \(\Delta f_{\text{RF}}\) and \(\text{NEP}\) such that the minimum practical temperature sensitivity of 1 K in (1) is achieved. Nowadays, detectors suitable for array implementation have been shown with room temperature NEPs as low as 12.6 \(\text{pW/}\sqrt{\text{Hz}}\) in CMOS technologies \([13, 22]\), 2.4 \(\text{pW/}\sqrt{\text{Hz}}\) for commercially available zero-bias Schottky-barrier diodes \([23]\) and even 0.48 \(\text{pW/}\sqrt{\text{Hz}}\) in III-V technologies \([24]\). Therefore, with this order of magnitude in \(\text{NEP}\), one can utilize a large operational bandwidth to achieve a practical sensitivity and imaging speed. Novel integrated wideband antenna concepts that can be exploited in this kind of scenario are connected arrays \([25]\) or leaky lens antennas \([26]\) which have been recently developed. The focus area of this work is indicated in Fig. 1, the work describes the FPA design used throughout this work is derived by following an electromagnetic approach assuming ideal and uniformly illuminated circular feeds placed in a square array in the focal plane of a symmetric quasi-optical system, as shown in Fig. 2. Figures of merit such as sensitivity, imaging speed, and noise equivalent power are introduced, which are used to analyze the optimal FPA configuration in terms of imaging speed in Section III and resolution in Section IV. In Section V, the analysis is substantiated by evaluating the imaging speed of leaky lens antennas operating from 200 to 600 GHz.

II. QUASI-OPTICAL RADIOMETRIC SYSTEM

Radiometry is the primary tool used to characterize the electromagnetic power spectrum of emission due to sources distributed in space. The key observable parameter is the spectral brightness distribution, \(B(f, \Omega)\), of the source that is defined over a source solid angle \(\Omega_S\), with a temperature distribution \(T_S(\Omega)\), as shown in Fig. 2. In fact, the spectral brightness

\[\frac{d^2P}{df d\Omega} = B(f, \Omega) \frac{d^2T}{d\Omega d\nu} = \frac{d^2P}{df d\Omega} \frac{d^\nu}{d\nu} = \frac{d^2P}{df d\Omega} \frac{d^\nu}{d\nu} \text{ at } \nu = \frac{f_{\text{max}}}{f_{\text{min}}} = 3\]
distribution characterizes an incoherent source at temperature $T$ and can be described by Planck’s law in units of [W m$^{-2}$ Sr$^{-1}$ Hz$^{-1}$] [29]

$$B(f, \Omega) = \frac{f^3}{c^2} e^{\frac{hf}{k_BT}} - 1$$

with $f$ the frequency, $c = 2.99 \times 10^8$ m/s the speed of light, and $h = 6.626 \times 10^{-34}$ Js being Planck’s constant. In the specific case that the radiometer is operating in the Rayleigh–Jeans region w.r.t. the source, i.e., when $hf \ll k_BT$ as such as in a security scenario, a convenient approximation of the spectral brightness can be used

$$B(f, \Omega)|_{hf \ll k_BT} \approx B^{RJ}(f, \Omega) = \frac{f^3}{c^2} 2k_BT(\Omega).$$

For sources with $T > 270$ K and $f < 1$ THz, the error in this approximation is less than 10%. It is worthwhile to mention that in Wien’s limit, i.e., when $hf \gg k_BT$, the spectral brightness can be approximated using $(e^{\frac{hf}{k_BT}} - 1)^{-1} \approx e^{-\frac{hf}{k_BT}}$; this term decreases exponentially with frequency and therefore only the lowest frequencies in the band contribute to the imaging speed. This implies that utilizing a large operational bandwidth is not very beneficial to increase the SNR when the radiometer is operating in Wien’s limit w.r.t. the source such as is the case for deep-space sensing. Accordingly, optimal FPA configurations in Wien’s limit can be studied as in [11]. In that case the spill-over efficiency will be

$$\eta_{SO} = \frac{D(f, \Omega)_{SO}}{D(f, \Omega)+D(f, \Omega)_{SO}}.$$
the Rayleigh–Jeans approximation from (3) in (5) leads to
\[ P_a = \frac{k_B}{4\pi} \int_{\Omega_S} T(\Omega) G_{\text{eff}}(\Omega) d\Omega \tag{7a} \]
\[ G_{\text{eff}}(\Omega) = \int_{f_{\text{max}}}^{f_{\text{min}}} \eta_{\text{opt}}(f) D^\Omega(f, \Omega) df. \tag{7b} \]

\( G_{\text{eff}} \), the effective gain pattern, is obtained by weighting the spectral directivity by the optical efficiency of the quasi-optical system and by integrating the resulting term over the operational bandwidth.

A signal can be resolved by the system if the received power \( P_a \) (7a) integrated over time results in an SNR larger than 1 (4). In order to have a convenient way to compare the SNR for different FPA architectures, we make some assumptions in the evaluation of (7a). First, the source is considered to have an effective, constant, temperature \( T_S \) over the entire solid angle such that it can be removed from the integral over source solid angle. Second, an important condition of the presented analysis is that the sources are distributed over a large solid angle w.r.t. intensity patterns after the optics, such that the angular integration in (7a) over the spectral directivity in (7b) is approximately 4\( \pi \). The average received power \( \bar{P}_a \) by each antenna feed can now be written as (8a)
\[ \bar{P}_a = k_B T_S \bar{\eta}_{\text{opt}} \Delta f_{\text{RF}} \tag{8a} \]
\[ \bar{\eta}_{\text{opt}} = \frac{1}{\Delta f_{\text{RF}}} \int_{f_{\text{min}}}^{f_{\text{max}}} \eta_{\text{opt}}(f) df. \tag{8b} \]

Here, we defined an average optical efficiency \( \bar{\eta}_{\text{opt}} \). A convenient definition to characterize the performance of the quasi-optical system is its effective bandwidth \( \Delta f_{\text{eff}} = \bar{\eta}_{\text{opt}} \Delta f_{\text{RF}} \). This definition of effective bandwidth is used in Fig. 1.

Let us consider an incoherent source with an effective temperature of \( T_S \). A temperature difference \( \Delta T \) in the incoherent source then results in a difference in average received power \( \Delta P_a \), according to \( P_a + \Delta P_a = k_B (T_S + \Delta T) \bar{\eta}_{\text{opt}} \Delta f_{\text{RF}} \). The purpose of detector integration is to average the fluctuating received power over time in order to detect \( \Delta P_a \). Substituting \( \Delta P_a = k_B \Delta T \bar{\eta}_{\text{opt}} \Delta f_{\text{RF}} \) in (4) and by forcing \( \text{SNR} = 1 \), we can solve for \( \Delta T \). This defines the sensitivity of the quasi-optical system as has been introduced in (1) [30]
\[ \text{NETD} \triangleq \Delta T|_{\text{SNR}=1} = \frac{\text{NEP}}{k_B \bar{\eta}_{\text{opt}} \Delta f_{\text{RF}} \sqrt{2\tau_{\text{int}}}}. \tag{9} \]

One should be careful in interpreting the NEP and its impact on sensitivity. Usually, the NEP can be divided into two contributions; \( \text{NEP} = \text{NEP}_\text{det} + \text{NEP}_b \). The detector noise \( \text{NEP}_\text{det} \) is intrinsic to the detector characteristics and strongly influenced by the read-out mechanism, while the background noise \( \text{NEP}_b \) is associated with the thermal fluctuations of the signals arriving at the input of the detector. For uncooled and fully passive systems, \( \text{NEP}_\text{det} \) is often a few orders of magnitude lower than \( \text{NEP}_b \), and can be neglected.\(^7\) Rearranging (9) and substituting \( \tau_{\text{int}} = \frac{\tau}{N_t} \) gives us an expression of the imaging speed \( s \) (10)
\[ s = \frac{1}{\tau_t} \left( \frac{\text{NETD} \cdot k_B \bar{\eta}_{\text{opt}} \Delta f_{\text{RF}}}{\text{NEP}} \right)^2 \frac{2}{N_t^2}. \tag{10} \]

Fig. 1, from the introduction, is generated using (10) as a function of \( \Delta f_{\text{eff}} = \bar{\eta}_{\text{opt}} \Delta f_{\text{RF}} \) and NEP, after fixing \( N_t = 1 \), and \( \text{NETD} = 1 \) K. When comparing two different configurations \( \alpha \) and \( \beta \), Griffin et al. [11] suggests focusing on the ratio in imaging speed (SR):
\[ SR = \frac{s^\alpha}{s^\beta} = \left( \frac{\bar{\eta}_{\text{opt}} \Delta f_{\text{RF}}}{\bar{\eta}_{\text{opt}} \Delta f_{\text{RF}}} \right)^2 \frac{N_t^\beta}{N_t^\alpha}. \tag{11} \]

The imaging speed ratio \( SR \) indicates how much faster configuration \( \alpha \) (with imaging speed \( s^\alpha \)) is relative to configuration \( \beta \) (\( s^\beta \)). With the introduction of such SR, the optimal configuration as a function of the bandwidth or the focal plane sampling can easily be investigated qualitatively. One should note the cancellation of the NEP in (11).\(^8\)

B. Focal Plane Sampling and Jiggling

The resolution of an imaging system \( \Delta \theta \) is defined by its ability to distinguish between multiple sources. Suppose that two point sources are angularly displaced by \( \Delta \theta \) in the far field of the quasi-optical system. The diffracted field arriving in the focal plane is a summation of two Airy discs,\(^3\) which are transversely displaced in the focal plane by \( \Delta \rho \) as is shown in Fig. 4(a). The geometrical relation between the angular displacement (\( \Delta \theta \)) of the point-sources and transverse displacement (\( \Delta \rho \)) of the Airy discs in the focal plane of an optical system with large \( F_\parallel \) can be shown to be \( \Delta \rho = F \Delta \theta \). The diffracted field is then sampled with a finite amount of antenna feeds in the focal plane. As is illustrated in Fig. 4(b), in order to be able to distinguish between the two incoherent sources in the focal plane image, one not only

\(^7\)In the case that the system is cooled or any amplification circuitry is present, the background noise \( \text{NEP}_\text{det} \) [7], [31], [32] can become dominant. In such scenario, the NEP will also depend on source temperature, operational bandwidth, and optical efficiency. The NETD (9) will then reduce to an expression that is commonly found in microwave radiometer literature \( \text{NETD} = \frac{T_S}{\sqrt{\Delta f_{\text{RF}} \tau_{\text{int}}}} \) [33], [34].

\(^8\)Since we focus the study on uncooled power detectors, the NEP is considered to be independent on the signal received by the feeds.

Airy disc = \( C \frac{\sin(2\pi \frac{\rho}{D})}{\pi \frac{\rho}{D}} \) for broad-side illumination, where \( C \) is an intensity constant, \( k \) is the wavenumber, and \( \rho \) is the transversal focal plane position.
needs to sample the peaks of the Airy discs but also needs to sample in between in order to actually see a decrease in intensity and discriminate between the two peaks. The relation between an angular source separation $\Delta \theta$ and the required sampling $d_f$ is then $d_f = \Delta \theta = 0.5F \Delta \theta$. By inverting this relationship, we can say that for a feed-size of $d_f$, the resolution of the imaging system is as follows:

$$\Delta \theta = 2 \frac{d_f}{F}.$$  \hfill (12)

It is not possible to continue decreasing the feed-size $d_f$ to achieve any resolution $\Delta \theta$ that is desired. The smallest resolution is indeed limited by the diffraction with the optics when the superposition of the two Airy discs only show one peak in the focal plane image. This minimal resolution $\Delta \theta_{\text{lim}}$ is approximately the $-3 \, \text{dB}$ beamwidth of the diffraction limited patterns [35]

$$\Delta \theta_{\text{lim}} \approx \frac{\lambda_c}{D},$$  \hfill (13)

where $\lambda_c$ is the wavelength at the center frequency $f_c = \lambda_c = f_{\text{det}}$, $f_{\text{det}}$, of the operational frequency band in which the antenna feeds are operating. Equating (13) with (12) gives rise to the sampling rule (14) stating that the diffraction-limited resolution $\Delta \theta_{\text{lim}}$ can be reached when the sampling points in the focal plane are separated by $[35]^{10}$

$$d_f = \Delta \theta_{\text{lim}} = 0.5F \lambda_c.$$  \hfill (14)

When the antenna feeds are sampled according to (14), the FPA is referred to as fully sampled. Sparser sampling (i.e., undersampling) will result in a loss in resolution while denser sampling (i.e., oversampling) is never useful since the quasi-optical system’s diameter limits the resolution. In this contribution, we will make use of undersampled arrays in order to maximize the imaging speed. The resolution will increase linearly with the undersampling factor $N$. The relationship between feed-size $d_f$ and resolution $\Delta \theta$ can be written as

$$d_f = N \cdot 0.5F \lambda_c \iff \Delta \theta = N \cdot \Delta \theta_{\text{lim}}.$$  \hfill (15)

The resolution of an undersampled array ($N > 1$) is not diffraction limited, but as we will explain in the next section.

\[10\]This sampling rule only applies when the signal is detected in intensity.
relatively small arrays (<100 elements), near real-time imaging speeds have been achieved. Therefore, with a maximum of only $N_t = 16$ scanning positions in this analysis, an additional time penalty associated with the mechanical scanner has been neglected.

### III. FPA DESIGN

In this section, we will compare the imaging speed for different sampling configurations. Subsequently, the increase in imaging speed that one can obtain by exploiting a large operational frequency band is quantified. In the last section, it is illustrated that the imaging speed can further be improved by trading off in resolution.

#### A. FPA Configurations

The three different sampling configuration from Fig. 5 are compared relatively to each other using the imaging speed ratio defined in (11). The different FPAs are assumed to be operating with the same operational bandwidth $\Delta f_R$ and are sampled $d_f = [0.5\,1\,2]F_R \lambda_c$ at the center frequency $f_c$. When including only the spill-over efficiency $\eta_{so}$ in the analysis ($\eta_{feed} = 1$), the average optical efficiency can be approximated to be the spill-over efficiency at $f_c$, i.e., $\eta_{opt} = 0.15\,0.46\,0.48$ for the sampling configurations $d_f = [0.5\,1\,2]F_R \lambda_c$ respectively, as can be seen in Fig. 3. In the comparison of the different sampling configurations, we define the imaging speed ratio, $SR = \frac{\eta_{opt}}{\eta_{opt}}$, as the speed ratio of the coherent- and maximum gain sampled FPAs, $s^\beta = s^1[1\,2]F_R \lambda_c$, w.r.t. to the reference case of a fully sampled FPA, $s^\beta = s^{0.5}F_R \lambda_c$. The imaging speed ratio approaches the limits as summarized in the fourth column of Table I.

It is beneficial to sample the feeds in the FPA coherently with $d_f = 1F_R \lambda_c$ and to accept a small integration time penalty of $N_t = 4$ to sample the full FoV. A coherently sampled array is approximately 2.35 times faster w.r.t. the fully sampled FPA ($d_f = 0.5F_R \lambda_c$). The fully sampled array is, in fact, the slowest focal plane architecture. The increase in spill-over efficiency for larger feed size is more significant than the introduced integration time penalty of $N_t = 4$.

#### B. Bandwidth Utilization

Most submillimeter-wave radiometric detectors are connected to integrated antennas that operate within a 20% fractional bandwidth [37]. A solution to make the detectors more sensitive is to increase the absolute bandwidth $\Delta f_R$ by using wideband antenna solutions [17], [25], [26], [38]. The imaging speed increases quadratically with the effective bandwidth

<table>
<thead>
<tr>
<th>$d_f$</th>
<th>$\eta_{opt}$</th>
<th>$N_t$</th>
<th>$SR$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.5F_R \lambda_c$</td>
<td>0.15</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$1F_R \lambda_c$</td>
<td>0.46</td>
<td>4</td>
<td>2.35</td>
</tr>
<tr>
<td>$2F_R \lambda_c$</td>
<td>0.84</td>
<td>16</td>
<td>1.96</td>
</tr>
</tbody>
</table>

Fig. 7. The effect on imaging speed when using a large operational frequency band is studied w.r.t. the reference case of 20% fractional bandwidth. (a) The three sampling configurations, $d_f = 0.5F_R \lambda_c$ (left), $d_f = 1F_R \lambda_c$ (middle), and $d_f = 2F_R \lambda_c$ (right) with $\nu = \frac{f_{\max}}{f_{\min}}$ relative bandwidth centered around $f_c$ and with their associated reference cases of 20% fractional bandwidth. (b) Imaging speed ratio, $SR(\nu) = \frac{s^\beta(\nu)}{s^\alpha(\nu=1.22)}$, as a function of the relative bandwidth w.r.t. the reference case.
C. Maximizing Imaging Speed With an Undersampled FPA

In the case that the detectors are not sufficiently sensitive at the desired imaging speed, two practical solutions exist to improve the sensitivity or imaging speed at the cost of (potential) resolution: by increasing the feed size $d_f$ (until $2F_{\#}\lambda_c$) or by translating the frequency band of operation to a higher portion in the spectrum (assuming the NEP is not affected). As explained in Section II-B, without performing any jiggling, the FPA becomes undersampled for $d_f > 0.5F_{\#}\lambda_c$ such that the optics do not offer a diffraction limited resolution (15).

In the first solution, the feed size $d_f$ is increased. Let us define the imaging speed ratio as a function of the feed size $d_f$.

$$SR(d_f) = \frac{s^a(d_f)}{s^a(d_f = 0.5F_{\#}\lambda_c)}.$$  

The reference case, with imaging speed $s^a(d_f = 0.5F_{\#}\lambda_c)$, is a fully sampled FPA as is illustrated in Fig. 8(a). The feed size is increased while the bandwidth is kept fixed with a 20% fractional bandwidth centered around $f_c$. A larger feed size results in an increase in spill-over efficiency and therefore imaging speed $s^a(d_f)$, as illustrated in Fig. 8(b) and quantified in Fig. 3.

The imaging speed ratio $SR(d_f)$ is shown in Fig. 8(c) with the black curve and left y-axis. For the same absolute bandwidth and without jiggling, the imaging speed ratio is increasing quadratically with the spill-over efficiency. When the feed size is $d_f = 2F_{\#}\lambda_c$, the FPA is faster with a factor of 32. This increase in imaging speed comes at the cost of resolution. Referring back to (15), for a feed size of $d_f = 2F_{\#}\lambda_c$, the resolution is not diffraction limited: $\Delta \theta = 4\Delta \theta_{\text{min}}$ since the pointing direction of the patterns associated with two adjacent feeds ($n = 0$ and $n = 1$) $d_f = 2F_{\#}\lambda_c$ in Fig. 6) is increased. The resolution, as a function of the feed size, is shown by the gray curve and the right axis in Fig. 8(c). Of course, one could compensate for this decrease in resolution by increasing the size of the optics.

The second option, if the technology and application allows for it, is to translate the frequency band of operation to a higher portion in the spectrum. This higher frequency band is defined with a new center frequency $f_c'$. Although the physical feed size remains unchanged, the electrical size (in terms of $\lambda_c$) grows linearly with $f_c'/f_c$, which contributes to the same increase in imaging speed as shown in Fig. 8(c). Furthermore, an additional increase of $(f_c'/f_c)^2$ in imaging speed can be expected since the absolute bandwidth increases with a factor $f_c'/f_c$ when the fractional bandwidth is kept fixed.

For example, if one initially operates with a fully sampled FPA $d_f = 0.5F_{\#}\lambda_c$ around $f_c$ but decides to increase in imaging speed by translating the frequency band to $f_c' = 4f_c$ (i.e., maximum gain sampling $d_f = 2F_{\#}\lambda_c$ with the same physical feed size) one expects a factor of $32 \times 16 = 512$ increase in imaging speed, where the factor 32 is the result of the increase in spill-over efficiency and the factor $(5/4)^2 = 16$ is the result of the increase in absolute bandwidth.

In the following section, we will see that in an imaging scenario an undersampled array is actually beneficial in terms of image quality.

IV. Image Quality in Undersampled Arrays

In the previous section, it was explained that a significant increase in imaging speed can be achieved when operating in a higher frequency band with center frequency $f_c' > f_c$. When the physical feed size is not altered, the resolution is not changed, although it is not anymore diffraction limited (15). In this section, we will see that in an imaging scenario the image quality of an undersampled array does improve w.r.t. a fully sampled array with the same resolution. For this study, we introduce a beam efficiency of the effective gain patterns $|G_{\text{eff}}(\Omega)|$ defined in (7b). The beam efficiency is, in essence, the spill-over efficiency of a pixel w.r.t. the angular region associated with that specific pixel. This angular region we refer to as the pixel region (referring to the centered feed $n = 0$ in Fig. 6, its associated pixel region subtends $\phi \in (0, 2\pi), \theta \in (0, \frac{\pi}{2})$).

Let us consider a fully sampled array as is illustrated in the left-hand side of Fig. 9(a). We consider both a narrowband ($\nu = 1.22$) and wideband ($\nu = 3$) FPA. The normalized effective gain patterns of three adjacent feeds in the FPA are shown on the left-hand side of Fig. 9(b). The beam patterns of adjacent feeds have a significant overlap. This means that a pixel shares a
significant portion of incident power with adjacent pixels, the pixels are not well restricted to their own pixel region. This pixel region is also indicated in Fig. 9(b) by the green bar.

Consider now the same FPA but operating in a frequency band around \( f' = 4f_c \), as illustrated in the right-hand side of Fig. 9(a). The normalized effective gain patterns of adjacent feeds are shown on the right-hand side of Fig. 9(b). Since the feed size remains the same, the pointing direction of the beam patterns is also unchanged. Both FPA configurations have therefore the same resolution. However, the FPA operating around \( f' = 4f_c \) operates in a higher frequency band, such that the beam patterns become more directive. The pixels are significantly better isolated in their pixel region.

Let us quantify this pixel isolation by introducing a beam efficiency, BE, that describes how well the effective gain pattern is isolated from the patterns associated with adjacent pixel regions. The beam efficiency can be defined by angularly integrating the beam pattern over the pixel region that is associated with that particular feed, which for the on-axis feed results in the following equation:

\[
BE = \frac{\int_0^{2\pi} \int_0^{\pi} G_{\text{eff}}(\theta) \sin \theta d\theta d\phi}{\int_0^{2\pi} \int_0^{\pi/2} G_{\text{eff}}(\theta) \sin \theta d\theta d\phi}.
\]  

The beam efficiency is shown in Fig. 10 as a function of the translated center frequency \( f'_c \). If the array is fully sampled for the frequency band of operation \((f'_c = f_c)\), the beam patterns are not well restricted to their own pixel region, with a BE \( \approx 15\% \) the feed will mainly integrate power from adjacent pixel regions. Instead, by undersampling the FPA to \( f'_c = 4f_c \), the beam patterns will be more directive and the beam efficiency is higher than 80\%, i.e., the beam patterns are well isolated from adjacent feeds. No significant difference in BE between narrowband and wideband systems can be observed.

The beam efficiency does impact the temperature sensitivity of the system. Despite the fact that the analysis is derived for distributed sources, it should be noted that a high BE implies that the system is less sensitive to isolated sources at the edges of a pixel region. Instead, for distributed sources, a high BE improves the sensitivity since the BE can, in fact, be included as an additional spill-over term in the evaluation of \( P_n^a \) (5), while simultaneously considering the power arriving from adjacent pixels to be contributing to the noise power.

Consider the following imaging example to investigate the effect of the beam efficiency on the quality of image reconstruction. The THz-sensing-group logo is defined with a source temperature \( T_S(\Omega) \) and is angularly distributed in space. The logo is projected in the \( u, v \)-domain \((u = \sin(\theta) \cos(\phi) \text{ and } v = \sin(\theta) \sin(\phi))\) in Fig. 11(a). The solid angle subtended by the source distribution is assumed so small, such that \( \sin \theta \approx \theta \) and the projection in the \( u, v \)-domain can be normalized to the angular pixel dimension, \( \lambda_c/2D \). The temperature distribution \( T_S(\Omega) \) is defined from 0 to 255 K, as is shown by the color bar. Indicated by the box is the FoV under investigation, which corresponds to the FoV of an undersampled FPA. A zoom-in on the letter S is shown, illustrating the pixel regions of \( 6 \times 6 \) elements in the FPA. The received power by the \( n \)-th feed \( P_n^a \) can be calculated by using (7a). Subsequently, by using the approximation on average received power \( \bar{P}_a \) (8a), the temperature distribution of the source can be reconstructed to be the following equation:

\[
T_{S}^\text{rec}(\Omega_n) = \frac{P_n^a}{k_B \bar{\eta}_{\text{opt}}} = \frac{\int_{\Omega} T_S(\Omega) G_{\text{eff}}(\Omega - \Omega_n) d\Omega}{4\pi \bar{\eta}_{\text{opt}} \Delta f_{\text{RF}}}.
\]  

The reconstructed thermal image of the source distribution \( T_{S}^\text{rec} \) is shown in the top two figures of Fig. 11(b) for the fully sampled FPA \((f'_c = f_c)\) with its associated effective gain pattern in the left subfigure in Fig. 9(b). By repeating the procedure...
Fig. 11. THz-sensing Group logo as source distribution in (a) and the reconstructed images in (b) for a fully sampled and undersampled array. (a) Source distribution projected in the $u,v$-domain using $u = \theta \cos(\phi)$ and $v = \theta \sin(\phi)$. Indicated is the FoV of a $100 \times 100$ element FPA and a zoom-in on the letter S showing the pixel regions of $6 \times 6$ elements. The color bar on the right shows the temperature distribution $T_s$ of the logo. (b) Reconstructed image of the source distribution shown in Fig. 11(a) in the case that the image is acquired using a fully sampled FPA and an undersampled FPA with $f_c' = 4f_c$. The reconstructed image is calculated by using (19). A zoom-in of the transitions between the white and blue regions are shown.

with the effective gain pattern [right subfigure in Fig. 9(b)] of the undersampled array ($f_c' = 4f_c$), one obtains the bottom two reconstructed images in Fig. 11(b). When both reconstructed images are compared, we can see that the letters Sens and the transition from white to blue are significantly sharper for the undersampled array due to a higher beam efficiency. Whereas the feeds in a fully sampled array, with a BE $\approx 15\%$, receives power from several pixel regions away, the undersampled array is better isolated with BE $\approx 80\%$, which results in an improved image reconstruction.

For this reason, the image quality can be qualified both in terms of the nominal resolution $\Delta \theta$ due to the angular discretization of the beams and in terms of the beam efficiency.

V. REALISTIC FEEDS

In the previous analysis, we made use of uniformly illuminated and ideal feeds. In this section, we will instead show the performance of a realistic antenna design. The modeling techniques that are used in this section are the same that have been used in the last years by some of the authors for the development and characterization of lens antenna FPAs for passive astronomical instruments. The predicted performance of the lens antennas show excellent agreement with measurements [3, 39].

In order to appreciate the applicability of this study, we will consider a FPA containing wideband leaky lens antenna feeds as is shown in Fig. 12(a) [26]. The radiation performance of a single leaky lens has been measured at low frequency in [40] and at high frequency in [41] and [42]. After introducing the quasi-optical system containing leaky lens feeds, a comparison in terms of imaging speed will be made with narrowband conical horn-antenna feeds.

The leaky lens feeds operate from $200$ to $600$ GHz ($\nu = 3$) and illuminate a parabolic reflector with $F_\# = 2.5$. In order to optimize the imaging speed, we place the feeds in a maximum gain sampled array ($d_f = 2F_\# \lambda_c$) at $f_c = 400$ GHz without mechanically jiggling the FPA. Note that this FPA is equivalent to the scenario discussed in Fig. 8 with $d_f = 2F_\# \lambda_c$; the array is undersampled to maximize the imaging speed. The resolution is not diffraction limited with $\Delta \theta = 4\Delta \theta_{\text{lim}} = 4\lambda_c/D$. The feeds are optimized and analyzed by numerical CST simulations [43], leading to the dimensions of the slot and elliptical lens as are shown in Fig. 12(b) and (c), respectively. The dimensioning of the elliptical lens is corrected for a phase center of $126 \mu m$ behind the slot and the $50^\circ$ truncation is at approximately the $-10$ dB beamwidth of the fields inside the lens at $200$ GHz. In Fig. 13, the patterns of the leaky lens antennas are plotted for $f = 250$ and $550$ GHz. As a reference, the patterns indicated with uniform are associated with uniformly illuminated feeds with the same feed size $d_f$, as those used throughout the previous sections. The pattern region that is subtended by the reflector is indicated by $\theta_0$. The reflector patterns are obtained by physical-optics in GRASP [44].

The feed efficiency $\eta_{\text{feed}}$, the spill-over efficiency w.r.t. the reflector $\eta_{\text{O}}$, and the optical efficiency $\eta_{\text{opt}} = \eta_{\text{feed}}\eta_{\text{O}}$ are shown in Fig. 14 by the solid lines. It is important to note that the feed efficiency is defined as the ratio of the power
radiated by the leaky lens $P_{\text{rad}}$, and the power presented to the port $P_{\text{in}}$, where any radiated power that is first reflected inside the lens or substrate is omitted. This is visualized in Fig. 12, where only the green, reflection free, rays contribute to the radiated power $P_{\text{rad}}$. Power leaving the leaky lens after the first reflection is not expected to be captured by the reflector. The average optical efficiency of the reflector system illuminated by the leaky lenses is $\eta_{\text{opt}} = 55.3\%$, leading to an effective bandwidth of $\Delta f_{\text{RF}} = 0.553 \times 400$ GHz = 221 GHz.

A significant advantage of the described leaky lens structure is that it is fully integrated. This means that it can be used in large FPAs. Furthermore, impedance matching over the full operational bandwidth is simple to achieve. Instead, feeding- and matching-architectures for a large integrated FPA based on horn antennas is proven to be a difficult task [45] when fractional bandwidths larger than 20% are desired. Let us compare the leaky lens FPA with an FPA containing conical horn antennas, sampled identically to the leaky lens antennas with $d_{\text{f}} = 2F_{\phi}/\lambda_c$. The horn antennas are operating in a 20% fractional bandwidth from 360 to 440 GHz, as indicated in Fig. 14. Their efficiencies are shown by the dashed lines. The horn antennas are simulated without any losses or feeding network. Instead, we define the feed efficiency optimistically at 90% over the full band. The average optical efficiency is 61.5% leading to an effective bandwidth of $\Delta f_{\text{RF}} = 49.2$ GHz. The imaging speed ratio (11) of the leaky lens-based FPA w.r.t. the horn-antenna-based FPA is $\text{SR} = \frac{f_{\text{imaging speed ratio}}}{}^{\text{leaky lens-antenna}}} = (\frac{0.553}{0.221})^2 = 20$.

This SR is indeed close to the predicted value in Fig. 7 of SR($\nu = 3$) = 21.5.

Referring back to Fig. 1, an effective bandwidth of 221 GHz is sufficient to achieve a 1 K sensitivity and a 10 Hz refresh rate imaging requirement, provided that the detectors have an NEP of 1.4 pW/√Hz. Recently conducted measurements [46] of a dual polarized leaky lens antenna show that the received power (5) from an incoherent source is increased by a factor of 2 compared to a single polarized version of the antenna. In that scenario, the requirement on the detector NEP is relaxed by the same factor. This demonstrates that a practical sensitivity and imaging speed can be achieved with an uncooled and fully passive integrated FPA. The horn-antenna-based FPA on the other hand, with an effective bandwidth of 49.2 GHz, will require cooling or amplification circuitry to achieve the practical imaging requirements.

VI. CONCLUSION

Uncooled submillimeter-wave direct detectors are characterized by moderate sensitivities due to high electronic noise and thus are not often proposed for real-time imaging. In this contribution, the design guidelines for focal plane architectures based on antenna-coupled direct detectors are proposed with greatly enhanced potential speeds, thanks to the efficient use of the available spectral bandwidth at submillimeter wavelengths. Relying on the guidelines and focal plane architectures introduced in this paper, it is shown that real-time imaging with high sensitivities can be achieved with already available practical detectors.

REFERENCES


[32] Sven van Berkel (S’16) received the B.Sc. and M.Sc. degrees (cum laude) in electrical engineering from the Delft University of Technology (TU Delft), Delft, The Netherlands, in 2012 and 2015, respectively. He is currently working toward the Ph.D. degree at the Terahertz Sensing Group, TU Delft. During his Master’s, he specialized in telecommunications engineering with a focus on antennas and electromagnetic wave theory.

[33] Ozan Yurduseven (S’11) received the B.Sc. and M.Sc. (Hons.) degrees in electronics and communications engineering from Yildiz Technical University, Istanbul, Turkey, in 2009 and 2011, respectively, and the Ph.D. degree in electrical engineering from the Delft University of Technology, Delft, The Netherlands, in 2016. His Ph.D. research concerned the development of dielectric lens antennas for THz space applications.

During his Ph.D., he spent six months as a Visiting Researcher with the Instituto de Telecomunicações, Instituto Superior Técnico, Lisbon, Portugal, where he was involved with double shell dielectric lenses. He has authored or co-authored more than 30 publications in peer-reviewed journals and conferences. His current research interests include dielectric lens antennas for millimeter and submillimeter-wave space applications, quasi-optical systems, numerical techniques in electromagnetics, and metamaterials.

Dr. Yurduseven is a member of the IEEE Antennas and Propagation Society and the European Association on Antennas and Propagation. He was the recipient of the Best Student Paper Award of the European Conference on Antennas and Propagation (EuCAP) in 2013. He was a co-recipient of the Best Paper on Electromagnetism and Antenna Theory of EuCAP 2016. He serves as a Reviewer for the IEEE TRANSACTIONS ON ANTENNAS AND PROPAGATION, IEEE ANTENNAS AND WIRELESS PROPAGATION LETTERS and IET Microwaves, Antennas and Propagation.
Andrea Neto (M’00–SM’10–F’16) received the Laurea (Doctors) degree in electronic engineering from the University of Florence, Florence, Italy, in 1994, and the Ph.D. degree in electromagnetics from the University of Siena, Siena, Italy, in 2000. Part of his Ph.D. degree was developed at the European Space Agency Research and Technology Center Noordwijk, The Netherlands.

He worked for the Antenna Section at the European Space Agency Research and Technology Center for over two years. From 2000 to 2001, he was a Postdoctoral Fellow with the California Institute of Technology, Pasadena, CA, USA. From 2007 to 2010, he was a Postdoctoral Fellow with the California Institute of Technology, working with the Sub-Millimeter Wave Advance Technology Group, Jet Propulsion Laboratory, Pasadena, CA, USA. From 2010 to 2012, she was a Ramón y Cajal Fellow with the Optics Department, Complutense University of Madrid, Madrid, Spain. In September 2012, she joined the THz Sensing Group, Technical University of Delft, Delft, The Netherlands, where she is currently an Associate Professor. She has co-authored more than 150 journal and international conference contributions. Her research interests include the analysis and design of planar antennas, periodic structures, reflector antennas, lens antennas, and waveguide structures with an emphasis on the THz range.

Dr. Llombart was the recipient of the H. A. Wheeler Award for the Best Applications Paper of 2008 of the IEEE TRANSACTIONS ON ANTENNAS AND PROPAGATION, the 2014 THz Science and Technology Best Paper Award of the IEEE Microwave Theory and Techniques Society, and several NASA awards. She was also the recipient of the 2014 IEEE Antenna and Propagation Society Lot Shafai Mid-Career Distinguished Achievement Award. She serves as a Board member of the IRMMW-THz International Society. In 2015, she was a recipient of a European Research Council Starting Grant.