Optimizing front metallization patterns: Efficiency with aesthetics in free-form solar cells

Deepak K. Gupta a,*, Matthijs Langelaar a, Marco Barink b, Fred van Keulen a

a Department of Precision and Microsystems Engineering, Faculty of 3mE, Delft University of Technology, 2628CD Delft, The Netherlands
b Department of Materials Solutions, TNO Eindhoven, The Netherlands

A R T I C L E   I N F O

Article history:
Received 12 June 2015
Received in revised form 20 August 2015
Accepted 28 September 2015

Keywords:
Front metallization
Free-form
Solar cells
Topology optimization
Optimal design

A B S T R A C T

Free-form solar cells are cells of unconventional shapes (e.g. hexagonal, leaf-shaped etc). Their flexible shape adds to the aesthetics of the surroundings as well as allows to place them over objects where conventional solar cells might not fit. Evidently, these cells need to be efficient as well, and one of the important factors that controls their performance is the front metallization design. In this paper, we present the application of topology optimization (TO) to optimize the front metallization patterns for free-form solar cells. TO distributes the electrode material on the solar cell front surface in an efficient manner, such that the total power output is maximized. To demonstrate the capability of the proposed methodology, we use it to optimize front metal grids for several complex solar cell shapes e.g. circular, hexagonal, leaf-shaped, motorbike fairings, etc. The results presented here demonstrate the capability of TO to generate efficient designs for these free-form shapes.

1. Introduction

With the rising global population and limited fossil fuel reserves, the availability of sufficient energy sources is increasingly uncertain. Solar energy is growing as one of the important sustainable energy sources for the future [1]. Within a few decades, photovoltaic (PV) technology has advanced significantly and solar panels are being installed across the deserts and fields around the world. Since a significant part of the generated energy is lost while transporting it from the remote areas where these PV systems are installed, people have started installing solar panels on rooftops and along their window panes.

For the installation of solar systems in residential areas, no additional space is needed. Mostly, these systems are used to generate electricity at the point of use, which leads to lower transmission costs. The reduced space requirements and lower transmission costs reduce the needed investments [2]. However, solar panels installed on the rooftops must add to the building’s aesthetics and blend into its architectural makeup. Many people, when buying homes or commercial building, give aesthetics a primary consideration in the purchasing decision. Since traditional looking houses and commercial buildings hold significantly high monetary value in the real estate market, owners try to ensure that the building’s appearance is not adversely affected by these solar panels [3]. Only a small part of the building might then be covered with solar panels leading to reduction in the total possible output power. Architects are therefore challenged to find the optimum balance between the traditional aesthetics and the energy efficiency.

The replacement of conventional building materials with building-integrated photovoltaic (BIPV) materials is one of the most efficient ways to integrate solar systems in the buildings without affecting their appearance [2]. Nowadays, solar panels of unconventional shapes are being used. Recently, for example, architects have designed hexagonal solar panels [5]. Proposals have been made to use these solar panels to design big structures such as parking lots, self-powered football stadiums etc. [5,7]. Another recent advancement for improved aesthetics is to design leaf-shaped solar cells [8]. Scientists have also shown that replicating the wrinkles and deep folds on solar cells as seen on leaves, helps to improve their performance [9]. Other than the improvement in performance, trees with leaf-shaped solar cells could contribute to neighbourhood’s aesthetics and are expected to be popular in residential areas. Fig. 1 shows three prototypes demonstrating the application of free-form solar panels. The solar tree shown in Fig. 1a consists of circular leaves made out of simple solar panels. Fig. 1b
and c correspond to a solar powered stadium and a solar bicycle, respectively. The aesthetics of these objects can be improved further by using solar cells of more complex shapes e.g. replacing the simple solar panels of the solar tree with natural leaf-shaped solar cells. There can be several other daily seen objects where solar cells of unconventional shapes can be integrated. Solar street lights could have the whole supporting pole designed out of solar cells. Solar-powered cars and boats already exist and researchers have been trying to improve their performance \cite{10-12}. By using unconventional shapes, not only could their look be improved, but the solar cells can also be placed more efficiently in a given space. Possibilities exist of putting thin solar cells over the aerodynamically designed complex parts of motorbikes and cars.

While designing arbitrarily-shaped solar cells, aesthetics should not be the only focus. One should also make sure that the solar system is efficient. Fig. 2 shows the schematic diagram for a simple solar cell. It consists of a semiconductor layer sandwiched between front and back metal electrodes. Sometimes, a transparent conductive oxide might also be added to further reduce the resistive losses. An important aspect that affects the efficiency of solar cells is the distribution of voltage on the front surface \cite{13}. There exists a voltage $V_{mp}$ at which the maximum local power is generated. Using a metal grid on the front surface helps to bring the voltage values on the entire front surface close to $V_{mp}$. However, deposition of metal grid blocks the sunlight and leads to shading losses. The metallization should be done in a way such that the sum of the resistive and the shading losses is minimized. Thus, for every solar cell shape, there exists an optimal design which leads to maximum power output. For rectangular solar cells, the conventional H-pattern is preferred. This pattern might be optimal or sufficiently good for rectangular solar cells \cite{13}. However, in general, there might be cases where the optimal pattern might not comprise of straight metal grid lines. As shown in Ref. \cite{13}, the optimized solution may consist of complex patterns which cannot be easily characterized using simple geometrical shapes. For shapes other than rectangular, the H-pattern might not necessarily be a good choice for front electrode design. For concentrator systems with III-V solar cells, circular-shaped solar cells are also used. For these shapes, radial distribution of electrode material leads to improved performance \cite{14}.

Several advanced techniques exist for fabricating very fine electrode line widths \cite{15-17}, which can eventually help in printing complex patterns. While printing such complex patterns is not a problem anymore, designing such patterns is still a challenge. For complicated shapes, it is not easy to intuitively come up with patterns that can lead to well-performing solar cells. In Ref. \cite{18}, the authors have shown that a topology optimization (TO) approach can be used to design optimal front electrode patterns for different solar cell shapes. Topology optimization is a mathematical approach to design the optimal structure in a provided design space with a given set of boundary conditions, with the objective of maximizing the performance of the system \cite{19}. For the study presented in Ref. \cite{18}, square-shaped, rectangular and circular solar cells were used. A detailed investigation of the application of TO for designing optimal front electrode patterns of solar cells has been provided in Ref. \cite{13}.

In this paper, we present the application of TO to design front electrode patterns for free-form solar cells to obtain maximum power output. We extend the application of TO to several complex solar cell geometries. The arbitrary design domains are discretized using the finite element method and the design is iteratively improved using gradient-based optimization. Section 2 provides a brief discussion on the TO approach and its applications. The details of the methodology and the geometries used here are provided in Section 3 and Section 4, respectively. Further, Section 4 provides in-depth discussions over each of the obtained results.
2. Topology optimization

We use topology optimization (TO) to design the optimal front electrode patterns for free-form solar cells. TO is a mathematical approach that helps to obtain the best performing design by optimizing the distribution of the material in a given space for a given set of boundary conditions [19]. TO is a different approach when compared to shape optimization. For clear understanding of shape and topology optimization, please see Fig. 3. Fig. 3a shows a square-shaped solar cell with the electrode material distributed using the H-pattern. Here, shape optimization would refer to finding the optimal width (w) of the electrode line and the optimal spacing (s) between the electrode lines. With this shape parametrization, the solution will necessarily consist of straight lines. When applying TO, there is no fixed parametrization and a much larger range of geometries can be explored.

TO has great implications in early conceptual and design phases where slight perturbations in the design significantly affect its performance. For the last two decades, TO has been broadly used by academia as well as industries to solve challenging real-world problems. It is no longer restricted to structural problems with linear responses. Rather, it has been used to solve problems related to designing structures with the combination of mechanics, heat transfer, acoustics, fluid flow, aeroelasticity, materials design, and other multiphysics problems. For in-depth discussions related to these applications, please see the review paper by Deaton and Grandhi [20]. Very recent applications of TO involve optimizing certain parts of the solar cells so that their performance can be improved. TO has been used to design efficient light trapping structures for solar cells [21,22]. In Refs. [13,18], TO has been used to optimize the front metallization patterns for solar cells. These studies involved applications on simple rectangular and circular grids. In this paper, we explore the possibility to use TO for free-form solar cells, rather than simply rectangular or circular solar cells.

In a solar cell front electrode design problem, the electrode material needs to be optimally distributed over the front surface of the solar cell. Generally, in a TO problem, a restriction is imposed on the maximum amount of electrode material that can be used. However, for the problem presented here, no such volume constraint is needed. For very low amount of material, the net electrical conductivity of the front surface will be very low, leading to higher voltage contrasts and increased power drop. Excessive electrode material leads to increased shading of the front surface, which again causes for a drop in the output power. Thus, the optimum amount of electrode material is obtained from the compromise between the resistive and shading losses.

The objective of our problem is to maximize the power output from solar cells. For this, we use Kirchhoff’s law according to which all the current generated in the domain has to pass through the busbar which is kept at a potential \(V_{\text{bus}}\). The busbar is the part of the solar cell from where all the current generated in the solar cell is collected. We compute the power output at the busbar using the following expression:

\[
P_{\text{out}} = V_{\text{bus}} \int_j dS
\]

where, \(j\) is the out-of-plane current density and \(\Omega\) is the solar cell front surface domain.

3. Methodology

The different steps involved in designing front electrode patterns for solar cells using TO approach can be understood from the flowchart shown in Fig. 4. Here, we provide a brief description of each of these steps. For detailed descriptions and mathematical formulations of each of these steps, see Ref. [13]. The first and foremost step is to define the physics of the problem and describe it through a mathematical model. The problem presented here can be expressed using the Poisson equation for electrical conductivity, given by

\[
\sigma \nabla^2 V = \frac{\partial \rho}{\partial t},
\]

where, \(\sigma\), \(V\) and \(\rho\) denote the material dependent conductivity, electric potential and enclosed charge density, respectively. The front surface domain is discretized into a number of finite elements and with every finite element is associated a material density \(x\). This density corresponds to the volume fraction of the element filled with the given material and is used as a design variable in the TO formulation. After the finite element discretization of Eq. (2), the matrix equations for the solar cell front electrode problem can be stated as

\[
G V = I(V),
\]
where $G$, $V$ and $I$ are the total conductivity matrix, voltage vector and current vector, respectively. This current is the current generated at any point in the active layer which then travels normal to the front surface. For the problem presented here, the current generated in the active layer and the voltage across it are nonlinearly related, resulting into a nonlinear physical model [23]. This nonlinear relation is as follows,

$$I = I_l - I_0 \left( e^{\frac{qV}{kT}} - 1 \right). \quad (4)$$

In Eq. (4), $I$, $I_l$ and $I_0$ are the net, photoilluminated and reverse bias currents, respectively, $q$ is the electric charge, $k_B$ is the Boltzmann constant and $T$ is the temperature at any point in the active layer. To account for the shading effect, $I_l$ in Eq. (4) is replaced by the corrected photoillumination current ($I_l^* = I_l(1 - x)^r$), given by

$$I_l^* = I_l^0 \left( 1 - \frac{r}{x} \right)^r, \quad (5)$$

where $r$ is a penalization factor [13]. Here, we take $r$ to be equal to 3. As a next step, we assume an initial electrode material distribution. This information is used to build the finite element model and the nonlinear problem is then iteratively solved using the Newton method. Then the performance of the solar cell is evaluated in terms of efficiency ($\eta$) using the following equation.

$$\eta = \frac{P_{out}/A_m}{p_{imp}} \times 100\% \quad (6)$$

where $A_m$ is the area of the solar cell and $p_{imp}$ is the input power density of the sun which is assumed to be 1000 Wm$^{-2}$ under standard conditions. Our problem is an unconstrained maximization problem and we solve it using a gradient-based optimization scheme [13]. At the start of the TO process, $\eta$ is set to 0. At every iteration, the new value of $\eta$ obtained from Eq. (6) is compared to its previous value. If a gain in performance is observed, the material distribution is updated.

The first step to update the design is to compute the design sensitivities, i.e. gradient information indicating the dependence of the cell efficiency on the electrode material distribution. An adjoint formulation is applied because of the high dimensionality of the optimization problem [24]. These design sensitivities are used by the optimizer to obtain a new design. The new design may consist of checkerboard formations which might lead to unrealistic performance values. To avoid this, we employ density-based smoothing as a post-processing step [25]. The resultant design is then again used to build a new finite element model and the whole process is repeated in a loop till no further improvements can be made to increase the power output. When the performance of the solar cell cannot be improved any further, the optimization process is stopped and the final density distribution is considered to be the optimized design.

### 4. Results

In this section, we demonstrate and evaluate the capability of TO to design optimized front electrode patterns for free-form solar cells. We start with some simple geometrical shapes and use TO to design optimized patterns for such domains. As more complex shapes, we use leaf-shaped domains as well as domains corresponding to bodyparts of a motorbike. For all these cases, the $J$–$V$ curve used is as follows [26],

$$j = 353 - 0.0029(e^{19.3V} - 1). \quad (7)$$

In Eq. (7), $j$ and $V$ are the current density and voltage values measured across the solar cell. The units of $j$ and $V$ are A/m$^2$ and volts, respectively. The front surface of the solar cell is assumed to consist of a transparent conductive oxide (TCO) layer and the metal electrode having thicknesses of 200 nm and 10 $\mu$m, respectively.

#### 4.1. Simple polygons

Simple polygonal shapes can be used to approximate most of the complex geometries. For example, in Fig. 1b, hexagonal shapes have been used to design the football stadium. The solar tree and the solar bike also make use of simple round-shaped solar cells (Fig. 1a and c). Thus, we present here, the application of TO on simple polygons. As examples, we use square-shaped, circular and hexagonal solar cells.

Fig. 5a and b shows a square-shaped solar cell and a circular solar cell, respectively. Square-shaped geometries are commonly used for conventional solar cells and circular shapes are used in concentrator solar cells [14]. The solar cell models presented in Fig. 5 are pin-up modules (PUM), where the busbar (denoted by red) is located at the centroid of the geometry. From the centroid, the busbar goes all the way through the cell towards the rear side and all the connections are done there. For both the cases, we observe that the optimized design consists of several electrode fingers starting from the center and diverging radially outwards. Another observation is that the performance of both the designs is almost the same. In general, the circular solar cells should be expected to perform better because, for circular solar cells, the distance of the farthest point from the busbar is smaller, leading to lower resistive losses than that in case of the square-shaped solar cells.

To demonstrate the applicability of TO on hexagonal solar cells, we use two different configurations. The busbar locations for these cases are denoted by red. In the first configuration, the busbar point is located at one of the vertices of the hexagon and we refer to this configuration as a single-busbar model. Fig. 6a shows the optimal

---

[Diagram Flowchart showing the steps involved in designing front electrode patterns for solar cells using topology optimization.]

Fig. 4. Flowchart showing the steps involved in designing front electrode patterns for solar cells using topology optimization.
front electrode pattern for this configuration. The efficiency of this solar cell is 13.55%. In the other configuration, the multi-busbar model, all the six vertices of the hexagon are assumed to be busbar points. The optimal electrode distribution for this configuration is shown in Fig. 6b. The multi-busbar model delivers an efficiency of 14.18%. Thus, we observe that increasing the number of busbar points helps to significantly improve the performance of the solar cell. This is because the average distance from any point on the front surface to the closest busbar point for the multi-busbar model is lower than that for the single busbar model, which has only one busbar point. Thus, for the single busbar model, the voltage drop on the front surface will be larger. For any J–V curve, there exists a maximum power point voltage ($V_{mp}$), at which the output power is maximized [23]. Due to the large voltage contrast on the front surface of the single busbar model, very few points have voltage values close to $V_{mp}$ leading to significantly lower power output compared to the multi-busbar model.

4.2. Leaf-shaped

We use two different leaf-shapes and design optimal front electrode configurations using TO. Fig. 7a shows a simple leaf where the lowermost end is assumed to be directly connected to the busbar. This cell delivers an efficiency of 13.25%. For the maple leaf domain (Fig. 7b), the efficiency of the cell is 14.39%. The efficiency for the maple leaf is higher compared to the simple leaf-shaped domain because the length-to-width ratio for this case is lower than for the simple leaf case. Thus, for the simple leaf, current from a large part of the front surface has to travel larger distance to reach the busbar, thereby leading to increased resistive losses. For both the cases, we observe that the optimized front electrode designs resemble to some extent to the venation networks seen on natural leaves. Contrary to natural leaves, we do not see very fine features in our design. This is due to the restriction imposed on the minimum allowed feature size in our design. Note that the venation network on natural leaves has a completely different role compared to the front electrode designs on solar cells. These networks are mainly optimized for efficient nutrient transportation and mechanical support to all parts of the leaf [27]. However, these patterns might not be optimal for other problems such as the nonlinear front electrode design problem in solar cells.

4.3. Motorbike parts

To illustrate the applicability of the proposed TO-based method to design optimized electrode patterns for free-form solar cells integrated over consumer products, we consider its application to parts of the bodywork of a motorcycle. Fig. 8 shows images of a Suzuki GSXR600 motorbike (side-view) and two parts of its bodywork. Fig. 8b and c correspond to the front head light cover and the right lower fairing, respectively. We present the idea that free-form solar cells can be integrated over these two body parts of the motorbike. For simplicity, we take 2D projections of these 3D parts of the bike and design optimal front electrode configurations for solar cells of such shapes. For each of these shapes, we use pin-up modules (PUM) with two different busbar configurations and try
to understand the difference between the two configurations through the corresponding front surface voltage distribution plots. Figs. 9 and 10 show the optimized front electrode designs and the corresponding front surface voltage distributions for the front headlight cover of the motorbike. The busbar point locations for both the configurations are shown using red circles. The TO optimized designs consist of dendritic patterns and look aesthetically pleasing. The efficiencies recorded for the two busbar models are 10.56% and 11.27%, respectively. Thus, we observe that having more busbar points helps to improve the performance of this solar cell design. This can be explained from the front surface voltage distribution plots shown in Figs. 9b and 10b. We observe that the front surface voltage contrast is higher for the single busbar model. This is because the average distance travelled by current from any point on the front surface of the cell to the closest busbar is higher for this case. This leads to increased resistive losses and lower power output for the case of single busbar model.

Fig. 11a and b shows the optimal front electrode designs for the lower fairing of the bike for PUM solar cells with 8 and 13 busbar points, respectively. The busbar points are denoted by red. Here...
also, we observe that the model with more busbar points performs better over the other one. However, the cases shown for multi-busbar model cannot be claimed to be the best configurations in terms of performance. The optimum number and locations of busbar points would vary based on the shape of the free-form solar cell and finding these is still an open research question.

4.4. Discussions

In this paper, we show all the results using the J–V curve of a thin-film solar cell. However, this does not restrict the applicability of the proposed methodology to only thin-film solar cells. This approach contains certain assumptions and simplifications and as long as these are valid for other solar cells (c-Si, CIGS etc.), our methodology can be used. A significant assumption in this methodology is that it assumes an active semiconductor layer sandwiched between a front electrode and a rear electrode. Because of the high conductivity of the electrodes, the current in the active layer is assumed to travel perpendicular to the front surface [13]. This assumption is expected to be valid for all solar cells which use a front metal electrode layer. Thus, this methodology is generalized and can be used for different types of solar cells. We do not have any experimental results so far and are currently involved in validating these designs through experiments.

The efficiency values for different cases discussed here vary from 11% to 14%. This difference is due to the different sizes of these cells combined with different busbar layouts. We tried to choose the sizes of these solar cells in a way that they are close to the realistic ones. Also, the reported efficiencies are only to give a rough indication of the performance of these solar cells. Since there are no standard shapes for these cells, no direct comparison can be made, but we have also evaluated the performance of the method on rectangular cells where comparative efficiencies were found [13]. In general, we observe that the designs obtained using TO contain very thin metal grids. Due to the limitations of the fabrication techniques which can lead to poor contact between the metal grids and the photovoltaic material, very thin lines may not be efficient. While we do not take these limitations into consideration in our model, we make sure that such problems are minimized by not allowing the grid thickness beyond a certain limit. This can be easily achieved in the proposed methodology by imposing a restriction on the minimum allowed grid thickness through the use of smoothing filters [29].

For all the examples presented in this paper, we assume that the illumination is uniform over the whole front surface of the solar cell. However, adapting the free-form designs to the illumination conditions is an important aspect. Non-uniform light intensity on the solar cell front surface can significantly reduce its performance. In conventional solar cells, the front surface is two-dimensional and can be easily aligned to face the sun. But, for the case of free-form solar cells, this might not be the case. For example, consider the fairing parts of the motorbike presented in Section 4.3. These are not flat but curved surfaces. Similarly, several other cases can be thought of where the front surface will be three-dimensional. In all
these cases, not the whole front surface of the cell can be aligned at the same time with the direction of the sun. This will cause for non-uniform illumination and can sometimes lead to a significant part of the solar cell not receiving any sunlight. The solar bike proposed by Ref. [6] overcomes this problem to some extent. The solar disks of the bike (Fig. 1c) can tilt up to a certain angle so as to maximize the incident solar light. For more complex front patterns, such an approach may not help. Another aspect to be given consideration is the angle of incidence of the sunlight. Here, we assume that the light input is constant. However, for curved free-form shapes e.g. bikes riding around, the angle of incidence will be significantly varying in time. For free-form solar cells to be prominently used, a method to reduce the effect of non-uniform illumination and varying angle of incidence has to be figured out. A direction of research could be to design a front electrode design in a way such that the sum total of power output for different angles of tilt is maximized. Here, tilt angle refers to the angle of rotation about any axis (vertical/horizontal).

5. Conclusions

In this paper, we explored the capability of topology optimization (TO) to optimize the front electrode patterns for free-form solar cells. While for conventional shapes, well performing front electrode patterns can be designed based on intuitive notions, designing efficient patterns for complex free-form shapes is not easy. It has been shown that TO is capable of optimizing the front electrode patterns for even very complex shapes such as leaves, motorbike fairing parts, etc. The designs obtained from TO exhibit a dendritic topology and are aesthetically pleasing. For the leaves, these designs tend to achieve to some extent the pattern seen on actual leaves. For the motorbike parts, these dendritic patterns give an impression of autographics which can make the motorbike look nicer. Also, we showed that the busbar location significantly affects the performance of the solar cells. From these examples we conclude that TO is capable of designing front patterns for even complex shapes and can be a good methodology in the direction of free-form solar cells.

Acknowledgements

This work is part of the Industrial Partnership Programme (IPP) ‘Computational sciences for energy research’ of the Foundation for Fundamental Research on Matter (FOM), which is part of the Netherlands Organisation for Scientific Research (NWO). This research programme (12CSER090) is co-financed by Shell Global Solutions International B.V. Thanks to Prof. Krister Svanberg for providing the MMA subroutine. We express our gratitude to Daniel Martins, Leon Kaye, M. Raeisi, Airtech, Ross Lovegrove Ltd. and Hight3ch for allowing us to use their images in this paper.

References