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Instability of an oscillator moving along a thin ring on a viscoelastic foundation

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Abstract

The stability of an oscillator uniformly moving along a thin ring that is connected to an immovable axis by a distributed viscoelastic foundation has been studied. The dynamic reaction of the ring to the oscillator is represented by a frequency and velocity dependent equivalent stiffness. The characteristic equation for the vibration of the oscillator is obtained. It is shown that this equation can have roots with a positive real part, which imply the exponential increase of the amplitude of the oscillator's vibration in time, i.e. instability. The critical velocity after which instability can occur is determined. With the help of the D-decomposition method, the instability domains are found in the space of the system parameters. Parametric study of the stability domains is carried out.

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Keywords: Thin ring; viscoelastic foundation; moving oscillator; instability

1. Introduction

The dynamic response of structures to moving loads has long been a subject of great interests especially in the field of railway engineering. It deals with interactions between moving loads/objects and elastic continua. Generally, two types of problems are of particular importance: i) the forced vibration of the supporting structures caused by moving loads or objects and ii) the stability of moving objects themselves. A considerable amount of literature has been published on the first problem, not to mention the classical and comprehensive monograph by Frýba [1]. The second problem is less extensively studied. Metrikine et al. [2–5] have systematically invistigated the stability problems of moving objects on elastically supported beams. It has been concluded that the moving object can be unstable when it moves faster than a critical speed because of the appearance of anomalous Doppler waves [6]. Other research on the stability issue include e.g. [7,8] and the recent studies conducted by Mazilu and his co-authors [9,10]. In the latter investigations, the nonlinearity of the wheel/rail contact is considered, which turns out to be important.

All the above-referenced studies which focus on stability of moving objects consider those to move along a straight system. Considering a moving mass on an infinite EB beam [2], the necessary condition for instability to occur is that it moves faster than the minimum phase speed of waves in the beam, which can be referred to as a resonant speed.

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In this paper, an elastic ring is considered as the structure supporting a moving oscillator. The closeness of the ring introduces infinitely many resonant speeds [11]. The stability of such a system has not been analysed in the past. After obtaining the complex-valued equivalent stiffness at the contact point, the oscillator-ring is reduced to a mass-spring system. The D-decomposition method is used to study the stability domains in the space of system parameters.

2. Model and characteristic equation

The study of an oscillator-ring system is motivated by modelling an elastic train wheel interacting with railway tracks. As shown in Fig. 1, the train wheel is modeled as a flexible ring attached to an immovable axis by visco-elastic springs in both radial and circumferential directions, with the stiffness per unit length k_r (viscosity σ_w) and k_c (viscosity σ_u), respectively. It is assumed that the mean radius of the undeformed ring is R. θ is the circumferential polar coordinate. Small displacements in radial and circumferential directions are denoted as $w(\theta, t)$ and $u(\theta, t)$. In addition, ρ denotes the density of the ring, E is the Young's modulus, A is the cross-sectional area and I is the cross-sectional moment of inertia. The track is represented by a point mass m. The contact between the track and wheel is simplified as a Hertz contact spring k_1, together with a dashpot σ_1 to account for dissipation at the contact area. The support of the track from the substructure is represented by a visco-elastic spring k_2 whose viscosity is characterized by σ_2. $w^{01}(t)$ and $w^{02}(t)$ denote the displacement of the ring at the contact point and the one of the mass, respectively.

The equations which govern vibrations of a thin ring can be found in [12]. The oscillator rotates at Ω (angular velocity). In order to analyse the problem, it is convenient to introduce the following dimensionless variables

$$
\begin{aligned}
&{t_0}^2 = \frac{\rho A R^4}{EI},\ \chi = \frac{EAR^2}{EI},\ (\bar{K}_r, \bar{K}_c) = \frac{(k_r, k_c)R^4}{EI},\ \varepsilon_{(w,u)} = \frac{\sigma_{(w,u)} t_0}{\rho A},\ (K_1, K_2) = \frac{(k_1, k_2)R^3}{EI},\ \varepsilon_{(1,2)} = \frac{\sigma_{(1,2)} t_0}{\rho A R},\\
&\tau = t/t_0,\ \bar{\Omega} = \Omega t_0,\ (w, u) = R(W, U),\ W^{(01,02)} = Rw^{(01,02)},\ M = m/(\rho A R).
\end{aligned} \tag{1}
$$

Thin rings are considered, thus shear deformation and rotatory inertia are not included. The dimensionless governing equations which describe vibrations of the ring-oscillator system in the moving reference $\{\phi = \theta - \bar{\Omega}\tau, \tau = \tau\}$ are

$$
\begin{aligned}
&\ddot{W} - 2\bar{\Omega}\dot{W}' + \bar{\Omega}^2 W + (W'''' - U''') + \chi(W + U') + \bar{K}_r W + \varepsilon_w(\dot{W} - \bar{\Omega}W') = \\
&-\sum_{n=-\infty}^{+\infty} \Big(K_1(W^{01} - W^{02}) + \varepsilon_1\big(\frac{dW^{01}}{d\tau} - \frac{dW^{02}}{d\tau}\big)\Big)\delta(\phi + 2n\pi),\\
&\ddot{U} - 2\bar{\Omega}\dot{U}' + \bar{\Omega}^2 U + (W''' - U'') - \chi(W' + U'') + \bar{K}_c U + \varepsilon_u(\dot{U} - \bar{\Omega}U') = 0,\\
&M\frac{d^2W^{02}}{d\tau^2} + \varepsilon_2\frac{dW^{02}}{d\tau} + \varepsilon_1\big(\frac{dW^{02}}{d\tau} - \frac{dW^{01}}{d\tau}\big) + K_2 W^{02} + K_1(W^{02} - W^{01}) = 0, W^{01}(\tau) = W(0, \tau)
\end{aligned} \tag{2}
$$

where n is an integer to account for the periodicity of the ring with the period 2π. Prime stands for the spatial derivative with respective to θ and overdot represents time derivative. Since k_1 and k_2 are large, the centrifugal force acting on the moving oscillator is relatively small comparing to the reaction force from the springs and thus is neglected.

The Fourier transform with respect to ϕ and Laplace transform with respect to τ are defined as ($\mathrm{i} = \sqrt{-1}$):

$$
\begin{Bmatrix} \tilde{W}_s(\phi, s) \\ \tilde{U}_s(\phi, s) \end{Bmatrix} = \int_0^{+\infty} \begin{Bmatrix} W(\phi, \tau) \\ U(\phi, \tau) \end{Bmatrix} \exp(-s\tau)d\tau,\quad \begin{Bmatrix} \tilde{\tilde{W}}_{k,s}(k, s) \\ \tilde{\tilde{U}}_{k,s}(k, s) \end{Bmatrix} = \int_{-\infty}^{+\infty} \begin{Bmatrix} \tilde{W}_s(\phi, s) \\ \tilde{U}_s(\phi, s) \end{Bmatrix} \exp(-\mathrm{i}k\phi)d\phi. \tag{3}
$$

Applying the above Fourier and Laplace transforms to equation (2) and eliminating $\tilde{\tilde{U}}_{k,s}$ from the first two equations of Eqs. (2) (the initial conditions are taken as trivial since they have no effect on stability of linear systems), we obtain

$$
\tilde{\tilde{W}}_{k,s}(k, s)D(k, s) = -(K_1 + \varepsilon_1 s)(\tilde{W}^{01}_s(s) - \tilde{W}^{02}_s(s)) \sum_{n=-\infty}^{+\infty} \exp(2\mathrm{i}kn\pi) \tag{4}
$$

where

$$
\begin{aligned}
&D(k, s) = \frac{A(k, s)B(k, s) + (C(k, s))^2}{A(k, s)},\quad A(k, s) = \big(1 - \bar{\Omega}^2 + \chi\big)k^2 + \big(-2\,\mathrm{i}\bar{\Omega}\,s - \mathrm{i}\bar{\Omega}\,\varepsilon_u\big)k + \bar{K}_c + s^2 + \varepsilon_u s,\\
&B(k, s) = k^4 - \bar{\Omega}^2 k^2 + \big(-2\,\mathrm{i}\bar{\Omega}\,s - \mathrm{i}\bar{\Omega}\,\varepsilon_w\big)k + \bar{K}_r + s^2 + \varepsilon_w s + \chi,\quad C(k, s) = \mathrm{i}k^3 + \mathrm{i}\chi\,k.
\end{aligned} \tag{5}
$$

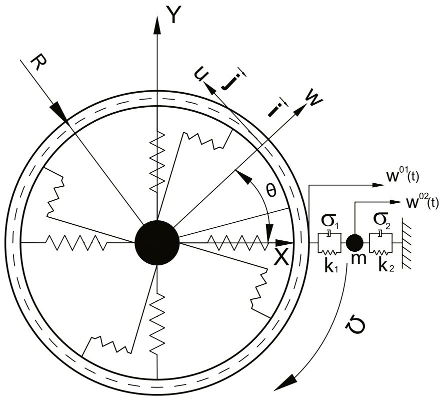

Fig. 1. Moving oscillator on a stationary thin ring.

Performing the inverse Fourier transform with respect to k in Eq. (4), the Laplace-displacement of the radial direction is

$$\begin{aligned}\tilde{W}_s(\phi, s) &= -\frac{(K_1+\varepsilon_1 s)(\tilde{W}_s^{01}(s)-\tilde{W}_s^{02}(s))}{2\pi}\int_{-\infty}^{+\infty}\frac{\sum_{n=-\infty}^{+\infty}\exp(2\mathrm{i}kn\pi)}{D(k,s)}\exp(\mathrm{i}k\phi)dk,\\ &= -\frac{(K_1+\varepsilon_1 s)(\tilde{W}_s^{01}(s)-\tilde{W}_s^{02}(s))}{2\pi}\sum_{n=-\infty}^{+\infty}\int_{-\infty}^{+\infty}\frac{\exp(\mathrm{i}k(\phi+2n\pi))}{D(k,s)}dk.\end{aligned} \tag{6}$$

By letting $\phi = 0$, a system of equations with respect to $\tilde{W}_s^{01}$ and $\tilde{W}_s^{02}$ are obtained

$$(K_1+\varepsilon_1 s+\chi_{eq})\tilde{W}_s^{01}-(K_1+\varepsilon_1 s)\tilde{W}_s^{02}=0,\ \ (Ms^2+K_2+\varepsilon_2 s+K_1+\varepsilon_1 s)\tilde{W}_s^{02}-(K_1+\varepsilon_1 s)\tilde{W}_s^{01}=0 \tag{7}$$

where

$$\chi_{eq}=\left(\frac{I_0}{2\pi}\right)^{-1}=\left(\frac{1}{2\pi}\sum_{n=-\infty}^{+\infty}\int_{-\infty}^{+\infty}\frac{\exp(2\mathrm{i}kn\pi)}{D(k,s)}dk\right)^{-1}. \tag{8}$$

This function determines the radial reaction of the ring to the moving oscillator at the contact point and is called equivalent stiffness. It can be evaluated using the contour integration similar to Ref. [12].

The characteristic equation for the oscillator dynamics can be obtained by setting the determinant of the coefficient matrix of Eq. (7) to zero. This gives

$$(K_1+\varepsilon_1 s+\chi_{eq})(Ms^2+K_2+\varepsilon_2 s+K_1+\varepsilon_1 s)-(K_1+\varepsilon_1 s)^2=0. \tag{9}$$

The system stability is determined by the roots of Eq. (9). The vibration of the oscillator is unstable if there is at least one root of s with a positive real part.

In next sections, the D-decomposition method [2–5] will be used to analyse the stability of the oscillator. It is customary to replace s by $s = \mathrm{i}\omega$ in the following analysis.

3. The equivalent stiffness

Analogous to an inifinitely long Euler beam whose critical speed, after which instability can happen, is the resonant speed (minimum phase speed) of a constant point load moving on the beam [2], we may expect that instability of a moving oscillator can occur when its velocity exceeds critical speeds at which resonances occur. For an elastic ring subjected to a moving point load, resonance occurs when the travelling speed Ω equals a natural frequency divided by the corresponding mode number [11,12]. The critical speeds are $\bar{\Omega}^n_{cr} = \omega_n/n$ in dimensionless form, where ω_n is the dimensionless natural frequency. The first critical speed is the minimum of $\bar{\Omega}^n_{cr}$.

The parameters are adopted from [12] for a steel ring with rectangular cross-section. These parameters are

$$E = 2.06\times10^{11}\mathrm{N/m^2},\ I = 2.83\times10^{-6}\mathrm{m^4},\ \rho = 7800\mathrm{kg/m^3},\ A = 1.5\times10^{-3}\mathrm{m^2},\ R = 0.3\mathrm{m},$$
$$k_r = 6\times10^{7}\mathrm{N/m^2},\ k_c = 0.3k_r,\ \sigma_w = \sigma_u = 6\times10^{3}\mathrm{Ns/m^2}. \tag{10}$$

For the parameters adopted here, the first three critical speeds are

$$\bar{\Omega}^{n=1}_{cr} \approx 0.736,\ \bar{\Omega}^{n=2}_{cr} \approx 1.371,\ \bar{\Omega}^{n=3}_{cr} \approx 2.455 \tag{11}$$

and they are corresponding to the 1 to 3 bending dominant modes. The minimum resonant speed is the one associated with mode 1 for the chosen parameters. Geometrically, the critical speeds are the ones at which the kinematic invariant intersect the dispersion curves at the natural frequencies [12].

The real and imaginary parts of the equivalent stiffness determine the elasto-inertial and viscous properties of the ring, respectively. The equivalent viscosity can be negative when the object is moving super-critically. It has been shown that a moving object might become unstable because of the ”negative radiation damping” [13]. Since the sign of the imaginary part of χ_{eq} represents viscosity, the boundary of positive and negative damping is

$$\mathrm{Im}(\chi_{eq}(\mathrm{i}\omega)) = 0. \tag{12}$$

$\omega = 0$ is always a root of Eq. (12) because of symmetry. The other frequencies which satisfy Eq. (12) are denoted as ω_{cr}. The number of ω_{cr} depends on the velocities of the moving oscillator and the ring properties.

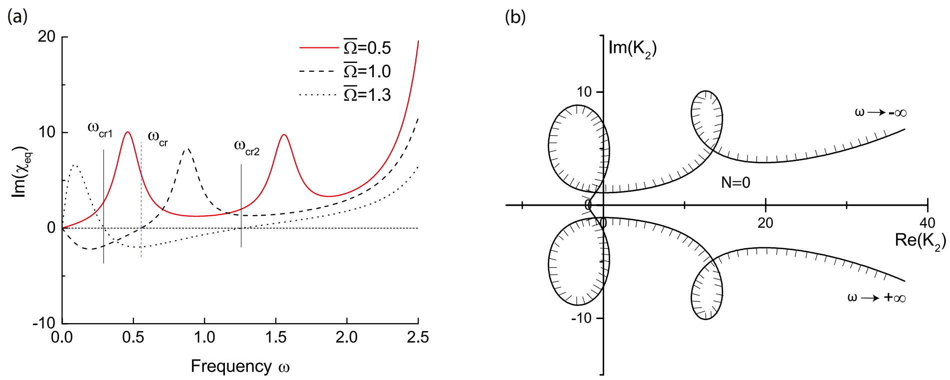

Fig. 2. (a) Imaginary part of the equivalent stiffness; (b) D-decomposition curves for $M = 5$ and travelling speed $\bar{\Omega} = 0.5 < \bar{\Omega}^{n=1}_{cr}$.

Fig. 2(a) shows the dependence of Im(χ_{eq}) on frequency. Here $\bar{\Omega} = 0.5$, 1 and 1.3 are chosen to illustrate the correlations. The first velocity is lower than the first critical speed whereas the latter two are between the first and second critical speeds. It is shown that the imaginary part of χ_{eq} is always positive when $\bar{\Omega} = 0.5$, indicating energy dissipation. There is one critical frequency ω_{cr} except $\omega = 0$ for $\bar{\Omega} = 1$. The equivalent damping is negative when $0 < \omega < \omega_{cr}$, implying energy gain and the oscillator may be unstable. When $\bar{\Omega} = 1.3$, there are two critical frequencies ω_{cr1} and ω_{cr2}, between which Im(χ_{eq}) < 0.

4. Stability analysis

The stiffness K_2 of the soil is chosen as our subject. Substituting $s = \mathrm{i}\omega$ into the characteristic equation Eq. (9), one obtains

$$K_2 = M\omega^2 - \chi'_{eq} - \mathrm{i}\varepsilon_2\omega \tag{13}$$

where

$$\chi'_{eq} = \frac{\chi_{eq}(K_1 + \mathrm{i}\varepsilon_1\omega)}{\chi_{eq} + K_1 + \mathrm{i}\varepsilon_1\omega}. \tag{14}$$

The equivalent stiffness χ_{eq} and K_1 act as two springs in parallel. It can be checked that the frequencies that correspond to $\mathrm{Im}(\chi_{eq}(\mathrm{i}\omega)) = 0$ are also the roots of $\mathrm{Im}(\chi'_{eq}(\mathrm{i}\omega)) = 0$ when $\varepsilon_1 = 0$.

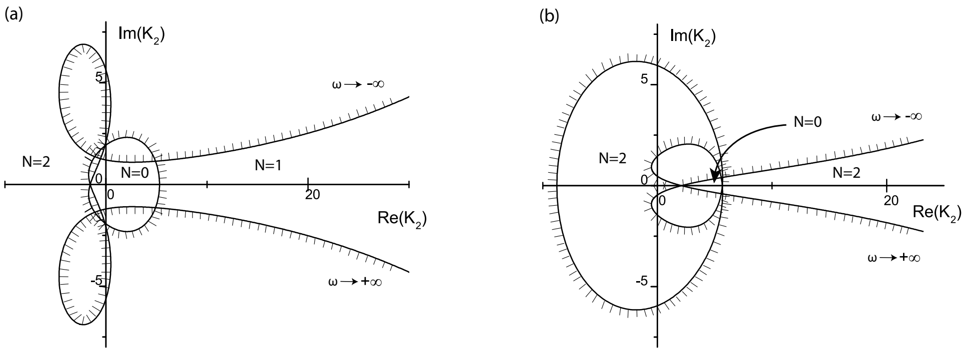

Fig. 3. D-decomposition curves for $M = 5$ and travelling speed : (a) $\bar{\Omega} = 1.0 \in \{\bar{\Omega}_{cr}^{n=1}, \bar{\Omega}_{cr}^{n=2}\}$; (b) $\bar{\Omega} = 1.3 \in \{\bar{\Omega}_{cr}^{n=1}, \bar{\Omega}_{cr}^{n=2}\}$.

The D-Decomposition curves are plotted for three velocities as illustrated in Fig. 2(b) and Fig. 3. The dashpots in the oscillator are neglected, namely $\sigma_1 = \sigma_2 = 0$. The stiffness of the Hertz contact spring is $k_1 = 1.4$GN/m which is taken from Ref. [14]. The crossing points (critical K_2, namely K_2^*) on the real axis are the points which divide domains with different number of unstable roots. The values of ω corresponding to the crossing points are given by Eq. (12). Crossing the D-decomposition line one time in the shading direction means that the number of unstable roots increases by one. The absolute numbers can be determined by either using the Cauchy's argument principle or following the way in the Appendix of Ref. [3]. The number of unstable roots is represented by N in each figure.

It shows that for speeds smaller than the first critical speed, no instability will occur (Fig. 2(b)). In Fig. 3(a), there is one unstable root when K_2 is greater than about 5. When the oscillator moves at velocity $\bar{\Omega} = 1.3$, two unstable regions exist on the positive real axis of K_2 as displayed in Fig. 3(b).

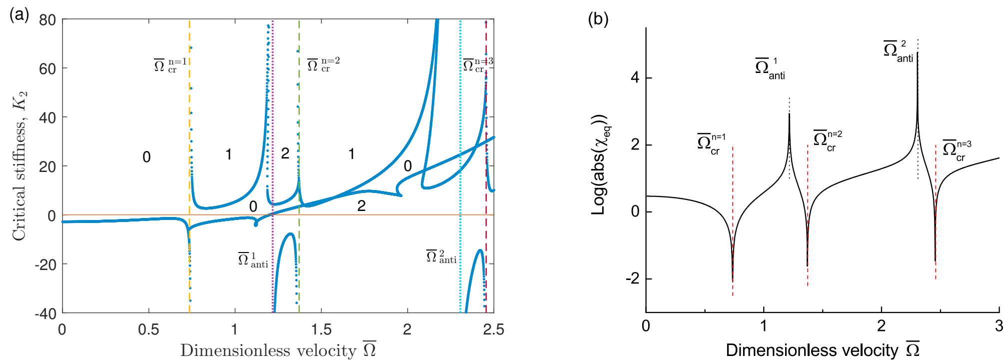

Fig. 4. (a) Critical K_2, namely K_2^* versus velocity, $\sigma_w = \sigma_u = 6000$Ns/m^2; (b) Equivalent stiffness versus velocity, $\sigma_w = \sigma_u = 10$Ns/m^2.

Fig. 4(a) shows the dependences of the stable and unstable regions on velocities. The number in different regions stands for the number of unstable roots of this region. At present we keep the regions where K_2 is negative as well in order to show a complete picture, although we are only interested in the regions where K_2 is positive eventually. The vertical dotted and dashed lines are velocities which separate the regions qualitatively. The stable region exists in the gap between the upright U-shape curve and the curve which is initially in the middle of the upright U-shape curve and the reverse U-shape curve. The originally middle curve has a tendency to increase with velocities, indicating that the stable region will disappear when this curve intersect one of the upright U-shape curve. However, extra stable regions may exist at higher speeds as Fig. 4(a) shows at about $\bar{\Omega} = 2$. As can be seen from Fig. 4(a), there exists one critical speed, namely the minimum resonant speed $\bar{\Omega}_{cr}^{n=1}$, below which the oscillator is always stable. Between two neighboring resonant speeds, there are two qualitatively different dependences of stability on moving velocities of the oscillator. For example, in the range of $\bar{\Omega} \in \{\bar{\Omega}_{cr}^{n=1}, \bar{\Omega}_{cr}^{n=2}\}$, $\bar{\Omega} \approx 1.22$ separates this domain into two parts. In the left domain, there are two curves of K_2^* (critical K_2), the upright U-shape curve correlates to $\omega = 0$ and the lower one to $\omega = \omega_{cr}$. In the right domain, three curves of K_2^* exist. The reverse U-shape curve in the negative plane of K_2 corresponds to $\omega = 0$ and the other two curves correspond to ω_{cr}. Similar conclusion holds for $\bar{\Omega} \in \{\bar{\Omega}_{cr}^{n=2}, \bar{\Omega}_{cr}^{n=3}\}$ in Fig. 4(a) although the values of K_2^* are located in a more complicated way. These velocities which demarcate the region among two adjacent resonant speeds are termed as $\bar{\Omega}_{anti}$ in Fig. 4(a), because after checking the equivalent stiffness, it is shown in Fig. 4(b) that these velocities correspond to anti-resonances.

5. Parametric study

The mass of the oscillator is a destabilising factor. The dissipation at the contact area has little influence on the stability of the oscillator. However the effect of the damping σ_2 is more profound. Although σ_2 serves as a stabilising factor for lower values, it destabilises the oscillator when it passes a certain magnitude. The most crucial parameter is the damping of the foundation of the ring. On one hand, the increasing of foundation damping shifts the first critical speed to a higher value and consequently moves the starting point of the unstable region to higher speeds. On the other hand, larger foundation damping expands the stable regions both to higher stiffness of K_2 and higher travelling speeds of the moving oscillator.

6. Conclusion

In this paper, the stability of a moving oscillator on a viscoelastically supported flexible ring has been investigated. The results have shown that exponential growth of the vibrational amplitude of the oscillator can occur when it moves super-critically. It has been found that the first critical speed after which the oscillator can be unstable is the minimum resonant speed of the ring under a moving load of constant magnitude. The influence of system parameters on stability has been analysed. The increasing mass shrinks the range of K_2 where the oscillator is stable. The damping of the ring foundation stabilises the oscillator. It increases the first critical speed and allows for higher stiffness of K_2. The influence of dissipation at the contact area on the stability of the oscillator is marginal. Lower values of σ_2 stabilise the oscillator, whereas σ_2 destabilises the oscillator when it reaches a certain magnitude. It should be mentioned that the critical speeds of the ring considered exceed the operational speeds of current trains. Although the initial motivation is to model elastic train wheels, the aim of the analysis in this paper is to show the existence and qualitative features of instability of such a ring-oscillator system which may find applications also in other engineering practice.

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