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## An optimization model for vehicle routing of automated taxi trips with dynamic travel times

Xiao Liang<sup>a,\*</sup>, Gonçalo Homem de Almeida Correia<sup>a</sup> and Bart van Arem<sup>a</sup>

<sup>a</sup> Department of Transport & Planning, Delft University of Technology, Stevinweg 1, Delft 2628CN, The Netherlands

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### Abstract

In this paper, we propose a method of automated vehicle operation in taxi systems that addresses the problem of associating trips to automated taxis (ATs) and assigning those vehicles to paths on an urban road network. This system is envisioned to provide a transport service within a city area with a seamless door-to-door connection for all passengers' origins and destinations. ATs can drive themselves on the roads with reduced direct human input, which allow taxis to satisfy the next trip or park themselves while waiting for a request if needed. We propose an integer programming model to define the routing of the vehicles according to a profit maximization function while depending on dynamic travel times which vary with the flow of the ATs. This will be especially important when the number of automated vehicles circulating on the roads is so high that will cause traffic congestion. The total profit involves the system revenue, vehicle fuel costs, vehicle depreciation costs, parking costs, penalties for unsatisfied trips and passengers' congestion delay. The model is applied to a small case study and the results allow assessing the impact of the ATs movements on traffic congestion and the profitability of the system. Even with a small case study, it is possible to conclude that having in consideration the effect of the vehicle flows on travel time leads to different results in terms of the system profit, the parking cost and the driving distance which points out the importance of this type of models.

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*Keywords:* Automated vehicles; Taxis; Vehicle routing; Dynamic travel time

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\* Corresponding author. Tel.: +31 (15) 27 85279.

E-mail address: [x.liang@tudelft.nl](mailto:x.liang@tudelft.nl)

## 1. Introduction

In the recent years, technology development has accelerated the process of vehicle automation. This technology is expected to have a beneficial impact on travel efficiency, especially on interurban roads. With respect to the impacts of these vehicles, studies have used micro-simulation tools or mathematical models to estimate the changes in road capacity and congestion under different levels of vehicle automation and cooperation. (Arem et al., 2006; Bose and Ioannou, 2003; Calvert et al., 2011).

Regarding the use of automated vehicles as transit systems, there are currently many projects which are testing the use of automated pods or buses in pilot experiments. One of the most relevant example being the CityMobil2 project (CityMobil2, n.d.) in which several field experiments are being run in Europe to test the possibilities of automated bus systems. In 2015 the province of Gelderland in the Netherlands started developing a project named WEpods with two self-driving vehicles, which are used between the Wageningen University and the train station (WEpods, 2015).

Several approaches have been proposed to test the effect of transit automation on mobility, especially looking at the combination between traditional taxis and shared use taxi fleets. The automated taxis (ATs) may provide a new type of door-to-door service competing with the traditional taxis or even the public transport because these new systems would be able to avoid extra manpower costs and make the AT service cheaper. Fagnant & Kockelman (2014) used a simulation method to study the implications of shared ATs and described the results of a case-study application. The results indicate that each shared automated vehicle can replace around 11 conventional ones, with up to 10% more travel distance. The International Transport Forum (ITF) built a model to test the introduction of 100% autonomous fleets of taxis to satisfy transport demand in a city (International Transport Forum, 2015). Results showed that with the subway still in operation each automated vehicle could remove 9 out of 10 cars, whilst without metro, the number goes down to 5. Spieser et al. (2014) used an analytical mathematical formulation to estimate the number of shared automated vehicles to replace all modes of personal transportation in the case-study city of Singapore. Using actual data, they are able to conclude that a shared-vehicle mobility solution could meet the personal mobility needs of the entire population with 1/3 of the number of passenger vehicles currently in operation.

A recent study addressed the traffic assignment of privately owned automated vehicles from user optimum perspective with the objective of minimizing each family's own transport costs (Correia and van Arem, 2016). This model proposed an equilibrium convergence where households with similar trips should have similar transport costs. Since system optimum vehicle assignment is a nonlinear problem which is difficult to solve, this was tackled by an iterative process where travel times do not change during each assignment of vehicles to trips.

In this paper, we are going to study a transport system with a fleet of automated vehicles used as taxis which provide a transport service within a city area with a seamless door-to-door connection for passengers' origins and destinations. Then a model is proposed to address the problem of assigning the vehicles to paths on an urban road network while satisfying the demand. Moreover, this model is expected to solve the problem with dynamic travel times which vary with the flow of the ATs.

## 2. Mathematical model

In this section, we describe a linear programming model whose objective is to determine the optimal vehicle routing of the AT system. We consider a private taxi service between any pair of nodes within the city road network, with no background traffic flow which means the flow is generated only by the ATs themselves. Clients use an online app to book an AT with the travel information including the origin, the destination and the desired departure time. The model works on the assumption that the taxi company can achieve total control of the system by being free to accept or reject requests according to a profit maximization objective.

*Sets:*

$N = \{1, \dots, i, \dots, N\}$ : set of the nodes in the network, where  $N$  is the total number of nodes.

$T = \{0, \dots, t, \dots, T\}$ : set of time instants in the service period, where  $T$  is the total number of time steps in the service period.

$T' = \{-T^r, \dots, 0, \dots, t, \dots, T, \dots, T + T^r\}$ : set of time instants in the operation time, including the service period

$\{0 \dots T\}$  and the buffer periods  $\{-T^r \dots 0\}$  and  $\{T \dots T + T^r\}$ , where  $T^r$  is the number of time steps in the buffer period.

$\mathbf{E} = \{1, \dots, e, \dots, E\}$ : set of trips, where  $E$  is the total number of all the trips in the service period.

$\mathbf{V} = \{1, \dots, v, \dots, V\}$ : set of vehicles, where  $V$  is the total number of taxis in the system.

$\mathbf{K} = \{0, \dots, k, \dots, K\}$ : set of breakpoints for dynamic travel time, where  $K$  is the total number of the break points in the flow-travel time curve.

$\mathbf{A} = \{\dots, (i, j), \dots\}$ : set of links of the road network where vehicles move,  $i \neq j$ .

$\mathbf{B} = \{\dots, (i_{t_1}, j_{t_2}), \dots\}$ : set of links in the time-space network,  $\forall (i, j) \in \mathbf{A}, \forall t_1, t_2 \in \mathbf{T}', t_1 < t_2, \delta_{ij}^{min} \leq t_2 - t_1 \leq \delta_{ij}^{max}$ .

*Data:*

$a^e$ : desired departure time for the  $e^{\text{th}}$  trip,  $e \in \mathbf{E}$ .

$b^e$ : latest arrival time for the  $e^{\text{th}}$  trip,  $e \in \mathbf{E}$ .

$ori^e$ : the origin node for the  $e^{\text{th}}$  trip,  $e \in \mathbf{E}$ .

$des^e$ : the destination node for the  $e^{\text{th}}$  trip,  $e \in \mathbf{E}$ .

$D_{ij}^e$ : equals to 1 if there is an  $e^{\text{th}}$  request from  $i$  to  $j$  who desires to start from time instant  $t$ ,  $i = ori^e, j = des^e, t = a^e, \forall i, j \in \mathbf{N}, \forall e \in \mathbf{E}, \forall t \in \mathbf{T}$ .

$\delta_{ij}^{max}$ : maximum travel time from node  $i$  to node  $j$ ,  $\forall (i, j) \in \mathbf{A}$ .

$\delta_{ij}^{min}$ : minimum travel time from node  $i$  to node  $j$ ,  $\forall (i, j) \in \mathbf{A}$ .

$\delta_{ij}^k$ : travel time from node  $i$  to node  $j$  at breakpoint  $k$ ,  $\forall (i, j) \in \mathbf{A}, k \in \mathbf{K}$ .

$d_{ij}$ : travel distance from node  $i$  to node  $j$ ,  $\forall (i, j) \in \mathbf{A}$ .

$Opt_{ij}$ : shortest travel time from node  $i$  to node  $j$ ,  $\forall i, j \in \mathbf{N}$  (computed with  $\delta_{ij}^{min}$ ).

$Cap_{ij}$ : Capacity of each link  $(i, j)$  which is the number of vehicles that go through the link at the highest travel time,  $\forall (i, j) \in \mathbf{A}$ .

*Parameters:*

$Pr$ : price rate of taxi per time step.

$C_f$ : fuel cost per driving distance.

$C_a$ : vehicle depreciation cost per vehicle.

$C_p$ : parking cost per time step.

$\rho$ : penalty cost per trip if a trip is rejected by AT and needs public transport.

$C_{del}$ : penalty cost per time step for congestion delay of passengers' requests.

*Decision variables:*

$x_{i_{t_1}, j_{t_2}}^v$ : binary variable equals to 1 if vehicle  $v$  drives from node  $i$  to  $j$  from time instant  $t$ ,  $\forall (i_{t_1}, j_{t_2}) \in \mathbf{B}, \forall v \in \mathbf{V}$ .

$y_{i_t}^v$ : binary variable equals to 1 if vehicle  $v$  parks at  $i$  from time instant  $t$  to  $t + 1$ ,  $\forall i \in \mathbf{N}, \forall v \in \mathbf{V}, \forall t \in \mathbf{T}', t \leq T + T^r - 1$ .

$S_{ij}^{ev}$ : binary variable equals to 1 if trip  $e$  from node  $i = ori^e$  to node  $j = des^e$  is done using vehicle  $v$ ,  $\forall e \in \mathbf{E}, \forall v \in \mathbf{V}$ .

$P_{ij}^{evt}$ : binary variable equals to 1 if trip  $e$  from node  $i = ori^e$  to node  $j = des^e$  done by vehicle  $v$  departs at time instant  $t$ ,  $\forall e \in \mathbf{E}, \forall v \in \mathbf{V}, \forall t \in \mathbf{T}, c^e \leq t \leq a^e + wait^e$ .

$A_{ij}^{evt}$ : binary variable equals to 1 if trip  $e$  from node  $i = ori^e$  to node  $j = des^e$  done by vehicle  $v$  arrives at time instant  $t$ ,  $\forall e \in \mathbf{E}, \forall v \in \mathbf{V}, \forall t \in \mathbf{T}', c^e + Opt_{ij} \leq t \leq b^e$ .

$L_v^t$ : binary variable equals to 1 if vehicle  $v$  is transporting a passenger from time instant  $t$  to  $t + 1$ ,  $\forall v \in \mathbf{V}, \forall t \in \mathbf{T}', t \leq T + T^r - 1$ .

$F_{ij}^t$ : flow of vehicles on link  $(i, j)$  starting from time instant  $t$ ,  $\forall (i, j) \in \mathbf{A}, \forall t \in \mathbf{T}', t \leq T + T^r - 1$ .

$\lambda_{ij}^{t,k}$ : binary variable equals to 1 if the traffic flow on link  $(i, j)$  starting from  $t$  is taking the value on breakpoint  $k$  of the flow-travel time curve,  $\forall (i, j) \in \mathbf{A}, \forall t \in \mathbf{T}', t \leq T + T^r - 1, \forall k \in \mathbf{K}$ .

$\delta_{ij}^t$ : travel time when travelling on link  $(i, j)$  starting from time instant  $t$ ,  $\forall (i, j) \in \mathbf{A}, \forall t \in \mathbf{T}', t \leq T + T^r - 1$ .

*Objective function:*

$$\begin{aligned} \max(\text{Profit}) = & Pr \cdot \sum_{\substack{e \in \mathbf{E}, v \in \mathbf{V} \\ i = \text{ori}^e \\ j = \text{des}^e}} S_{ij}^{ev} \cdot \text{Opt}_{ij} - C_f \cdot \sum_{\substack{(i_{t_1}, j_{t_2}) \in \mathbf{A}, \\ v \in \mathbf{V}}} x_{i_{t_1}, j_{t_2}}^v \cdot d_{ij} - C_d \cdot V - C_p \cdot \sum_{\substack{i \in \mathbf{N}, v \in \mathbf{V} \\ t \in \mathbf{T}', t \leq T + T^r - 1}} y_{i_t}^v \\ & - \rho \cdot \sum_{e \in \mathbf{E}} \left( D_{ij}^e - \sum_{\substack{v \in \mathbf{V} \\ i = \text{ori}^e \\ j = \text{des}^e}} S_{ij}^{ev} \right) - C_{del} \cdot \sum_{\substack{e \in \mathbf{E}, v \in \mathbf{V} \\ i = \text{ori}^e \\ j = \text{des}^e}} \left( \sum_{t \in \mathbf{T}'} A_{ij}^{evt} \cdot t - (a^e + \text{Opt}_{ij}) \cdot S_{ij}^{ev} \right) \end{aligned} \tag{1}$$

The objective function is considered from both the AT company and the passengers' perspective. For the AT company, it aims to maximize the total profit which includes the total revenue from the customers, the vehicle fuel costs, the vehicle depreciation costs and the parking costs. The revenue is only charged for the shortest travel time of each request in order to avoid detour problems. Regarding the passengers, when a request is rejected, they would use public transport as an alternative to finish this trip and the AT system should pay a fixed charge for the users as a penalty. Late arrivals of AT service are also penalized in order to involve the system performance for the clients.

Constraints:

$$S_{ij}^{ev} \leq \sum_{\substack{i_{t_1}, j_{t_2} \in \mathbf{B} \\ t_1 \geq a^e, t_2 \leq b^e}} x_{i_{t_1}, j_{t_2}}^v \quad \forall e \in \mathbf{E}, v \in \mathbf{V}, i = \text{ori}^e, j = \text{des}^e \tag{2}$$

$$S_{ij}^{ev} \leq \sum_{\substack{i_{t_1}, j_{t_2} \in \mathbf{B} \\ t_1 \geq a^e, t_2 \leq b^e}} x_{i_{t_1}, j_{t_2}}^v \quad \forall v \in \mathbf{V}, e \in \mathbf{E}, i = \text{ori}^e, j = \text{des}^e \tag{3}$$

$$\sum_{\substack{t \in \mathbf{T} \\ t = a^e}} P_{ij}^{evt} = S_{ij}^{ev} \quad e \in \mathbf{E}, v \in \mathbf{V}, i = \text{ori}^e, j = \text{des}^e, t \leq a^e \tag{4}$$

$$\sum_{\substack{t \in \mathbf{T}' \\ a^e + \text{Opt}_{ij} \leq t \leq b^e}} A_{ij}^{evt} = S_{ij}^{ev} \quad e \in \mathbf{E}, v \in \mathbf{V}, i = \text{ori}^e, j = \text{des}^e \tag{5}$$

$$\sum_{\substack{t \in \mathbf{T} \\ t = a^e}} P_{ij}^{evt} \leq 1 \quad e \in \mathbf{E}, v \in \mathbf{V}, i = \text{ori}^e, j = \text{des}^e \tag{6}$$

$$\sum_{\substack{t \in \mathbf{T}' \\ a^e + \text{Opt}_{ij} \leq t \leq b^e}} A_{ij}^{evt} \leq 1 \quad e \in \mathbf{E}, v \in \mathbf{V}, i = \text{ori}^e, j = \text{des}^e \tag{7}$$

$$\sum_{\substack{t \in \mathbf{T} \\ t = a^e}} (P_{ij}^{evt} \cdot t) \leq \sum_{\substack{t \in \mathbf{T}' \\ a^e + \text{Opt}_{ij} \leq t \leq b^e}} (A_{ij}^{evt} \cdot t) \quad e \in \mathbf{E}, v \in \mathbf{V}, i = \text{ori}^e, j = \text{des}^e \tag{8}$$

$$\sum_{v \in \mathbf{V}} S_{ij}^{ev} \leq 1 \quad \forall e \in \mathbf{E}, i = \text{ori}^e, j = \text{des}^e \tag{9}$$

$$\sum_{i \in \mathbf{N}} y_{i_t}^v \leq 1 \quad \forall v \in \mathbf{V}, t \in \mathbf{T}', t \leq T + T^r - 1 \tag{10}$$

$$\sum_{i_{t_1}, j_{t_2} \in \mathbf{B}} x_{i_{t_1}, j_{t_2}}^v \leq 1 \quad \forall i \in \mathbf{N}, v \in \mathbf{V}, t_1 \in \mathbf{T}' \tag{11}$$

$$\sum_{i_t, j_{t_1} \in B} x_{i_t, j_{t_1}}^v + \sum_{i \in N} y_{i_t}^v = 1 \quad v \in V, t \in T', t \leq T + T^r - 1 \quad (12)$$

$$\sum_{i_t, j_{t_1} \in B} x_{i_t, j_{t_1}}^v + y_{i_t}^v = \sum_{i_{t_2}, j_t \in B} x_{i_{t_2}, i_t}^v + y_{i_{t-1}}^v \quad \forall i \in N, t \in T', t \leq T + T^r - 1, v \in V \quad (13)$$

$$\sum_{i_0, j_{t_1} \in B} x_{i_0, j_{t_1}}^v + \sum_{v \in V} y_{i_0}^v = V \quad i = \text{central parking station} \quad (14)$$

$$F_{ij}^{t_1} = \sum_{v \in V} x_{i_{t_1}, j_{t_2}}^v \quad \forall (i, j) \in A \quad (15)$$

$$F_{ij}^t \leq Q_{ij} \quad \forall (i, j) \in A, \forall t \in T', t \leq T + T^r - 1 \quad (16)$$

$$F_{ij}^t = \sum_{k \in K} \lambda_{ij}^{t,k} \cdot k \quad \forall (i, j) \in A, \forall t \in T', t \leq T + T^r - 1 \quad (17)$$

$$\delta_{ij}^t = \sum_{k \in K} \delta_{ij}^k \cdot \lambda_{ij}^{t,k} \quad \forall (i, j) \in A, \forall t \in T', t \leq T + T^r - 1 \quad (18)$$

$$\sum_{k \in K} \lambda_{ij}^{t,k} = 1 \quad \forall (i, j) \in A, \forall t \in T', t \leq T + T^r - 1 \quad (19)$$

$$\delta_{ij}^{t_1} \leq (t_2 - t_1) \cdot x_{i_{t_1}, j_{t_2}}^v + t_{ij}^{\max} \cdot (1 - x_{i_{t_1}, j_{t_2}}^v) \quad \forall (i_{t_1}, j_{t_2}) \in B, v \in V \quad (20)$$

$$\delta_{ij}^{t_1} \geq (t_2 - t_1) \cdot x_{i_{t_1}, j_{t_2}}^v + t_{ij}^{\min} \cdot (1 - x_{i_{t_1}, j_{t_2}}^v) \quad \forall (i_{t_1}, j_{t_2}) \in B, v \in V \quad (21)$$

$$L_t^v = \sum_{\substack{i=ori^e, j=des^e \\ e \in N, \forall t_1 \in T, t_1 \leq t}} P_{ij}^{evt_1} - \sum_{\substack{i=ori^e, j=des^e \\ e \in N, \forall t_2 \in T', t_2 \leq t}} A_{ij}^{evt_2} \quad \forall t \in T, t \leq T' - 1, v \in V \quad (22)$$

$$L_t^v + \sum_{i \in N} y_{i_t}^v \leq 1 \quad \forall t \in T, t \leq T + T^r - 1, v \in V \quad (23)$$

Constraints (2) impose that trip  $e$  from node  $i$  to node  $j$  can only be satisfied by vehicle  $v$  if that vehicle has passed through node  $i$  after the earliest departure time  $a^e$ . Constraints (3) impose that trip  $e$  from node  $i$  to node  $j$  can only be satisfied by vehicle  $v$  if that vehicle has pass through node  $j$  before the latest arrival time  $b^e$ . Constraints (4) and (5) assure that a satisfied trip must have only one departure time and only one arrival time or this trip is not satisfied at all. Constraints (6) and (7) guarantee that one trip can only have one departure time and only one arrival time when it is served or this trip is not satisfied by any vehicle. Constraints (8) impose that the departure time instant of a satisfied trip must happen before the arrival time instant. Constraints (9) yield that a satisfied trip must be served by only one AT. Constraints (10) impose that for a specific time step  $t$  to  $t + 1$  one automated taxi should only park at one node or not park at all. Constraints (11) guarantee that one trip with specific origin and departure time can only have one destination and one arrival time. Constraints (12) ensure that each vehicle can only have one status at time instant  $t$ : either going to the next destination or parking at that place. Constraints (13) are flow conservation constraints which make sure that the number of taxis leaving from node  $i$  and parking there at time instant  $t$  is equal to the number of vehicles arriving or parking at the same node ending at time instant  $t$ . Constraint (14) describes the initial status of the AT fleet. At the beginning of the operation period, all vehicles are stocked at the central parking station. Constraints (15) compute the flow of vehicles on each road link  $(i, j)$  from time instant  $t_1$ . Constraints (16) limit flow by the capacity of each link. Constraints (17)-(19) define the travel time on road link  $(i, j)$  starting at time instant  $t$  as a function of the congestion level represented by the breakpoint  $k$ . Constraints (20) and (21) impose the travel time is not greater than the maximum travel time or smaller than the minimum travel time. Meanwhile they also guarantee that when vehicle  $v$  is travelling on link  $(i_{t_1}, j_{t_2})$ , the travel time must be

$\delta_{ij}^{t_1}$ . Constraints (22) and (23) impose that when the vehicle is transporting a passenger, it should not stop or park at any node.

### 3. Case study and results

In this section, we apply the model to a small toy network to illustrate how it works and the results that can be obtained. We consider a road network with 9 nodes and 12 links (Figure 1). The length of each link is 1 km and the central parking station is located at node 5. The capacity of each link is 3 vehicles per direction per time step. The minimum travel time ( $\delta_{ij}^{min}$ ) on each link is considered as 1 time step and the assumed maximum speed is 30km/h. With a simplified volume-delay function, the travel time values are 1, 2 and 4 time step respectively, when the traffic flows are 1, 2 and 3 vehicles respectively (Figure 2). When ATs parking at the central node 5, it is free to charge while parking at other nodes should be paid.

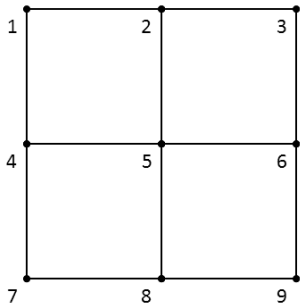


Figure 1: Road network with 9 nodes and 12 links

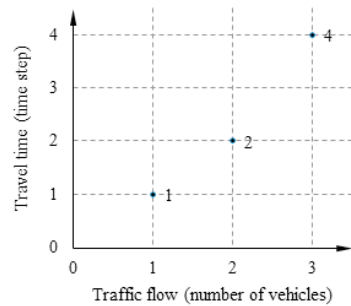


Figure 2: Traffic flow and travel time value

There are 10 time steps from time instant 0 to 10 in the service period. Each time step of the optimization is 2 minutes. A short time period with 2 time steps is set in order to be possible to satisfy the early trips and late trips in the service period, which makes the operation time run from -2 to 12 time instant. Using Monte Carlo simulation, we generate 20 requests each of which has an origin, a destination and a desired departure time based on an uneven distribution with a higher probability for node 5 to be an origin. In this case study, the number of ATs serving in the system is 5.

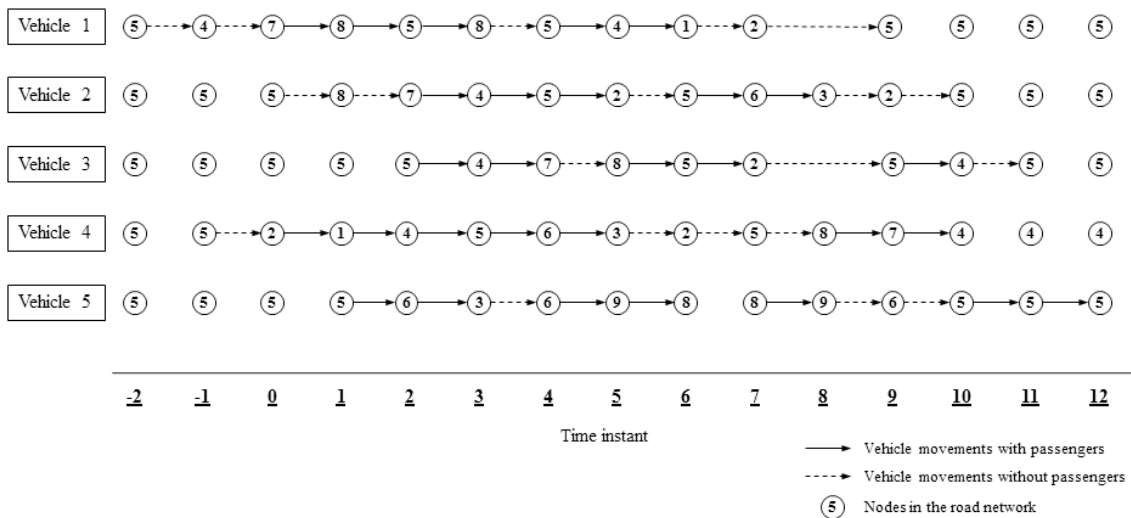


Figure 3 AT movement results

For this case study, we consider the following parameters: price rate  $Pr = 2\text{€}/\text{time step}$ , vehicle fuel cost  $C_f = 0.1\text{€} / \text{km}$ , vehicle depreciation cost  $C_d = 5 \text{€}/\text{vehicle}$ , parking cost, penalty cost for using public transport  $\rho = 2\text{€} / \text{trip}$ , congestion delay penalty  $C_{del} = 0.05\text{€}/\text{time step}$ .

Firstly a scenario with the parking cost  $C_p = 0.05\text{€}/\text{time step}$  is tested and the graphical results of every vehicle’s movements ( $x_{t_1, j, t_2}^v$ ) in the optimal solution are shown in Figure 3. In this figure, each solid line represents the AT’s movement when it is satisfying passengers’ demand while the dashed line represents an empty AT. Each vehicle’s position at a specific time instant is shown by the number with a circle in this figure. Before the service period the ATs, are allowed to travel from central node 5 to any node in the network if it is needed. In this case, two ATs (vehicle 1 and 4) are relocated before time instant 0 in order to pick up passengers in time.

According to constraints(15)-(19), when more than one AT is travelling on the same link starting at the same time instant, traffic congestion must be formed. This can be seen in Figure 3. When only assigning vehicle 1 to road link (2,5) starting from time instant 1, the travel time would be 1 time step. Then vehicle 3 is also assigned to the same link at the same departure time hence this leads to a more crowded road link. Therefore the travel time for both vehicles is delayed.

Table 1 Optimization model results

	Total profit (€)	Parking price (€/time step)	Number of satisfied requests (out of 20)	Total idle time (time step)	Paid parking time (time step)	% of congestion	Congestion delay (time step)	Total travel distance (km)	Computational time (s)
Dynamic travel time	15.00	0	14	14	12	0%	0	40	34.4
Static travel time	15.00		14	13	10	8%	-	40	1.3
Dynamic travel time	14.15	0.05	15	5	1	2%	0	47	23.9
Static travel time	14.15		15	11	7	10%	-	44	4.45
Dynamic travel time	14.00	0.10	15	5	2	2%	0	40	12.6
Static travel time	14.10		15	7	1	12%	-	40	5.4

A comparison between AT routing optimization with dynamic and static travel time was done and Table 1 presents the AT service results under different parking price rate scenarios. The column “Total idle time” represents the parking time at all the nodes in the road network while column “Paid parking time” is the total parking time at all the nodes except node 5. “Congestion delay” is computed as the total number of time steps between the real travel time and the shortest travel time for all the AT trips while “Number of delay requests” is only for the trips when ATs are serving requests. The computational time is within one minute for all the scenarios and the static travel time scenario has the shorter computational time than the dynamic one when the parking price is the same.

When parking at any node of the road network is free, there is no difference of the optimal solution between the scenario with dynamic and static travel time. This is because the system is free to park anywhere at any time to avoid traffic congestion so more than one AT travelling on the same link is not needed. When travel time is static, the system is free to generate congestion on each link which brings 8% congested links in this scenario and this could also be seen in the other parking rate scenarios. Moreover, the total idle time is the highest among all parking rates.

With regards to 0.05€/time step for parking price, two scenarios have the same value of profit but the optimal solutions are different which could be seen from the column “Paid parking time” and the “Total travel distance”. The static travel time scenario pays 0.3€ more for 6 time-step parking and the dynamic one pays 0.3€ more because of the extra 3 km driving distance. This also indicates that in the dynamic scenario the model leads to a higher driving distance to avoid congestion and also avoid paying parking time in order to get a higher system profit.

When the parking price is 0.1 €/time step, the system decides to serve 15 out of 20 requests for both of the scenarios. But the values of profit are different because of the parking cost. When ATs park at nodes except the



central node 5, the system should pay for it. As a result, the dynamic travel time scenario paid 0.1€ more. In addition to this, there are 2% congestion links but no congestion delay for the passengers with dynamic travel time. This is due to the objective function which proposes the penalty for passengers' delay but no extra costs when empty ATs are travelling slower because of the congestion.

#### 4. Conclusion

This paper proposed a mathematical model to study a dynamic travel time based AT system to provide transport service within the city area. The contribution of this paper is to consider the travel time on each road link according to the number of vehicles travelling on and do system optimization with the objective to maximize the total profit of such system by deciding on each AT's routing selection. This model is able to include traffic congestion when computing the vehicle routing to the ATs to achieve system optimum in the vehicle routing solutions. The model was then applied to a case study with 10 time-step service period and 20 travel requests generating from the road network with 9 nodes and 12 links. From that application we were able to take the following conclusions:

Traffic congestion happens when vehicles travelling on the network and this leads to arrival delay for the trips compared with the scenario when considering the travel times static and never change on that link. The system profit is sensitive to the parking price and the penalty cost for arrival late because traffic congestion may generate benefit loss and no congestion leads to more parking costs. Sometimes the system would like to take a longer route and drive more distance in order to avoid traffic congestion and parking costs according to the profit maximization objective.

The next step will be applying the model in a real-size case with a rolling-horizon method which intends to divide one day into several horizons and make it possible to solve the real-time demand.

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#### Reference

- Arem, B. Van, Driel, C.J.G. Van, Visser, R., 2006. The Impact of Cooperative Adaptive Cruise Control on Traffic-Flow Characteristics. *IEEE Transactions on Intelligent Transportation Systems* 7, 429–436.
- Bose, A., Ioannou, P. a., 2003. Analysis of traffic flow with mixed manual and semiautomated vehicles. *IEEE Transactions on Intelligent Transportation Systems* 4, 173–188. doi:10.1109/TITS.2003.821340
- Calvert, S.C., Van Den Broek, T.H.A., Van Noort, M., 2011. Modelling cooperative driving in congestion shockwaves on a freeway network. *IEEE Conference on Intelligent Transportation Systems, Proceedings, ITSC* 614–619. doi:10.1109/ITSC.2011.6082837
- CityMobil2, n.d. CityMobil2 - Cities demonstrating automated road passenger transport [WWW Document]. URL <http://www.citymobil2.eu/en/>
- Correia, G.H. de A., van Arem, B., 2016. Solving the User Optimum Privately Owned Automated Vehicles Assignment Problem (UO-POAVAP): A model to explore the impacts of self-driving vehicles on urban mobility. *Transportation Research Part B: Methodological* 87, 64–88. doi:10.1016/j.trb.2016.03.002
- Fagnant, D.J., Kockelman, K.M., 2014. The travel and environmental implications of shared autonomous vehicles, using agent-based model scenarios. *Transportation Research Part C: Emerging Technologies* 40, 1–13. doi:10.1016/j.trc.2013.12.001
- International Transport Forum, 2015. Urban mobility system upgrade: How shared self-driving cars could change city traffic.
- Spieser, K., Treleaven, K., Zhang, R., Frazzoli, E., Morton, D., Pavone, M., 2014. Toward a Systematic Approach to the Design and Evaluation of Automated Mobility-on-Demand Systems: A Case Study in Singapore. *Road Vehicle Automation* 229–245. doi:10.1007/978-3-319-05990-7\_20
- WEpods, 2015. WEpods project DAVI – Dutch Automated Vehicle Initiative [WWW Document]. URL <http://davi.connekt.nl/wepods-project/>