Fast rotor loss calculations in fractional-slot permanent magnet machines

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Fast Rotor Loss Calculations in Fractional-Slot Permanent Magnet Machines

F. Wani, J. Dong and H. Polinder

Abstract -- Permanent magnet machines with fractional slot windings are popular because of their shorter end windings, low cost and ease of manufacturing. On the other hand, fractional slot machines have higher eddy current losses in the rotor magnets and sleeve. An accurate calculation of these losses usually involves a time-stepped finite-element analysis with moving rotor, which is time consuming. This paper compares different methods of evaluating these losses using finite element analysis based on their time-cost. The models are mainly divided into two types: time-domain and frequency-domain models. The paper proposes an equivalent frequency-domain analysis in the stator frame for rotor loss calculations, using the Lorentz force term. This method yields similar results to the time-stepped finite element model with a significant time saving.

Index Terms -- Eddy currents, Finite element analysis, Frequency-domain analysis, Lorentz force, Permanent magnet machines, Time-domain analysis.

I. NOMENCLATURE

\begin{align*}
A_z & \text{ Magnetic vector potential (Wb/m), z-component} \\
J_{ext} & \text{ External current density (A/m}\text{\textsuperscript{2}}) \\
K_c & \text{ Linear current density (A/m)} \\
\sigma & \text{ Material Conductivity (S/m)} \\
\lambda & \text{ Lagrange multiplier} \\
p & \text{ Number of pole-pairs} \\
v & \text{ Space harmonic order, such that } v = p \text{ is the main field component} \\
u & \text{ Time harmonic order, such that } u = l \text{ is the fundamental time harmonic} \\
N_{ph} & \text{ Number of stator turns per phase} \\
I & \text{ Phase current} \\
o_r & \text{ Mechanical rotational speed (rad/s)} \\
K_{wph} & \text{ Winding factor for space harmonic of order } 'u' \\
K_{sph} & \text{ Slot opening factor for space harmonic order } 'v' \\
\end{align*}

II. INTRODUCTION

PERMANENT magnet (PM) machines with fractional slot concentrated windings are popular because of their shorter end windings, high efficiency, low cost and ease of manufacturing, etc. compared to the integer slot windings [1]. Due to the fractional number of slots per pole per phase, these machines contain a higher magnitude of the space harmonics in their stator magnetomotive force (MMF). Space harmonics in the stator MMF cause eddy current losses in the rotor. A good estimation of the rotor losses is necessary for a good design of a PM machine with fractional slot windings. Furthermore, the measurement of rotor eddy current losses is also not straightforward, which further emphasizes the need for estimation of these losses. This paper compares different methods to quickly estimate the eddy current losses in the rotor based on their time cost.

Various analytical, finite element (FE), and a combination of analytical and FE models have been proposed for the eddy current loss modeling in PM machines. A detailed survey of these loss calculation methods is given in [2].

Analytical models are fast and provide a good insight into the loss characteristics and trends. However, analytical models are difficult to formulate, and suffer from various inaccuracies resulting from the non-linear behavior of the materials, complex geometry of the PM machine (stator slotting, circumferential segmentation of the magnets, etc.), and various other assumptions that are made to simplify the model [2]–[5].

On the other hand, FE models are easy to setup, can deal with complex geometries, segmentation of the magnets, and non-linear permeability of the iron, etc. [6]–[8]. The main drawback of the FE models is that they are time-consuming [2]. Computing memory is no longer an issue, unless large scale 3D modelling is necessary [9]. A time-stepped FE simulation, solved over both the stator and the rotor domains, is considered to be the most accurate method to calculate the rotor losses. Such models are time-consuming, which makes them undesirable in situations where speed is preferred over precision/accuracy; for instance, in a machine design optimization problem.

A good approach to speed up the loss calculation is to model only the rotor domains and the airgap, and impose a linear current density or a magnetic field boundary condition on the stator inner radius, as shown in Fig.1. This template has been adopted widely both in analytical as well as FE methods [3], [10], [4], [11]; consequently, these models are referred to as ‘rotor-only’ models. Following assumptions are commonly employed in the rotor-only models:

- Stator and rotor irons are assumed to have a linear BH-curve.
- Rotor iron is laminated, and has negligible losses.

Nonetheless, the models presented in this paper can also predict losses in the solid rotor iron, provided accurate values of the permeability in the iron are.

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known.

- Eddy current loss in the rotor owing to the airgap permeance variations because of the stator slotting is neglected. Normally, this gives rise to losses in the rotor even under no-load conditions. However, these losses are usually significant only in high-speed machines.

- End effects in the permanent magnets and the rotor sleeve are neglected. Induced currents only have an axial component.

- Eddy current field in the rotor does not influence the stator field at the stator inner radius.

The main advantage of the rotor-only models is their faster speed because of the reduced mathematical complexity involved; reduced problem size further enhances the speed of calculation. However, these models require some prior analytical or FE calculations to appropriately define the boundary conditions. As mentioned earlier, the boundary condition at the stator inner radius can be of different types, for example, current density, or magnetic field intensity. Magnetic field boundary conditions may yield more accurate results under certain conditions, such as when including saturation or stator slotting effects [6], [12]. Furthermore, analytical calculation of magnetic field/potential boundary conditions is rather difficult, and may require some sort of Magnetostatic FE calculations [6]. For linear iron, a current density boundary condition is sufficiently accurate, and easy to calculate analytically. For these reasons, we employ a current density boundary condition in our models.

This paper compares three different approaches to solve a rotor-only model, using time-domain and frequency domain analyses. The results from these three models are compared to the benchmark model. The benchmark model comprises a full 2D cross-section (including the stator) of the PM machine solved using a time-stepped finite element model. The main drawback of the benchmark model is its time cost.

This paper shows that frequency domain models can yield nearly the same results as the time-domain models within a fraction of a time taken by the latter. Two different frequency-domain methods are presented. First, a commonly applied frequency-domain analysis in the rotor frame is explained. This is followed by a more novel frequency-domain analysis in the stator frame, using the Lorentz force term. The latter of the frequency-domain methods, proposed in this paper, is shown as the fastest method to compute the rotor losses.

Section III gives the design details of the PM machine on which the eddy current loss modeling has been performed. Section IV and V entails the mathematical formulations regarding the time-domain and frequency-domain models for eddy current loss calculations in the rotor, respectively. Section VI presents the eddy current losses obtained from the time-domain and the frequency domain models, along with their time cost. Conclusions from this paper are given in section VII.

III. SYSTEM DESCRIPTION

If a sheet of conducting material is enclosing the magnets, the losses in this sheet material are usually more critical than the losses in the magnets themselves. This is also because the magnets are usually segmented, which increases the overall resistance to the eddy currents. The material sheet could be present for different reasons: retaining magnets in high-speed machines, preventing contamination of the magnets or windings in motors which operate in unclean environments [13], or in Canned motors [14]. For this reason, a PM machine with conductive rotor can/sleeve is analyzed.

Usually in addition to the higher order space harmonics, fractional slot machines also have sub-harmonics in their stator MMF. Higher order harmonics have shorter pole-pitch compared to the main field, and hence, are less likely to cross the airgap. Conversely, sub-harmonics of the main field penetrate deeper into the rotor. This means that sub-harmonics could be more critical from the eddy current point of view. However, this assertion is dependent on the slot-pole combination of the PM machine.

In addition to the space harmonics, eddy current losses in the rotor may also arise because of the time harmonics in the stator current. However, this paper mainly addresses the losses caused by the space harmonics in the stator MMF. Nevertheless, the models can be easily extended to include the time harmonics as well.

In this paper, we shall compare the eddy current losses in the magnets and the rotor sleeve of a fractional slot PM machine with a 9/8 (slot-pole) double-layer winding. This winding is chosen because of the multiple dominant rotor frequencies that arise from its stator MMF. In other words, more than one space harmonic contributes significantly to the rotor eddy current losses. The machine dimensions and other parameters are listed in Table I.

IV. TIME DOMAIN MODELS

A. Full Transient Model: Benchmark model (TD-FU)

A more accurate calculation of the eddy current losses usually involves a time-stepped finite-element analysis (TSFEA) involving mesh coupling in the airgap to model the rotor motion [12], [15]; see Fig. 2. Following equations are used in this model:

\[ \nabla \times \left( \frac{1}{\mu} \nabla \times A \right) + \sigma \frac{\partial A}{\partial t} = J_{\text{ext}} \]

\[ \nabla \cdot \left( -\sigma \frac{\partial A}{\partial t} + J_{\text{ext}} \right) = 0 \]

\[ \oint_{\text{airgap}} \left( A_{\text{stator}} - A_{\text{rotor}} \right) \, dl = 0 \]
### TABLE I
DESIGN PARAMETERS FOR PM MACHINE

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated Power</td>
<td>(P_0)</td>
<td>0.8 MW</td>
</tr>
<tr>
<td>Rated rotor speed</td>
<td>(\omega_r)</td>
<td>30 rpm</td>
</tr>
<tr>
<td>Remanent density in magnets</td>
<td>(B_r)</td>
<td>1.25 T</td>
</tr>
<tr>
<td>Magnet conductivity</td>
<td>(\sigma_{pm})</td>
<td>5.5 (\times) 10^5 S/m</td>
</tr>
<tr>
<td>Airgap radius</td>
<td>(R_g)</td>
<td>1.2 m</td>
</tr>
<tr>
<td>Airgap radial length</td>
<td>(g)</td>
<td>7 mm</td>
</tr>
<tr>
<td>Rotor sleeve thickness</td>
<td>(b_r)</td>
<td>2 mm</td>
</tr>
<tr>
<td>Rotor sleeve conductivity</td>
<td>(\sigma_{sl})</td>
<td>1.4 (\times) 10^6 S/m</td>
</tr>
<tr>
<td>Magnet-arc to pole-pitch ratio</td>
<td>(\beta_m)</td>
<td>0.70</td>
</tr>
<tr>
<td>No. of pole-pairs</td>
<td>(p)</td>
<td>80</td>
</tr>
<tr>
<td>No. of slots</td>
<td>(Q_s)</td>
<td>180</td>
</tr>
<tr>
<td>Slot depth</td>
<td>(h_s)</td>
<td>80 mm</td>
</tr>
<tr>
<td>Slot width</td>
<td>(b_s)</td>
<td>25 mm</td>
</tr>
<tr>
<td>Magnet height</td>
<td>(h_m)</td>
<td>24 mm</td>
</tr>
<tr>
<td>Current density in the stator slot</td>
<td>(J_s)</td>
<td>5 (\times) 10^5 A/m^2</td>
</tr>
<tr>
<td>Slot fill factor</td>
<td>(k_{fill})</td>
<td>0.6</td>
</tr>
<tr>
<td>Stack length</td>
<td>(l_s)</td>
<td>0.7 m</td>
</tr>
<tr>
<td>Stator and rotor yoke heights</td>
<td>(h_{yoke})</td>
<td>20 mm</td>
</tr>
</tbody>
</table>

Equation (1) is the quasi-static Ampere-Maxwell equation, coupled with the current conservation over each domain which is electrically insulated from other domains. In other words, it is used to ensure that the total current in each magnet segment adds up to zero at all time instants. Equation (2) is used for mesh coupling in the airgap, and ensures that the magnetic vector potential is continuous across the stator-rotor interface in the airgap. A simple periodic boundary condition at the edges of the 2D sector, and a magnetic insulation boundary conditions at rotor inner and stator outer radius suffice for this model.

This model yields time-variant rotor losses, which converge about the steady state value after some time. A case example is shown in Fig. 3. Thus, some sort of judgement is necessary to determine if the model has converged satisfactorily after certain time. This time is sensitive to the machine design parameters, operating point, and material properties; hence, undesirable from the viewpoint of automated calculations [16]. Full transient model includes the modelling of the stator domains as well. Consequently, if finer mesh is to be used, the problem size increases much more compared to the rotor-only models.

The advantages of this model include implicit inclusion of the stator slotting, saturation of iron, and the simplicity of its boundary conditions. In addition to this, the beauty of this method lies in its ability to simultaneously include all the space as well as time-harmonics. For this reason, despite its aforementioned drawbacks, full 2D transient model is used as a benchmark for the rest of the models described in this paper.

### B. Time-Stepped Rotor-Only Model (TD-RO)

The model is based on (1) as the previous model. Since it is a rotor-only model, there is no need for any mesh-coupling in the airgap. This model uses the linear current density boundary condition at the stator inner radius, as shown in Fig. 1. The current density is given by [3]:

\[
K_s = \frac{3N_{ph}}{2\pi R_s} \sum_n \sum_v I_n \sum K_{sv} \sin(u \omega_i t + \nu \theta_i + \theta_s) \tag{3}
\]

where \(R_s\) is the stator inner radius. The equations for calculating the slot opening and winding factors can be found in [17]. When slot opening width is negligible compared to the airgap diameter, then \(K_{sow}\) approaches 1.

The corresponding boundary condition is then given by:

\[
n \times H = K_s \tag{4}
\]

where \(n\) is the normal at the boundary, and \(H\) is the magnetic field intensity just inside the airgap. Besides this boundary condition, the rest of the boundary conditions remain the same as in the TD-FU model.

Because the rotor is kept stationary, the boundary condition needs to be transformed from the stator frame to the rotor frame using the following transformation in (3):

\[
\theta_s = \theta_i + \omega_i t \tag{5}
\]

The model just like the previous time-domain model can include all the time harmonics as well as space harmonics simultaneously. Normally, only a few dominant space harmonics need to be included. A quick analytical calculation of the relative magnitudes of the MMF due to different space harmonics can be calculated. From this, it is possible to predict which harmonics should be included for a good rotor loss estimation.
time-variant rotor losses also converge faster to the steady-state value. Speed enhancement is thus twofold, because of the smaller problem size and shorter time to convergence. Just like the previous model, total time cost and solver time step depends on the material properties and speed of the rotor. Higher frequencies imply smaller time steps, whereas low frequencies need to be modelled for longer time to transcend the transient period.

V. FREQUENCY DOMAIN MODELS

Eddy current loss calculation using the frequency-domain analysis is much faster than using time-domain models. And for a single frequency model, should in principle give identical results. Contrary to time-domain models which have to wait for transients to die down, frequency-domain analysis straightaway provides with the steady state results. However, these models are preferable only when dealing with a single frequency component. For systems where the superposition principle holds, the total eddy current loss is obtained by summing up the contributions from each frequency component. Since the assumption of linear BH-curve has been made for the iron, the superposition principle is applicable.

Utilizing the speed advantage of the frequency-domain analysis, the time-domain rotor-only models can be easily transferred to the frequency domain to speed up the rotor loss calculation. In this section, two frequency-domain models: in the rotor frame, and in the stator frame, are explained.

A. Rotor Reference Frame (FD-RORF)

In literature, this model is also referred to as the ‘time-harmonic’ model. Space harmonics corresponding to the same time harmonic of the stator current induce eddy currents of different frequencies in the rotor. As a result, an independent frequency domain analysis is required for each frequency component. The time cost for each frequency domain analysis is almost independent of the frequency. The total time cost is the time cost of a single frequency domain analysis times the number of frequency components. However, it is possible to solve for each frequency component in parallel, which can significantly reduce the total time-cost, albeit at the expense of more computing resources/complexity.

The governing equations remain the same as in TD-RO model, which are expressed in the frequency-domain as:

$$\nabla \times \left( \frac{1}{\mu} \nabla \times A_i \right) + j\sigma\omega A_i = J_{est}$$

$$\nabla \cdot (-j\sigma\omega A_i + J_{est}) = 0$$

(6)

The boundary conditions also remain exactly the same as in TD-RO model. The frequency associated with the space harmonic of order ‘\(v\)’ in the rotor frame is given by the following equation:

$$\omega_r = \omega_1 \left( 1 - \text{sgn}(v) \frac{\nu}{p} \right)$$

(7)

where, \(\omega_1\) is the fundamental electrical frequency associated with the time-harmonic in the stator; ‘\(\text{sgn}(v)\)’ should be taken as ‘+1’ or ‘-1’, depending upon whether the MMF wave corresponding to ‘\(v^{th}\)’ harmonic is moving in the same or opposite direction compared to the main field, respectively.

The total eddy current loss is then given by:

$$P_{ed, total} = \sum_{v} P_{ed,v}$$

(8)

where, \(P_{ed,v}\) is the loss corresponding to the each space harmonic.

B. Stator Reference Frame (FD-ROSF): Proposed method

In the stator frame, space harmonics of the MMF have the same electrical frequency, therefore, if the loss calculation is performed in the stator frame then a single frequency-domain analysis should suffice. The following method is based on this principle.

To model the rotor motion, the rotating domains equations are modified with the Lorentz force term \((v \times B)\), as shown below:

$$\nabla \times \left( \frac{1}{\mu} \nabla \times A_i \right) + j\sigma\omega A_i - \sigma v \times (\nabla \times A_i) = J_{est}$$

$$\nabla \cdot (-j\sigma\omega A_i + \sigma v \times (\nabla \times A_i) + J_{est}) = 0$$

(9)

where, \(v = r \times \omega\) is the linear velocity at any point in the rotor domain, and \(B = \nabla \times A_i\). Clearly, the velocity term is zero for any stationary domain.

Even in the presence of multiple dominant rotor frequencies, only one frequency-domain analysis is necessary to calculate the eddy-current losses in the rotor. If other time harmonics are included, then the total time cost is equal to the time cost of one frequency-domain analysis times the number of dominant stator time harmonics, irrespective of the number of dominant rotor frequencies. Again parallel computing will bring down the total time cost.

Furthermore, this method neither requires any transformation of the current density boundary condition from the stator to the rotor frame, nor does it require calculation of the rotor frequencies.

VI. RESULTS AND DISCUSSION

In this section, the eddy current losses calculated from the time-domain and the frequency-domain models for the PM machine with parameters listed in Table I are compared. Furthermore, to verify the accuracy of the rotor-only models over the range of material properties and rotor velocities, results from the rotor-only models are compared to the TD-FU model over a range of rotor sleeve conductivity values and rotor speeds.

A. Comparing Rotor Losses from Time-Domain and Frequency-Domain Models

As stated earlier, full transient model with moving rotor (TD-FU) forms the benchmark on which the results from other models, namely, time-domain rotor-only (TD-RO), frequency-domain rotor-only in rotor frame (FD-RORF) and frequency-domain rotor-only in stator frame (FD-ROSF), shall be judged.

Eddy current losses for the PM machine as a function of speed are shown in Fig. 4. It is evident that not only do the rotor-only models give reasonably accurate results at rated
values, but that excellent agreements are also seen at higher speeds. At higher speeds, the current loading is assumed to be the same as at the lower speeds, and thus, the power rating is assumed to vary linearly with the speed. It is observed that the eddy current losses predicted from the TD-FU models is lower than what is predicted from the rotor-only models. A possible reason is that in the TD-FU model, some of the MMF does not cross into the rotor because of the slot-leakage, which is more prominent in a vertical double-layer winding assumed in this paper. Some inaccuracies also occur because of the tolerances set in the time-domain models, especially in the current conservation equation, in order to facilitate the convergence towards the steady-state solution. Tight setting of the tolerance will most likely result in more accurate agreement.

In Fig. 5, the accuracy of the rotor-only models is shown with different values of the conductivity in the rotor sleeve. Slight deviation is seen as the conductivity increases, this is because the Lorentz force term is multiplied by the conductivity, and slight error in the estimation of the magnetic field are reflected in the results. Same happens when the speeds start getting higher.

PM losses are seen to decrease at higher rotor sleeve conductivities because of the shielding effect of the sleeve, as shown in Table II. The effect is seen in all four models, although for illustration, only the results from TD-RO model are shown. Although it is unlikely that a rotor sleeve with electrical conductivity of the order of 1e9 (S/m) shall be used, Fig. 5 shows that models work excellently even at extremely high conductivity values. Also, even though rotor losses are shown to decrease after 1e7 (S/m), it’s only a matter of discretization. The losses may increase until a certain point above 1e7 before they start declining due to the shielding effect. It also shows that using a material of very high conductivity will actually decrease losses rather than increase them.

B. Comparing Time Cost of Time-Domain and Frequency-Domain Models

It is well known that frequency domain models take far less than equivalent time-domain models to be solved. Also, for time-domain models, the time cost depends on the time for which simulation is carried out, that is, until transient effects almost die out. Therefore, some sort of definition needs to be adopted to determine the time cost of the time-domain models. In this paper, we define the time cost of the time-domain models as the time it takes to model 5 electrical cycles of the lowest frequency induced in the rotor. Clearly, this is an arbitrary definition, but it is hardly relevant to the time cost trend observed. Table III shows the time cost of the four rotor eddy loss methods explained in this paper. The simulations were performed on Intel Xeon 3.5 GHz processor with 32 GB RAM, using the COMSOL Multiphysics software. The trend for the time cost is as follows:

Full transient (~minutes) > TD-RO (~seconds/minutes) > FD-RORF (~seconds) > FD-ROSF (~seconds).

\[
\begin{array}{ccc}
\text{Rotor-can} & \text{Can loss (W)} & \text{PM losses (W)} \\
\text{conductivity (S/m)} & & \\
1.4e6 & 2674 & 992 \\
1.4e7 & 24587 & 947 \\
1.4e9 & 11439 & 3.36 \\
\end{array}
\]

\[
\begin{array}{ccc}
\text{Model type} & \text{Time Cost (s)} \\
\text{Full Transient (TD-FU)} & 8m 10s \\
\text{Time-domain Rotor only (TD-RO)} & 1m 50s \\
\text{Frequency domain Rotor only in Rotor frame (FD-RORF)} & 12s \\
\text{Frequency domain Rotor only in Stator frame (FD-ROSF)} & 4s \\
\end{array}
\]

VII. CONCLUSIONS

This paper highlighted two main approaches to calculate the eddy current losses in the permanent magnet rotor: time-domain analysis and frequency domain analysis. In general, frequency domain methods are faster than the time-domain methods. As a benchmark, time-stepped finite element model modeling stator as well as rotor domains was chosen. The results from the rotor-only models were compared to the time-stepped finite element model, which showed good
agreement. Sensitivity analysis was done to show that the methods hold good even for high speeds and high rotor conductivities.

The proposed frequency domain method in the stator frame was shown to be the fastest method. Despite being a rotor-only model, and involving no rotor motion using the moving mesh, this model required no transformation of the current density boundary condition, and no prior calculation of rotor frequencies. Furthermore, this model calculated losses from all space harmonics simultaneously using a single frequency domain analysis.

VIII. REFERENCES


IX. BIOGRAPHIES

Faisal Wani graduated with master’s degree in Electrical engineering (2016) from Delft University of Technology, Netherlands, and a master’s degree in Wind Technology from Norwegian University of Science and Technology (NTNU), Norway. He obtained his bachelor’s degree in Electrical and Electronics engineering from National Institute of Technology, Trichy, India in 2012. Since October 2016, Wani has been working as a PhD candidate at Delft University of Technology. Prior to this, he worked as an Engineer at Power grid Corporation of India Limited. His research areas include tidal and wind energy conversion systems, with emphasis on electrical machine modelling and reliability of electrical drives.

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Henk Polinder holds a PhD (1998) in electrical engineering from Delft University of Technology, the Netherlands. Since 1996, he has been an assistant or associate professor at Delft University of Technology in the field of electrical machines and drives. He worked part-time at Lagerwey (1998/99), at Philips (2001) and at ABB Corporate Research (2008). He was a visiting scholar at the universities of Newcastle-upon-Tyne (2002), Quebec (2004), Edinburgh (2006) and Itajubá (2014). His main research interests are electric drives for renewable energy (power take off systems in ocean energy, drive trains for wind energy) and maritime applications. He (co) authored over 250 papers.