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DOI

[10.1016/j.ifacol.2017.08.809](https://doi.org/10.1016/j.ifacol.2017.08.809)

Publication date

2017

Document Version

Final published version

Published in

IFAC-PapersOnLine

Citation (APA)

Ferrari, R., Dibowski, H., & Baldi, S. (2017). A message passing algorithm for automatic synthesis of probabilistic fault detectors from building automation ontologies. *IFAC-PapersOnLine*, 50(1), 4184-4190. <https://doi.org/10.1016/j.ifacol.2017.08.809>

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A Message Passing Algorithm for Automatic Synthesis of Probabilistic Fault Detectors from Building Automation Ontologies

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Abstract:

Model-based fault diagnosis has been indicated as a fundamental method in enabling optimal maintenance of buildings, thus leading to important energy savings. However, accurate models of buildings and their technical equipment are seldom available, and this lack of knowledge applies even more to descriptions of modeling and measurement uncertainties, which are needed for developing robust fault detection thresholds with given performances in terms of False Alarm Rates. In the present paper we propose to overcome both limitations, by introducing: 1) a methodology for the automatic synthesis of a global model of a building and its equipment, leveraging a purposely built ontology-based Building Information Model; and 2) a novel message passing algorithm called MP-BUP for automatically propagating the effects of uncertainties in interconnected bilinear systems and derive robust probabilistic thresholds.

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Keywords: Building automation, Fault detection and diagnosis, Lumped-parameter modeling, Building information model

NOMENCLATURE

Decorations and general notation

| | |
|----------------------------|---|
| x, X | A scalar or array, a matrix |
| \mathcal{X}, \mathcal{X} | A set, another mathematical object |
| \bar{x} | Mean value |
| Var | Variance |
| Cov | Covariance |
| x, \hat{x} | Actual value, estimated value |
| \tilde{x} | Difference between nominal and actual value |
| $m[x]$ | Measurement of x |

Parameters

| | |
|--------|--|
| ρ | Density of the transfer medium |
| A | Contact area for convection/conduction |
| c | Specific heat capacity of the transfer medium |
| h | Convective/conductive heat transfer coefficient |
| p | Fraction of solar radiation directly impinging on the thermal zone |
| V | Volume of the transfer medium |

Variables

| | |
|-----------|---------------------------------------|
| P | Power provided from outside |
| R_{sol} | Solar radiation |
| T | Temperature of the transfer medium |
| u | Control input |
| w | Mass flow rate of the transfer medium |

Indexes

| | |
|-----------|---|
| 1, 2, ... | In/outlet of multiway mixer/splitter |
| a_D | Air supplied by the fan to AHU ductworks |
| B | Boiler |
| br | Boiler room |
| e | Building envelope |
| ma_D | Air mixing inside the AHU ductworks |
| p | Piping transferring water to the AHU coil |
| R | Radiator |
| rw | Boiler return water |
| sa_D | Supply air to the AHU |
| sw | Boiler supply water |

| | |
|-----|----------------|
| v | Splitter valve |
| z | Thermal zone |

1. INTRODUCTION

Commercial and residential buildings are estimated to account for as much as 40% of global CO₂ emissions (Eurostat, 2012), and as such any improvement to their energy efficiency will lead to important environmental benefits. In particular, advanced fault diagnosis (FD) methods have been indicated as a key technology for maintaining optimal operation of energy intensive technical equipment in buildings (Hensen and Lamberts, 2012), such as Heating, Ventilation and Air Conditioning (HVAC) components and the related Building Automation System (BAS). Model-based FD techniques are a powerful and effective solution (Blanke et al., 2006), but their applicability is hindered by the lack of accurate dynamical models, especially in the building sector. Furthermore, robust FD approaches do also require some deterministic or probabilistic knowledge of all the sources of uncertainties affecting the system being modeled (Ding, 2008).

While basic FD methods may be embedded inside BAS equipment itself, more advanced FD require a precise and detailed building model. Developing such models, however, can be done manually only and is thus a rarely justified activity (Bonvini et al., 2014; Bruton et al., 2014). Hence, there is a need for automatic methods for synthesizing models of complex systems, such as an entire building, starting from a description of its components and their interconnections. Works in this direction, in different domains, are documented in (Cellier and Elmquist, 1993; Blanke et al., 2006; Carpanzano and Maffezzoni, 1998). In the building case, it would be natural to exploit the decades long efforts which lead to the modeling approach called *Building Information Model* (BIM) (Conover et al.,

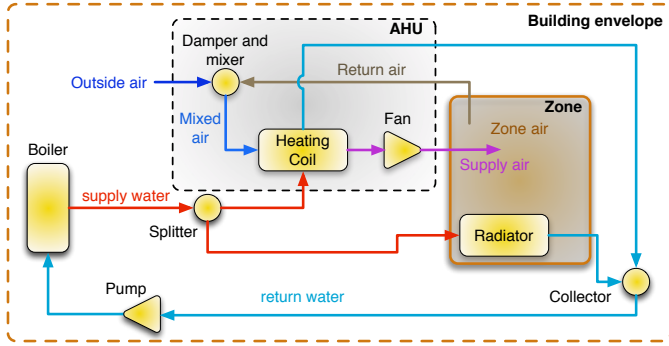


Fig. 1. Building automation system example used as test case (details such as bypass piping are not shown).

2009). The opportunity to leverage BIMs has been recognized for instance by Hensen and Lamberts (2012), and inspired the Annex 60 project¹ and the recent works of Wetter et al. (2016) and Jeong et al. (2016). Although such works solved the problem of automatic translation of BIM data using an *Object-Oriented Physical Modeling* (OOPM) paradigm, still they did require human intervention for defining its *semantics*, and correctly map it to relevant physical parameters and models. Furthermore, the problem of automatically assessing and propagating modeling and measurement uncertainties is yet to be addressed in the literature, to the best of the authors' knowledge.

To overcome both limitations, we propose to introduce: a semantic rich *ontology* for coupling existing BIM data to a library of physical components models which can then be automatically instantiated and connected; and a novel algorithm, called *Message Passing Bilinear Uncertainty Propagation* for the automatic propagation of uncertainties, thus allowing to compute probabilistically robust fault detection thresholds. The rest of the paper is organized as in the following: first principles models covering the thermal dynamics of common HVAC components are introduced in Sect. 2, while Sect. 3 deals with how to instantiate and parameterize them automatically using an ontology-based BIM; their integration in an overall model-based building FD architecture is covered in Sect. 4, and the novel automatic uncertainty propagation algorithm in Sect. 5, while concluding remarks are given in Sect. 6.

2. HEAT AND MASS TRANSFER MODELS OF AN HVAC SYSTEM

Throughout the paper a HVAC test case is used to guide the reader in the development of the ideas and methods (Figure 1). Hot water from a boiler is pumped and then split into two flows, feeding a radiator and an *Air Handling Unit* (AHU) in order to heat a single thermal zone. The AHU is supplied with a mixture of fresh outside air and return air from the zone. A coil fed by hot water is used to heat the mixed air flow, which finally is blown by a fan through ducts and delivered to the zone. The splitter valve controls the ratio between the AHU and radiator heating.

Nominal dynamical equations, that is ignoring uncertainties, are obtained using the approach in (Satyavada and Baldi, 2016) and are reported in (1) for all components.

3. ONTOLOGY-DRIVEN AUTOMATIC GENERATION OF HVAC SYSTEM MODELS

The following steps should be implemented for building an overall model of a given BAS for the purpose of FD: first individual models should be *instantiated* according to the actual specific components present in the BAS; then they should be connected. An ontology-based BIM is in our opinion the cornerstone for solving such a task (semi) automatically. Ontologies have been used in (Pauwels and Deursen, 2012; Zhang and Issa, 2013) for automatically formalizing and semantically enriching BIM data. *BASont*, an ontology that formalizes a BAS-specific vocabulary with the *Web Ontology Language* (OWL), was proposed in Ploennigs et al. (2012). OWL is an expressive ontology language with formal syntax and semantics based on description logics (DL) theory (World Wide Web Consortium, 2012). Instead of just being an information model, i.e. data container, OWL technically turns a BIM into a knowledge base thus enabling advanced features such as semantic retrieval and DL logical reasoning.

3.1 Instantiation of Components Models

A domain specific ontology such as *BASont* represents a given physical component, i.e. a boiler, through a class called *Boiler* and some subclasses *CondensingBoiler*, *Non-CondensingBoiler*, etc., for example. We propose to extend the existing class attributes by encoding its first principles equations, through an annotation property of type *string*. For the boiler, assuming the usual case in which the BAS includes a sensor for measuring the supply water temperature, we would thus encode both the following equations

$$\begin{cases} \dot{T}_{sw} &= (c_w \rho_w V_B)^{-1} (P_B + c_w w_B (T_{rw} - T_{sw}) \\ &+ h_B A_B (T_B - T_{sw})) + \chi_B \\ m[T_{sw}] &= T_{sw} + \xi_B \end{cases}, \quad (2)$$

where, in addition to (1), we introduced the process and measurement uncertainties χ_B and ξ_B , respectively.

Following Blanke et al. (2006), we will further describe each component through a structural bipartite directed graph. For instance, the boiler structural graph $\mathcal{G}_B = (\mathcal{N}_B, \mathcal{E}_B)$ is represented in Fig. 2, where \mathcal{E}_B is the edge set and $\mathcal{N}_B \triangleq \mathcal{X}_B \cup \mathcal{C}_B$ is the node set, partitioned in a variables set $\mathcal{X}_B \triangleq \{P_B, T_{rw}, T_B, w_B, \dot{T}_{sw}, T_{sw}, m[T_{sw}], \chi_B, T_{sw}^0, \xi_{T_{sw}}\}$ and a constraints set $\mathcal{C}_B \triangleq \{C_B, \int, \mathcal{M}_B\}$. C_B is the differential equation in (2), \int denotes the integration of \dot{T}_{sw} which leads to T_{sw} , and \mathcal{M}_B the measurement equation of T_{sw} in (2), which leads to $m[T_{sw}]$. The additional variable T_{sw}^0 represents the uncertain initial conditions of the integrator. We notice now that (2) is still a general equation for all boilers, whose symbols act as placeholders for actual quantities related to the boilers present in the BAS. We thus need a mechanism that can determine 1) the actual physical variables corresponding to such placeholders in (2), and 2) a probabilistic description of the physical parameters and the uncertainties χ_B and ξ_B in terms of their mean and variance. The rationale for this last requirement is that while the mean value of a parameter will represent its nominal known value, the variance will be used to account for the parametric uncertainty of the model. The mean of the non-parametric uncertainties χ_B and ξ_B will be assumed to be null, for well-posedness. In order to build such mechanism, we will assume that each variable and parameter of interest for

¹ "New generation computational tools for building and community energy systems based on the Modelica and Functional Mockup Interface standards", <http://www.iea-annex60.org>.

$$\begin{aligned}
[\text{pump}] \quad & w_B = u_w \\
[\text{boiler}] \quad & \dot{T}_{sw} = (c_w \rho_w V_B)^{-1} [P_B + c_w w_B (T_{rw} - T_{sw}) + h_B A_B (T_B - T_{sw})] \\
[\text{splitter}] \quad & w_R = u_v w_B \\
& w_p = (1 - u_v) w_B \\
[\text{radiator}] \quad & \dot{T}_{rwR} = (c_w \rho_w V_R)^{-1} [c_w w_R (T_{swR} - T_{rwR}) + h_R A_R (T_z - T_{rwR})] \\
[\text{fan}] \quad & w_{aD} = u_a \\
[\text{damper, mixer}] \quad & T_{maD} = u_D T_o + (1 - u_D) T_z \\
[\text{AHU heating coil}] \quad & \dot{T}_{rw_p} = (c_w \rho_w V_p)^{-1} [c_w w_p (T_{swB} - T_{rw_p}) + h_h A_p (T_{en} - T_{rw_p})] \\
[\text{AHU air duct}] \quad & \dot{T}_{saD} = (c_a \rho_a V_D)^{-1} [c_a w_{aD} (T_{maD} - T_{saD}) + h_h A_D (T_{en} - T_{saD})] \\
[\text{collector}] \quad & T_{rw} = u_v T_{rwR} + (1 - u_v) T_{rw_p} \\
[\text{zone}] \quad & \dot{T}_z = (c_a \rho_a V_Z)^{-1} [h_e A_e (T_{en} - T_z) + h_R A_R (T_{rwR} - T_z) + h_D A_D (T_{saD} - T_z)] \\
[\text{envelope}] \quad & \dot{T}_e = (c_e \rho_e V_e)^{-1} [h_e A_e (T_z - T_{en}) + h_h A_p (T_{rw_p} - T_{en}) + h_h A_D (T_{saD} - T_e)]
\end{aligned} \tag{1}$$

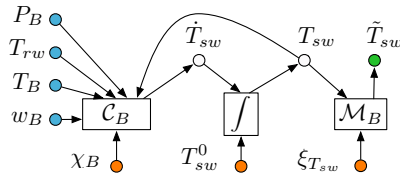


Fig. 2. The structural bipartite graph of the boiler component (blue, orange, white, green nodes represent respectively inputs, uncertainties, unmeasured and measured variables).

the components models have been defined in the ontology-based BIM through an OWL individual, which is defined by a unique *Internationalized Resource Identifier* (IRI), and their mean and variance have been associated to them. Such values can either come from existing knowledge of manufacturers and constructors, or be the result of suitable system identification experiments. With this assumption in place, we then propose a graph pattern matching on the ontology itself², i.e. a search of the ontology graph in order to identify the proper semantically right variables and constants. It can be realized as a SPARQL graph pattern that is stored as annotation property to the classes in the ontology, e.g. to the *Boiler* class. A successful query returns for each placeholder symbol the IRI of its corresponding individual, through which the needed numerical values of their probabilistic description can be retrieved from the *Building Management System* (BMS). By applying the same approach to other components as well, the first principle equations for all the components making up the whole HVAC system can be instantiated at the ontology level.

3.2 Combining Components into a Complex HVAC Model

By introducing similar graphs for the remaining components, and connecting together variable nodes with the same name, we can obtain an overall structural graph $\mathcal{G}^* \triangleq (\mathcal{N}^*, \mathcal{E}^*)$, with \mathcal{N}^* again partitioned in a variables set \mathcal{X}^* and a constraints set \mathcal{C}^* , which is illustrated in Fig. 3. In order to make it possible to use such graph for setting up automatically a probabilistically robust fault diagnosis architecture, we will require \mathcal{G}^* to satisfy the following rules, where each of them can be checked through another SPARQL query at the ontology level:

² Here we refer to the graph usually employed for representing the ontology itself, not to be confused with the structural graph representing the physical components.

- R1** Loops should occur only through an integration equation
- R2** The outbound degree of every node in \mathcal{C}^* must be equal to 1
- R3** Every node in \mathcal{X}^* must have an inbound degree equal to either 0, if it represents a manipulated input or an uncertainty, or 1, in all the other cases
- R4** Every node in \mathcal{X}^* with an inbound degree equal to 1 must be reachable from a node in \mathcal{X}^* with inbound degree equal to 0

The rationale for R1 is to avoid algebraic loops, while R2 simply requires every equation to be causal and solving for only a single variable. Finally, R3 and R4 will lead to every variable node to either be known, albeit in a probabilistic sense such as in the case of uncertainties, or computable from other variables. In the test case, it is easy to verify that all rules hold, so that we can confidently proceed forward in addressing the fault detection problem.

4. PROBABILISTIC MODEL-BASED FAULT DETECTION

By traversing \mathcal{G}^* it is possible to obtain, by symbolic manipulation and successive time discretization, a formulation of the overall plant nominal dynamics as a discrete time bilinear system

$$\begin{cases} x(k+1) &= (\bar{A} + \tilde{A})x(k) + (\bar{B} + \tilde{B})u(k) + \\ &\quad \sum_{i=1}^p (\tilde{N}_i + \tilde{N}_i)x(k)u_{(i)}(k) + \chi(k) , \\ y(k) &= (\bar{C} + \tilde{C})x(k) + \xi(k) \end{cases} \tag{3}$$

where $x \in \mathbb{R}^n$, $y \in \mathbb{R}^m$, $u \in \mathbb{R}^p$ are respectively the state, output and input vectors. $\bar{A} \in \mathbb{R}^{n \times n}$, $\bar{B} \in \mathbb{R}^{n \times p}$ and $\bar{C} \in \mathbb{R}^{m \times n}$ are the nominal values of the state, input and output matrices, while the p matrices $N_i \in \mathbb{R}^{n \times n}$ represent the nominal bilinear influence of the i -th input component $u_{(i)}$ on the state dynamics. It should be remembered that a tilde is used to denote parametric uncertainties. For well-posedness, we need the following assumption

Assumption 1. \tilde{A} , \tilde{B} , \tilde{N}_i , \tilde{C} and χ , ξ are, respectively, matrix and vector-valued random variables defined on some probability spaces. While elements of \bar{A} , \bar{B} , \bar{N}_i , \bar{C} in general are not independent, they are independent from either χ and ξ , and the last two are mutually independent as well. While the distribution of every component of said variables is unknown, the knowledge of their mean and variance, and of the covariance with respect to all the other components is assumed to be known.

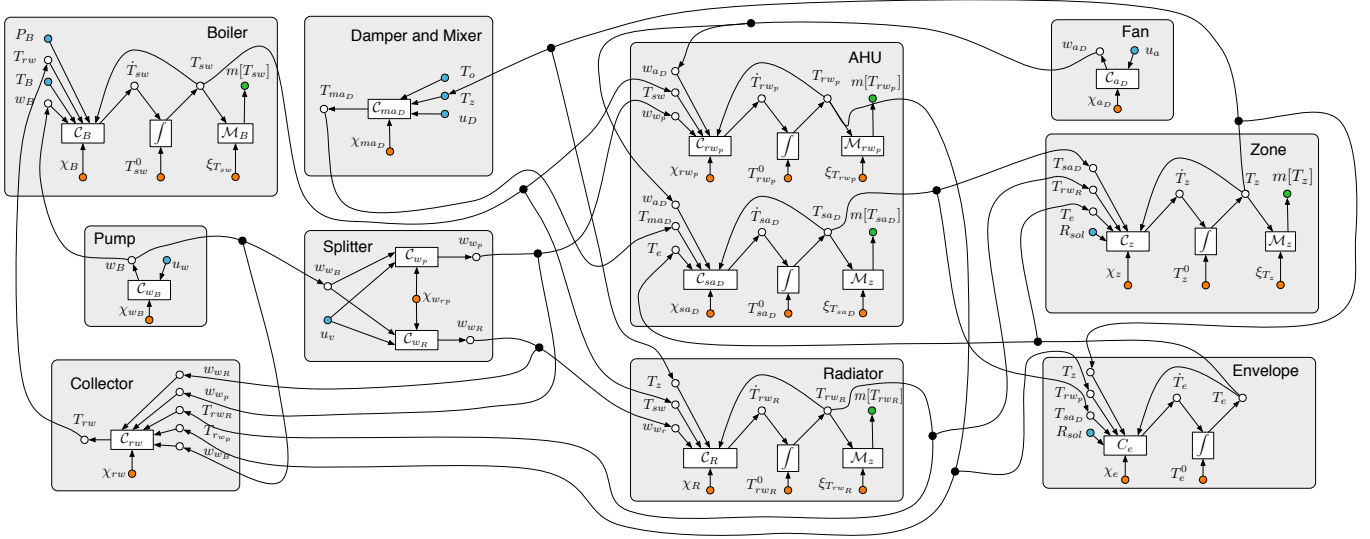


Fig. 3. Overall structural graph of the test case (blue, orange, white, green nodes represent respectively inputs, uncertainties, unmeasured and measured variables).

We will also assume that x is ordered such that $x = \text{col}(x', x'')$, where x' contains the measured state variables, that in the test case are T_{sw} , T_{rwR} , T_{rwP} , T_{saD} and T_z , and x'' the unmeasured ones, that is T_e . This will lead to $\bar{C} = [I_m \ 0]$ with I_m being the $m \times m$ identity matrix. Finally, we will assume that the only measuring uncertainty is due to ξ , so that $\bar{C} = 0$.

4.1 Fault Detection Estimator

By proceeding along an approach similar to Boem et al. (2015); Ferrari et al. (2008), it is possible to derive the following *Fault Detection Estimator* (FDE)

$$\begin{cases} \hat{x}(k+1) &= \bar{A}\hat{z}(k) + \bar{B}u(k) + \sum_{i=1}^p \bar{N}_i \hat{z}(k) u_{(i)}(k) \\ &+ L(\hat{y}(k) - y(k)) \\ \hat{y}(k) &= \hat{x}'(k) \\ \hat{z}(k) &= \text{col}(y(k), \hat{x}''(k)) \end{cases}, \quad (4)$$

where $L \in \mathbb{R}^{n \times m}$ is chosen such that the dynamics for the residual $r_x(k) \triangleq x(k) - \hat{x}(k)$ are stable. By subtracting (4) from (3) we get

$$\begin{aligned} x(k+1) - \hat{x}(k+1) &= (\bar{A} + \tilde{A})x(k) - \bar{A}\hat{z}(k) + (\bar{B} + \tilde{B})u(k) \\ &- \bar{B}u(k) + \sum_{i=1}^p \left[(\bar{N}_i + \tilde{N}_i)x(k)u_{(i)}(k) - \bar{N}_i\hat{z}(k)u_{(i)}(k) \right] \\ &+ \chi(k) + L[C(x(k) - \hat{x}(k)) - \xi(k)] \end{aligned}$$

which leads to

$$r_x(k+1) = A_0 r_x(k) + \gamma(k), \quad (5)$$

where $A_0 \triangleq L\bar{C}$ and we introduced the *total uncertainty term*

$$\begin{aligned} \gamma(k) \triangleq & \tilde{A}x(k) + \tilde{A}\hat{x}(k) + \tilde{B}u(k) + \sum_{i=1}^p \left(\tilde{N}_i x(k) \right. \\ & \left. + \tilde{N}_i \hat{x}(k) \right) u_{(i)}(k) + \chi(k) + L\xi(k) \end{aligned} \quad (6)$$

and the difference term $\tilde{x}(k) \triangleq x(k) - \hat{z}(k)$. While $r_x(k)$ can be defined for analysis purposes, it cannot be directly used for fault detection. We can instead notice that, by partitioning r_x into two subvectors r'_x and r''_x , as we did with the state x , it turns out that

$$r_y(k) \triangleq y(k) - \hat{y}(k) = \bar{C}r_x(k) + \xi(k) = r'_x(k) + \xi(k). \quad (7)$$

As $r_y(k)$ is a measurable quantity, we can use it for fault detection. Due to the Ass. 1 and to eqs. (5) and (7) we have that while at time k the residual $r_y(k)$ is a deterministic quantity, as it can be computed, its future value $r_y(k+1)$ is a random variable, as it depends on the random variables $r_x(k)$, $\gamma(k)$ and $\xi(k+1)$. We can so assume the existence of its mean $\bar{r}_y(k+1) \triangleq \mathbb{E}[r_y(k+1)] \in \mathbb{R}^n$ and covariance matrix $\Sigma_{r_y}(k+1) \triangleq \text{Cov}[r_y(k+1)] \in \mathbb{R}^{n \times n}$, and define the *probabilistically α -robust ellipsoid* \mathcal{E}_α

$$\mathcal{E}_\alpha \triangleq \left\{ r_y \in \mathbb{R}^n \mid (r_y - \bar{r}_y) \Sigma_{r_y}^{-1} (r_y - \bar{r}_y) \leq \frac{n}{\alpha} \right\}, \quad \alpha \in (0, 1], \quad (8)$$

whose name reflects the fact that, thanks to the Multivariate Chebyshev Inequality (see Chen (2007)), in healthy conditions it holds

$$\text{Prob}[r_y \in \mathcal{E}_\alpha] \geq \alpha. \quad (9)$$

We propose, thus, to use the set-membership condition in the left hand side of (9) as a mean to detect faults (such as, e.g. boiler degradation or a stuck valve), thus letting \mathcal{E}_α to act as a set-based fault detection threshold. As α is a user defined constant, it allows to guarantee a desired probability of false alarms.

5. UNCERTAINTY PROPAGATION ALGORITHM

Checking online the eventual violation of the condition in (9) requires the computation, at time instant k of the ellipsoid parameters $\bar{r}_y(k+1)$ and $\Sigma_{r_y}(k+1)$ for the next instant $k+1$. In order to pursue that, we assume the knowledge of the initial mean and covariance $\bar{r}_x(0) \triangleq \mathbb{E}[r_x(0)]$ and $\Sigma_{r_x}(0) \triangleq \text{Cov}[r_x(0)]$ for $k=0$, and then use eqs. (5) and (7) to iteratively propagate them to the next time instant. This will require the computation, at each time instant, of the mean $\bar{\gamma}(k)$ and covariance $\Sigma_\gamma(k)$ of the total uncertainty $\gamma(k)$, which in turn depends on the mean and covariances of the individual uncertainty sources in (6). In the following, we will refer to updating in time the ellipsoid parameters as the *uncertainty time propagation problem*, and to computing $\bar{\gamma}(k)$ and $\Sigma_\gamma(k)$ as the *uncertainty model propagation problem*.

5.1 The Uncertainty Time Propagation Problem

Let us introduce for analysis purposes the vector $\varepsilon(0) \triangleq \text{col}(r_x(0), \gamma(0))$. While it is unknown and not measurable, we can nevertheless compute its mean and covariance, thanks to Ass. 1, as $\bar{\varepsilon}(0) \triangleq \mathbb{E}[\varepsilon(0)] = \text{col}(\bar{r}_x(0), \bar{\gamma}(0))$ and $\Sigma_\varepsilon(0) \triangleq \text{Cov}[\varepsilon(0)] = \text{diag}(\Sigma_{r_x}(0), \Sigma_\gamma(0))$. In these expressions we took advantage that, at time $k = 0$, r_x and γ are independent as the uncertainty had no chance to influence the state dynamics yet.

By introducing the block diagonal matrix $A_\varepsilon \triangleq \text{diag}(A_0, I)$, with I an identity matrix of suitable size, we can simply write the following equations for the propagation in time

$$\bar{\varepsilon}(k+1) = A_\varepsilon \bar{\varepsilon}(k), \quad \Sigma_\varepsilon(k+1) = A_\varepsilon \Sigma_\varepsilon(k) A_\varepsilon^\top. \quad (10)$$

Applying eq. (10) at the start time index $k = 0$ it is possible to compute the mean and covariance of the state residual r_x at time $k+1 = 1$. Using eq. (7) the mean and covariances for $r_y(k+1)$ can thus be obtained, which allow to determine the set-based threshold $\mathcal{E}_\alpha(k+1)$. Anyway, in order to proceed to the time index $k = 2$ and further, we need to compute the new values $\bar{\gamma}(1)$ and $\Sigma_\gamma(1)$, which both depend on $\bar{r}_x(1)$ and $\Sigma_x(1)$ and on the uncertainty sources \tilde{A} , \tilde{B} , \tilde{N}_i , χ and ξ . How to update the moments of γ is the subject of the next subsection.

5.2 The Uncertainty Model Propagation Problem

As eq. (6) is dependent on the not fully measured state x , it cannot be readily used for uncertainty propagation. We can anyway rewrite it as

$$\begin{aligned} \gamma(k) \triangleq & \tilde{A}\tilde{x}(k) + \tilde{A}\tilde{z}(k) + \tilde{A}\tilde{x}(k) + \tilde{B}u(k) + \sum_{i=1}^p \left(\tilde{N}_i\tilde{x}(k) \right. \\ & \left. + \tilde{N}_i\tilde{z}(k) + \tilde{N}_i\tilde{x}(k) \right) u_{(i)}(k) + \chi(k) + L\xi(k) \end{aligned} \quad (11)$$

where we eliminated x through the identity $x = \tilde{x} + \tilde{z}$. As it follows, from the definition of \tilde{x} and eq. (7), that $\tilde{x} = \text{col}(r'_x + \xi, r''_x)$, we can compute its mean and covariance from the last known values of the mean and covariance of r'_x and r''_x , obtained from the last uncertainty time propagation step, and those for ξ , which are known due to Ass. 1. Now it can be seen that the total uncertainty term γ depends on the sum of mixed products of the kind $\theta\tilde{a}$ and $\tilde{a}\tilde{b}$, where θ is a known deterministic quantity and \tilde{a} and \tilde{b} are random variables, representing either a parametric uncertainty such as \tilde{A} , an estimation error such as r_x or an uncertain term such as χ or ξ .

While terms such as $\theta\tilde{a}$ are trivial, terms like $\tilde{a}\tilde{b}$ require more care. In the case of scalar \tilde{a} and \tilde{b} , exact formulas for their mean and covariances are given in Bohrnstedt and Goldberger (1969)

$$\mathbb{E}[\tilde{a}\tilde{b}] = \mathbb{E}[\tilde{a}]\mathbb{E}[\tilde{b}] + \text{Cov}[\tilde{a}, \tilde{b}] \quad (12)$$

$$\begin{aligned} \text{Var}[\tilde{a}\tilde{b}] = & \text{Cov}[\tilde{a}^2, \tilde{b}^2] + (\text{Var}[\tilde{a}] + \mathbb{E}[\tilde{a}]^2)(\text{Var}[\tilde{b}] + \mathbb{E}[\tilde{b}]^2) \\ & - (\text{Cov}[\tilde{a}, \tilde{b}] + \mathbb{E}[\tilde{a}]\mathbb{E}[\tilde{b}])^2 \end{aligned} \quad (13)$$

$$\text{Cov}[\tilde{a}\tilde{b}, \tilde{c}] = \mathbb{E}[\tilde{a}]\text{Cov}[\tilde{b}, \tilde{c}] + \mathbb{E}[\tilde{b}]\text{Cov}[\tilde{a}, \tilde{c}] + \mathbb{E}[\Delta\tilde{a}\Delta\tilde{b}\Delta\tilde{c}] \quad (14)$$

where we allow in general for means such as $\mathbb{E}[\tilde{a}]$ to be nonzero and introduced the notation $\Delta\tilde{a} \triangleq \tilde{a} - \mathbb{E}[\tilde{a}]$ for the generic random variable \tilde{a} .

The reason for needing eq. (14) is that in general the product term $\tilde{a}\tilde{b}$ will not be independent from other uncertain contributions denoted by \tilde{c} . To better understand this let us consider the interconnection of the pump, collector and boiler (see Fig. 3). When computing the components of the total uncertainty affecting the boiler dynamics, it should be taken into consideration that the uncertainties affecting the physical variables T_{r_w} and w_B are correlated, as they are both depending on the uncertainties affecting the algebraic equation for the pump, which defines w_B .

This consideration, together with the fact that eqs. (12-14) are difficult to generalize to matrix and vector terms, suggest to solve the uncertainty model propagation problem one component γ_i at a time, by proceeding step by step along the structural bipartite graph \mathcal{G}^* of Fig. 3. A novel algorithm leading to such a solution is presented next.

A Message Passing Bilinear Uncertainty Propagation Algorithm We will now introduce a novel Message Passing Bilinear Uncertainty Propagation (MP-BUP) algorithm, inspired by the well known Belief Propagation algorithm by J. Pearl (see Kschischang et al. (2001) for a tutorial). It will be first described for the case of the following class of systems of bilinear equations

$$\begin{aligned} a_l = & \sum_{i=1}^n \theta_{l,(i)} a_i + \sum_{i=1}^m \kappa_{l,(i)} b_i + \sum_{i=1}^n \sum_{j=1}^m \psi_{l,(i,j)} a_i b_j \\ & + \sum_{i=1}^m \sum_{j=1}^m \omega_{l,(i,j)} b_i b_j, \quad \forall l \in \{1, \dots, n\} \end{aligned} \quad (15)$$

where $a \in \mathbb{R}^n$ are quantities to be solved for, $b \in \mathbb{R}^m$ is a vector of random variables acting as inputs to the equation, $\kappa_l, \theta_l \in \mathbb{R}^m$ and $\psi_l, \omega_l \in \mathbb{R}^{m \times m}$ are vectors and matrices of known coefficients, $\forall l \in \{1, \dots, n\}$. We assume the constraint $\theta_{l,(l)} = 0$, $\omega_{l,(j,j)} = 0$ and $\psi_{l,(l,j)} = 0$ for every j and l , in order to make (15) an explicit equation and avoid quadratic terms in b_j . Now, assuming the knowledge of the mean \bar{b} and covariance Σ_b of the input variables, the goal of the MB-BUP algorithm is to iteratively compute the mean and covariance of a .

The key point in the BP, and in the present MP-BUP algorithm as well, is to recognize that eq. (15) can be written as the following sum-of-products (SoP) expression

$$a_l = \sum_{h=1}^{n_f} \varphi_{(l,h)} f_h(a, b), \quad \forall l \in \{1, \dots, n\} \quad (16)$$

where $f_h : \mathbb{R}^n \times \mathbb{R}^m \mapsto \mathbb{R}$ is a so called *factor* that is obtained as the product of individual components of a and b . From (15) it follows that there are $n_f = (m+n)(m+1)$ possible factors, which for convenience will be numbered in lexicographic order. With this convention, it is straightforward to define the new coefficient matrix $\varphi \in \mathbb{R}^{n \times n_f}$ by taking in the right order the elements of the previous coefficients κ, θ, ψ and ω . The relation between factors, the variables to which they concur and the variables on which they depend, can be conveniently expressed by yet another directed bipartite graph $\mathcal{G} \triangleq (\mathcal{D}, \mathcal{E})$, where we introduced the node set $\mathcal{D} \triangleq \mathcal{C} \cup \mathcal{F}$, and $n_{\mathcal{E}} \triangleq \text{dim}(\mathcal{E})$ is the total number of edges. The sets $\mathcal{C} \triangleq \{a_1, \dots, a_n, b_1, \dots, b_m\}$ and $\mathcal{F} \triangleq \{f_1, \dots, f_{n_f}\}$ define the two partitions of nodes, respectively into variables and factors. An edge $e = (f_h, a_l)$ belongs to \mathcal{E} iff $\varphi_{h,l} \neq 0$,

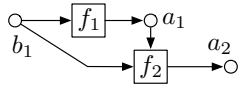


Fig. 4. A simple example of a bipartite graph representing the bilinear system $a_1 = \varphi_{(1,1)}f_1(a, b) = \varphi_{(1,1)}b_1$ and $a_2 = \varphi_{(2,2)}f_2(a, b) = \varphi_{(2,2)}a_1b_1$.

so that f_h concurs to the sum for a_l . Similarly, an edge $e = (c_j, f_h)$ belongs to \mathcal{E} , with $c_j \in \mathcal{C}$, iff c_j is a term being part of the product f_h (see Fig. 4).

Finally, we need to introduce the vector $d \in \mathbb{R}^{n_d}$, defined as the concatenation of all the elements of \mathcal{D} taken in lexicographic order. The algorithm will iteratively compute the mean and covariance matrix of d , whose values at iteration $q \in \{0, 1/2, 1, \dots\}$ will be denoted by $\bar{d}(q)$ and $\Sigma_d(q)$, by exchanging exactly one message over each directed edge in \mathcal{E} . To this end two further auxiliary and varying sets will be needed, $\mathcal{V}(q)$ and $\mathcal{T}(q)$, containing respectively the already *visited* nodes and *traversed* edges at iteration q . The algorithm working variables are initialized as $\bar{d}(0) \triangleq \text{col}(0_a, \bar{b}, 0_f)$ and $\Sigma_d(0) \triangleq \text{diag}(0_{\Sigma_a}, \Sigma_b, 0_{\Sigma_f})$, where the symbols 0_* denote a vector or matrix of zeros the same size as the variable $*$, and as $\mathcal{V}(0) = \{b_1, \dots, b_m\}$ and $\mathcal{T}(0) = \emptyset$. This reflects the initial knowledge of the central moments of the inputs b only. The algorithm then alternates at every major step q the following two minor steps, where a \neg placed before the symbol of a set denotes its complement, until all nodes and edges result visited and traversed, that is $\mathcal{V} = \mathcal{D}$ and $\mathcal{T} = \mathcal{E}$:

Variables→factors message passing: for every untraversed edge $e = (j, h) \in (\neg\mathcal{T})$ connecting a visited variable node $c_j \in \mathcal{C} \cap \mathcal{V}$ to an unvisited factor node $f_h \in \mathcal{F} \cap (\neg\mathcal{V})$, a message is transmitted to f_h along e containing \bar{c}_j and the covariances $\text{Cov}[c_j, d_i]$ between c_j and all the visited nodes $d_i \in \mathcal{V}$; e is added to \mathcal{T} to mark it as traversed, but f_h is added to \mathcal{V} to mark it as visited only if all its incident edges belong to \mathcal{T} .

Factors→variables message passing: for every untraversed edge $e = (h, j) \in (\neg\mathcal{T})$ connecting a visited factor node $f_h \in \mathcal{F} \cap \mathcal{V}$ to an unvisited variable node $c_j \in \mathcal{C} \cap (\neg\mathcal{V})$, a message is transmitted to c_j along e containing \bar{f}_h and the covariances $\text{Cov}[f_h, d_i]$ between f_h and all the visited nodes $d_i \in \mathcal{D}$; e is added to \mathcal{T} to mark it as traversed, but c_j is added to \mathcal{V} to mark it as visited only if all its incident edges belong to \mathcal{T} .

To denote quantities computed during the first minor step, fractional values of q will be used, while integer values will be used for the second minor step. At the end of both the above minor steps, when a node is marked as visited its mean and covariance are updated using all the messages received until that instant. In the case of the variables→factor substep, after a factor node f_h is marked as visited formulas (12–14) are used to compute \bar{f}_h and all the covariances of the form $\text{Cov}[f_h, d_i]$, $d_i \in \mathcal{V}$, which will be used to populate corresponding elements of the matrix Σ_d . It should be noted that due to the way \mathcal{G} has been defined, covariances relating variables appearing in f_h to non visited nodes must be zero, so the fact they have not been received by the node f_h will not alter the end result. Instead, in the case of the factor→variables substep, after a variable node a_l is marked as visited we should remember that a_l is a linear combination of factors (see eq. (16)). We can so update mean and covariances as $\bar{d}(q) = \Phi\bar{d}(q-1)$, $\Sigma_d(q) = \Phi\Sigma_d(q-1)\Phi^T$, where $\Phi \in \mathbb{R}^{n_d \times n_d}$ and it holds that $\Phi_{(i,i)} = 1$ for every i such

that $d_i \in \mathcal{V}(q-1)$ and $\Phi_{(l,h)} = \varphi_{(l,h)}$ for every h such that the edge $e = (h, l) \in \mathcal{T}$.

In order to better understand the presently proposed algorithm, a step-by-step illustration for the simple graph in Fig. 4 is provided.

A simple example The system of equations can be written as $a_1 = \varphi_{(1,1)}f_1 = \varphi_{(1,1)}b_1$ and $a_2 = \varphi_{(2,2)}f_2 = \varphi_{(2,2)}a_1b_1$.

The nodes partition is defined by $\mathcal{C} = \{a_1, a_2, b_1\}$ and $\mathcal{F} = \{f_1, f_2\}$, and the working variables are initialized to $\bar{d}(0) = [0, 0, \bar{b}_1, 0, 0]^T$, $\Sigma_d = \text{diag}(0, 0, \sigma^2[b_1], 0, 0)$, $\mathcal{V}(0) = \{b_1\}$, $\mathcal{T}(0) = \emptyset$. The notation $\sigma^2[\cdot]$ will be used as a shorthand to either variance or covariance.

$q = 1/2$, **variables→factors** The only node that can transmit is b_1 , and will send the values \bar{b}_1 and $\sigma^2[b_1]$ to f_1 and f_2 , along edges (b_1, f_1) and (b_1, f_2) . The visited and traversed sets are updated to $\mathcal{V}(1/2) = \{b_1, f_1\}$ and $\mathcal{T}(1/2) = \{(b_1, f_1), (b_1, f_2)\}$. As f_1 was marked as visited, we compute the values $f_1 = \bar{b}_1$ and $\sigma^2[b_1, f_1] = \sigma^2[f_1, b_1] = \sigma^2[b_1]$ and update relevant components of the mean and covariances as $\bar{d}_{(4)}(1/2) = \bar{b}_1$ and $\Sigma_{d,(4,4)}(1/2) = \Sigma_{d,(3,4)}(1/2) = \Sigma_{d,(4,3)}(1/2) = \sigma^2[b_1]$.

$q = 1$, **factors→variables** Now only f_1 can transmit, as f_2 is waiting for its incoming edge (a_1, f_2) to be traversed. So a message is transmitted from f_1 to a_1 along the edge (f_1, a_1) carrying the values f_1 , $\sigma^2[b_1, f_1]$ and $\sigma^2[f_1, b_1]$. The visited and traversed sets are updated to $\mathcal{V}(1) = \{a_1, b_1, f_1\}$ and $\mathcal{T}(1) = \{(b_1, f_1), (b_1, f_2), (f_1, a_1)\}$. As a_1 was marked as visited, we can build the matrix Φ whose non-zero entries are $\Phi_{(3,3)} = \Phi_{(4,4)} = 1$ and $\Phi_{(1,4)} = \varphi_{(1,1)}$ and proceed to the last update of this iteration: $\bar{d}(1) = \Phi\bar{d}(1/2)$ and $\Sigma_d(1) = \Phi\Sigma_d(1/2)\Phi^T$. This results in $\bar{d}_{(1)}$, $\Sigma_{d,(1,1)}$, $\Sigma_{d,(1,4)}$ and $\Sigma_{d,(4,1)}$ being non-zero too.

$q = 1 + 1/2$, **variables→factors** As the node a_1 was marked visited in the last minor step, now it can transmit its mean and corresponding covariance components to the factor node f_2 . The visited and traversed sets are updated to $\mathcal{V}(1 + 1/2) = \{a_1, b_1, f_1, f_2\}$ and $\mathcal{T}(1 + 1/2) = \{(a_1, f_2), (b_1, f_1), (b_1, f_2)\}$. f_2 mean and covariance can be now computed, by remembering that $f_2 = a_1b_1$ and applying formulas (12–14) to obtain $\mathbb{E}[a_1b_1]$, $\sigma^2[a_1b_1, a_1]$, $\sigma^2[a_1b_1, b_1]$ and $\sigma^2[a_1b_1, f_1]$.

$q = 2$, **factors→variables** In this last step we should send a last message through the edge $e = (f_2, a_2)$, after which the visited and traversed sets are complete, that is $\mathcal{V}(2) = \mathcal{D}$ and $\mathcal{T}(2) = \mathcal{E}$. Updating \bar{d} and Σ_d is trivial as the mean of a_2 is $\varphi_{(2,2)}$ times the mean of f_2 , while the covariance components involving a_2 are $\varphi_{(2,2)}^2$ times those of f_2 .

Application to the present case In order to apply the MP-BUP algorithm to the present case, each component of eq. (11) should be rewritten in the form described in eq. (15). This can be done either by symbolic manipulation, or by traversing the structural bilinear directed graph \mathcal{G}^* and expanding all the bilinear equations in \mathcal{C}^* . In doing so, the input terms b_i in (11) are understood to be constituted by all the relevant components of the parametric uncertainties, the estimation error r_x or the uncertain terms χ or ξ , numbered for convenience in lexicographic order. The output terms a_l are understood

to be the value of the right hand sides of the equations (1) describing all the HVAC components.

6. CONCLUDING REMARKS

In the present paper a methodology for the automatic synthesis of a global model of a building HVAC system for FD purposes, leveraging a purposely built ontology-based-BIM such as BASont, has been proposed. A novel algorithm called MP-BUP has been presented for propagating the effects of single sources of stochastic uncertainties to the FD residuals, in order to derive robust detection thresholds with probabilistic performances.

The first advantage of the proposed approach is interoperability and modularity, as it can be extended to arbitrary BAS without sizeable manual intervention. In addition, the limitations of (quasi-)steady-state or RC modeling approaches (Yang et al., 2012; Wu and Sun, 2011; Wang et al., 2010; Underwood and Yik, 2004; Yu and van Paassen, 2004) and the risk of introducing algebraic loops (Wetter et al., 2015) were avoided, thanks to the use of bipartite graphs for describing the structure of the overall system and to the use of dynamic models.

Future works will evaluate the proposed approach on data provided by the AMBI project (FP7 grant no. 324432) simulator. Furthermore, the need to encode extensive knowledge in the ontology-based-BIM could be lessened by methods identifying both components parameters and their interconnection structure. For instance symbolic regression tools could be used, where primitives are represented by the classes of HVAC components, and an evolutionary algorithm is implemented to identify the optimal BAS structure.

ACKNOWLEDGEMENTS

The research leading to these results has received funding from European Union's FP7 under grant agreement no. 324432 (AMBI: Advanced Methods for Building Diagnostics and Maintenance). Patent Pending.

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