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Prediction of Strength for Inhomogeneous - Defective Glass Elements Based on the Sequential Partitioning of the Data and Weibull Statistical Distribution

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An analytical approach based on the on the sequential partitioning of the data and Weibull Statistical Distribution for inhomogeneous - defective materials is proposed. It allows assessing the guaranteed strength of glass structures for the low probability of fracture with a higher degree of reliability. Parameters of equations for the piecewise linear approximation for Weibull statistical distribution have been defined on the example of processing of bending tests results for float glass. The advisability of using this approach to structural elements of different size is proved. It was shown that excluding the minimum values from the sample does not lead to the uni-modal distribution. A group of values, forming the lower branch of the distribution, appears again. Statistical analysis of the distribution curves made it possible to identify groups of defects, the technological removal of which would ensure an increase in the guaranteed level of strength. The results are the basis for solving optimization problems when you need to get a guaranteed level of strength for a given probability of fracture with minimal costs for glass element manufacture and treatment.

Keywords: Glass, Weibull Distribution, Bending, Guaranteed Strength, Inhomogeneous - defective materials

Strength is as an important characteristic of structural materials. In the analysis of experimental data statistical methods have been widely used to increase the reliability of strength values. For inhomogeneous - defective materials a significant statistical spread of the test results and the complex nature of the statistical distribution are observed. Inhomogeneous - defective materials as glass, ceramics, glass ceramics, are increasingly used as structural materials (Rodichev 2003; Rodichev 2005). Surface technological and operational defects have a drastic effect on the strength and character of the statistical distribution (Rodichev and Veer 2010; Veer and Rodichev 2011). This leads to the problem of a reasonable prediction of guaranteed level of strength for critical structures.

Reliability of the guaranteed strength level assessment for the preset low probability of failure is actual and important for load-bearing structures in aircraft industry, transport glazing, building and architecture (Shupikov and Ugrimov 2013; Veer and Rodichev 2011). Unfortunately manufacturers of glass don't give specific strength characteristics. The European standards EN 1288-3: 2000 and EN 12150-2: 2004 regarding the mechanical properties of of architectural glass call for the testing of standard specimens of the appropriate thickness with dimensions of 1100 mm x 360 mm in an amount of 10 pieces. To assess the guaranteed strength level for a given probability of failure in the interval 0.1 ... 0.001% a uni-modal approximation of the experimental Weibull distribution curves is used (Pisarenko et al. 1979; Standard EN 1288-3:200011; Standard EN 12150-2:2004). In the aircraft industry special plate-type specimens are tested under axisymmetric bending, and the confidence probability of failure may be taken still less, depending on the requirements for the reliability of glazing.

To obtain more reliable assessment of glass elements carrying capacity the quantity of tested specimens is increased. Analysis of experimental results showed that, in many cases, the uni-modal statistic distribution leads to significant error of the guaranteed strength level (Rodichev and Veer 2010; Veer and Rodichev 2011; Veer and Riemslag 2009). Analytical descriptions for multimodal distributions of glass strength are not used in practice. In addition, when testing small specimens, the probability of detecting large defects and real assessment of the guaranteed level of structural glass strength decreases. This significantly reduces the reliability of the obtained results (Veer and Rodichev 2011; Veer and Riemslag 2009).

Glass is a linearly elastic brittle material. Due to its almost ideal elasticity, fracture toughness of glass is the lowest compared to other brittle materials. Typical values of the stress intensity factor (KIC) for a building sheet float glass are 0.5 ... 0.7MPa√m, whereas for heat-resistant glass ceramics and solid technical ceramics up to 2.0 ... 6,0MPa√m (Rodichev 2005; Standard EN 12150-2:2004; Veer and Riemslag 2009). As a result, very small critical defects such as microcracks determine the limiting state of the glass elements under different types of thermomechanical loading. The depth of critical microcracks depends on the structure of the material, the degree of its homogeneity and technological defects. It varies from 10…100 microns for annealed glass with homogeneous nanosized amorphous structure up to 200 ... 500 microns for polycrystalline ceramic structures.

The specific cracked surface layer has predominant influence on fracture resistance of glass elements. This surface layer has an unexplored and difficultly controlled defective structure. It includes a system of microcracks as well as different technological and operational surface defects that are much larger and more significant than defects of the internal structure (Fig. 1).

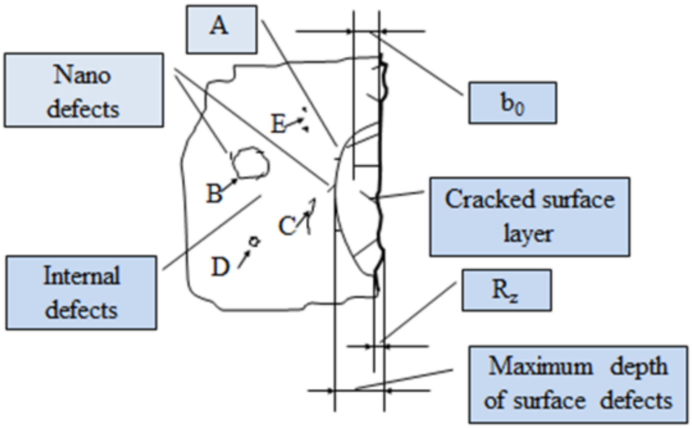


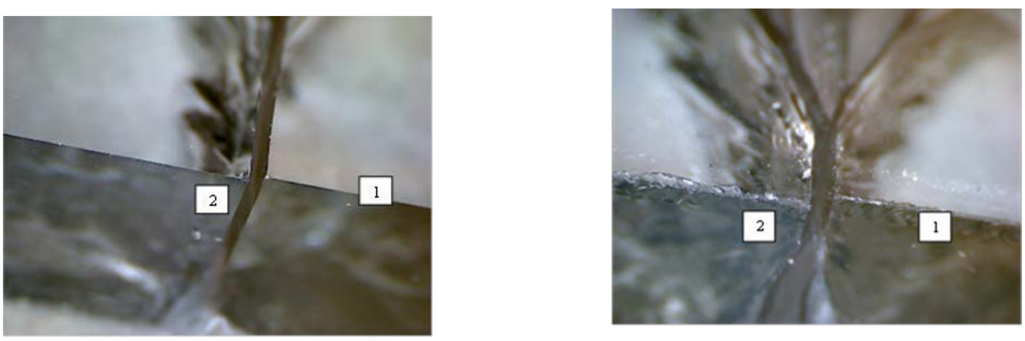
Fig.1 Surface and internal defects of glass and ceramics (b_0 – depth of initial microcracks in float glass; R_z – depth of microrelief; A – maximum surface defect; B – gas bubble; D, S– micro inclusions

The cracked surface layer of glass without mechanical treatment does not contain significant microcracks. Their maximal initial depth b_0 does not exceed 50…100μ. More significant surface defects A are formed as a result of cutting and abrasive treatment under the influence of abrasive grains of tools. The analysis of fracture sources in glass beams under bending shows that internal defects of glass elements are not critical in the destruction of structures made of glass.

Modern technology of machining allows increasing the strength of glass products due to the proper choice of tools and cutting modes as well as the use of experimental data concerning the influence of surface defects on the glass strength level. To increase the lower values of glass strength, it is important to control the dimensions of the largest defects such as cracks, which are sources of fracture.

The aim of the paper is to develop analytical approach on the basis of the results of glass specimens bending tests which allows to increase the validity and reliability for the prediction of the guaranteed level of strength at a given fracture probability taking into account the influence of technological factors.

Statistical processing and analysis of the experimental results presented in the works of the authors (Veer and Rodichev 2011; Veer and Riemslag 2009). Specimens with a size of 400 mm×50 mm × 6 mm were tested under pure bending. They were cut from the a large-sized “Jumbo” plate with a size 6 m × 3.21 m × 6 mm When machining the edge of the samples, microcracks are formed on the surface of the ends and edges of the facets. The dimensions of these microcracks exceed other surface defects. The tests were conducted both under the conditions of the horizontal position of the plate and in its vertical position. In vertical position all possible large defects of the mechanical treatment such as microcracks, chips, and point defects are exposed to the maximum stress level. Examples of defects and the nature of failure are shown in Fig 2, 3.



a) b)

Fig.2 a) Sharp edge 1 and source of fracture - micro-crack 2 under bending, b) Defects in cut edge 1 and micro-crack 2 - the focus of fracture under bending

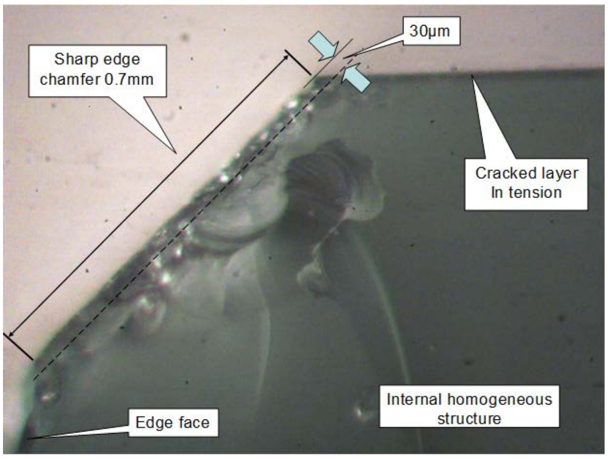


Fig.3 The source of fracture - a long microcrack depth of 30 microns on a ground surface of a chamfer

The tables 1, 2 show the values of the fracture load F and the bending strength σ for tested specimens. When processing experimental results for specimens in horizontal position, the data when cut edge is in tension zone are combined when it is in compression zone, since in practice, there is a simultaneous positioning of the notch both in the tension zone and in the compression zone. For the specimens in vertical position, the data obtained for left and right positions of the notch are combined, since they are at the same stress level.

Table 1 The results of bending tests in horizontal position of flat specimen

Sample	Fracture load F, [kN] notch in the zone of tension	Bending strength σ, [MPa] notch in the zone of tension	Fracture load F, [kN] notch in the zone of compresion	Bending strength σ, [MPa] notch in the zone of compresion
1	357	52.06	259	37.77
2	335	48.85	447	65.19
3	351	51.19	413	60.23
4	382	55.71	373	54.40
5	405	59.06	414	60.38
6	352	51.33	365	53.23
7	364	53.08	463	67.52
8	356	51.92	467	68.10
9	386	56.29	470	68.54
10	347	50.60	400	58.33
11	328	47.83	431	62.85
12	248	36.17	360	52.50
13	340	49.58	381	55.56
14	350	51.04	361	52.65
15	377	54.98	398	58.04
16	368	53.67	375	54.69
17	382	55.71	408	59.50
18	380	55.42	432	63.00
19	375	54.69	343	50.02
20	396	57.75	301	43.90
21	340	49.58	442	64.46
22	361	52.65	290	42.29
23	343	50.02	369	53.81
24	383	55.85	467	68.10
25	366	53.38	512	74.67
26	305	44.48	275	40.10

27	341	49.73	485	70.73
28	397	57.90	434	63.29
29	286	41.71	554	80.79
30	384	56.00	683	99.60
31	303	44.19	402	58.63
32	354	51.63	656	95.67
33	395	57.60	302	44.04
34	238	34.71	364	53.08
35	329	47.98	470	68.54
36	378	55.13	349	50.90
37	337	49.15	396	57.75
38	327	47.69	532	77.58
39	335	48.85	383	55.85
40	379	55.27	539	78.60
41			559	81.52
42			418	60.96
43			455	66.35
44			257	37.48
45			469	68.40
46			440	64.17
47			544	79.33
48			505	73.65
49			517	75.40
50			446	65.04
σ_{mean}, [MPa]		51.6		61.4
Variation coefficient		10.6%		20.5%
σ_{max}, [MPa]		59.1		99.6
σ_{min}, [MPa]		34.7		37.5

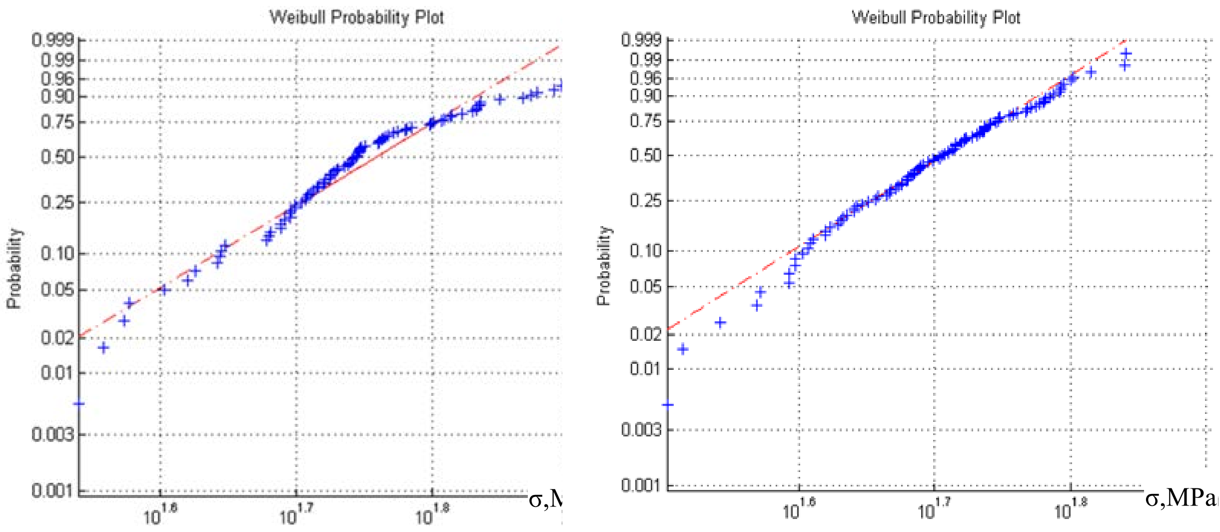
Table 2 The results of bending tests in the vertical position of flat specimen

Sample	Fracture load F, [kN] notch left	Bending strength σ, [MPa] notch left	Fracture load F, [kN] notch right	Bending strength σ, [MPa] notch right
1	2770	48.48	3730	65.28
2	2800	49.00	3620	63.35
3	2450	42.88	3450	60.38
4	2430	42.53	3550	62.13
5	2860	50.05	3010	52.68
6	2830	49.53	2890	50.58
7	3290	57.58	2780	48.65
8	2970	51.98	2900	50.75
9	2860	50.05	2530	44.28
10	2750	48.13	3560	62.30
11	2260	39.55	2740	47.95
12	2740	47.95	2950	51.63
13	1820	31.85	2500	43.75
14	2510	43.93	3020	52.85
15	2560	44.80	2790	48.83

16	2500	43.75	3010	52.68
17	2970	51.98	2730	47.78
18	2710	47.43	2130	37.28
19	2240	39.20	3120	54.60
20	2590	45.33	3350	58.63
21	1990	34.83	2650	46.38
22	3120	54.60	3950	69.13
23	3190	55.83	3110	54.43
24	2240	39.20	2710	47.43
25	2470	43.23	3050	53.38
26	3080	53.90	3340	58.45
27	2440	42.70	3190	55.83
28	2800	49.00	2930	51.28
29	2320	40.60	2760	48.30
30	2310	40.43	2260	39.55
31	3150	55.13	3420	59.85
32	2890	50.58	3140	54.95
33	2930	51.28	3370	58.98
34	2600	45.50	3250	56.88
35	2290	40.08	1870	32.73
36	2400	42.00	3960	69.30
37	2660	46.55	3400	59.50
38	3080	53.90	3440	60.20
39	3610	63.18	3480	60.90
40	3460	60.55	3200	56.00
41	2680	46.90	3540	61.95
42	2640	46.20	3270	57.23
43	3500	61.25	3180	55.65
44	2120	37.10	2380	41.65
45	3120	54.60	3090	54.08
46	2970	51.98	2810	49.18
47	2330	40.78	3000	52.50
48	2690	47.08	3550	62.13
49	3120	54.60	2910	50.93
50	2380	41.65	2960	51.80
σ_{mean}, [MPa]		47.9		54.3
Variation coefficient		13.5%		14.3%
$\sigma_{max,}$ [MPa]		61.3		69.3
σ_{min}, [MPa]		31.9		32.7

For the test results given in the tables, Weibull distributions of the bending strength are done (Fig. 4). An analysis of the results shows that both types of distribution are characterized by a significant deviation from the uni-modal distribution, especially in the area of low bending strength values. These low values of bending strength are critical for the bearing capacity of glass elements.

To increase the accuracy of the analytical approximation of these results, the following approach is proposed. It is assumed that the reason for the deviation of the lower values from the uni-modal distribution is a specific nature of the largest defects formation in the manufacturing of glass elements. In this regard, the parameters of their statistical distribution can fundamentally differ from the parameters characteristic for the majority of the experimental results.



a) b)

Fig.4 Experimental Weibull plots for bending strength distribution a) the plates in the horizontal position, b) the plates in the vertical position

It is proposed to use piecewise-linear approximation of this kind of multimodal dependencies by separating a group of lower values deviating from the uni-modal distribution. Thus, we realize partitioning of the data based on appear slope change. Determining the strength at a given level of probability, we can compare the values obtained for the lower branch of the graph with ones when using the uni-modal distribution. A significant difference between these values will indicate the need to apply the proposed approach. The offered approach is realized on an example of processing of experimental results using Weibull distribution of bending strength for the specimens tested in vertical position (Fig. 5).

In the coordinate system y0z, for the Weibull distribution we write the equation of the straight line (Dobson 2006):

$$z = \alpha y - \beta \quad (1)$$

where

$z = ln(-1(ln - F(x))), F(x)$ - Weibull distribution function;

y = lnx, x – a certain value taken by the random quantity X (the value of the bending strength);

α – form parameter;

β = αlnλ, λ – scale parameter.

Equation (1) is the equation of a straight line and the estimation of the parameters α and λ can be done according to the least square procedure. Ordinate is defined as:

$$F = \frac{i - 0.3}{N + 0.4} \quad (2)$$

where

i - – the sequence number of the value, sorting in ascending order of the sample of random quantity;

N – total number of sample elements.

Approximation by the line z = α y - β according to the least square procedure for the whole sample, taking into account the fact that we plot the values of the decimal logarithm lgx along the abscissa, gives the equation:

$$z = 17.7 \lg x - 30 \quad (3)$$

Then the value of guaranteed glass strength at the probability of fracture 0.001%, determined on the basis of equation (3), is equal 15MPa. This value of probability is accepted in the design of the critical load bearing architectural structures with increased reliability.

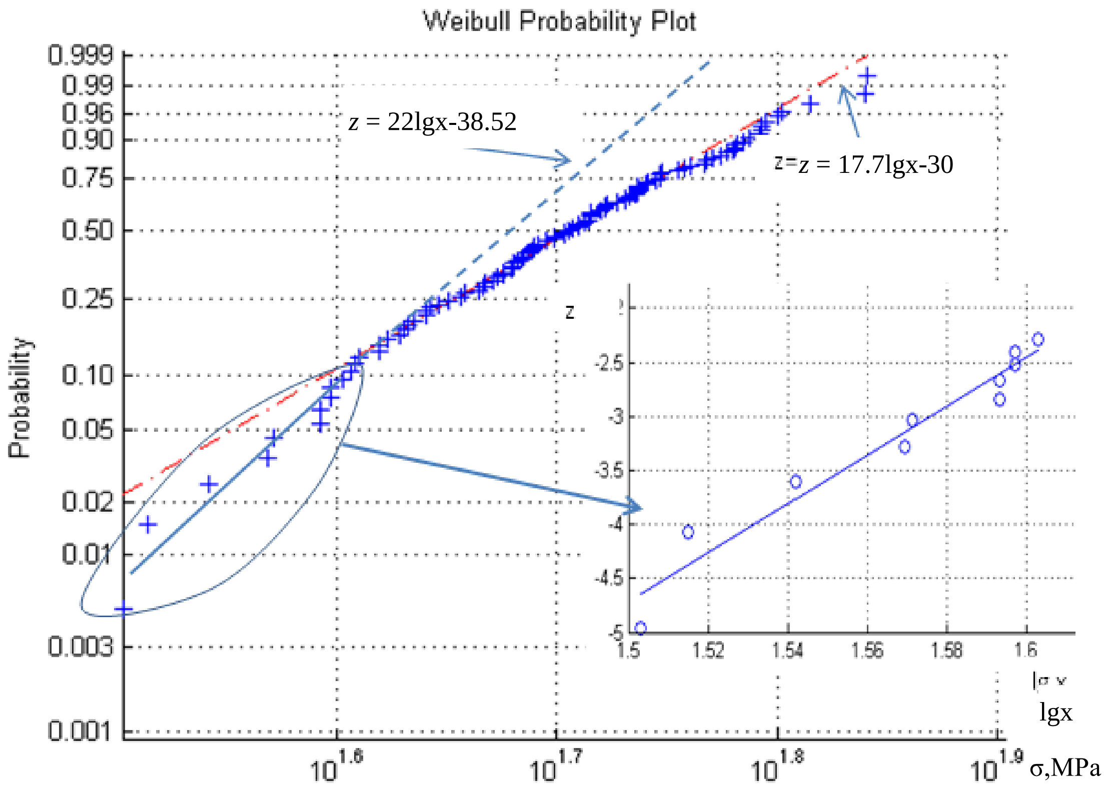


Fig.5 Bi-modal Weibull approximation of the experimental bending tests results

Approximation by the line z = α y - β the lower values (Fig. 5) gives the equation:

$$z = 20.0\lg x - 38.5 \tag{4}$$

The guaranteed ultimate strength based on this equation with a fracture probability of 0.001% is $\sigma_{0.0001}$ = 21.5 MPa. Using equation (4), extrapolation of the curve for minimum strength values to the probability of failure of 0.99 (dashed line) was also carried out. This straight line characterizes influence of very large defects with the specific nature, associated with a certain part of mechanical treatment and possible major defects of a different origin. Analysis of the dependence (4) allows obtaining a range of strength values characteristic for this type of processing, which is 21.5 ... 48 MPa. This kind of Weibull curve may occur in the case of treating the entire surface of the specimen by a similar treatment. The upper limit of the analytically obtained strength range is significantly lower (by 30%) than the maximum values of glass strength caused by the technology of glass formation on the float line.

Thus, suggested approach shows the possibility of increasing the accuracy of the guaranteed level of strength at a low fracture probability, and also allows separately assessing the influence of different production stages on the bearing capacity of the product.

For the assessment of the possible level and nature of the statistical distribution of higher strength values, two samples were analyzed (Fig. 6). These samples were obtained by excluding from the original total sample of 100pcs consistently 10 and 20 minimum values. It is 10 and 20 percent of the volume of the original sample, respectively. Such excluding can correspond to improving of mechanical treatment and/or other methods for reducing defects, for example the 100% testing of crucial parts with abandoning ones with larger defects.

Based on the analysis of the obtained results, it was shown that excluding from the sample minimum values does not lead to the uni-modal distribution (Fig. 6). The group of values that form the lower branch of a bimodal distribution appears again.

A piecewise linear approximation shows that the value of the guaranteed strength for a given low fracture probability obtained from the equation of a straight line constructed on the lower values is higher than in the previous case and corresponds to less gross defects. Thus, for a sample with 10 excluded minimal values of strength, corresponding to the grossest defects (Fig. 6a) the lower branch is approximated by the equation:

$$z = 76.1 \lg x - 126.5 \quad (5)$$

The guaranteed ultimate strength value, determined on the basis of this equation, with the failure probability 0.001% increased to $\sigma_{0.0001}$ = 37 MPa.

The subsequent removal of 10 more minimal defects and the approximation of the lower branch of the obtained sample lead to the equation:

$$z = 71.0 \lg x - 120.9 \quad (6)$$

The guaranteed value of bending strength at a fracture probability of 0.001%, determined on the basis of (6) – $\sigma_{0.0001}$ =39 MPa.

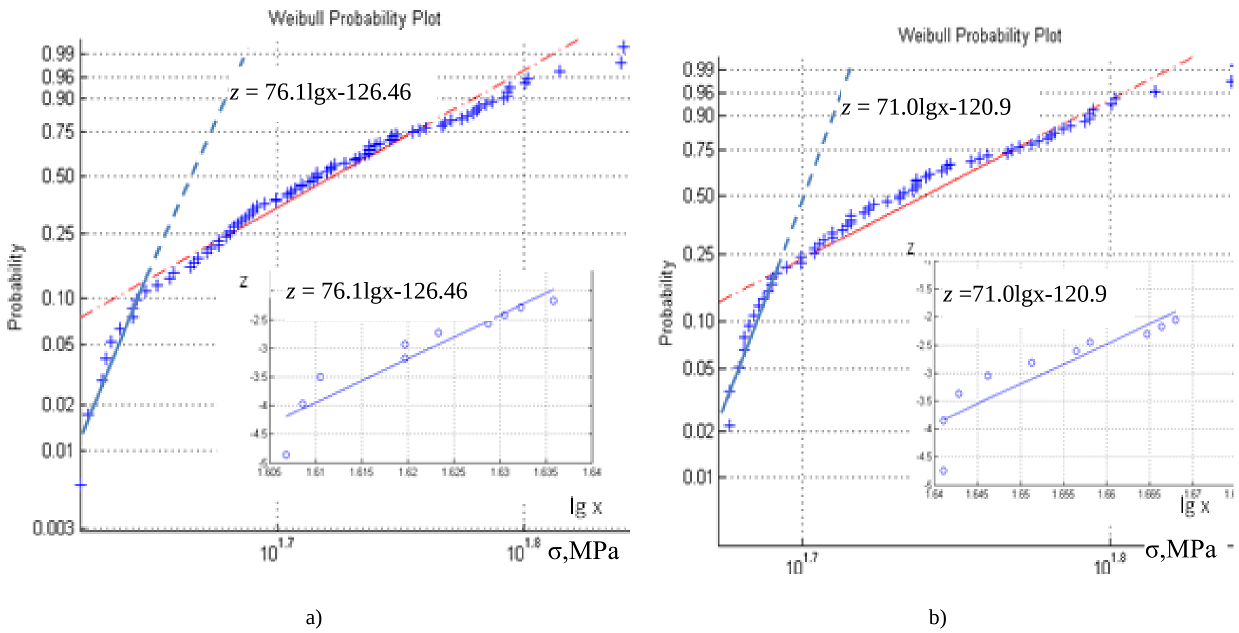


Fig.6 Weibull approximation of the experimental bending tests results after excluding from the original total sample of 100pcs a)10 and b) 20 minimum values

Distribution in Figure 6, b corresponds to the excluding of 20 lower strength values from the total sample. Its analysis shows that the increase in the guaranteed level of strength from $\sigma_{0.0001}$ = 37 MPa to $\sigma_{0.0001}$ = 39 MPa is no longer significant. It is due to the fact that the remaining defectiveness is largely determined by manufacturing process on the float line. This is also confirmed by the constructed Weibull distribution functions (Fig. 7) for the form and scale parameters obtained from equations (3) - (6).

It was concluded that uni-modal approximation for statistical distribution of bending strength causes insufficient accuracy in assessment the guaranteed strength level of glass for a given low probability of failure. The assessment of strength level at the failure probability 0.001% (taken in statistical strength analysis of glass building structures), showed that the error, when the uni-modal Weibull law is used, is up to 30% .

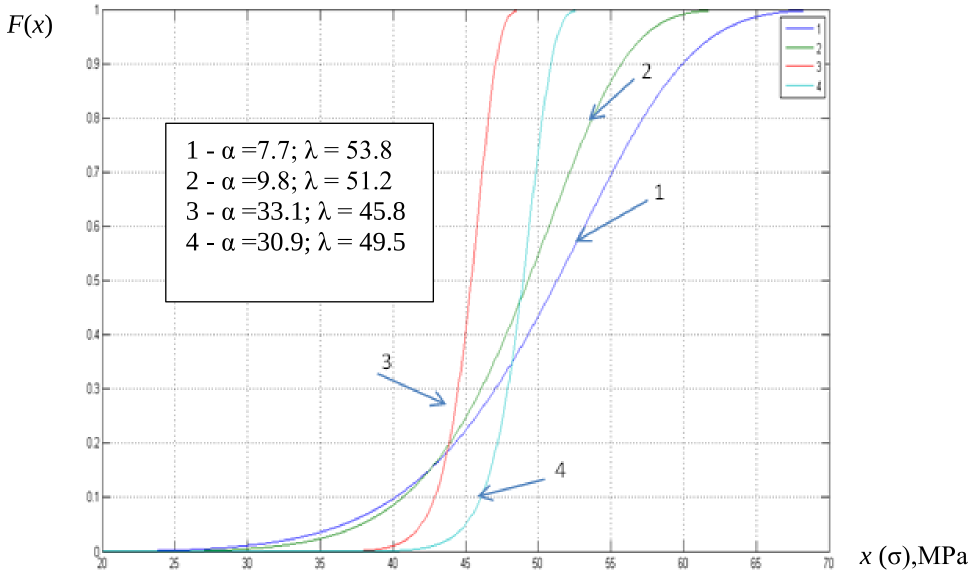


Fig.7 The Weibull distribution function F(x) for the form parameter α and the scale parameter λ determined from equations (3) - (6): (3) - curve 1; (4) - curve 2; (5) - curve 3, (6) - curve 4

It is also clear that a reduction of glass defectiveness due to enhancement of technological process in glass manufacture and treatment can give not only an increase in the average strength values, but also a noticeable increase in the guaranteed level of strength at a given low probability of fracture.

Conclusion

- Statistical analysis of the experimental results of float glass bending strength showed that applying of the uni-modal Weibull distribution leads to significant errors in the guaranteed level of glass strength at a low probability of fracture (up to 0.001%).
- The use of the proposed analytical approach of partitioning the data based on appear slope change and the analytical description of the lower branch of experimental data allowed significantly improve the value of the guaranteed strength level at a given low probability of the fracture, as well as to establish a range of strength values for explored type of glass processing.
- Statistical analysis of the distribution curves made it possible to identify groups of defects, the technological removal of which would ensure an increase in the guaranteed level of strength.
- The results the basis for solving optimization problems when, depending on the glass element sphere of application, you need to get a guaranteed level of strength for a given probability of failure with minimal costs of glass manufacture and treatment.

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