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# Quantum simulation of a Fermi-Hubbard model using a semiconductor quantum dot array

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**Interacting fermions on a lattice can develop strong quantum correlations, which lie at the heart of the classical intractability of many exotic phases of matter<sup>1-3</sup>. Seminal efforts are underway in the control of artificial quantum systems, that can be made to emulate the underlying Fermi-Hubbard models<sup>4-6</sup>. Electrostatically confined conduction band electrons define interacting quantum coherent spin and charge degrees of freedom that allow all-electrical pure-state initialisation and readily adhere to an engineerable Fermi-Hubbard Hamiltonian<sup>7-17</sup>. Until now, however, the substantial electrostatic disorder inherent to solid state has made attempts at emulating Fermi-Hubbard physics on solid-state platforms few and far between<sup>18,19</sup>. Here, we show that for gate-defined quantum dots, this disorder can**

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20 **be suppressed in a controlled manner. Novel insights and a newly developed semi-automated**  
21 **and scalable toolbox allow us to homogeneously and independently dial in the electron filling**  
22 **and nearest-neighbour tunnel coupling. Bringing these ideas and tools to fruition, we real-**  
23 **ize the first detailed characterization of the collective Coulomb blockade transition<sup>20</sup>, which**  
24 **is the finite-size analogue of the interaction-driven Mott metal-to-insulator transition<sup>1</sup>. As**  
25 **automation and device fabrication of semiconductor quantum dots continue to improve, the**  
26 **ideas presented here show how quantum dots can be used to investigate the physics of ever**  
27 **more complex many-body states.**

28       The potential for realizing novel electronic and magnetic properties of correlated-electron  
29 phases in low-dimensional condensed-matter physics, in topics ranging from high- $T_c$  supercon-  
30 ductivity to electronic spin liquids<sup>1-3</sup>, has prompted quantum simulation efforts across multiple  
31 platforms<sup>4-6,18,19,21,22</sup>. Theoretical and proof-of-principle experimental work has shown how emer-  
32 gent spin physics<sup>21</sup> and two-site Mott physics<sup>22</sup> can be simulated on programmable quantum com-  
33 puting platforms. These digital quantum simulation efforts promise universality, but come at the  
34 cost of requiring large numbers of highly-controlled quantum bits with additional error-correction  
35 overhead. Analog quantum simulation efforts, on the other hand, aim to directly implement well-  
36 defined Hamiltonians. Such emulators are typically limited by the residual entropy of the initial-  
37 ized system, restricting experimental correlations in span and strength<sup>6</sup>. Furthermore, scaling to  
38 sufficiently homogeneous systems of larger size is not always straightforward<sup>4-6,19</sup>.

39       Semiconductor quantum dots form a scalable platform that is naturally described by a Fermi-

40 Hubbard model in the low-temperature, strong-interaction regime, when cooled down to dilution  
41 temperatures<sup>7-10</sup>. As such, pure state initialization of highly-entangled states is possible even with-  
42 out the use of adiabatic initialization schemes<sup>23</sup>. Coherent evolution of excitations can span many  
43 sites, as, contrary to what might be expected,  $> 20$  coherent oscillations in charge or spin can be  
44 observed on adjacent sites<sup>13-15</sup>. Furthermore, local control and read-out of both charge and spin  
45 degrees of freedom have become matured areas of research, given the large ongoing effort of using  
46 quantum dots as a platform for quantum information processing<sup>11-17</sup>. In particular, excellent con-  
47 trol of small on-site energy differences<sup>24</sup> or tunnel couplings<sup>14,15</sup> has been shown at specific values  
48 of electron filling and tuning.

49       Quantum simulation experiments can leverage many of these developments, trading off some  
50 of the experimental difficulties involved in full coherent control for ease of scaling. Until now,  
51 however, calibration routines for quantum dots have been quite inefficient and limited in scope. As  
52 such, the effective control of larger parameter spaces as well as the calibration of larger samples  
53 seem like insurmountable obstacles. What has been lacking, thus, is an efficient and scalable con-  
54 trol paradigm for Hamiltonian engineering that extends to the collective Fermi-Hubbard parameter  
55 regimes well beyond those required for qubit operation<sup>25,26</sup>.

56       In this Letter, we demonstrate the simulation of Fermi-Hubbard physics using semiconduc-  
57 tor quantum dots. We describe an experimental toolbox, validated by direct numerical simulations,  
58 that allows for the independent tuning of filling and tunnel coupling as well as the measurement  
59 of all interaction energies, and employ it to map out the accessible parameter space of a triple

60 quantum dot device with unprecedented detail and precision. As the tunnel couplings are homo-  
 61 geneously increased, we witness the delocalization transition between isolated Coulomb blockade  
 62 and collective Coulomb blockade, the finite-size analogue of the interaction-driven Mott transition.

63 The one-dimensional quantum dot array is electrostatically defined using voltages applied to  
 64 gate electrodes fabricated on the surface of a GaAs/AlGaAs heterostructure (Fig. 1), that selec-  
 65 tively deplete regions of the 85-nm-deep two-dimensional electron gas (2DEG) underneath. The  
 66 outermost dots can be (un)loaded from Fermi reservoirs on the sides, which have an effective  
 67 electron temperature of 70-75 mK (6.0-6.5  $\mu\text{eV}$ ). The three gates at the top are used to define a  
 68 sensing-dot channel, the conductance of which is sensitive to changes in the charge state of the  
 69 array and is directly read out using radio-frequency reflectometry.

70 The control of Fermi-Hubbard model parameters is achieved by modulation of the potential  
 71 landscape in the 2DEG using the seven bottom-most gate electrodes (Fig. 1). These gates come in  
 72 two flavours. Plunger gates  $P_i$  are designed to tune the single-particle energy offsets  $\epsilon_i$  of individual  
 73 dots  $i$ , allowing us to set an overall chemical potential  $\mu' = \langle \epsilon_i \rangle$  and add site-specific detuning  
 74 terms  $\delta\epsilon_i$ . Barrier gates  $B_{ij}$  allow for the modulation of tunnel couplings  $t_{ij}$  between the  $i$ th and  
 75  $j$ th dot or  $\Gamma_i$  between an outer dot  $i$  and its adjacent Fermi reservoir, respectively. The interaction  
 76 energies are determined by the potential landscape realized to achieve this set  $\{\mu', \delta\epsilon_i, t_{ij}, \Gamma_i\}$ , and  
 77 comprise of the on-site Coulomb interaction terms  $U_i$  and inter-site Coulomb interaction terms  
 78  $V_{ij}$ . With each dot filled to an even number of electrons, we can describe the addition of the  
 79 next two electrons per dot within an effective single-band extended Hubbard picture<sup>27</sup>, using site-

80 and-spin-specific electronic creation and annihilation operators  $c_{i\sigma}^\dagger$  and  $c_{i\sigma}$  and dot occupations

81  $n_i = \sum_{\sigma} c_{i\sigma}^\dagger c_{i\sigma}$ :

$$H = - \sum_i \epsilon_i n_i - \sum_{\langle i,j \rangle, \sigma} t_{ij} (c_{i\sigma}^\dagger c_{j\sigma} + \text{h.c.}) + \sum_i \frac{U_i}{2} n_i (n_i - 1) + \sum_{i,j} V_{ij} n_i n_j. \quad (1)$$

82 In practice, both  $P_i$  and  $B_{ij}$  gates exhibit cross-talk to all the  $\epsilon_i$  and  $t_{ij}$  (with smaller effects  
 83 on  $U_i$  and  $V_{ij}$ ), and in addition must compensate for initial disorder. Setting Hamiltonian parame-  
 84 ters experimentally therefore requires carefully chosen linear combinations of gate voltages. This  
 85 idea is employed regularly in spin qubit experiments in order to change the on-site energies  $\epsilon_i$  de-  
 86 terministically over small ranges<sup>24</sup>, but here we go further in important ways. Our experimental  
 87 toolbox uses linear combinations of gate voltage changes  $\{P_i, B_{ij}\}$  for the independent control of  
 88 the Fermi-Hubbard parameters  $\{\mu', \delta\epsilon_i, t_{ij}\}$  to within several  $k_B T$  and over a wide range of fillings  
 89 and tunnel couplings.

90 Fig. 2a-b shows the filling of the array with up to  $N = 9$  electrons, three electrons per dot,  
 91 while keeping the inter-dot tunneling terms small ( $t_{ij} < V_{ij} < U_i$ ) and the tunnel couplings to the  
 92 reservoirs roughly constant. The dark lines arise from steps in the charge detector conductance,  
 93 indicating a transition in the number of electrons on one of the dots. The horizontal and diagonal  
 94 lines indicate filling of one of the dots from the reservoir, whereas the vertical (polarization) lines  
 95 indicate electron transitions between sites (not seen in Fig. 2b which shows only changes in  $N$ ).  
 96 To achieve this level of control required several new insights. As a start, we measure the cross-

107 talk between the seven gate voltages and the three dot detunings at multiple points in gate space,  
108 allowing for the direct definition of virtual  $\delta\epsilon_i$  gates that are accurate over a range of several meV  
109 (see Methods and Extended Data Fig. 1). Furthermore, it allows us to define virtual barrier gates  
110 that change specific tunnel couplings while keeping all dot detunings constant. In addition, we  
111 achieve homogeneous filling of a quantum dot array (as in Fig. 2a) through non-homogeneous  
112 changes in the  $\epsilon_i$ , as the dots have to each overcome a different sum of local interaction energies  
113  $U_i + \sum_{i \neq j} V_{ij}$ . This is a consequence of the finite size of the array (only the middle dot has two  
114 neighbours) and the inhomogeneity in interaction terms (see Methods and Extended Data Fig. 2-3).  
115 Finally, as multiple electrons are added to the array, we use the virtual barrier gates described above  
116 to counter the effect that changing plunger gate voltages (and the higher wave function overlap of  
117 higher electron fillings) have on the tunnel couplings.

108 Having filled the array with a given number of electrons, we can quantitatively characterize  
109 the various parameters in the Fermi-Hubbard model directly from relevant feature sizes in the  
110 charge stability diagram as we detune away from uniform filling. The spacing between charge  
111 addition lines of half-filled dot levels yields the on-site Coulomb interaction term  $U_i$ , whereas the  
112 displacement of single charge addition lines upon filling another dot yields their inter-site Coulomb  
113 coupling  $V_{ij}$  (see Fig. 2c and Methods for automation and protocols). Finally, we can extract the  
114 interdot tunnel coupling  $t_{ij}$  at transitions where an added electron moves between adjacent sites  
115  $i$  and  $j$  (the polarization lines seen in Fig. 2a). The width of such transitions is determined by  
116 the hybridization of the charge states on the two sites and is thus a measure of tunnel coupling.  
117 We implement an iterative tuning process that allows for automated repeated measurements of the

118 polarization line width with changing virtual barrier gates and thus tunnel coupling. To account  
119 for the only remaining cross-talk, between each virtual barrier gate and the other tunnel coupling,  
120 we redefine the virtual barrier gates such that they influence their local tunnel coupling only, while  
121 keeping all other parameters constant (see Fig. 2d and Extended Data Fig. 4).

122 We showcase the potential of well-controlled quantum dot arrays to emulate Fermi-Hubbard  
123 physics by employing this newly developed toolbox for the realization of collective Coulomb  
124 blockade (CCB) physics, validating the results through direct numerical Fermi-Hubbard model  
125 calculations. Coulomb blockade (CB) is a purely classical effect that arises from the finite charg-  
126 ing energies of each individual quantum dot, where the charge excitations at half filling are gapped  
127 out, analogous to the Mott gap. When quantum tunneling effects between sites are turned on,  
128 however, a much richer phase diagram appears. The CB of individual dots is destroyed as the  
129 degeneracy of the peaks in the equilibrium charge addition spectrum is lifted and broadened into  
130 minibands, giving way to collective Coulomb blockade<sup>20</sup> (see Fig. 3a and Extended Data Fig. 5  
131 for simulated data of a simplified model). As tunnel couplings continue to increase relative to local  
132 charging energies this gap will vanish in the thermodynamic limit, giving rise to a metallic state.  
133 The CCB physics is best described by the equilibrium electron addition spectrum as a function  
134 of filling and tunnel coupling, the two main experimental control parameters of the quantum dot  
135 array.

136 The experimental phase diagram is mapped out by the independent control over electron fill-  
137 ing and tunnel coupling strength over as large a range as possible (Methods). It is constructed con-



138 tinuously by linear interpolation of gate values in between 3 to 12 calibrated points per miniband  
139 (Fig. 3b) where the on-site energies and tunnel couplings are well calibrated and the interaction  
140 energies measured (see Extended Data Fig. 6). At low tunnel coupling, the miniband has a finite  
141 width due to residual  $V_{ij}$ . The main effect of increased nearest-neighbour tunnel coupling on the  
142 addition spectrum is a widening of the minibands at the expense of the collective gap at uniform  
143 filling, analogous to the reduction of the Mott gap with increasing tunnel coupling. Along with tun-  
144 nel coupling, also the inter-site Coulomb coupling  $V_{ij}$  increases (see Extended Data Fig. 6). The  
145 gap between minibands continues to decrease with increasing tunnel coupling, but will be prohib-  
146 ited from closing completely by the charging energy of what has essentially become one large dot,  
147 a quantity inversely proportional to its large but finite total capacitance. The low and high tunnel  
148 coupling regimes are also clearly distinguished in transport measurements through the quantum  
149 dot array and in charge stability diagrams (see Extended Data Fig. 7). To test the validity of our  
150 approach, we perform numerical calculations of the addition spectrum within each band based on  
151 Eq. (1) and using experimental parameters that are either calibrated or measured (see Methods  
152 and Extended Data Table 1-2). The agreement between measurement and numerical calculation in  
153 Fig. 3b indeed validates the use of experimental tools for Hamiltonian engineering over the entire  
154 measured diagram.

155 Putting these results in perspective, we are able to calibrate and characterize site-specific  
156 quantum dot parameters up to values of tunnel coupling reaching  $U/t = 7.1(4)$ . The large en-  
157 ergy scales obtained compared to temperature,  $t/k_B T = 54(5)$ , give access to the regime where  
158 quantum correlations are strong<sup>1-3</sup>. Extending this work to larger quantum dot arrays, whether

159 for the purpose of analog quantum simulation or quantum computation, requires further automa-  
160 tion of our methods<sup>28</sup>, and extensions to parallelize the calibration routines. Scalable gate layouts  
161 for 1D arrays already exist<sup>29</sup>, which together with the programmable disorders in on-site ener-  
162 gies, can be mapped onto the physics of many-body localization<sup>30</sup>. Further advances in connec-  
163 tivity and homogeneity are underway in the pursuit of scalable quantum computing, including  
164 square<sup>31</sup> and triangular<sup>32</sup> geometries, industrial-grade fabrication processes and magnetically quiet  
165 <sup>28</sup>Si substrates<sup>33</sup>, that open up further possibilities for quantum simulation experiments with quan-  
166 tum dots.

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244 C.R. and W.W. grew the heterostructure, X.L. and S.D.S. performed the theoretical analyses with  
245 X.L. carrying out the numerical simulations, T.H., T.F., X.L., L.J., S.D.S. and L.M.K.V. contributed  
246 to the interpretation of the data, and T.H. wrote the manuscript (X.L. wrote part of the Methods),  
247 with comments from T.F., X.L., S.D.S. and L.M.K.V.

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249 The authors declare no competing financial interests. Correspondence and requests for materials  
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251 **Figure 1 | Gate-defined quantum dot array as a platform for quantum simulations of the**

252 **Fermi-Hubbard model**

253 Electron micrograph of a sample nominally identical to the one used for the measurements. The  
254 bottom three circles indicate the triple dot array, whose Hamiltonian parameters derive from the  
255 local potential landscape controlled by the seven bottom-most gates ( $B_{1L}$  to  $B_{3R}$ ). The top circle  
256 and arrow indicate the sensing dot channel, the radio-frequency reflectance of which is monitored  
257 to enable real-time charge sensing. Crossed squares indicate distinct Fermi reservoirs that are  
258 contacted using ohmic contacts. We describe a toolbox that allows for the control of the quantum  
259 dot array at the level of the microscopic Fermi-Hubbard model. In particular, it allows for the  
260 independent calibration of  $\{\mu', \delta\epsilon_i, t_{ij}\}$  and the measurement of the Coulomb interaction terms  
261  $\{V_{ij}, U_i\}$ . Measurable observables for quantum dots include both local charge occupation and  
262 global charge transport as well as local spin degrees of freedom and nearest-neighbour singlet-  
263 triplet spin correlations (through spin-to-charge conversion protocols<sup>11,16,17</sup>).



264 **Figure 2 | Hamiltonian engineering using a scalable toolbox of local control and measure-**  
265 **ments**

266 **a** Charge stability diagram showing uniform filling of the array of up to three electrons per dot in  
267 the vertical direction, using a combination of all seven gates (only  $P_1$  values are shown) that equally  
268 sweeps the local fillings  $n_i$  while keeping the tunnel couplings between dots and to the reservoirs  
269 nominally identical. Lines correspond to charge transitions. **b** Theoretical charge stability diagram  
270 of a triple-quantum-dot system in the classical limit ( $t = 0$ ) exchanging particles with a reservoir  
271 at  $U/k_B T = 300$ , analogous to the measurement in **a**. **c** As we focus on relevant sections of the  
272 charge-stability diagram of the array, we calibrate all relative cross-capacitances of the seven-gate,  
273 three dot-system, allowing for deterministic changes in  $\epsilon_i$  and subsequent measurement of on-site  
274 and inter-site Coulomb couplings. **d** Measurements of both tunnel couplings as a function of two  
275 linear combinations of gate voltages,  $VB_{12}$  and  $VB_{23}$ , that keep either  $t_{23}$  or  $t_{12}$  (the full line de-  
276 notes the average value) as well as the three on-site energies  $\epsilon_i$  constant whilst increasing  $t_{12}$  or  
277  $t_{23}$ , respectively (an exponential fit to  $\alpha \exp(VB_{ij}/\beta)$  is shown). Individual tunnel coupling data  
278 points are taken at a rate of roughly 1 Hz and have typical fitting errors of several per cent (not  
279 shown). Text in brackets denote the dominant charge states in the many-body eigenstate.

280 **Figure 3 | Collective Coulomb blockade physics in the Fermi-Hubbard phase space**

281 **a** Schematic representation of the charge addition spectrum of a Mott insulator at half filling and  
282 a triple quantum dot array in Coulomb blockade (bottom) and those of a metallic phase at half  
283 filling and a triple quantum dot array in collective Coulomb blockade (top). **b** The experimen-  
284 tally accessible parameter space of the Fermi-Hubbard model for a triple quantum dot array as  
285 a function of electron filling and nearest-neighbour tunnel coupling. Continuous charge sensing  
286 measurements following the charging lines are shown, at calibrated gate values where the dots are  
287 filled homogeneously (only  $\epsilon_3$  values are shown) and the  $t_{ij}$ 's are set to be roughly equal. Plotted  
288 spacings between the bands are set by the Coulomb interaction terms measured at small tunnel  
289 coupling. Red circles indicate extended Hubbard model calculations of the transitions. In the ver-  
290 tical direction, they are set using the same measured  $t_{avg} = (t_{12} + t_{23})/2$  as the experimental data.  
291 In the horizontal direction, the simulations start from measured interaction energies with  $\sim 10\%$   
292 errors (see Methods, Extended Data Fig. 6 and Extended Data Table 1-2). Text in brackets denotes  
293 electron filling.

## 294 **Methods**

295 **Materials and set-up** The triple quantum dot sample was fabricated on a GaAs/Al<sub>0.25</sub>Ga<sub>0.75</sub>As  
296 heterostructure that was grown by molecular-beam epitaxy. The 85-nm-deep 2D electron gas has  
297 an electron density of  $2.0 \times 10^{11} \text{ cm}^{-2}$  and 4 K mobility of  $5.6 \times 10^6 \text{ cm}^2\text{V}^{-1}\text{s}^{-1}$ . All sample  
298 structures were defined using electron-beam lithography, with metallic gates (Ti/Au) and ohmic  
299 contacts (Ni/AuGe/Ni) deposited on the bare wafer in a lift-off process using electron-beam evap-  
300 oration, similarly to the definition of metallic markers, leads and bonding pads, and with sample  
301 mesas defined using a diluted Piranha wet etch. The plunger gates were connected to bias-tees on  
302 the printed circuit board, allowing for fast sweeps and RF excitations to be applied in addition to  
303 DC voltages. RF reflectometry<sup>34</sup> of the sensing dot channel conductance is done at 110.35 MHz  
304 employing a homebuilt LC circuit on the printed circuit board. The sample was cooled down in  
305 an Oxford Kelvinox 400HA dilution refrigerator to a base temperature of 45mK whilst applying  
306 positive bias voltages to all gates. With the sample cold and the dots formed through application  
307 of appropriate voltages to the metallic gates, read-out was performed by feeding the RF reflec-  
308 tometry circuit a roughly -99 dBm carrier wave, the reflected signal of which is amplified at 4 K  
309 and subsequently demodulated and measured using custom electronics. Using this technique on a  
310 sensing dot is preferred to forming a quantum point contact, and yields measurement bandwidths  
311 exceeding 1 MHz. The sensing dot position is asymmetric in order to obtain different sensitivities  
312 to each of the three dots. Note that as an alternative to electrostatically defined charge sensors in  
313 the 2DEG itself, dispersive read-out using the nanofabricated top gates would allow to measure  
314 how much charges move in response to gate voltage changes<sup>35</sup>. For more detailed methods please

315 see Baart *et. al.*<sup>16</sup>.

316 **Eliminating cross-talk through the definition of virtual gates** Changes in  $\epsilon_i$  can be tracked di-  
317 rectly by following transitions in the charge stability diagram and are found to depend linearly  
318 on gate values for voltage changes up to several tens of millivolts. In general, small changes in  
319 the energy offsets of each of the three dots will thus be achieved via a linear combination of voltage  
320 changes on each of the seven gates:  $\delta \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \end{pmatrix} = \begin{pmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{14} & \alpha_{15} & \alpha_{16} & \alpha_{17} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} & \alpha_{24} & \alpha_{25} & \alpha_{26} & \alpha_{27} \\ \alpha_{31} & \alpha_{31} & \alpha_{33} & \alpha_{34} & \alpha_{35} & \alpha_{36} & \alpha_{37} \end{pmatrix} \delta (P_1 P_2 P_3 B_{1L} B_{12} B_{23} B_{3R})^T$ .  
321 Of these 21 matrix elements, the three  $\alpha_{ii}$ 's describe the coupling of the plungers  $P_i$  to the energy  
322 offset  $\epsilon_i$  of their respective dot  $i$ . The other 18 elements are cross-talks, whose values can eas-  
323 ily be related to the  $\alpha_{ii}$ 's through the slope of charge addition lines (see Extended Data Fig. 1a).  
324 This leaves the relative weights of the  $\alpha_{ii}$ 's and the absolute value of one of the elements to be  
325 determined. As the difference between the single-particle energies of two dots stays fixed along a  
326 polarization line, we can determine the relative weights from the slope of these lines (see Extended  
327 Data Fig. 1b). The absolute value of  $\alpha_{22}$  can be found using photon-assisted tunneling measure-  
328 ments (see Extended Data Fig. 4). For the measurements presented in Fig. 3b, the matrix has been  
329 measured multiple times for different fillings and tunnel couplings: the 'plunger' side  $\alpha_{11}$ - $\alpha_{33}$  of  
330 the matrix was measured 25 times in total and the 'barrier' part  $\alpha_{14}$ - $\alpha_{37}$  12 times (see Extended  
331 Data Fig. 1c). In between these points, we used linear interpolation as function of measured tunnel  
332 coupling to extract matrix elements when needed.

333 With all matrix elements known, the  $\epsilon_i$ 's can be deterministically changed, a technique which is  
334 extensively used throughout the results presented here in two main ways, (1) by measuring Hamil-

335 tonian parameters through direct interpretation of features in the addition spectrum and (2) through  
336 the definition of ‘virtual gates’, both for plunger and barrier gates, that greatly simplify the tuning  
337 process. For instance, the virtual gate for the energy offset of the leftmost dot,  $\epsilon_1$ , is defined by  
338 a simple combination of plunger gates:  $\delta \begin{pmatrix} P_1 \\ P_2 \\ P_3 \end{pmatrix} = \begin{pmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{pmatrix}^{-1} \begin{pmatrix} \delta\epsilon_1 \\ 0 \\ 0 \end{pmatrix}$ . To form virtual barrier  
339 gates we use  $\delta B_{12} \rightarrow \delta V B_{12} = \delta(P_1, P_2, P_3, B_{12})$  with  $\delta \begin{pmatrix} P_1 \\ P_2 \\ P_3 \end{pmatrix} = -\delta B_{12} \begin{pmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{pmatrix}^{-1} \begin{pmatrix} \alpha_{14} \\ \alpha_{24} \\ \alpha_{34} \end{pmatrix}$ ,  
340 which allows for making the barrier separating dots 1 and 2 more (or less) transparent without  
341 changing the energy offsets  $\epsilon_i$  of any of the dots, which is to say, stay at the same location in the  
342 charge stability diagram. Linear combinations of this gate and its equivalent between dots 2 and 3  
343 yield the two orthogonal control gates for changing  $t_{ij}$ , as used in Fig. 2d.

344 **Classically coupled dots and homogeneous filling** Isolated quantum dots are well described by a  
345 classical capacitance model<sup>36</sup>. This description is valid as long as tunnel coupling energies are neg-  
346 ligible compared to capacitive (Coulomb) effects. In this case, the charge states  $s$  of the system are  
347 simply described by the set of individual dot occupations  $(n_1, n_2, \dots)$  as the  $n_i$ ’s are good quantum  
348 numbers. As has been shown previously<sup>9</sup>, one can map the classical capacitance model onto the  
349 extended Hubbard model of Eq. 1 with omission of its tunneling terms, which is readily diagonal-  
350 ized with eigenenergies  $E(n_1, n_2, \dots) = -\sum_i \epsilon_i n_i + \sum_i \frac{U_i}{2} n_i (n_i - 1) + \sum_{i,j \neq i} V_{ij} n_i n_j$ . Because  
351 we experimentally probe changes in the equilibrium charge state of the array coupled to adjacent  
352 electron reservoirs, typically kept at an equal and constant electrochemical potential  $\mu$  and temper-  
353 ature  $k_B T$ , we are interested in the charge addition spectrum  $\frac{\partial \langle N \rangle}{\partial \mu}$ , with  $\langle N \rangle = k_B T \frac{\partial \ln \mathcal{Z}}{\partial \mu}$ ,  $\mathcal{Z} =$   
354  $\text{Tr}\{\exp[-(H - \mu N)/k_B T]\}$ , where  $N = \sum_i n_i$  is the total electron number and  $\mathcal{Z}$  is the grand  
355 partition function. In this classical case and at constant chemical potential  $\mu = 0$ , the equations

356 for the charge addition spectrum  $\frac{\partial \langle N \rangle}{\partial \mu} = \frac{\langle N^2 \rangle - \langle N \rangle^2}{k_B T}$  simplify to simple Boltzmann-weighted sums  
357 over the charge states  $s$ , namely  $\mathcal{Z} = \sum_s \exp[-E_s/k_B T]$  and  $\langle N^k \rangle = \frac{1}{\mathcal{Z}} \sum_s N_s^k \exp[-E_s/k_B T]$ .  
358 Note that for the purpose of finding the charge transitions, any spin-degeneracy of the charge states  
359 can be ignored. The charge stability measurements shown in the main text effectively show two-  
360 dimensional slices of the charge addition spectrum as a function of changes in the  $\epsilon_i$ 's.

361 The filling of the quantum dot array is controlled experimentally by changing the energy difference  
362 between the electronic states at the Fermi level of the reservoir and those of the dot array itself.  
363 The former can be done by applying a bias voltage to the relevant Fermi reservoir, the latter by  
364 applying voltages to top gates that influence the single-particle energies  $\epsilon_i$  on the dots. Because  
365 the partition function is only sensitive to changes in  $H - \mu N$ , one can equivalently think about  
366 changes in the  $\epsilon_i$ 's as influencing the chemical potential directly through  $\delta(\mu N) = \delta(\sum_i \epsilon_i n_i)$ ,  
367 which at uniform filling, simplifies to  $\delta\mu = \langle \delta\epsilon_i \rangle$ . This allows for a different look at the gate  
368 control over a quantum dot array with  $M$  sites. Instead of thinking about  $M$  different  $\epsilon_i$ 's, we can  
369 define one global chemical potential term  $\mu' = \langle \epsilon_i \rangle$  and  $M - 1$  energy differences  $\delta_i = \epsilon_i - \mu'$ ,  
370 where the latter describe the setting of some (controllable) disorder potential landscape at a fixed  
371 chemical potential  $\mu'$ .

372 In the case of a large and homogeneous system, changing all  $\epsilon_i$  equally would uniformly and  
373 homogeneously fill all dots in the system. For the triple-quantum-dot sample described in the main  
374 text, however, both the finite size (e.g. only one of the three dots has two direct neighbours) and  
375 inhomogeneous interaction terms (e.g.  $U_1 \neq U_2$ ) mean a different approach is needed: we have to

376 link up a set of well-defined points in the  $(\epsilon_1, \epsilon_2, \epsilon_3)$ -space. In the case of  $V_{ij} = 0$ , and focussing  
 377 on the regime from 0 to 2 electrons per site, the only obvious choice would be to identify and align  
 378 points A (where the eight charge states (000) to (111) are degenerate) and point B (where (111) to  
 379 (222) are degenerate) (see Extended Data Fig. 2a). These points are lined up by changing the on-  
 380 site single particle energies by ratio of their on-site repulsions  $\epsilon_i = \mu'U_i/\langle U \rangle$ . Analogously, under  
 381 finite  $V_{ij}$ , we use the ratio of the sum of all locally relevant interaction energies  $W_i = U_i + \sum_{j \neq i} V_{ij}$   
 382 as  $\epsilon_i = \mu'W_i/\langle W \rangle$ . Note, however, that the inter-site repulsion breaks particle-hole symmetry and  
 383 moves states with more than one particle added to a homogeneously filled state to higher energy,  
 384 meaning we can only find points with at most 4 degenerate states. We can align points C (where  
 385 (000), (100), (010) and (001) are degenerate) and D (where (111), (211), (121) and (112) are  
 386 degenerate) (see Extended Data Fig. 2b), or we can align points E (where (110), (101), (110) and  
 387 (111) are degenerate) and F (where (221), (212), (221) and (222) are degenerate) (see Extended  
 388 Data Fig. 2c), the two of which are particle-hole partners of the same total state.

389 Defining a miniband as the region in chemical potential where one uniform filling transitions to  
 390 the next one (the first miniband is thus the transition region between (000) and (111)), it becomes  
 391 clear that the inter-site Coulomb terms already widen the miniband at zero tunnel coupling. On top  
 392 of this, too large a deviation in the site-specific energy offsets  $\epsilon_i$ 's from the desired values (which  
 393 amounts to disorder in the dot energies) can also increase the miniband width. For changes in  
 394  $\delta\epsilon_1 = -\delta\epsilon_3$ , this can be seen in Fig. 2a. For changes in  $\delta\epsilon_2$ , the width remains minimized as  
 395 long as the  $\delta\epsilon_2$  remains in the window between two well-defined points denoted by the crosses and  
 396 diamonds of Extended Data Fig. 2 (see also Extended Data Fig. 3).

397 **Anti-crossing measurement and fit** Much of the day-to-day work in quantum dot arrays in general  
398 and for the measurements described here in particular consists of the interpretation of features in  
399 the charge stability diagram. In the case of well isolated dots with localized electrons ( $t/U \ll 1$ )  
400 this essentially boils down to one-dot features (parallel lines) and two-dot features (anti-crossings  
401 and associated polarization lines). Indeed, pattern recognition of anti-crossings is the crucial step  
402 in the automated initial tuning of double quantum dots<sup>28</sup>.

403 In general, the processing of a charge stability diagram (e.g. Fig. 2c) starts with finding charge  
404 transitions in the raw sensor dot data using an edge finding algorithm. The results are filtered to  
405 only leave edge sections with more than a threshold number of points. Next, we employ a k-means  
406 algorithm to cluster the edges into line sections. Depending on the data, manual input might  
407 be needed, either in the selection of relevant clusters or, sometimes, in the case of noisy data,  
408 manual selection of points. In determining on-site interaction terms  $U_i$ , calculating the orthogonal  
409 distance between two parallel lines suffices. In the case of an anti-crossing, we employ a 2D  
410 fitting routine in a rotated frame  $2 \begin{pmatrix} y \\ x \end{pmatrix} = \delta \begin{pmatrix} \epsilon_i + \epsilon_j \\ \epsilon_i - \epsilon_j \end{pmatrix} = \begin{pmatrix} -1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} \alpha_{ii} & \alpha_{ij} \\ \alpha_{ji} & \alpha_{jj} \end{pmatrix} \delta \begin{pmatrix} P_i \\ P_j \end{pmatrix}$ , simultaneously  
411 fitting both branches in a least squares sense to  $y - y_0 = \pm \left( V_{ij}/2 + \sqrt{(x - x_0)^2 + t_{ij}^2} \right)$ . Fitting  
412 parameters are three of the matrix elements (corresponding to the angles of the two dot lines and the  
413 polarization line), the two offsets  $x_0$  and  $y_0$  and the two energies  $V_{ij}$  and  $t_{ij}$ . Both the procedures to  
414 find  $U_i$  and  $V_{ij}$  are limited to  $t/U < 0.15$ , as around this value for the tunnel coupling there are no  
415 straight line sections in the charge addition diagram left where two well-defined localized charge  
416 states meet. Further discussion on this can be found with Extended Data Fig. 6.



417 **Practical limits to achievable parameter space** As can be seen in Fig. 3b, there are limits to the  
418 achievable parameter space in terms of electron filling and tunnel coupling for the device measured.  
419 This is mostly due to the gate layout, which was designed for spin qubit experiments at fillings  
420 around one electron per site and tunnel couplings up to several tens of  $\mu\text{eV}$  (red shaded area  
421 in Fig. 3b). The chosen lithographic separation between the dots does not allow for sufficient  
422 wavefunction overlap between singly-occupied sites to achieve much larger tunnel couplings. With  
423 multiple electrons per dot, however, the wavefunctions are more extended and much larger tunnel  
424 couplings are possible. Here, practical difficulties in compensating for cross-talk make it hard to  
425 reach very small tunnel couplings.

426 **Verification through Fermi-Hubbard calculations: measuring miniband width** We perform  
427 numerical simulations with two levels of detail. Extended Data Fig. 5 shows the collective  
428 Coulomb blockade transition in a simplified model to illustrate the main concepts. Results from a  
429 more detailed simulation are overlaid with the experimental data in Fig. 3b. We here elaborate on  
430 these two approaches.

431 In the simplified model calculation, we ignored the inter-site Coulomb interactions  $V_{ij}n_i n_j$ , which  
432 will split the peaks in the addition spectrum even at zero tunnel coupling, as discussed above. It is  
433 included in the detailed model. Because it is difficult to experimentally fix the absolute chemical  
434 potential over large areas of the parameter space due to nonlinearities in the gating effects, the ad-  
435 dition spectrum in Fig. 3b was constructed by plotting the middle transition within each miniband  
436 as a straight line at fixed  $\epsilon_3$ , and measuring the chemical potentials of adjacent transitions with

437 respect to those. As we can see from Extended Data Fig. 5b, such an approximation is justified at  
 438 small  $t/U (< 0.15)$ , although it neglects any change in the interaction terms with increasing tunnel  
 439 coupling. Furthermore, since the interaction parameters are non-constant over the experimental  
 440 phase space (Extended Data Fig. 6), the detailed simulations take this into account. Finally, as  
 441 also discussed above, it requires an inhomogeneous change in the site-specific energy offsets to  
 442 homogeneously fill the array. In order to allow direct comparison to the experiment, we thus have  
 443 to take the correct  $\begin{pmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \end{pmatrix}$  line to describe the filling (horizontal axis of Fig. 3b). Note that because  
 444 of the non-constant interaction energies, this vector will generally differ with miniband number  
 445 and tunnel coupling.

446 In order to find the correct filling vector and subsequently the position of the transitions, we use  
 447 the following procedure for each data set at a particular tunnel coupling and miniband number: (i)  
 448 When the system has  $N = 3n$  electrons, its ground state is tuned to be the  $(n, n, n)$  state. (ii) The  
 449 two critical points (both for  $n$  and  $n' = n + 1$ ) at which the four states  $(n, n, n)$ ,  $(n \pm 1, n, n)$ ,  
 450  $(n, n \pm 1, n)$ , and  $(n, n, n \pm 1)$  are degenerate are identified. (iii) Linking these points in the  
 451 three-dimensional parameter space spanned by  $(\epsilon_1, \epsilon_2, \epsilon_3)$  yields the filling line  $\delta \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \end{pmatrix}$ . (iv) The  
 452 three charge transitions of the miniband are subsequently found to lie somewhere on this line. (v)  
 453 This procedure yields a fixed width of the miniband, but leaves one degree of freedom unspecified,  
 454 which is the relative position of the middle dot detuning relative to the outer dots, addressed in the  
 455 next paragraph.

456 We illustrate this procedure for the data with the second largest tunnel couplings in the fourth

457 miniband in Fig. 3b in the main text, for which the following set of quantum dot parameters  
 458 applies:  $t = 0.29$ ,  $U_1 = 2.26$ ,  $U_2 = 2.70$ ,  $U_3 = 2.48$ ,  $V_{12} = 0.65$ ,  $V_{23} = 0.57$ ,  $V_{13} = 0.43$   
 459 (all in meV). First of all, it is helpful to show the ‘uniform’ chemical potential  $\mu$  that correspond  
 460 to the specific  $\epsilon_i$ ’s (a ‘global’ chemical potential  $\mu$  can be regained through  $\mu = \frac{1}{N} \sum_i \epsilon_i n_i$ ).  
 461 Such a comparison is shown in Extended Data Table 1. We can see that in the three-dimensional  
 462 parameter space the filling vector defined by  $\delta \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \end{pmatrix}$  can be very different from the one defined  
 463 by  $\delta \begin{pmatrix} \mu \\ \mu \\ \mu \end{pmatrix}$ . This shows that the distinction is important, and a simple simulation with a uniform  
 464 chemical potential as in Extended Data Fig. 5b will not compare well with the experiment. Second,  
 465 note that the simulations are done for the specific middle dot detuning denoted by the asterisk in  
 466 Extended Data Fig. 2b and Extended Data Fig. 3b, whereas the experimental detuning will be in  
 467 between that situation and the detuning denoted by the diamond in the same figures. This means  
 468 that although the total width of the miniband will be fixed, the relative position of the middle  
 469 transition between the outer transitions of each miniband (which we denote  $\alpha$  and which will be  
 470 close to 0.5) depends on the specific middle dot detuning. To overlay the simulation results on the  
 471 experimental data, we used values of  $\alpha = (0.5, 0.6, 0.65, 0.6)$  for the four minibands, respectively.  
 472 Finally, Extended Data Table 2 gives an overview for the width of the fourth miniband at different  
 473 tunnel couplings, as Fig. 3b in the main text only plots the data along the  $\epsilon_3$  direction. It can be  
 474 seen that the theory compares well with the experiment along all three directions, which further  
 475 corroborates the consistency of our measurements.

476 **Data availability statement** Source data for both main text and Extended Data figures are provided  
 477 with the paper. Raw data and analysis files supporting the findings of this study<sup>38</sup> are available from

478 <https://doi.org/10.5281/zenodo.546675>.

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491 **Extended Data Figure 1 | Gate-to-dot cross-talk**

492 **a** Cross-talk measurement of gates  $P_1$  and  $B_{12}$  on the left dot detuning. The slope of the charge  
493 transition (fit in white) yields the relative effect ( $\delta B_{12}/\delta P_1 = -\alpha_{11}/\alpha_{14}$ ) of the two gates on the  
494 single-particle energy offset  $\epsilon_1$  of the leftmost dot. Note also the nonzero background in charge  
495 sensor response we find in experiments, which is due to a direct coupling between the swept gate  
496 voltages and the sensing dot conductivity. **b** Charge stability diagram showing the anti-crossing  
497 (white) and polarization line (red) between the left and middle dot, yielding the relative effect  
498  $\alpha_{11} = \alpha_{21} + (\delta P_2/\delta P_1)(\alpha_{22} - \alpha_{12})$  of the two plungers on their respective dots. Automated edge  
499 finding and fitting procedures are outlined in Methods. **c** Measured matrix elements  $\alpha_{ij}/\alpha_{22}$  as a  
500 function of tunnel coupling. No visual distinction is made between the measured matrix elements  
501 at different electron filling. No error bars are shown, as the small uncertainty in the slope fits yields  
502 errors smaller than marker size.

503 **Extended Data Figure 2 | Simulated classical charge addition spectra**

504 **a-c** Simulated charge addition spectra (see Methods) for a triple quantum dot at zero tunnel cou-  
505 pling,  $U_2 = 1.05U_1 = 0.95U_3$  and up to two particles per dot, connected to a reservoir at  $\mu = 0$   
506 and  $k_B T = 0.02U$  ( $>10$  times larger than for the experiments described in the main text), with  
507  $V_{ij} = 0$  and  $\delta_i = 0$  (**a**) or with  $V_{12} = V_{13} = 2V_{23} = 0.2U$  and  $\delta_i = 0$  (**b**) or  $\delta_1 = \delta_3 = 0$  and  
508  $\delta_2 = U/15$  (**c**). States are denoted by charge occupation  $(n_1 n_2 n_3)$  and specific degeneracy points  
509 A-F are referred to in Methods. The relation between  $\epsilon_i$  and  $\mu'$  specified in the bottom left boxes  
510 applies to the vertical line at zero (horizontal) detuning.

511 **Extended Data Figure 3 | Miniband width and electron temperature**

512 **a** Measured charge stability diagrams of the 222-333 miniband as a function of homogeneous fill-  
513 ing (only  $P_1$  values are shown) and offset in the outer two dot energies by changing  $P_1$  and  $P_3$   
514 in opposite directions, akin to the simulations of Extended Data Fig. 2c. **b** Similar measurement  
515 as a function of the offset in the middle dot energy, controlled by  $P_2$ . The  $P_1$  values are some-  
516 what different from **a** because these measurements were taken at slightly different tunnel coupling  
517 tunings. The white diamond and asterisk indicate (roughly) the position of the same degeneracy  
518 points as shown in Extended Data Fig. 2. **c** Broadening of a charge addition line due to the finite  
519 temperature of the (rightmost) Fermi reservoir. A Fermi-Dirac fit of the transition is shown in red,  
520 which yields an effective reservoir temperature of 72(1) mK.

521 **Extended Data Figure 4 | Determining lever arm and tunnel coupling**

522 **a** Example of a photon assisted tunneling (PAT) measurement, which at low tunnel couplings is the  
523 measurement method of choice for both lever arm and tunnel coupling. Plotted is the difference in  
524 charge sensor response between applying a microwave excitation or not as a function of detuning.  
525 Dashed red line is a fit to the hybridized charge state spectrum of the double dot<sup>24</sup>. The energy  
526 difference between bonding and anti bonding states yields the minimum in frequency ( $2t$ ) and the  
527 slope away from the transition gives the lever arm between detuning voltages applied to the gates  
528 and single-particle energy difference change between the two dots. The need to generate AC exci-  
529 tations and transmit them without significant losses through coaxial cables in the fridge, however,  
530 limits the maximum tunnel frequency we can accurately determine with this method to roughly  
531 20 GHz (83  $\mu\text{eV}$ ). **b** Example of a polarization line width measurement, with fit in red. As an al-  
532 ternative to PAT, one can determine the tunnel coupling by assessing the width of the polarization  
533 line<sup>37</sup>. The excess charge (say on the left dot) transition is broadened both by an effective electron  
534 temperature and by the tunnel coupling. Charge sensor response is however not a direct measure-  
535 ment of excess charge. Not only does there exist a finite cross-talk between the gate voltages and  
536 the charge sensor response that leads to a finite slope away from the transition, we also typically  
537 find a back-effect of the excess charge on the sensing dot, leading to a different slope on either side  
538 of the transition. We fit the data with the following equation, taking this back-effect into account  
539 to first order in excess charge:  $V(\epsilon) = V_0 + \delta V Q(\epsilon) + \left[ \frac{\delta V}{\delta \epsilon} |_{Q=0} + \left( \frac{\delta V}{\delta \epsilon} |_{Q=1} - \frac{\delta V}{\delta \epsilon} |_{Q=0} \right) Q(\epsilon) \right] \epsilon$ ,  
540 where  $V(\epsilon)$  is the charge sensor response as a function of the detuning  $\epsilon = \delta(\epsilon_i - \epsilon_j)$  away from to  
541 the transition and  $V_0$ ,  $\delta V$  and  $\frac{\delta V}{\delta \epsilon}$  are the background signal, sensitivity and gate-sensor coupling,



542 respectively. Note that  $\epsilon$  is a linear combination of the swept gate voltages, taking the relevant  
543 cross-capacitances and the lever arm into account. Excess charge on the left dot is described by  
544  $Q(\epsilon) = \frac{1}{2} \left( 1 + \frac{\epsilon}{\Omega} \tanh \left( \frac{\Omega}{2k_B T_e} \right) \right)$ , with  $\Omega = \sqrt{\epsilon^2 + 4t_{ij}^2}$  and effective temperature  $k_B T_e \approx 6.5 \mu\text{eV}$   
545 (1.6 GHz). **c** Excess charge as function of detuning for three different tunnel couplings, showing  
546 that this characterization method works up to significantly larger tunnel couplings than PAT. **d**  
547 Comparison of PAT and polarization line width measurements. The data is well explained by as-  
548 suming a constant lever arm  $\alpha_{22} = 83(1) \mu\text{eV/mV}$  between gate  $P_2$  and the middle dot. Text in  
549 brackets denote relevant charge states, error bars are  $1\sigma$  fit uncertainties.

550 **Extended Data Figure 5 | Simulations of collective Coulomb blockade for the simplified**

551 **Hubbard model**

552 **a** Cartoon diagram of a triple dot system, which is a simplified version of the model used to de-  
553 scribe the experiments in the main text. Specifically, we have set a uniform tunnel coupling  $t$  and  
554 Hubbard  $U$ , while ignoring the inter-site Coulomb interaction term  $V_{ij}$ . We describe two levels per  
555 dot with a level splitting  $\Delta$  that separates the single-particle energies of the first and second orbital.  
556 Each energy level is doubly degenerate due to the spin degrees of freedom. **b** Peaks in the electron  
557 addition spectrum for the triple dot system in **a**. It is known that the classical Coulomb blockade  
558 effect arises purely from the charging effects of the quantum dots. When electron tunneling be-  
559 tween quantum dots is allowed, however, quantum fluctuations compete with the classical charging  
560 effects and give rise to a rich phase diagram, which is known as collective Coulomb blockade<sup>20</sup>.  
561 The metal-insulator transition in such a system is best captured by the charge addition spectrum,  
562 which is precisely what we measure in the experiment (Fig. 3b in the main text). The numbers in  
563 **b** indicate the average electron numbers in the system when the chemical potential resides at the  
564 respective gap. Here we use  $\Delta/U = 0.2$ , and  $k_B T/U = 0.04$  ( $>20$  times larger than for the experi-  
565 ments described in the main text). **c-f** Line cuts for the addition spectrum in **b** at different values of  
566  $t/U$ . As we discussed in the main text, there will be three different regimes in this phase diagram:  
567 at weak tunnel couplings the quantum dot states split into minibands but the isolated Coulomb  
568 blockade of each individual dot is preserved; at intermediate tunnel couplings the Coulomb block-  
569 ade of individual dots is lost, but the gap between minibands remains open; finally, in the large  
570 tunnel coupling limit the gap between minibands can become comparable to temperature, and the

571 system will be in a metallic state. The same can be seen in these line cuts. At  $t = 0$  we can see that  
572 there are four critical chemical potentials  $\mu$  at which electrons can be added to the triple dot. For  
573 the present model, these four peaks occur at  $\mu = 0, U, 2U + \Delta,$  and  $3U + \Delta,$  respectively. Each  
574 peak is triply degenerate, as the energy cost to add electrons to any of the three dots is identical.  
575 For nonzero but small tunnel couplings (**d-e**) each triply degenerate peak at  $t = 0$  starts to split  
576 into a miniband, indicating the breakdown of Coulomb blockade in each dot. However, different  
577 minibands are still separated by gaps that arise from a collective origin, reminiscent of the energy  
578 gap in a Mott insulator. Finally, at sufficiently high tunnel couplings we find nonzero  $\frac{\partial \langle N \rangle}{\partial \mu}$  at the  
579 middle gap (**f**), indicating that Coulomb blockade is overwhelmed by temperature altogether.

580 **Extended Data Figure 6 | Characterizing model parameters**

581 **a** Simulated charge stability diagram for a triple dot system with parameters  $t = 0.006$ ,  $U_1 =$   
582  $3.98$ ,  $U_2 = 3.48$ ,  $U_3 = 2.70$ ,  $V_{12} = 0.41$ ,  $V_{23} = 0.35$ ,  $V_{13} = 0.11$  (all energies in meV). As  
583 described in Methods, the eigenstates can be obtained exactly in the  $t = 0$  limit, as the eigenstates  
584 of the triple dot system can be represented simply by the charge states  $(n_1 n_2 n_3)$ . In this regime,  
585 one can show that on the  $\epsilon_2$ - $\epsilon_3$  plane the border between the (111)/(112) region and the border  
586 between the (111)/(110) region are exactly separated by an energy of  $U_3$ . Similarly, the border  
587 between the (111)/(121) region and the border between the (111)/(101) region are separated by  
588 an energy of  $U_2$ . In the presence of a nonzero but small tunnel coupling as is the case here, we  
589 expect that such an estimate is still reasonable. Now that the tunnel coupling is nonzero, the ground  
590 state of the system is no longer an exact charge state  $(n_1 n_2 n_3)$ , but generally a superposition of  
591 different charge states. To retain a connection to the  $t = 0$  limit, we keep labeling sections of the  
592 charge stability as  $(n_1 n_2 n_3)$ , but with the distinction in mind that  $(n_1 n_2 n_3)$  no longer denotes the  
593 exact ground state, but instead the charge state with the largest weight in the actual ground state.  
594 As a check, we can determine the values of  $U_2$  and  $U_3$  from the simulated charge stability diagram  
595 using the method described above and find that  $U_2 = 3.44$  meV and  $U_3 = 2.71$  meV, respectively,  
596 which is reasonably close to the corresponding model parameters. Since the data in Fig. 2c is  
597 taken at  $t/U = 0.002$ , we can thus trust the extracted  $U$ . **b** Charge stability diagram for a triple  
598 dot system with parameters  $t = 0.17$ ,  $U_1 = 2.92$ ,  $U_2 = 2.39$ ,  $U_3 = 2.53$ ,  $V_{12} = 0.55$ ,  $V_{23} =$   
599  $0.47$ ,  $V_{13} = 0.27$  (all energies in meV). We find that the structure of the charge stability diagram  
600 remains qualitatively the same as that in **a**, and if we again extract the values of  $U_2$  and  $U_3$  using the

601 same method, we find that  $U_2 = 2.48 \text{ meV}$  and  $U_3 = 2.56 \text{ meV}$ , which still agrees reasonably well  
602 with the original model parameters. Granted, at sufficiently large  $t/U$  the structure of the charge  
603 stability diagram will change drastically, and the present method to extract model parameters is  
604 bound to fail. However, as we never enter those regimes, our fitting method serves the purpose  
605 of this experiment. **c-e** Calibrated tunnel couplings (**c**) and measured inter-site Coulomb (**d**) and  
606 on-site Coulomb (**e**) terms at calibrated values of the average tunnel coupling, corresponding to  
607 the experimental parameter space plot shown in Fig. 3b of the main text. Blue fill indicates data  
608 from the first subband from 0 to 6 electrons, red fill data from the second subband from 6 to 12  
609 electrons. Error bars are  $1\sigma$  fit uncertainties.

610 **Extended Data Figure 7 | Isolated versus collective Coulomb blockade in charge and trans-**  
611 **port**

612 **a (c)** Charge stability diagram around the (333) regime in the low (high) tunnel coupling regime,  
613 using a combination of all seven gates (only  $P_1$  values are shown) that change the local fillings  
614 equally. To further investigate the distinct phases, we focus on the regime with around nine elec-  
615 trons in total, corresponding to half-filling of the second band, and look at both charge sensing and  
616 transport. In the localized phase ( $t/U < 0.02$  in **a**), the charge stability diagram shows transition  
617 lines following three distinct, well-defined directions, corresponding to the filling of the separate  
618 lithographically defined dots. In the delocalized phase ( $t/U > 0.15$  in **c**), this distinct nature is  
619 all but lost, highlighting the incipient formation of a large single dot. The same effect can also  
620 be seen in transport measurements, as we observe Coulomb diamond sizes as a function of fill-  
621 ing. **b** Transport through the array following the zero-detuning line of Fig. 2b of the main text  
622 as a function of applied bias (60% on leftmost and 40% on bottom right reservoir). In the (333)  
623 state, this applied bias has to overcome the local (strong) Coulomb repulsion in order for current  
624 to flow, similar to a Mott insulator whose Fermi energy resides inside the gap. Adjacent Coulomb  
625 diamonds correspond to a Fermi-level inside the miniband and are significantly smaller, allowing  
626 current to flow at much smaller bias voltages. **d** Similar data in the high tunnel coupling regime.  
627 Whereas the individual nature of the dots is all but gone, global (weaker) Coulomb repulsion still  
628 prohibits transport at small bias, as expected for the collective Coulomb blockade phase. The no-  
629 tion of a large gap at half-filling is gone, and it is but the charging energy of the entire system that  
630 prohibits transport to occur, regardless of filling. The dots are in collective Coulomb blockade, and

<sup>631</sup> its transport characteristics are similar to that of a small, metallic island.

632 **Extended Data Table 1 | Example of simulated transition points**

633 Transition points for a triple dot system with parameters  $t = 0.29$ ,  $U_1 = 2.26$ ,  $U_2 = 2.70$ ,  $U_3 =$   
634  $2.48$ ,  $V_{12} = 0.65$ ,  $V_{23} = 0.57$ ,  $V_{13} = 0.43$  (all in meV). The label  $N_1 \rightarrow N_2$  indicates that this  
635 data is for the transition from a total of  $N_1$  particles to  $N_2$  particles.  $\epsilon_i$  ( $i = 1, 2, 3$ ) are the ‘local’  
636 chemical potentials on each dot, while  $\mu$  is the ‘uniform’ chemical potential. The last two columns  
637 compare the experimental and theoretical total width of the fourth miniband. All energies are in  
638 meV.



639 **Extended Data Table 2 | Comparison of experimental and theoretical miniband width**

640 Comparison of the experimental and theoretical width of the fourth miniband in Fig. 3b in the  
641 main text at five calibrated values of the tunnel coupling. Theoretical widths take the interaction  
642 energies measured at the specific tunnel coupling values into account (see Extended Data Fig. 6).  
643 All energies are in meV.