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FIBRE MODEL IDENTIFICATION FOR NONLINEAR FOURIER TRANSFORM-BASED TRANSMISSION

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Abstract

Fibre-optic transceivers based on nonlinear Fourier transforms approximate the link by a lossless path-average model. We propose a new method that identifies a suitable lossless model from time-domain input-output data when fibre parameters are unknown. No special training signals are needed.

1 Introduction

In the past few years, novel fibre optical communication systems have been developed which explicitly take nonlinear effects in optical fibres into account by embedding data in the nonlinear Fourier spectrum [1–4], where the combined effect of the Kerr nonlinearity and dispersion reduces to simple phase shifts (similar to linear systems). These communication systems require that the fibre link can be approximated by a lossless fibre model. The usual way to obtain this model is by path-averaging [5, Ch. 9.2.2], [6], which will be described later. However, path averaging requires knowledge about the true fibre parameters throughout the whole link, which is often not the case in practice [7–11]. In this paper, we provide a novel method that identifies the parameters of a lossless fibre model as required by nonlinear Fourier transform (NFT)-based systems directly from input-output data. To the best of our knowledge, this is the first such method.

The paper is structured as follows. In Sec. 2, the fibre models, path-averaging, and the nonlinear Fourier spectrum are presented. In Sec. 3, our novel method to fit the lossless fibre model to actual input-output data is presented. In Sec. 4, the method is evaluated numerically. Sec. 5 concludes the paper.

2 Fiber models & nonlinear Fourier transform

An optical single-mode fibre with anomalous dispersion with loss, amplification and noise can be modelled by the focusing nonlinear Schrödinger equation (NLSE) [5, Ch. 5.3.1],

$$i\hat{q}_{\hat{z}} + \beta\hat{q}_{\hat{t}\hat{t}} + 2\gamma\hat{q}|\hat{q}|^2 = -i\alpha\hat{q}, \qquad (1)$$

in which $\hat{z}$ and $\hat{t}$ respectively denote the position and retarded time coordinate, $\hat{q}\left(\hat{t},\hat{z}\right)$ the complex field envelope, β the dispersion coefficient, γ the nonlinearity coefficient, α the loss coefficient and i is the unit imaginary number. The subscripts indicate partial derivatives. The total link of length L is divided into n equidistant fibre spans of length $L_{span} = L/n$, and – assuming lumped amplification – at the end of each span the signal is amplified with a factor $e^{\alpha L_{span}}$ to compensate the loss. We remark that the amplification scheme is not important for our algorithm, and that our simulations will also contain amplified spontaneous emission (ASE) noise.

In order to be able to apply NFTs, the optical fibre link with loss is approximated by a lossless path-average model [5, 6], which accounts for the loss by introducing a change of coordinates $\hat{q}(\hat{t},\hat{z}) = e^{-\alpha\hat{z}}\hat{u}\left(\hat{t},\hat{z}\right)$. This results in

$$i\hat{u}_{\hat{z}} + \beta\hat{u}_{\hat{t}\hat{t}} + 2\gamma e^{-2\alpha\hat{z}}\hat{u}|\hat{u}|^2 = 0. \qquad (2)$$

The varying nonlinearity coefficient is approximated by the path average coefficient $\gamma_1 = L_{span}^{-1}\int_0^{L_{span}}\gamma e^{-2\alpha\hat{z}}\,\mathrm{d}\hat{z}$, resulting in a lossless NLSE with constant coefficients,

$$i\hat{u}_{\hat{z}} + \beta\hat{u}_{\hat{t}\hat{t}} + 2\gamma_1\hat{u}|\hat{u}|^2 = 0. \qquad (3)$$

Next, consider the normalisation of the path-averaged NLSE by scaling the time and space coordinates by

$$u = \hat{u}, \quad t = c_t\hat{t} = \sqrt{\gamma_1/\beta}\,\hat{t}, \quad z = c_z\hat{z} = \gamma_1\hat{z}, \qquad (4)$$

which finally results in the normalised NLSE

$$iu_z + u_{tt} + 2u|u|^2 = 0. \qquad (5)$$

The normalised NLSE can be solved explicitly by considering its nonlinear Fourier transform (NFT) [12, 13], which is not possible in general for Equation 1. The NFT is found by association with the following scattering problem,

$$\frac{d}{dt}\phi(t,\lambda) = \begin{bmatrix} -i\lambda & u(t) \\ u^*(t) & i\lambda \end{bmatrix}\phi(t,\lambda), \qquad (6)$$

$$\phi(t,\lambda) = \begin{bmatrix} e^{-i\lambda t} \\ 0 \end{bmatrix} + o(1), \quad t \to -\infty. \qquad (7)$$

Under the condition that $u \to 0$ as $t \to \pm\infty$ sufficiently fast, we can define scattering coefficients from $\phi = (\phi_1, \phi_2)^T$,

$$a(\lambda) := \lim_{t\to\infty} e^{i\lambda t}\phi_1(t,\lambda), \quad b(\lambda) := \lim_{t\to\infty} e^{-i\lambda t}\phi_2(t,\lambda). \qquad (8)$$

The nonlinear Fourier spectrum consists of a continuous spectrum and a discrete spectrum. The continuous spectrum consists of the real line $\mathbb{R}$ and can be represented by the values of

$b(\lambda)$ for λ real. The discrete spectrum consists of eigenvalues, defined as the zeros λ_j of $a(\lambda)$ in the upper half of the complex plane, and can again be specified by the values of $b(\lambda_j)$. It is also possible to use the values of $b(\lambda)/a(\lambda)$ and $b(\lambda_j)/\frac{da}{d\lambda}(\lambda_j)$ instead. The spectra are invariant during propagation, while the evolution of the a- and b-coefficient is trivial as long as (5) is satisfied [12],

$$a(z, \lambda) = a(0, \lambda), \quad b(z, \lambda) = b(0, \lambda)e^{4i\lambda^2 z}. \tag{9}$$

3 Identification algorithm

In this section, a new method that identifies the parameters β and γ_1 of the lossless fibre model (4) is proposed. We consider a scenario in which some rough initial estimates of these values are used to operate the NFT-based transceiver. We provide the resulting input signals $\hat{q}(\hat{t}, 0)$ and measured output signals $\hat{q}(\hat{t}, L)$ to the identification algorithm in real-world units. The basic idea of the algorithm is to vary the parameters until the NFT of the input matches that of the output – subject to (9) – as good as possible. The algorithm first determines the constant c_t in (4) by matching the eigenvalues. Then, it determines the constant c_z using (9) in a second stage. Once c_t and c_z are known, β and γ_1 can be recovered with Eq. 4.

In this paper, we will only match the discrete spectra, even though our approach can be extended to take the continuous spectra into account as well. An implicit assumption therefore is that the fibre input has a non-empty discrete spectrum around the optimal normalisation point. This is a reasonable assumption e.g. in multi-soliton transmitters such as [14].

3.1 Stage 1: Finding c_t by matching the eigenvalues λ_j

Remember that the discrete spectrum consists of the so-called eigenvalues λ_j, which are identical for input and output under ideal conditions due to the space invariancy in Eq. 9. Hence, they are independent of the space scaling c_z. Therefore, by only varying the time scaling c_t for the input and output signal and matching their eigenvalues, the optimal time normalisation may be determined. The effect of different time normalisations c_t on the input-output spectra is shown in Figure 1.

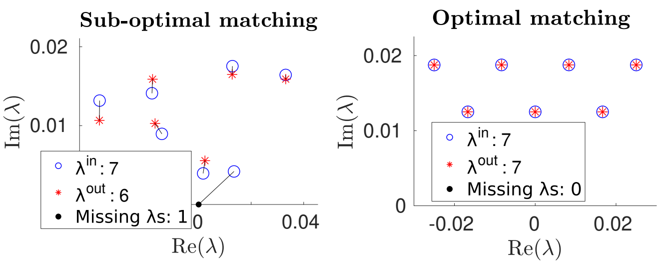


Fig. 1: The matching for input and output eigenvalues, for a single input-output signal normalised with a sub-optimal and an optimal c_t.

To quantify the error between an input and output spectrum for a certain c_t, first a matching problem is solved using the Hungarian algorithm [15], which creates pairs of eigenvalues of the input and output spectrum, $\left(\lambda_j^{in}, \lambda_j^{out}\right)$, such that the sum of the Euclidean distances in all pairs is minimised, see Figure 1. The matching error for each pair is then defined as

$$E_j = \begin{cases} \dfrac{|\lambda_j^{in} - \lambda_j^{out}|}{\max\limits_k(|\lambda_k^{in}|, |\lambda_k^{out}|)}, & \text{for j} \leq \min\left(N^{in}, N^{out}\right) \\ 1, & \text{for j} > \min\left(N^{in}, N^{out}\right) \end{cases} \tag{10}$$

It may occur that the number of input eigenvalues N^{in} and the number of output eigenvalues N^{out} are not equal, leaving unmatched eigenvalues in the larger spectrum, $(\lambda_j^{in}, -)$ or $(-, \lambda_j^{out})$, as is also the case for the bottom-right eigenvalue in the sub-optimal matching in Figure 1. The matching error for such 'half-pairs' is then set to 1. The total matching error is

$$E = \frac{\sum_{j=1}^{N} E_j}{N}, \qquad N = \max(N^{in}, N^{out}). \tag{11}$$

The error E is evaluated for different values of c_t on a logarithmic grid. The minimum error point provides an estimate for c_t. This estimate is iteratively refined using finer grids.

3.2 Stage 2: Finding c_z by matching the $b(\lambda_j)$

After the time scaling c_t has been fixed, the space scaling c_z may be determined by comparing the b-coefficients at input and output, $b^{in}(\lambda_j)$, $b^{out}(\lambda_j)$. The b-coefficients of the time-normalised input and output signal are related according to Eq. 9, which we use to find the normalised length $c_z L$,

$$\begin{aligned} |b(c_z L, \lambda_j)| &= |b(0, \lambda_j)| e^{\text{real}\left(4i\lambda_j^2\right) c_z L} \\ \Rightarrow c_z L &= \frac{\log|b^{\text{out}}(\lambda_j)| - \log|b^{\text{in}}(\lambda_j)|}{\text{real}\left(4i\lambda_j^2\right)}. \end{aligned} \tag{12}$$

Assuming that the real fibre length L is known, this relation immediately provides a method for determining c_z. Three aspect need to be further considered. First, each input-output eigenvalue pair contains two eigenvalues, whereas the above formula requires a consensus on the value. Our method takes the average of the input and output eigenvalue for each pair, $\lambda_j = \left(\lambda_j^{in} + \lambda_j^{out}\right)/2$. Half pairs are discarded due to the lack of one b. Second, each eigenvalue pair will individually yield an estimate for c_z, and therefore only a single eigenvalue pair is required for an estimate. In case more pairs are present, our method takes the average of all estimates of c_z. Third, when an eigenvalue is purely imaginary or has a small real part, real$(\lambda_j) \ll |\lambda_j|$, the denominator in Equation 12 may become close to zero, yielding unstable estimations for c_z. This can be overcome by discarding these estimates when they do not correspond with the other estimates.

After c_z has been determined from the moduli of the b-coefficients, the phases may then be compared to determine the reliability of the estimation. Once c_z and c_t have been determined, β and γ_1 may be recovered from Equation 4.

4 Results

In this section, we first validate the method for a lossless, noiseless optical fibre. Second, we identify an optimal lossless fibre model for a non-ideal fibre optical communication system.

4.1 Baseline: Identification of an ideal fibre model

For a first validation of the proposed method, a loss- and noiseless optical fibre link is considered. The link has length $L = 360$ km, divided into $n = 10$ fibre spans of $L_{span} = 36$ km each, and parameters $\beta = 5.75 \cdot 10^{-27}\text{s}^2\text{m}^{-1}$ and $\gamma = 1.6 \cdot 10^{-3}(\text{Wm})^{-1}$, which corresponds to space and time normalisation constants $c_t = 5.275 \cdot 10^{-27}$ and $c_z = 1.6 \cdot 10^{-3}$. We used the software *NFDMLab* [16] for generating and propagating input signals with 7 discrete eigenvalues with the same parameters as in [14]. The algorithm proposed in this paper was applied to first identify the normalisation constant c_t. The matching error of Eq. 11 is shown in Fig. 2, the minimum value appears at the expected c_t. The sudden jumps are due to the appearance of an eigenvalue in one spectrum, but not in the other. The algorithm recovered the fibre parameters β and γ up to 4 digits, validating the method.

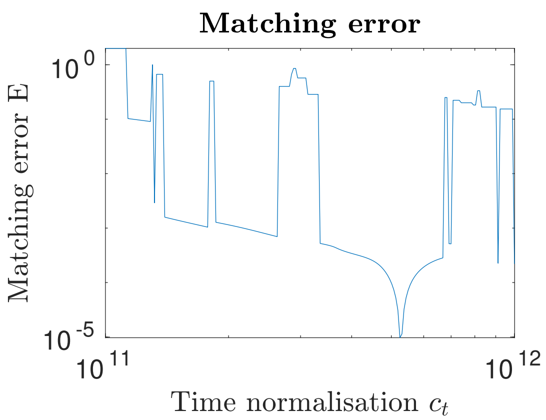


Fig. 2: The matching error between input and output spectrum.

4.2 Identification in a link with lumped amplification

We consider the same setup as above, but with an additional loss term $\alpha = 4.6052 \cdot 10^{-4}\,\text{m}^{-1}$ that is compensated by EDFA amplifiers with 6 dB noise figures [5, Ch. 7.2.3] after each span. The transceiver initially uses the incorrect parameters $\beta = 5 \cdot 10^{-27}$ and $\gamma_1 = 1 \cdot 10^{-3}$ for the normalisation. According to the path average model discussed in Sec. 2, the system should instead be approximated using $\gamma_1 = 0.465 \cdot 10^{-3}$ and β as in the previous subsection. Fig. 4a depicts the constellation diagram for the initial configuration. By applying our proposed algorithm, better normalisation constants c_t and c_z are identified. Fig. 3 shows the β and γ_1 parameters that correspond to the identified normalisation constants from 30 different signals. The average of these values were used to determine new normalisation constants. Fig. 4b shows the constellation diagram after the normalisation constants have been updated. A significant improvement can be observed. The error vector magnitude (EVM) improved from approximately 32 dB to -11 dB. The latter is similar to the EVM achieved by the path average model with fully known parameters.

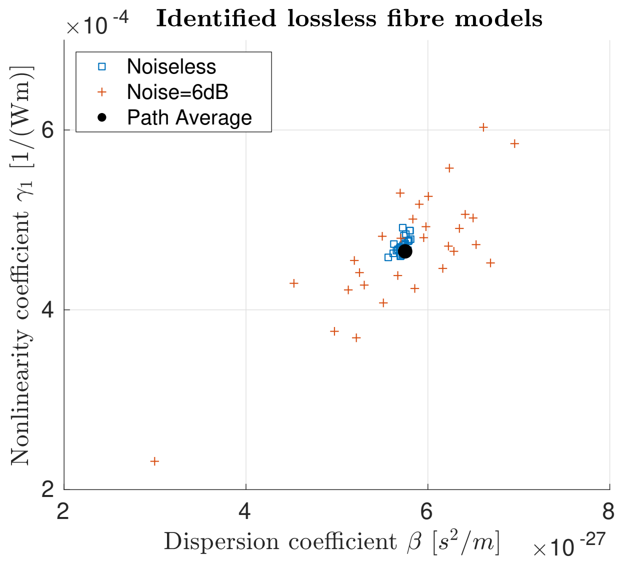
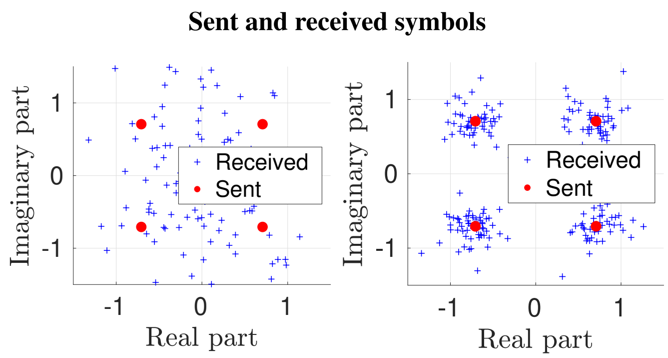


Fig. 3: Identified β and γ from 30 noiseless transmissions and 30 transmission with 6 dB ASE-noise.



(a) Initially guessed model. (b) Identified model.

Fig. 4: Symbol transmission for two models. The sent symbols all lie at locations $(\pm\frac{\sqrt{2}}{2} \pm i\frac{\sqrt{2}}{2})$, the dots are the received symbols.

5 Conclusion

We have presented a method to directly identify a lossless fibre model for nonlinear Fourier transform based communication by matching the discrete nonlinear Fourier spectra of input and output signals. The only requirement on the signals is that their discrete spectra are non-empty. Using signals generated with incorrect fibre parameter estimates and their transmitted outputs, the method identified a lossless fibre model which improved the error vector magnitude in the transmitted symbols from 32 dB to -11 dB, which is similar to the best path average model for the fully known system.

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