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EXACT NONLINEAR FREQUENCY DIVISION MULTIPLEXING IN LOSSY FIBERS

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Abstract

The path-average approximation penalizes NFDM transmission over lumped amplified fiber links. We investigate suitably tapered lossy fibers to overcome the approximation error induced by the path average, making the NFDM transmission exact. Error vector magnitude gains up to 4.8 dB are observed.

1 Introduction

Nonlinear Fourier transform (NFT) based transmission techniques were proposed in optical fiber communication to overcome the nonlinear behavior of the fiber. In an ideal lossless fiber, the complex interplay between the fiber Kerr nonlinearity and the dispersion during the propagation of the signal in optical fiber translates into simple phase rotations in the nonlinear Fourier spectrum [1]. This recently motivated a new information transmission technique called nonlinear frequency division multiplexing (NFDM) that modulates the nonliner Fourier spectrum [2, 3]. NFDM techniques utilize a lossless fiber model and are thus challenged by the loss in real fibers. The power variations due to losses are typically taken into account using a path-average model [4, 5]. After a suitable change of coordinates the fiber propagation equation become lossless, but nonlinear parameter now varies along the fiber. Averaging the nonlinear parameter, a lossless model suitable for NFDM is obtained. Many NFDM transmission systems have been demonstrated in the past years by using the path-averaged approach [5–7]. The impact of the path-average approximation was investigated in optical links with lumped amplification in [5] and it was found that the approximation accuracy depends upon the signal bandwidth, the transmitted power and distance.

In order to avoid the approximation errors caused by the path averaged method, we investigate an idea from classical single soliton systems, where dispersion decreasing fiber (DDF) was proposed to overcome the soliton broadening due to loss in the the optical fiber [8–10]. These fibers are made such that the balance between the dispersive and nonlinear effects is preserved along the fiber in the presence of loss. It has been pointed out that the propagation in such fibers can be solved exactly with NFTs even though there is loss [11]. However, it seems that this idea has never been tested so far for NFDM.

In this paper, we present the first simulative demonstration of a 7-eigenvalue multisoliton NFDM transmission system over DDF and compare its performance with a constant dispersion fiber (CDF) through simulations. This paper is structured as follows. In Sec. 2, we introduce the dispersion decreasing fiber considered in this case together with the corresponding NFT. Then, in Sec. 3, we introduce the simulation setup and present simulation results. The paper is concluded in Sec. 4.

2 Methodology

The propagation of the slowly-varying complex optical field envelope $Q(\ell, t)$ in a single mode fiber (SMF) can be modelled by the nonlinear Schrödinger equation (NLSE) [4, Ch. 2.6.2]

$$\frac{\partial Q}{\partial \ell} + i\frac{\beta_2}{2}\frac{\partial^2 Q}{\partial t^2} - i\gamma|Q|^2Q = -\frac{\alpha}{2}Q, \tag{1}$$

where ℓ represents the propagation distance and t is retarded time. The parameters α, β_2 and γ are the loss, dispersion and nonlinear coefficients respectively. Here, we consider the anomalous dispersion case $\beta_2 < 0$. Inspired by classical soliton systems [8–10], we propose to use a DDF for NFDM systems in order to avoid the errors induced by the path-averaged method. A practical method to achieve a decreasing dispersion profile is by tapering the optical fiber during the draw process [4, Ch. 9.3.1]. We assume a simplified approximate relation between the effective core radius r (in μm) and the dispersion parameter β_2 that was given in [8],

$$r(\beta_2) = (\beta_2/\kappa + 20)/8, \tag{2}$$

where $\kappa = \lambda_0^2/2\pi c$ and λ_0 and c are the wavelength and speed of light in free space respectively. The nonlinear parameter depends on the effective core radius as follows [4, Ch. 2.6.2]

$$\gamma = 2\pi n_2/(\pi r^2), \tag{3}$$

where n_2 is nonlinear-index coefficient. As a result, the fiber will have a variable dispersion parameter $\beta_2(\ell) = \beta_2(0)D(\ell)$ and a variable nonlinear parameter $\gamma(\ell) = \gamma(0)R(\ell)$. The propagation of the complex envelope of the field $Q(\ell,t)$ in such a fiber is then given by [11]

$$\frac{\partial Q}{\partial \ell}+i\frac{\beta_2(0)D(\ell)}{2}\frac{\partial^2 Q}{\partial t^2}-i\gamma(0)R(\ell)|Q|^2Q=-\frac{\alpha}{2}Q. \quad (4)$$

By a change of variables $q=Q/\sqrt{P}$, $z=\ell/L_D$, $\tau=t/T_o$, where $L_D=T_o^2/|\beta_2(0)|$, $P=1/(\gamma L_D)$ and T_o is a free parameter, the above equation can be transformed into the normalized form [4]

$$\frac{\partial q}{\partial z}-i\frac{D(z)}{2}\frac{\partial^2 q}{\partial \tau^2}-iR(z)|q|^2q=-\frac{\alpha L_D}{2}q. \quad (5)$$

It was shown in [11] that the above equation can be solved exactly via NFT if

$$\alpha L_D=-\frac{[R(z)D'(z)-R'(z)D(z)]}{R(z)D(z)}, \quad (6)$$

where the prime denotes differentiation. In order to satisfy the above condition, the required dispersion profile has to satisfy

$$\frac{\beta_2(\ell)}{\gamma(\ell)}=ae^{-\alpha\ell}, \quad (7)$$

where $a=\beta_2(0)/\gamma(0)$. By combining (2), (3) and (7), we arrive at

$$8r^3(\ell)\kappa-20\kappa r^2(\ell)-2\pi n_2 ae^{-\alpha\ell}=0. \quad (8)$$

The real-valued solution to this equation provides us the effective core radius. Once the radius is known we can find $\beta_2(\ell)$ and $\gamma(\ell)$ from (2) and (3). The parameters of a DDF are shown

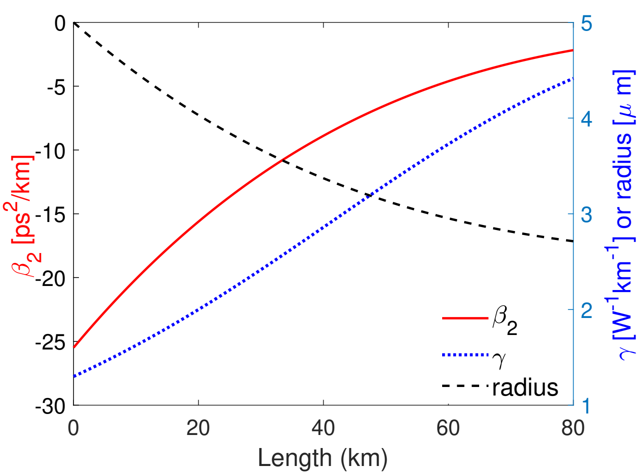

Fig. 1 DDF parameters for a single span as used in the simulations.

in Fig. 1 for the loss coefficient $\alpha=0.2$ dB/km.
Next, we introduce an appropriate NFT for (5) subject to the condition (6). For the standard NLSE that is used normally,

$$\frac{\partial u}{\partial z}-i\frac{1}{2}\frac{\partial^2 u}{\partial \tau^2}-i|u|^2u=0, \quad u=u(z,\tau), \quad (9)$$

the forward NFT requires the solution of the so-called Zakhrov-Shabat scattering problem [1]

$$\frac{\partial}{\partial \tau}\begin{pmatrix}\vartheta_1(\tau,z)\\ \vartheta_2(\tau,z)\end{pmatrix}=\begin{pmatrix}-j\lambda & u(\tau,z)\\ -u^*(\tau,z) & j\lambda\end{pmatrix}\begin{pmatrix}\vartheta_1(\tau,z)\\ \vartheta_2(\tau,z)\end{pmatrix} \quad (10)$$

with the boundary condition

$$\begin{pmatrix}\vartheta_1(\tau,z)\\ \vartheta_2(\tau,z)\end{pmatrix}\to\begin{pmatrix}1\\ 0\end{pmatrix}\exp(-j\lambda\tau)\ \text{for}\ \tau\to-\infty. \quad (11)$$

The Jost scattering coefficients are defined as

$$\begin{aligned}a(\lambda,z)&=\lim_{\tau\to+\infty}\vartheta_1(\tau,z)\exp(j\lambda\tau),\\ b(\lambda,z)&=\lim_{\tau\to+\infty}\vartheta_2(\tau,z)\exp(-j\lambda\tau).\end{aligned} \quad (12)$$

The NFT of $u(z,\tau)$, for fixed z, consists of the reflection coefficient $\rho(\lambda)=b(\lambda)/a(\lambda)$, $\lambda\in\mathbb{R}$, and the discrete spectrum

$$\left(\lambda_j,\ \rho_j:=b(\lambda_j)/\frac{da}{d\lambda}(\lambda_j)\right),$$

where eigenvalues λ_j are the zeros of $a(\lambda,z)$ in the complex upper half-plane. The evolution of these Jost scattering coefficients with respect to the standard NLSE (9) is given by

$$a(\lambda,z)=a(\lambda,0), \quad b(\lambda,z)=b(\lambda,0)e^{2i\lambda^2 z} \quad (13)$$

The NFT of $q(z,\tau)$ with respect to (5) is now defined as the conventional NFT of the signal $u(z,\tau)=\sqrt{\frac{R(z)}{D(z)}}q(z,\tau)$. If $a(\lambda,z)$ and $b(\lambda,z)$ are the Jost scattering coefficients of $u(z,\tau)$, then their evolution with respect to (5) is given by

$$a(\lambda,z)=a(\lambda,0), \quad b(\lambda,z)=b(\lambda,0)e^{2i\lambda^2\int_0^z D(\varsigma)d\varsigma}. \quad (14)$$

This equation allows us to recover the NFT of fiber input from that of the fiber output in a simple way.

3 Results

The simulation setup is close to the multi-soliton transceiver presented in [6], in which the spectral amplitudes ρ_j of 7 eigenvalues were modulated independently with QPSK. The system in [6] was carefully designed to reduce the path-average error which imposes extra limitations on the data-rate, signal-bandwidth, and the span-length. To illustrate the potential advantage of using DDF in NFDM system, we changed several transmission parameter summarized in Tab. 1. Due to the

Table 1 Parameters of NFDM system

Fiber type	CDF	DDF
α [dB/km]	0.2	0.2
β_2 [ps^2/km]	-10.53	-25 to -2.17
γ [1/W/km]	2.86	1.3 to 4.4
Span length [km]	80	80
Normalized pulse duration	18π	18π

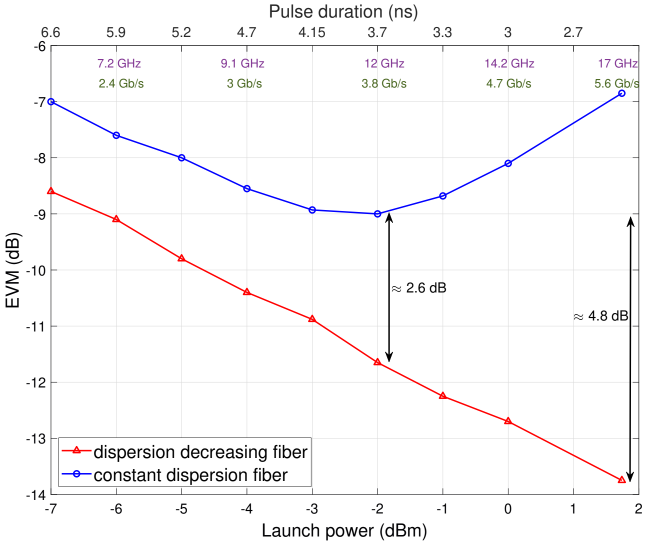

Fig. 2 EVM variation with launch power after 800 km transmission.

different fiber characteristics, it is not obvious how the two systems should be compared. In our simulations, we swept the launch power by changing the normalization parameter T_o. The eigenvalues and spectral amplitudes of the NFDM system with CDF were chosen as in [6]. For the NFDM sytem with DDF, the imaginary parts of the eigenvalues were first scaled such that the launch power of both systems match. Then, the real parts were scaled to match the bandwidth. The duration of the normalized multi-soliton pulse was chosen as 18π to avoid truncation effects. The multi-soliton pulses were unnormalized before transmission through a 10×80 km link of optical fiber. The fiber propagation was simulated using a split-step Fourier method. After every 80 km, the span loss of 16 dB was compensated using lumped amplification with noise figure of 6 dB. Third order dispersion parameter was not implemented since realistic values around $\beta_3 = 0.05\text{ps}^3$/km are expected to be insignificant in our case due to the high average dispersion parameter and nanosecond pulse durations [12]. Since the fiber diameter varies slowly over distance, the radiation loss is expected to be negligible as well [12]. Thus, it is ignored in the numerical simulations.

At the receiver, the received signal was filtered to remove out of band noise. After normalization, the signal was then back propagated in the nonlinear Fourier domain using (9) and then the QPSK symbols were demodulated from the spectral amplitudes of the eigenvalues. Finally, the error vector magnitudes (EVM) were computed using the received QSPK symbols and the transmitted ones.

In order to visualize the error due to path-averaged approximation, the EVM is plotted against the launch power for 800 km transmission in Fig. 2. The pulse duration, bandwidth and data rate of the system for the corresponding launch power are also mentioned in the figure. For the NFDM system with CDF, the EVM initially decreases with launch power due to increase in effective signal to noise power ratio (SNR). After launch power of -2 dBm, the approximation error due to path-average model dominates and hence, EVM starts rising with launch power. However, for the NFDM scheme that uses DDF there is no such limitation and the EVM drops with the launch power due to improved SNR. We remark that even though it is not yet visible in the plot, EVMs will eventually starts increasing also for NFDM with DDF because of inter-symbol interference.

We observe a large gain of $\approx$ 2.6 dB in the EVM for transmission over DDF in comparison to CDF at launch power of -2 dBm which corresponds to data rate of 3.8 Gb/s. We also observe that the EVM gain is even larger at higher launch power which also correspond to higher data rates of 5.6 Gb/s.

4 Conclusion

We presented the first results for an exact NFDM transmission in lossy fiber. We showed that by considering a suitable dispersion decreasing fiber and a suitably adapted nonlinear Fourier transform, the limiting path-average approximation error can be avoided. As a result, we obtained a gain of up to 4.8 dB in the EVM for 800 km transmission over DDF in comparison to fiber with constant dispersion. The next step is to include the continuous part of nonlinear Fourier spectrum.

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