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# Markets for Direct Current Distribution Systems: Towards Energy-based vs Flex-based Designs

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**Abstract**—Direct Current Distribution System (DCDS), a promising alternative to existing AC systems, connect customers to DC energy sources without AC/DC conversion. The unique features of DC, including the power-voltage coupling effect, impose different requirements to DCDS operation compared to AC. Addressing a liberalized energy market, this paper investigates the significant impact of market design on DCDS operation with an empirical analysis of electric vehicle charging. With an empirical analysis on EV charging, we investigate the level of efficiency a centralized market may theoretically reach, then compare it with a market based on prosumers' local decision.

**Index Terms**—electricity market design, direct current, distribution system, flexibility, electric vehicle

## I. INTRODUCTION

The power system is embracing increasing numbers of distributed renewables and electric vehicles (EVs). Consumers are becoming prosumers, and many of their devices have a DC nature but are connected to traditional AC networks via extra AC/DC conversions. These conversions limit the power capacity, energy efficiency and operational flexibility of the network. DC distribution systems (DCDS), by contrast, remove AC/DC conversions and connect DC prosumers directly, thereby becoming a promising alternative to AC systems [1].

A DCDS's technical features require different coordination schemes from AC systems. It has much lower system inertia, stricter power limits, and a stronger power-voltage coupling effect [2]. The operational security of DCDS, including voltage stability, is highly dependent on prosumer flexibility. A market for DCDS should respect DC technical features and prosumers' roles. In a liberalized energy market, simply applying AC market designs to a DCDS cannot guarantee the latter's supply security and voltage stability.

This paper evaluates two promising electricity markets for DCDSs out of those who meet the technical requirements [3]. Both markets follow the systematic design framework of reviewing goals, listing options, making choices and evaluating performance [4]. Although distinguished by market architecture [5], they both reward customers for energy production but also for providing flexibility at a specific location. With an empirical analysis of EV charging, we investigate the efficiency level a centralized market may reach, then compare it with a decentralized market based on local decisions.

## II. PROPOSED MARKET DESIGNS

Previous work has explored the broad design space of DCDS electricity markets [2]. This paper evaluates two promising designs out of those who meet DCDS requirements: the *integrated market (IM) design*, and the *locational Flex market (LFM) design*. These markets, although different in architecture, both remunerate three primary commodities of a DCDS: energy, network capacity and voltage regulation service [6]. Below we briefly introduce their architecture and evaluate their features, referring to [3] for further information.

### A. Integrated Market (IM) Design

This design defines energy as an integrated commodity. It remunerates power generation but also the provision of network capacity and voltage regulation [7], [8]. A market operator acquires sufficient information from the DCDS operator and from prosumers (who submit complex bids including energy needs, constraints and preferences), then optimally allocates energy and other resources to maximize prosumers' profit. This market performs security-constrained economic dispatch with real-time centralized optimization. Prosumers are paid/charged in terms of energy based on their marginal contribution to the total economic welfare, resulting in real-time locational energy prices. Prosumer flexibility is not rewarded explicitly but implicitly via highly dynamic energy prices.

### B. Locational Flex Market (LFM) Design

The other design, aiming to facilitate prosumer participation in wholesale energy markets, dispatches flexibility (Flex) resources to support reliable DCDS operation. It allows barrier-free energy trading among local and wholesale market players—local energy price is coupled to the wholesale market. Hence, the market's task is to provide location-specific flexibility services that resolve DCDS network issues [9], especially voltage deviations. Notably, this market defines Flex as an explicit, standard commodity: Flex is an option to adjust prosumers' power in real-time [3]. This market attracts Flex investments where congestion and voltage deviation occur. Flex resources are paid differently depending on their locational value to a DCDS. For participation, prosumers place Flex bids in advance to get activated in real-time DCDS dispatch.

### III. OPTIMIZATION MODEL

Recall that we aim to demonstrate the impact of market design on the economic efficiency of DCDS operation. A centralized optimization approach allows us to focus on our aim. We adopt an EV charging scenario to investigate the impact of the market design on the overall DCDS operation. Indeed, DCDS of future power systems will serve large numbers of distributed renewables (PV panels), steadily increasing household loads, and numbers of EVs plugged into the network. Without coordination, the simultaneous charging of many EVs may result in new load peaks that are even higher than the existing peaks.

#### A. Objective and Decision Variables

The objective is to maximize prosumers' revenue from energy sales.

$$\max_{p_t^e, p_t^c} W = \sum_{t \in \mathcal{T}} (-p_t^w) \lambda_t^w \quad (1)$$

The decision variables are the power imported from the wholesale market  $p_t^w$  and the EV charging dispatch  $p_t^c$ . The objective function (1) is subject to the constraints regarding the network (2–9) and prosumers (10–15). Readers may refer to Table I for the full list of indices, variables and parameters.

#### B. Network Constraints

##### Nodal Power Expression

$$p_t^n = i_t^n v_t^n \quad \forall t, \forall n \quad (2)$$

This quadratic equality constraint is used for power-based measurement and settlement in a DCDS.

##### Nodal Power Injection

$$p_t^{n=1} = p_t^w \quad \forall t \quad (3)$$

$$p_t^n = \sum_{g \in \mathcal{G}^n} p_t^g + \sum_{l \in \mathcal{L}^n} p_t^l + \sum_{e \in \mathcal{E}^n} p_t^e \quad \forall t, \forall n \neq 1 \quad (4)$$

where  $\mathcal{L}^n, \mathcal{G}^n, \mathcal{E}^n$  are loads, generators and EVs at node  $n$ .

##### Nodal Current Balance

$$i_t^n = \sum_{m|(n,m) \in \mathcal{A}} f_t^{(n,m)} - \sum_{m|(m,n) \in \mathcal{A}} f_t^{(m,n)} \quad \forall t, \forall n \quad (5)$$

##### DC Line Current Flow

$$\omega^{(m,n)} f_t^{(m,n)} = (v_t^m - v_t^n) \quad \forall t, \forall (m,n) \in \mathcal{A} \quad (6)$$

##### DC Nodal Voltage

$$0 < \underline{v} \leq v_t^n \leq \bar{v} \quad \forall t, \forall n \quad (7)$$

##### Capacity of Substation

$$-\overline{p^w} \leq p_t^w \leq \overline{p^w} \quad \forall t \quad (8)$$

##### Capacity of Lines

$$-\overline{f^a} \leq f_t^a \leq \overline{f^a} \quad \forall t, \forall a \quad (9)$$

#### C. Prosumer Constraints

This model assumes that both household consumers and PV panels are non-flexible and may not be curtailed. Hence, EVs are the only flexible prosumers of the DCDS.

##### EV Battery Charging Power (negative value)

$$p_t^e = 0 \quad \forall t \in [0, t_a^e) \cup [t_d^e, T], \forall e \quad (10)$$

$$\underline{p^e} \leq p_t^e \leq 0 \quad \forall t \in [t_a^e, t_d^e), \forall e \quad (11)$$

##### EV Battery State-of-Charge (SOC) Increase

$$(r_{t+1}^e - r_t^e) c^e = \eta^e (-p_t^e) \Delta t \quad \forall t \neq T, \forall e \quad (12)$$

##### EV Battery Overall / Initial / Target SOC

$$0 \leq r_t^e \leq 1 \quad \forall t, \forall e \quad (13)$$

$$r_{t_a^e}^e = r_a^e \quad \forall e \quad (14)$$

$$r_{t_d^e}^e \geq r_d^e \quad \forall e \quad (15)$$

#### D. Flex Constraints

If a Flex device exists, we should replace Constraint 4 with:

$$p_t^n = \sum_{g \in \mathcal{G}^n} p_t^g + \sum_{l \in \mathcal{L}^n} p_t^l + \sum_{e \in \mathcal{E}^n} p_t^e + \sum_{f \in \mathcal{F}^n} p_t^f \quad \forall t, \forall n \neq 1 \quad (16)$$

where  $\mathcal{F}^n$  is a set of Flex devices at node  $n$ .

The optimization problem is further subject to:

##### Flex Battery Charging Power

$$\underline{p^f} \leq p_t^f \leq \overline{p^f} \quad \forall t, \forall f \quad (17)$$

##### Flex Battery SOC Increase (assuming no losses)

$$(r_{t+1}^f - r_t^f) c^f = (-p_t^f) \Delta t \quad \forall t \neq T, \forall e \quad (18)$$

##### Flex Battery SOC Limits

$$0 \leq r_t^f \leq 1 \quad \forall t, \forall f \quad (19)$$

$$r_{t=1}^f = r_{t=T}^f \quad \forall f \quad (20)$$

## IV. EMPIRICAL ANALYSIS

We evaluate the two market designs of Section II using numerical simulation. The simulated DCDS represents a low-voltage distribution network of a small European neighborhood. 55 households build up the baseload (54kWp in total), while 25 of them own rooftop PV arrays (67kWp in total). Independently from PV installation, 25 households own an EV (7kW, 24kWh) that demands overnight charging. In the LFM design, we consider two Flex batteries at the end of the two longest lines of the DCDS network. The simulation with 1-minute resolution starts at noon and lasts for 24 hours.

### A. EV Charging Strategies

We test the following four EV charging strategies for their influence on DCDS operation:

- 1) S1–Dumb charging: EVs are charged with maximum power until full (base case without a market).
- 2) S2–Individually optimized charging: prosumers charge their EVs when the expected wholesale energy price is the lowest, potentially resulting in network issues (an inefficient market design).
- 3) S3–Centrally optimized charging: prosumers communicate EV charging needs with the market, then leave the charging decision to the latter for central optimization (the integrated market design).
- 4) S4–Individually optimized + Flex-supported charging: prosumers charge EVs when the expected wholesale energy price is low, whereas two Flex batteries support the voltage (the locational Flex market design).

Our first hypothesis is that given complete and truthful information, a coordination scheme (and the corresponding market design) can serve much more EVs than in uncoordinated cases. Here we compare Strategy S1 and S3, where EV charging is centrally optimized with complete information.

Our second hypothesis is that market designs that give wrong incentives may result in less optimal system operation—sometimes worse than the case without market coordination. To justify this, we compare Strategy S1, where there is no coordination, and S2, where EV owners individually ‘optimize’ charging based on expected wholesale energy prices.

### B. Data

The simulation is based on the IEEE European Low Voltage (EULV) Distribution Test Feeder [10], a low-voltage radial distribution network representing residential areas in Europe. We assume that the 3-phase 400V AC grid has been upgraded to a 350V unipolar DC network, where a 100kW substation converter replaces the AC transformer. Existing AC lines are still used for DC power distribution. To reduce the problem scale, we simplified the 906-node EULV network into a 7-node illustrative network based on potentially congested lines.

The EULV feeder also provides 55 household load profiles with per-minute measurements within 24 hours. The generation profiles of 25 PV arrays, representing a clear summer day, are based on the per-minute measurements of Virginia Tech & EPRI [11]. The driving patterns of 25 EVs base themselves on a synthetic driver profile [12] obtained from K-means clustering algorithm. For real-time energy prices, we refer to the TenneT imbalance settlement price [13] of a summer day.

## V. SIMULATION RESULTS

Below we demonstrate the simulation results for each of the EV charging strategies. During the simulation, the two limiting factors of DCDS operation are substation converter congestion and nodal voltage deviations, whereas no line congestion was observed in all simulations. These simulations imply that congestion of a DCDS generally happens at substations rather than lines; we conclude that for distribution level market

designs, zonal pricing may already be sufficient for congestion management instead of the complex nodal pricing.

### A. S1–Dumb charging

Prosumers charge their EVs immediately upon arrival. The driving profile indicates that most of the EVs arrive during the rush hours after work [12]; hence, the EV charging load adds to the existing peak load during the evening, as can be seen in Figure 1 between 19:00 and 20:00. Nodal voltage also drops during that time, but it still stays within an acceptable range of 5%. With Strategy 1, only 23 out of 25 EVs can be served due to simultaneous charging.

### B. S2–Individually optimized charging

Prosumers foresee wholesale energy prices and schedule their EV charging ‘optimally’ during the lowest price hours, such as 00:00-01:00 and 07:00-08:00. Although ‘optimal’ for prosumers, the simultaneous charging in these hours pushed the DC substation to congestion. Consequently, only 13 out of 25 EVs get served with this strategy, which is even worse than the dumb charging case with Strategy S1. This simulation illustrates the case where a less careful local market design (in our cases, purely based on wholesale market prices) may lead to less efficient system operation than if there is no market.

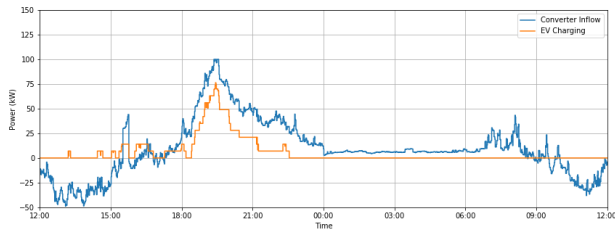
### C. S3–Centrally optimized charging

This charging Strategy corresponds to our integrated market (IM) design. The market operator receives practically all necessary information from prosumers, the DCDS and the wholesale market, then optimally operates prosumer devices to maximize their profit. As shown in Figure 3, the market maximizes the use of network capacity, by coordinating EV charging, during the periods where wholesale market prices are the lowest. Although some prosumers pay slightly higher energy prices compared to Strategy S2, the market serves all 25 EVs and maximizes their profit.

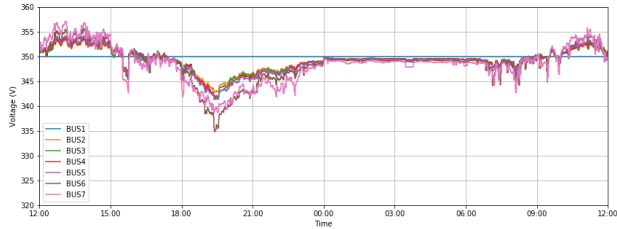
In theory, as shown in Figure 4, the centralized coordination has the potential of serving up to 275 EVs (with the same driving patterns)—11 times of the base case scenario—which is far more than enough to serve the local residents. Coordinated EV charging also facilitated local consumption of renewable generation. Nevertheless, such a large charging load inevitably result in volatile nodal voltages. In case this becomes a concern, we may consider installing Flex devices at critical nodes for voltage regulation.

### D. S4–Individually optimized + Flex-supported charging

Strategy S2 is intended to offer prosumers free access to the wholesale market. However, this individual-oriented market design cannot accommodate all of the EVs, so 25 EVs must compete with each other for cheaper charging opportunities. Such competitions inevitably challenge DCDS supply security. To guarantee security while supporting prosumer autonomy, we may introduce small Flex batteries into the branches that face congestion or voltage deviations. Strategy S4 corresponds to our LFM Design.

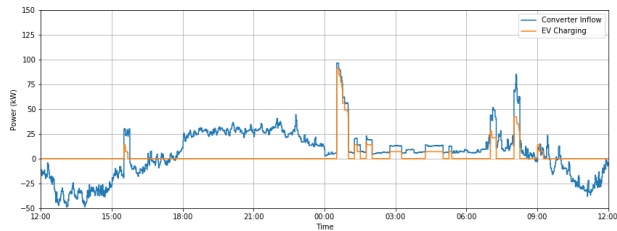


(a) total EV charging load vs. converter power intake

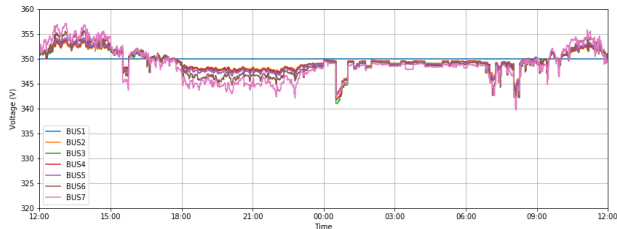


(b) nodal voltage

Fig. 1. Strategy S1 with only 23 EVs served



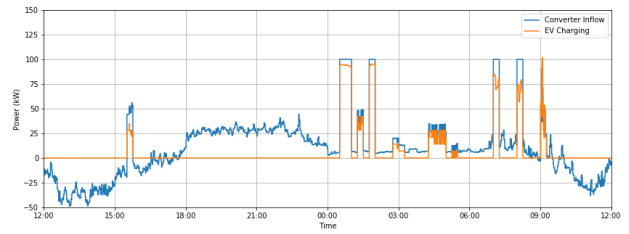
(a) total EV charging load vs. converter power intake



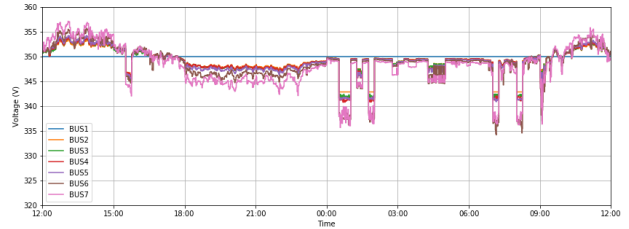
(b) nodal voltage

Fig. 2. Strategy S2 with only 13 EVs served

In our simulation, two small Flex batteries with 41kW power capacity and 10kWh energy capacity are already sufficient to accommodate the individually optimal charging of the 25 EVs. In practice, such a Flex service can be easily provided by two EVs connected to fast charging stations (50kW or more) with vehicle-to-grid capability. Hence, we strongly recommend that fast EV charging should be centrally coordinated, not only for system optimization but also to avoid the creation of new problems. In this case, the market operator does not necessarily require detailed prosumer data, because it may estimate upcoming peak loads based on historical data and wholesale market price forecasts. The corresponding market design, LFM, can serve flexible local loads without affecting prosumer autonomy and privacy. With a modest investment in Flex devices, the LFM design guarantees reliable connection

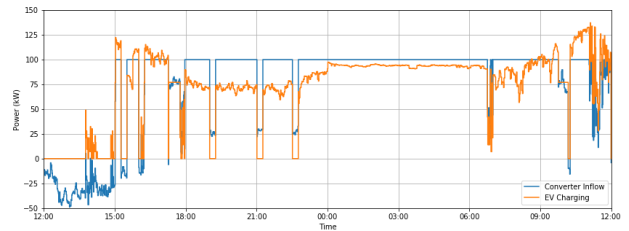


(a) total EV charging load vs. converter power intake

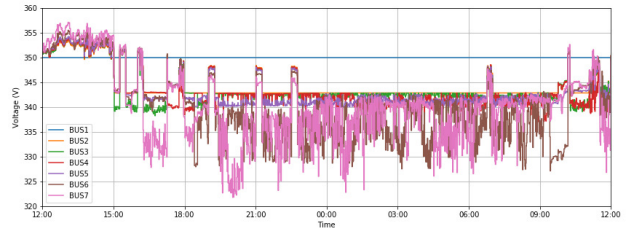


(b) nodal voltage

Fig. 3. Strategy S3 with 25 EVs



(a) total EV charging load vs. converter power intake



(b) nodal voltage

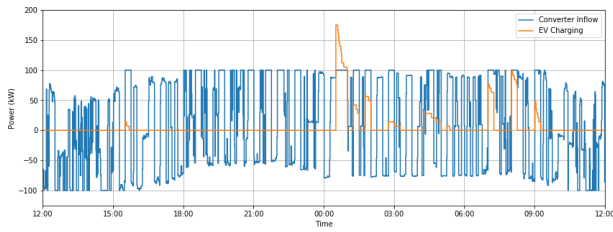
Fig. 4. Strategy S3 with 275 EVs

while not demanding immediate upgrades in DCDS networks.

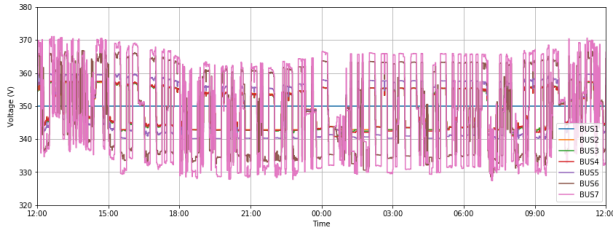
Explicit Flex markets, including the LFM, create new business models for flexible devices but also generate new challenges. As shown in Figure 5, Flex batteries are not only used to cover EV charging peak loads; the optimization scheme steadily uses these batteries to arbitrage from wholesale energy price fluctuations. As a result, the Flex batteries and the DC network are heavily exploited for profit, leading to a distorting, highly volatile power flow of the DCDS. Careful market rules and regulations should resolve such problems.

## VI. CONCLUSION

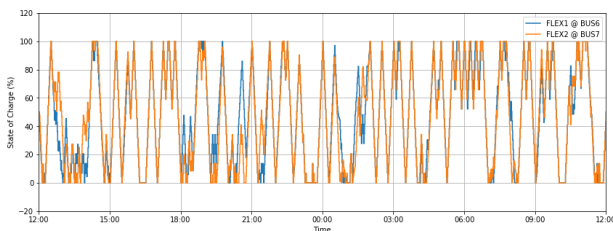
This paper evaluated two promising electricity market designs for DC distribution systems (DCDSs) with an empirical analysis of electric vehicle charging. The first market design is an integrated energy market that only rewards prosumer



(a) total EV charging load vs. converter power intake



(b) nodal voltage



(c) SOC of Flex Batteries

Fig. 5. Strategy S4 with 25 EVs

flexibility implicitly. Such a design demands extensive private information from prosumers. The second design is combined locational flexibility (Flex) and energy market, which guarantees secure DCDS operation while not hindering prosumers' autonomous decision.

As simulation results suggest, an inefficient market design, such as those solely relying on wholesale markets, may lead us to a situation even worse than one without a market. Moreover, with sufficient prosumer data, a centrally operated market has the potential of serving much more prosumers than the base case; in our cases, the market can accommodate 11 times the number of EVs needed by the local households. Finally, in case a market does not expect prosumers to submit private data truthfully, flexibility-based market designs such as LFM can still guarantee supply security without the need for massive network reinforcement. Such a market may generate new challenges, including pricing issues and arbitrage prevention.

This paper represents our first effort to study DCDS market design quantitatively. However, many questions remain unanswered and require future study. First, this paper mainly focuses on the optimal allocation, but future study should propose fair, incentive-compatible pricing schemes for local energy and Flexibility trading. Second, attention is warranted to the locational Flex market design in order to find a balance between supply security (how much Flex in kW and kWh is needed for a certain level of reliability) and economic efficiency

(fair prices). Third, the optimization problem is solved with linearized power flow constraints in this paper; future work should improve the dispatch accuracy by implementing exact (nonlinear) power flow and solving the optimization problem efficiently. Fourth, the proposed empirical analysis is based on deterministic operation, yet the markets should be improved to be able to handle the uncertainties from wholesale markets and local power prosumption. Last but not least, the proposed market designs should be thoroughly tested by more detailed prosumer models, with agent-based simulations that better describe prosumers' preferences or strategic behavior.

#### ACKNOWLEDGMENT

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#### REFERENCES

- [1] L. Mackay, N. H. van der Blij, L. Ramirez-Elizondo, and P. Bauer, "Towards the Universal DC Distribution System," *Electric Power Components and Systems*, vol. 45, no. 10, pp. 1032–1042, 2017.
- [2] L. Piao, L. de Vries, M. de Weerd, and N. Yorke-Smith, "Electricity Market for Direct Current Distribution Systems: Exploring the Design Space," in *15th International Conference on the European Energy Market (EEM)*, 2018.
- [3] —, "Electricity Markets for DC Distribution Systems: Design Options," *Energies*, vol. 12, no. 14, p. 2640, 7 2019. [Online]. Available: <https://www.mdpi.com/1996-1073/12/14/2640>
- [4] P. M. Herder and R. M. Stikkelman, "Methanol-based industrial cluster design: A study of design options and the design process," *Industrial and Engineering Chemistry Research*, vol. 43, no. 14, pp. 3879–3885, 2004.
- [5] S. Stoft, "Market Architecture," in *Power System Economics: Designing Markets for Electricity*. Wiley-IEEE Press, 2002, pp. 82–92.
- [6] L. Piao, M. de Weerd, and L. de Vries, "Electricity Market Design Requirements for DC Distribution Systems," in *IEEE Second International Conference on DC Microgrids (ICDCM)*, 2017, pp. 95–101.
- [7] L. Mackay, L. Ramirez-Elizondo, and P. Bauer, "Current pricing: Avoiding marginal losses in locational marginal prices for DC grids," *2017 IEEE Manchester PowerTech, Powertech 2017*, 2017.
- [8] S. Karambelkar, L. Mackay, S. Chakraborty, L. Ramirez-Elizondo, and P. Bauer, "Distributed Optimal Power Flow for DC Distribution Grids," *IEEE Power and Energy Society General Meeting*, vol. 2018-Augus, pp. 1–5, 2018.
- [9] A. Ramos, "Coordination of Flexibility Contracting in Wholesale and Local Electricity Markets," Ph.D. dissertation, Katholieke Universiteit Leuven, 2017.
- [10] IEEE PES AMPS DSAS Test Feeder Working Group, "European Low Voltage Test Feeder," 2017. [Online]. Available: [http://sites.ieee.org/pes-testfeeders/files/2017/08/European\\_LV\\_Test\\_Feeder\\_v2.zip](http://sites.ieee.org/pes-testfeeders/files/2017/08/European_LV_Test_Feeder_v2.zip)
- [11] Virginia Tech/EPRI, "Measurement Data from Field Monitoring Sites: Single Modules on Poles and PV Plants," 2012. [Online]. Available: [https://dpv.epri.com/media/high\\_pen\\_pv\\_site\\_info\\_for\\_2012\\_june\\_16-24\\_data.pdf](https://dpv.epri.com/media/high_pen_pv_site_info_for_2012_june_16-24_data.pdf)
- [12] R. Verzijlbergh, "The Power of Electric Vehicles: Exploring the value of flexible electricity demand in a multi-actor context (Appendix D: Synthetic driver profile)," Ph.D. dissertation, Delft University of Technology, 2013.
- [13] TenneT, "Settlement Prices." [Online]. Available: [https://www.tennet.org/english/operational\\_management/System\\_data\\_relating\\_processing/settlement\\_prices/index.aspx](https://www.tennet.org/english/operational_management/System_data_relating_processing/settlement_prices/index.aspx)

## APPENDIX

TABLE I  
INDICES, VARIABLES AND PARAMETERS

$t$	(subscript) dispatch interval in set $\mathcal{T}$
$l$	(superscript) inelastic load in set $\mathcal{L}$
$g$	(superscript) photovoltaic (PV) generator in set $\mathcal{G}$
$e$	(superscript) electric vehicle (EV) in set $\mathcal{E}$
$f$	(superscript) Flex resources (batteries) in set $\mathcal{F}$
$w$	(superscript) real-time wholesale energy market
$n$	(superscript) power node of the DCDS in set $\mathcal{N}$
$a$	(superscript) index set for lines in set $\mathcal{A} \subset \mathcal{N} \times \mathcal{N}$
$p_t^e$	power consumption (negative) of EV $e$ charging at time $t$
$p_t^f$	power discharged from Flex $f$ at time $t$
$p_t^w$	power imported from wholesale market at time $t$
$p_t^n$	net power injection (generation) of node $n$ at time $t$
$i_t^n$	net current injection (generation) of node $n$ at time $t$
$r_t^e$	state of charge (SOC) of EV $e$ at time $t$
$r_t^f$	state of charge (SOC) of Flex battery $f$ at time $t$
$v_t^n$	voltage at node $n$ at time $t$
$\lambda_t^n$	real-time (cost-causal) local energy price at time $t$
$f_t^a$	current flow of line $a = (m, n) \in \mathcal{A}$ at time $t$
$\Delta t$	length of each dispatch interval
$c^e$	battery energy capacity of EV $e$
$c^f$	battery energy capacity of Flex $f$
$\eta^e$	charging efficiency of EV $e$
$r_a^e$	initial SOC of EV $e$ upon arrival
$r_d^e$	target SOC of EV $e$ upon departure
$t_a^e$	time of arrival of EV $e$
$t_d^e$	time of departure of EV $e$
$\lambda_t^w$	real-time wholesale energy price at time $t$
$p_t^l$	power consumption (negative) of inelastic load $l$ at time $t$
$p_t^g$	power production of inflexible PV generator $g$ at time $t$
$\omega^a$	line resistance of line $a = (m, n) \in \mathcal{A}$