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ABSTRACT

This contribution puts forward a flexible approach to model the decision-making or design controller for automated driving systems, where tactical-level lane change decisions and control-level accelerations are jointly evaluated based on iteratively solving an online optimization problem. The key idea is that automated vehicles determine lane change times and accelerations in the predicted future to minimize an objective function representing multiple criteria of driving safety, efficiency and comfort. The interactions between controlled vehicles and surrounding vehicles are captured in the objective function. The approach can be applied to model non-cooperative decision-making of autonomous vehicles with optimization of own cost and cooperative behavior of connected vehicles with joint optimization of the collective cost. The problem is formulated as a differential game where automated vehicles make decisions based on the expected behavior of surrounding vehicles. An efficient numerical solution algorithm is used to solve problem. The proposed model performance is demonstrated via numerical examples. The results show that the proposed approach can produce efficient lane-changing maneuvers while obeying safety and comfort requirements. Particularly, the approach generates optimal lane change times and accelerations in the predicted future, including strategic overtaking and cooperative merging scenarios.

INTRODUCTION

Automated driving systems automate partial or full driving tasks depending on the level of automation. Different classes are categorized with respect to sensing and decision-making characteristics. Based on the differences in sensing the driving environment, we distinguish between *autonomous vehicles* and *connected vehicles*. Autonomous vehicles rely solely on on-board sensors based on cyber-physical sensing technologies, such as radar, lidar, machine vision (*1*, *2*). Connected vehicles exchange (state and control) information with each other via Vehicle-to-Vehicle (V2V) communication or with road infrastructure via Vehicle-to-Infrastructure (V2I) communication to improve situation awareness (*3*, *4*, *5*, *6*, *7*, *8*). With respect to control, the controllers or decision-making systems of IVs can be either non-cooperative or cooperative. IVs with *non-cooperative control* strategies make control decisions for their own sake, i.e. they do not consider the responses of surrounding vehicles to the control actions and there is no negotiation nor consensus in the decision-making process (*2*). IVs with *cooperative control* strategies coordinate their behavior and take into account the expected response of other vehicles when making decisions (*7*).

Considerable efforts have been devoted to partial automation that automate longitudinal driving tasks in both autonomous vehicle systems (*1*, *9*, *10*) and connected vehicle systems (*3*, *4*, *5*, *7*, *11*). Automotive studies focused on steering control, given that a desired lane change trajectory based on constant vehicle speed has been decided by the high-level controller (*12*, *13*). But the expected response of surrounding vehicles to the actions of the subject vehicle is not considered and the current approaches are not tractable to connected vehicles. Although many attempts have been made, the issue of when and where it is optimal to change lane remains largely unresolved. The discrete lane change events relative to the continuous vehicle positions and the coupled nature between the lateral and longitudinal vehicle dynamics render the problem much more complex compared to the longitudinal control (*14*, *15*, *16*, *17*, *18*, *19*, *20*).

In this article, we generalize previous work on acceleration control (*2*, *7*) to a flexible mathematical framework to model *fully automated* Lane-changing and Car-following Control Systems (LCCS), where discrete and continuous control variables, i.e. lane change times and accelerations, are jointly evaluated. The approach entails that controlled vehicles make decisions to minimize predicted cost that reflects undesirable situations. The interaction between controlled vehicles and surrounding vehicles is captured in the cost function by including proximity costs. The approach is applied to construct decision-making models or controllers for both *non-cooperative control* systems where controlled vehicles only optimize their own cost and *cooperative control* systems where controlled vehicles coordinate their decisions to optimize the joint cost. To determine the controller behavior, the problem is formulated as a *dynamic game* where controlled vehicles make decisions based on the expected behavior of other vehicles. A problem decomposition technique is employed to reduce the dimensionality of the original problem by introducing a finite number of continuous sub-problems. An iterative numerical solution algorithm based on Pontryagin's Principle is used to solve the formulated problem. The proposed control framework is applied to derive optimal lane change decisions and accelerations for both autonomous and cooperative controllers. The proposed controller properties and their performance are verified at the microscopic level via numerical examples.

In the remainder, we first present the necessary assumptions regarding the proposed model. Then we formulate the control problem under a mathematical framework. After that, we decompose the original problem into a finite number of sub-problems and derive the optimal control strategies. Numerical examples are shown to verify the performance of the proposed approach, followed by a discussion on some properties of the approach. Concluding remarks are presented in the end.

MODELING ASSUMPTIONS

Let us consider the *system* from the perspective of an LCCS controlled vehicle i, of which the state **z** can be described by the positions and speeds of the controlled vehicle and the surrounding vehicles. At time instant t_0, the estimator estimates the positions and speeds of surrounding vehicles from observations either made by its on-board sensors, i.e. autonomous vehicle systems (*2*), or transmitted from other sensors through V2V and/or V2I communications (*7*), i.e. connected vehicle systems. The optimization-based controller uses a system dynamics model to predict the future state in a time horizon T_p, with the current system state as the initial conditions. The optimal control input/variable **u**, e.g. lane change sequences (i.e. time and direction of lane changes) and accelerations, is determined to optimize some criterion or cost reflecting undesirable situations. The optimal control variable determines a unique path, and the low-level vehicle steering, propulsion and braking systems are automatically controlled to follow the path. In a *receding horizon* implementation, only the first sample of the control input is executed by the actuators. As the vehicle maneuvers, the *system* state changes, and the optimal control signal **u** is recalculated with the newest information regarding the system state at regular time intervals, e.g. Δt.

The main assumptions are listed below:

- LCCS scan the driving environment via on-board sensors, e.g. radar or camera, identifying positions and speeds of other vehicles in the vicinity. The problem of state estimation is not covered in this paper.
- LCCS predict the future evolution of the system state, including the expected behavior of surrounding vehicles, using a system dynamics model. We emphasize that the prediction of behaviors of surrounding vehicles needs not to be perfect.
- LCCS make control decisions, i.e. lane choices and accelerations, to fulfill control objectives and the vehicle actuators execute the control decisions automatically.

MATHEMATICAL FRAMEWORK FOR OPTIMAL DECISIONS

In this section, we describe the mathematical control framework that is used to formulate the control problem for LCCS controllers.

System state prediction

For controlled vehicle i, let us define the continuous state vector **z** as $\mathbf{z} = (x_i, v_i, y_i)^T$, where x_i and v_i denotes the longitudinal position and speed of vehicle i and y_i denotes its lateral position. Lateral velocity and heading angle are omitted since lateral vehicle dynamics can be ignored in non-emergency lane change conditions. The continuous lateral position y_i is predicted and implemented based on representative lane change trajectories from driving experiments (*19*), cf. Eq. (5). This enables us to obtain feasible and efficient lane change trajectories under linear representation of system dynamics.

The control vector **u** is defined as $\mathbf{u} = (a_i, \boldsymbol{\tau}_i, \boldsymbol{\delta}_i)^T$, where a_i denotes the longitudinal acceleration. The vector $\boldsymbol{\tau}_i = (\tau_{i,1}, \tau_{i,2}, ..., \tau_{i,n})^T$ with $\tau_{i,k} \in [0, T_p]$ includes the time instants when the controlled vehicle changes its desired lane σ_i, i.e. the time when initiating a lane change. $\boldsymbol{\delta}_i = (\delta_{i,1}, \delta_{i,2}, ..., \delta_{i,n})^T$ with $\delta_{i,k} \in \{-1, 0, 1\}$ denoting the direction of lane change, i.e. $\{-1, 0, 1\} :=$

$\{$change left, no lane change, change right$\}$. Note the desired lane sequence σ_i can be *uniquely* determined by the vector pair $(\boldsymbol{\tau}_i, \boldsymbol{\delta}_i)^T$ with

$$\sigma_i(t) = \begin{cases} \sigma_i(t_0) + \delta_{i,1} & \text{if } t_0 \le t \le t_0 + \tau_{i,1} \\ \sigma_i(t_0) + \sum_{q=1}^{k} \delta_{i,q} & \text{if } t_0 + \tau_{i,k-1} < t \le t_0 + \tau_{i,k}, \forall k \in \{2, ..., n\} \\ \sigma_i(t_0) + \sum_{q=1}^{n} \delta_{i,q} & \text{if } t_0 + \tau_{i,k} < t < t_0 + T_p \end{cases} \quad (1)$$

and hence the vector $\mathbf{u} = (a_i, \boldsymbol{\tau}_i, \boldsymbol{\delta}_i)^T$ is equivalent to $(a_i, \sigma_i)^T$. Equation (1) implies that the desired lane σ_i may change (instantaneously) at time $\tau_{i,k}$.

The controller uses the following kinematic vehicle model to predict its state dynamics:

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{z} = \frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} x_i \\ v_i \\ y_i \end{pmatrix} = \begin{pmatrix} v_i \\ a_i \\ g(\boldsymbol{\tau}_i, \boldsymbol{\delta}_i) \end{pmatrix} = \mathbf{f}(\mathbf{z}, \mathbf{u}) \quad (2)$$

with initial condition of $\mathbf{z}(t_0) = (x_0, v_0, y_0)^T$.

The lateral position of the controlled vehicle is determined by the control input pair $(\boldsymbol{\tau}_i, \boldsymbol{\delta}_i)$ under heuristic rules. It is assumed that the controlled vehicle finishes a lane change in a fixed time window t^{lc}, which is underpinned by the average lane change duration from empirical observations (*21, 22*) In addition, the controlled vehicle maintains at least a short interval t^{inter} on the new desired lane after a lane change before initiating a subsequent lane change. This adds a constraint to the choice of subsequent lane change time instants with:

$$\tau_{i,k+1} \ge \tau_{i,k} + t^{lc} + t^{\text{inter}}, \forall k \in \{1, 2, ..., n-1\} \quad (3)$$

To ensure closed-loop stability on lane choices, the controller predicts and plans a complete lane change trajectory within the prediction horizon, which adds another constraint on the choice of lane change time instant as:

$$\tau_{i,k} \le T_p - t^{lc}, \forall k \in \{1, 2, ..., n\} \quad (4)$$

The lateral trajectory is assumed to follow a trigonometric (cosine) function of time (*19*) and its dynamics can be described by:

$$\frac{\mathrm{d}}{\mathrm{d}t} y_i = g(\boldsymbol{\tau}_i, \boldsymbol{\delta}_i) = \begin{cases} \frac{\delta_{i,k} w \pi}{2t^{lc}} \sin\left(\frac{t - t_0 - \tau_{i,k}}{t^{lc}}\pi\right) & \text{if } t_0 + \tau_{i,k} \le t \le t_0 + \tau_{i,k} + t^{lc}, \forall k \in \{1, 2, ..., n\} \\ 0 & \text{if otherwise} \end{cases} \quad (5)$$

$$\frac{\mathrm{d}}{\mathrm{d}t} y_i = g(\boldsymbol{\tau}_i, \boldsymbol{\delta}_i) = \frac{\delta_{i,k} w \pi}{2t^{lc}} \sin\left(\frac{t - t_0 - \tau_{i,k}}{t^{lc}}\pi\right) \quad (6)$$

where w is the lane width and y_0 is the initial lateral position of the vehicle in the middle of the current lane.

The controlled vehicle uses the same model and the decision-making algorithm described in the following section to predict the behavior of other vehicles j $(j \ne i)$ in its vicinity. Note that this prediction needs not to be perfect, i.e. there can be errors in the predicted behavior in the open loop and the actual vehicle behavior in the closed loop. The feedback nature of the receding horizon framework corrects the mismatch in the prediction (*2*).

As we will show in the ensuing, this model allows the efficient derivation of optimal lane-changing strategies and car-following laws that can anticipate the changes when traveling in different lanes in the future.

Cost optimization

The optimal control problem for LCCS at time t_0 is to seek the optimal control $\mathbf{u}^*$ over a prediction horizon T_p that drives the system along a trajectory such that the predicted cost J is minimized, i.e.:

$$\mathbf{u}^*_{[t_0,t_f)} = \arg\min_{\mathbf{u}} J(\mathbf{z}, \mathbf{u}, t|\mathbf{z}(t_0)) \tag{7}$$

$$J(\mathbf{z}(t), \mathbf{u}(t), t|\mathbf{z}(t_0)) = \int_{t_0}^{t_f} \mathcal{L}(\mathbf{z}(t), \mathbf{u}(t), t)\mathrm{d}t \tag{8}$$

subject to the state dynamic equation of (2) and the initial condition $\mathbf{z}(t_0) = \hat{\mathbf{z}}_0$, and other constraints on state and control variables. In Eq. (8), $t_f = t_0 + T_p$ denotes the terminal time. $\mathcal{L}$ denotes the running cost, describing the costs incurred during an infinitesimal period $[t, t + \Delta t)$. Other constraints on the state and control vectors include:

- Speed constraint: the controlled vehicle drives under a maximum speed of v_{max}.

$$v_i \in [0, v_{\text{max}}] \tag{9}$$

- Acceleration constraint: the accelerations are limited to some admissible bounds.

$$a_i \in [a_{\text{min}}, a_{\text{max}}] \tag{10}$$

Cost specification for autonomous vehicles

For the non-cooperative decision-making of autonomous vehicles, the running cost function for longitudinal movement on a certain lane is specified as:

$$\begin{aligned}\mathcal{L}(\mathbf{z}(t), \mathbf{u}(t), t) \quad = \quad & \underbrace{\frac{\beta_{\text{safe}}}{s_{i,\sigma_i}} \Delta v^2_{i,\sigma_i} \Theta(-\Delta v_{i,\sigma_i})}_{\text{safety}} + \underbrace{\beta_{\text{eq}}(v^e(s_{i,\sigma_i}) - v_i)^2}_{\text{equilibrium}} + \underbrace{\beta_{\text{ctrl}} a_i^2}_{\text{control}} \\ & + \underbrace{\beta_{\text{eff}}(v^d - v^a_{\sigma_i})^2}_{\text{travel efficiency}} + \underbrace{\beta_{\text{route}} \exp(d^0/d^{\text{end}}_{i,\sigma_i})}_{\text{route}} + \underbrace{\beta_{\text{pref}} h(\sigma_i)}_{\text{lane preference}} + \underbrace{\beta_{\text{swit}} L_{\text{swit}}(\sigma_i)}_{\text{lane switch}}\end{aligned} \tag{11}$$

where

$$s_{i,\sigma_i} = x^l_{\sigma_i} - x_i - l \tag{12}$$

$$\Delta v_{i,\sigma_i} = v^l_{\sigma_i} - v_i \tag{13}$$

$x^l_{\sigma_i}$ and $v^l_{\sigma_i}$ are the position and speed of the lead vehicle on lane σ_i. s_{i,σ_i} and $\Delta v^l_{\sigma_i}$ are the longitudinal gap and relative speed with respect to the lead vehicle on lane σ_i. l is the vehicle length.

v^d is the free/desired speed and s^0 is the minimum gap at standstill conditions. v^e denotes the local equilibrium speed determined by local density, which is a non-decreasing function of gap:

$$v^e(s_{i,\sigma_i}) = \begin{cases} v^d & \text{if } s_{i,\sigma_i} > s_f \\ \frac{s_{i,\sigma_i} - s^0}{t^d} & \text{if } s_{i,\sigma_i} \le s_f \end{cases} \tag{14}$$

t^d denotes the desired time gap and $s_f = v^d t^d + s^0$ is a gap threshold defining the *spatial range of interaction* with predecessor. β_{safe}, β_{eff}, β_{eq}, β_{route}, β_{pref}, β_{swit}, β_{ctrl} are positive weight factors. $v^a_{\sigma_i}$ is the anticipated speed on lane σ_i, which is the minimum of predecessor speed $v^l_{\sigma_i}$ and the speed limit $v^{\text{lim}}_{\sigma_i}$ on lane σ_i:

$$v^a_{\sigma_i} = \min(v^{\text{lim}}_{\sigma_i}, v^l_{\sigma_i}) \tag{15}$$

The running cost function (11) implies the controller makes some trade-off among *multi-criteria*:

• The *safety* cost stems from approaching the predecessor, i.e. when $\Delta v_{i,\sigma_i} < 0$, and it ensures a large penalty when the controlled vehicle approaches its preceding vehicle at small gaps.

• The *equilibrium* cost implies that the vehicle tends to relax to the local equilibrium speed determined by the distance to the predecessor. The equilibrium and safety costs are the proximity cost terms reflecting the interaction with the preceding vehicle.

• The *control* cost is represented by penalizing large accelerations or decelerations. It also reflects driving comfort to some extent.

• The *travel efficiency* cost stems from deviation of the preceding vehicle speed from the free/desired speed v_0. Note that when the preceding vehicle is driving at a speed higher than the desired speed, the efficiency cost equals zero.

• The *route* cost stems from driving on a lane that does not lead to the destination, i.e. approaching a dead end to the destination. $d^{\text{end}}_{i,\sigma_i}$ is the distance between the location of the controlled vehicle to the dead end position x_{end}:

$$d^{\text{end}}_{i,\sigma_i} = x_{\text{end}} - x_i \tag{16}$$

This cost increases exponentially as distance to the dead end decreases to below d^0. The route cost only occurs when the $d^{\text{end}}_{i,\sigma_i}$ is smaller than a certain distance d_r, which is around 1 km as indicated by traffic signs or navigation systems.

• The *lane preference* cost can be described as a function of the lane number. We start with a simple formulation that captures the keep-right directive in most European countries:

$$h(\sigma_i) = n_{\text{total}} - \sigma_i \tag{17}$$

where n_{total} denotes the total lane number. σ_i increases from left to right with $\sigma_i = 1$ for the leftmost lane. Equation (17) reflects the keep-right directive, since the cost increases when moving from right to left.

• The *lane switch* cost stems from the change in the desired lane. It reflects some discomfort caused by lateral accelerations or the driver's unwillingness to change lane. It also guarantees that the controlled vehicle will not change lane due to small incentives such as marginal speed gains.

$$L_{\text{swit}}(t) = \begin{cases} 1 & \text{if } \lim_{\Delta t \to 0} \sigma_i(t+\Delta t) \neq \lim_{\Delta t \to 0} \sigma_i(t-\Delta t) \\ 0 & \text{if } \lim_{\Delta t \to 0} \sigma_i(t+\Delta t) = \lim_{\Delta t \to 0} \sigma_i(t-\Delta t) \end{cases} \tag{18}$$

Cooperative control of connected vehicles

The aforementioned basic formulation of the controller pertains to non-cooperative control systems for autonomous vehicles. It can be easily extended to cooperative control of connected vehicles where controlled vehicles exchange information and cooperate with each other using V2V communication. In the remaining of this section, we show how to formulate the cooperative control problem with two vehicles, though the extension to more vehicles is straightforward (7). The designed connected vehicles exchange their state and control information and make control decisions together to minimize a joint cost function representing the collective situations of all vehicles.

Expansion of state and control space

Consider two connected vehicles i and j interacting with each other with the possibility of changing lanes. We consider a joint state vector of $\mathbf{z} = (x_i, v_i, y_i, x_j, v_j, y_j)^T$ and a joint control vector of $\mathbf{u} = (a_i, \boldsymbol{\tau}_i, \boldsymbol{\delta}_i, a_j, \boldsymbol{\tau}_j, \boldsymbol{\delta}_j)^T$. The state and control space are expanded. The system dynamics model becomes:

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{z} = \frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} x_i \\ v_i \\ y_i \\ x_j \\ v_j \\ y_j \end{pmatrix} = \begin{pmatrix} v_i \\ a_i \\ g(\boldsymbol{\tau}_i, \boldsymbol{\delta}_i) \\ v_j \\ a_j \\ g(\boldsymbol{\tau}_j, \boldsymbol{\delta}_j) \end{pmatrix} = \mathbf{f}(\mathbf{z}, \mathbf{u}) \tag{19}$$

The desired lane sequence is determined by Eq. (1) for both vehicles and the heuristics lateral position dynamics (5) and constraints on the lane change time instants (3, 4) are the same for both vehicles.

Joint cost function

The optimal control problem for the cooperative controller remains the same as in Eqs. (7,8), except that the dimension of the problem is expanded compared to the non-cooperative controller. The joint running cost function is formulated as:

$$\mathcal{L}(\mathbf{z}(t), \mathbf{u}(t), t) = \sum_{q=i,j} \left(\begin{array}{l} \underbrace{\frac{\beta_{\text{safe}}}{s_{\sigma_q}} \Delta v_{\sigma_q}^2 \Theta(-\Delta v_{\sigma_q})}_{\text{safety}} + \underbrace{\beta_{\text{eq}}(v^e(s_{\sigma_q}) - v_q)^2}_{\text{equilibrium}} + \underbrace{\beta_{\text{ctrl}} a_q^2}_{\text{control}} + \\ \underbrace{\beta_{\text{eff}}(v^d - v_{\sigma_q}^a)^2}_{\text{travel efficiency}} + \underbrace{\beta_{\text{route}} \exp(d_0 / d_{q,\sigma_q}^{\text{end}})}_{\text{route}} + \underbrace{\beta_{\text{pref}} h(\sigma_q)}_{\text{lane preference}} + \underbrace{\beta_{\text{swit}} L_{\text{swit}}(\sigma_q)}_{\text{lane switch}} \end{array} \right) \tag{20}$$

Equation (20) shows that the connected vehicles aim at minimizing a joint cost function consisting of the safety, equilibrium, control, efficiency, route, lane preference, and lane switch costs of all vehicles.

DERIVATION OF OPTIMAL ACCELERATIONS AND LANE CHANGE SEQUENCES

In this section, we present details of the problem decomposition scheme and the optimization algorithms.

Problem decomposition

In general there is an *infinite* number of possible $(\boldsymbol{\tau}_i, \boldsymbol{\delta}_i)$ pairs of lane change strategies for controlled vehicle i during the prediction horizon, which makes the problem difficult to solve. The dimensionality of the problem is reduced by differentiating the update frequency of lane change decisions and accelerations. Although the lane change decisions and longitudinal accelerations are predicted jointly, the lane change decisions take place at a lower frequency compared to accelerations. The pair $(\boldsymbol{\tau}_i, \boldsymbol{\delta}_i)$ is evaluated at $p\ Hz$, while the accelerations are evaluated at a frequency 5 to 10 times higher than the lane change decisions. Hence the choice of $\tau_{i,k} \in \boldsymbol{\tau}_i$ is limited to a *finite* set of $\tau = \{0, \frac{1}{p}, \frac{2}{p}, ..., \frac{m}{p}\}$, with m the largest integer less than or equal to $p(T_p - t^{lc})$. The choice of $\delta_{i,k}$ is constrained to the set of $\Psi = \{-1, 0, 1\}$. With the constraints of (3,4), this significantly reduces the dimensionality of the considered problem. In addition, for each chosen feasible pair $(\boldsymbol{\tau}_i, \boldsymbol{\delta}_i)$ with n lane change decision points, a lane sequence $\sigma(t)$ can be determined with Eq. (1). Hence, the original problem of Eqs. (7,8,11) is translated into several equivalent sub-problems of

$$a^*_{i,[t_0,t_f)} = \arg\min_{a_i} J(x_i, v_i, a_i|\bar{\mathbf{z}}(t_0), \boldsymbol{\tau}_i, \boldsymbol{\delta}_i) \tag{21}$$

$$\begin{aligned} J(\bar{\mathbf{z}}, \bar{\mathbf{u}}|\bar{\mathbf{z}}(t_0), \boldsymbol{\tau}_i, \boldsymbol{\delta}_i) \quad = \quad & \int_{t_0}^{\tau_{i,1}} \mathcal{L}(\bar{\mathbf{z}}, \bar{\mathbf{u}}|\bar{\mathbf{z}}(t_0), \tau_{i,1}, \delta_{i,1})\mathrm{d}t + \int_{\tau_{i,1}}^{\tau_{i,2}} \mathcal{L}(\bar{\mathbf{z}}, \bar{\mathbf{u}}|\bar{\mathbf{z}}(\tau_{i,1}), \tau_{i,2}, \delta_{i,2})\mathrm{d}t + ... \\ & + \int_{\tau_{i,n-1}}^{\tau_{i,n}} \mathcal{L}(\bar{\mathbf{z}}, \bar{\mathbf{u}}|\bar{\mathbf{z}}(\tau_{i,n-1}), \tau_{i,n}, \delta_{i,n})\mathrm{d}t + \int_{\tau_{i,n}}^{t_f} \mathcal{L}(\bar{\mathbf{z}}, \bar{\mathbf{u}}|\bar{\mathbf{z}}(\tau_{i,n}))\mathrm{d}t \end{aligned} \tag{22}$$

where $\bar{\mathbf{z}} = (x_i, v_i)^T$ denotes the sub-state variable and $\bar{\mathbf{u}} = a_i$ denote the sub-control variable of the controlled vehicle. The sub-state dynamic equation becomes:

$$\frac{\mathrm{d}}{\mathrm{d}t}\bar{\mathbf{z}} = \frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} x_i \\ v_i \end{pmatrix} = \begin{pmatrix} v_i \\ a_i \end{pmatrix} = \bar{\mathbf{f}}(\bar{\mathbf{z}}, \bar{\mathbf{u}}) \tag{23}$$

Equation (22) implies that the original problem under a certain lane change strategy $(\boldsymbol{\tau}_i, \boldsymbol{\delta}_i)$ with n lane change decisions is transcribed into $n+1$ sub-problems. The sequential sub-problems are coupled via the terminal and initial conditions for sub-state and sub-control variables, i.e. the terminal conditions of the sub-state and sub-control variables of one sub-problem is the initial conditions of the sub-state and sub-control variables of the following one. The transcribed sub-problems are continuous in the sub-state and sub-control variables and hence are finite horizon optimal control problems with known parameters and exogenous variables, including lane sequence and surrounding vehicles' state.

With this problem decomposition technique, we can compute the optimal acceleration for each sub-problem and integrate the running cost function over the complete prediction horizon $[t_0, t_f]$ to calculate the predicted cost for each lane change strategy. After that, we can search the optimal lane change strategy or equivalently desired lane sequence among a *finite* number of possibilities that gives the minimum cost. The evaluation frequency p determines the number of possible lane change trajectories within the prediction horizon from current time and consequently the total number of sub-problems proportionally. A lower p leads to fewer feasible lane change strategies but may result in sub-optimal solutions to the original problem, while increasing the evaluation frequency of lane change decisions increases the number of sub-problems *finitely*.

In the following, we present an efficient numerical solution to the sub-problems.

Solution approach to sub-problems based on Pontryagin's Principle

Without going too much into details, the solution approach based on Pontryagin's Principle entails defining the Hamiltonian $\mathcal{H}$ as follows:

$$\mathcal{H}(\bar{\mathbf{z}}, \bar{\mathbf{u}}, \boldsymbol{\lambda}) = \mathcal{L}(\bar{\mathbf{z}}, \bar{\mathbf{u}}) + \boldsymbol{\lambda}^T \cdot \bar{\mathbf{f}}(\bar{\mathbf{z}}, \bar{\mathbf{u}}) \tag{24}$$

where $\boldsymbol{\lambda}$ denotes the so-called *co-state* or *marginal cost* of the state $\bar{\mathbf{z}}$, which reflects the relative extra cost incurred due to a small change on the state.

Using the Hamiltonian, we can derive the *necessary condition* for optimality with $\bar{\mathbf{u}}^* = \arg\min_{\bar{\mathbf{u}}} \mathcal{H}(\bar{\mathbf{z}}, \bar{\mathbf{u}}, \boldsymbol{\lambda})$, s.t. $\bar{\mathbf{u}} \in \mathcal{U}$, where $\mathcal{U}$ denotes the *admissible control space*. This requirement will enable expressing the optimal control input $\bar{\mathbf{u}}^*$ as a function of the state $\bar{\mathbf{z}}$ and the co-state $\boldsymbol{\lambda}$. Furthermore, the co-state has to satisfy the following dynamic equation:

$$-\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{\lambda} = \frac{\partial \mathcal{H}}{\partial \bar{\mathbf{z}}} = \frac{\partial \mathcal{L}}{\partial \bar{\mathbf{z}}} + \boldsymbol{\lambda}^T \cdot \frac{\partial \bar{\mathbf{f}}}{\partial \bar{\mathbf{z}}} \tag{25}$$

subject to the terminal conditions of $\boldsymbol{\lambda}(t_f)$ (*23*, *24*).

The optimal control problem now is relaxed to solve the set of ordinary differential equations of (2) and (25). An iterative algorithm has been proposed to solve the two-point boundary value problem efficiently (*24*).

Optimal acceleration control law and non-cooperative lane change decision

Applying the aforementioned solution approach based on Pontryagin's Principle yields the following optimal acceleration control law for vehicle i employing non-cooperative control strategy:

$$a_i^* = -\frac{\lambda^{v_i}}{2\beta_{\text{ctrl}}} \tag{26}$$

where λ^{v_i} denotes the marginal cost of speed v_i. The optimal acceleration shows the direction in which the cost decreases *fastest*. The co-state dynamics of the controller satisfies the following differential equation:

$$-\frac{\mathrm{d}}{\mathrm{d}t}\lambda^{x_i} = \frac{\partial \mathcal{H}}{\partial x_i} = \frac{\beta_{\text{route}} d_0}{(d_{i,\sigma_i}^{\text{end}})^2}\exp(d_0/d_{i,\sigma_i}^{\text{end}}) + \frac{\beta_{\text{safe}}}{s_{i,\sigma_i}^2}\Delta v_{i,\sigma_i}^2 \Theta(-\Delta v_{i,\sigma_i}) - \frac{2\beta_{\text{eq}}}{t_d}\left(v_e(s_{i,\sigma_i}) - v_i\right) \tag{27}$$

$$-\frac{\mathrm{d}}{\mathrm{d}t}\lambda^{v_i} = \frac{\partial \mathcal{H}}{\partial v_i} = -\frac{2\beta_{\text{safe}}}{s_{i,\sigma_i}}\Delta v_{i,\sigma_i}\Theta(-\Delta v_{i,\sigma_i}) + 2\beta_{\text{eq}}(v_i - v_e(s_{i,\sigma_i})) + \lambda^{x_i} \tag{28}$$

Using the iterative numerical schemes can solve the state dynamic equation and co-state dynamic equation efficiently (*24*).

Solving the state and co-state dynamic equations yields the optimal acceleration a_i^* and the state trajectory $\bar{\mathbf{z}}^*$ for the sub-problem. Hence the predicted (optimal) cost for a given $(\boldsymbol{\tau}_i, \boldsymbol{\delta}_i)$ pair can be calculated with:

$$J(a_i^*(t)|\boldsymbol{\tau}_i, \boldsymbol{\delta}_i) = \int_{t_0}^{t_f} \mathcal{L}(\bar{\mathbf{z}}^*(t), a_i^*(t), t|\bar{\mathbf{z}}(t_0), \boldsymbol{\tau}_i, \boldsymbol{\delta}_i)\mathrm{d}t \tag{29}$$

and the optimal lane sequence is determined by:

$$(\boldsymbol{\tau}_i^*, \boldsymbol{\delta}_i^*) = \arg \min_{\tau_{i,k}\in\tau,\ \delta_{i,k}\in\Psi} J^*(a_i^*(t)|\bar{\mathbf{z}}(t_0), \boldsymbol{\tau}_i, \boldsymbol{\delta}_i) \tag{30}$$

Equation (30) entails that the LCCS controller evaluates the predicted costs on all possible lane change scenarios (including remaining on the same lane) and chooses the optimal lane change decision points or equivalently the optimal desired lane sequence with the minimum predicted cost. Due to the limited number of lane change possibilities, exhaustive enumeration is used to find the optimal lane change strategy that minimizes the predicted cost (29).

Cooperative lane change strategies

Using the same problem decomposition technique, we arrive at the problem reformulation for the cooperative controller under the lane change strategy of $(\boldsymbol{\tau}_i, \boldsymbol{\delta}_i, \boldsymbol{\tau}_j, \boldsymbol{\delta}_j)$ as:

$$(a_i^*, a_j^*)_{[t_0,t_f)} = \arg\min_{a_i,a_j} J(x_i, v_i, a_i, x_j, v_j, a_j|\bar{\mathbf{z}}(t_0), \boldsymbol{\tau}_i, \boldsymbol{\delta}_i, \boldsymbol{\tau}_j, \boldsymbol{\delta}_j) \tag{31}$$

$$J(\bar{\mathbf{z}}, \bar{\mathbf{u}}|\bar{\mathbf{z}}(t_0), \boldsymbol{\tau}_i, \boldsymbol{\delta}_i, \boldsymbol{\tau}_j, \boldsymbol{\delta}_j) = \int_{t_0}^{t_f} \mathcal{L}(\bar{\mathbf{z}}, \bar{\mathbf{u}}|\bar{\mathbf{z}}(t_0), \boldsymbol{\tau}_i, \boldsymbol{\delta}_i, \boldsymbol{\tau}_j, \boldsymbol{\delta}_j)\mathrm{d}t \tag{32}$$

where the expanded sub-state variable $\bar{\mathbf{z}} = (x_i, v_i, x_j, v_j)^T$ and the sub-control variable of $\bar{\mathbf{u}} = (a_i, a_j)^T$ satisfy the following dynamic equation:

$$\frac{\mathrm{d}}{\mathrm{d}t}\bar{\mathbf{z}} = \frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} x_i \\ v_i \\ x_j \\ v_j \end{pmatrix} = \begin{pmatrix} v_i \\ a_i \\ v_j \\ a_j \end{pmatrix} = \bar{\mathbf{f}}(\bar{\mathbf{z}}, \bar{\mathbf{u}}) \tag{33}$$

and the same initial conditions and constraints on state and control variables. The transcribed sub-problems are continuous in the sub-state $\bar{\mathbf{z}} = (x_i, v_i, x_j, v_j)^T$ and sub-control vector of $\bar{\mathbf{u}} = (a_i, a_j)^T$ of the controlled vehicle, and hence are finite horizon optimal control problems with known parameters and exogenous variables.

The sub-problem can be solved using Pontryagin's Principle. Taking the necessary conditions, we arrive at the optimal acceleration laws for both vehicles as:

$$a_i^* = -\frac{\lambda^{v_i}}{2\beta_{\text{ctrl}}} \text{ and } a_j^* = -\frac{\lambda^{v_j}}{2\beta_{\text{ctrl}}} \tag{34}$$

Equation (34) shows the optimal accelerations of the two connected vehicles are negatively proportional to the co-state or marginal costs of their speeds respectively, and the co-state dynamic equations can be determined in the same way as the non-cooperative case with Eqs. (25,20).

The following mathematical programme shows how to compute the optimal lane change strategy for connected vehicles. Solving the state and co-state dynamic in each sub-problem gives us the optimal trajectory driven by the optimal accelerations (a_i^*, a_j^*) under the corresponding lane change strategy $(\boldsymbol{\tau}_i, \boldsymbol{\delta}_i, \boldsymbol{\tau}_j, \boldsymbol{\delta}_j)$. This allows us to calculate the predicted cost under this lane change strategy with

$$J(a_i^*, a_j^*|\bar{\mathbf{z}}(t_0), \boldsymbol{\tau}_i, \boldsymbol{\delta}_i, \boldsymbol{\tau}_j, \boldsymbol{\delta}_j) = \int_{t_0}^{t_f} \mathcal{L}(a_i, a_j|\bar{\mathbf{z}}(t_0), \boldsymbol{\tau}_i, \boldsymbol{\delta}_i, \boldsymbol{\tau}_j, \boldsymbol{\delta}_j)\mathrm{d}t \tag{35}$$

and the optimal lane change sequences for connected vehicles can be determined by:

$$(\boldsymbol{\tau}_i^*, \boldsymbol{\delta}_i^*, \boldsymbol{\tau}_j^*, \boldsymbol{\delta}_j^*) = \arg \min_{\tau_{i,k},\tau_{j,k}\in\tau \quad \delta_{i,k},\delta_{j,k}\in\Psi} J^*(a_i^*(t)|\bar{\mathbf{z}}(t_0), \boldsymbol{\tau}_i, \boldsymbol{\delta}_i, \boldsymbol{\tau}_j, \boldsymbol{\delta}_j) \tag{36}$$

Note that the problem of (35,36) forms a *dynamic cooperative game*, where connected vehicles, also known as *players*, coordinate their behavior and seek the optimal strategy to minimize the *joint* cost (25). The cooperative formulation also shows the inter-dependency of lane change decisions and accelerations, since the predicted cost is determined by both the desired lane sequence and the acceleration trajectory. The desired lane sequence determines *whether*, *when*, and *how* the controlled vehicles interact with each other, e.g. whether they will travel on the same lane.

MODEL PERFORMANCE VERIFICATION WITH NUMERICAL EXAMPLES

To verify the performance of the proposed approach, we carry out several numerical experiments for representative scenarios in this section. The main objective is to show that the proposed approach leads to working algorithms and generates plausible lane change decisions and accelerations. While acknowledging the chosen scenarios are a subset of reality, they demonstrate the fundamental workings of the approach and show face validity in the model/controller behavior.

The following model/controller parameters are used in the numerical experiments unless stated otherwise: $v^d = 30$ m/s, $t^d = 1.2$ s, $s^0 = 2$ m, $t^{lc} = 5$ s, $t^{\text{inter}} = 2$ s, $p = 1$ Hz, $T_p = 8$ s, $(\beta_{\text{safe}}, \beta_{\text{eq}}, \beta_{\text{eff}}, \beta_{\text{pref}}, \beta_{\text{swit}}, \beta_{\text{ctrl}})^T = (2, 0.02, 0.1, 1, 1, 0.5)^T$, $l = 4$ m, $d^0 = 350$ m, $v_{\max} = 40$ m/s, $a_{\max} = 2$ m/s^2, $a_{\min} = -8$ m/s^2.

Numerical experimental design

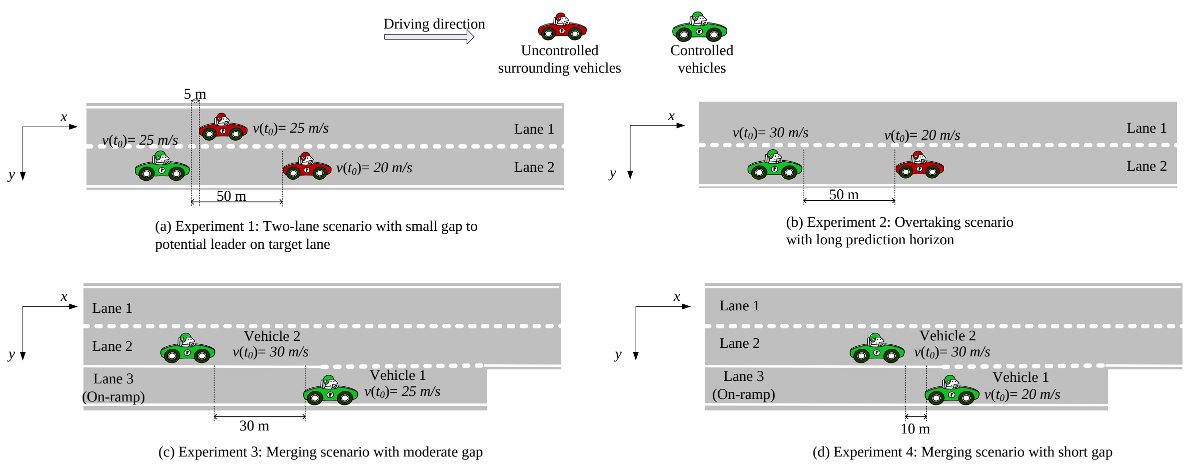

FIGURE 1 Experimental scenarios

In Experiment 1, we verify whether the proposed model can correctly predict lane changes that are preferred in the future. The scenario is shown in Figure 1(a). The controlled vehicle starts with an initial speed of 25 m/s on the right lane, 50 m behind the leader who travels at a constant speed of 20 m/s. On the left lane, another uncontrolled surrounding vehicle travels at a constant speed of 25 m/s but with a short gap of only 5 m in front of the controlled vehicle. It is ideal for the controlled vehicle to travel on the left lane to gain speed, but an immediate lane change will result in a very close distance to the leader on the left lane. Hence it is expected that a delayed lane change leads to better situation for the controlled vehicle.

In Experiment 2, we verify whether the approach can predict multiple lane change decisions of one controlled vehicle in a longer prediction horizon. We increase the prediction horizon to 15 seconds to allow two lane change decisions in the prediction horizon under the constraints (3,4). The scenario is shown in Figure 1(b). The controlled vehicle starts at the same location of $x_1 = 0$ m on the right lane, with an initial speed of 30 m/s. The leader on the right lane travels with a constant speed of 20 m/s. No surrounding vehicles prevail on the left lane. The controlled vehicle is expected to change left to reduce the risk of collision and after it passes the right lane vehicle it should change right under the *keep-right* traffic rule.

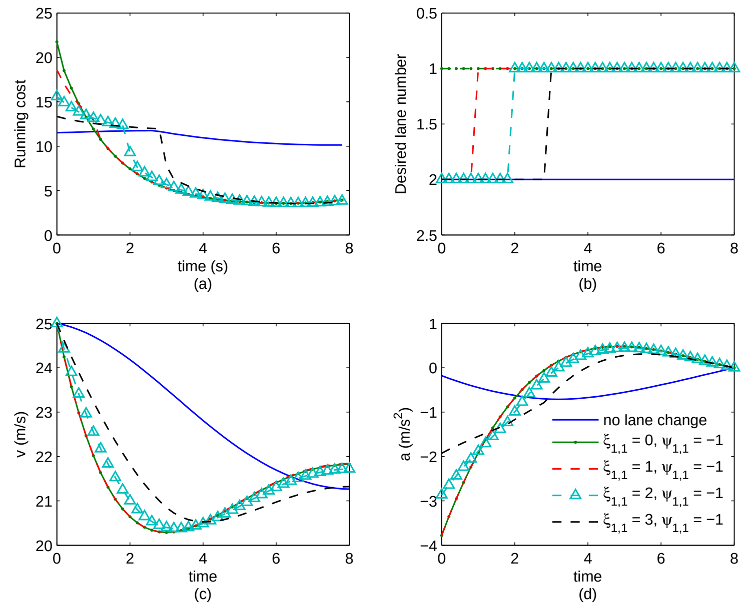

FIGURE 2 Experiment 1: (a) running cost; (b) desired lane number; (c) speed and (d) acceleration.

In Experiment 3 and 4, we examine the cooperation between two controlled vehicles under merging scenarios, where interaction and cooperation between merging vehicles and mainline vehicles are observed in real traffic (*15, 18*). The network is a two-lane highway with an on-ramp. In Experiment 3, the merging vehicle (vehicle 1) starts at the on-ramp at $x_1 = 0$ m with an initial speed of 25 m/s, as shown in Figure 1(c). The end of the on-ramp is at $x_{\text{end}} = 350$ m. The mainline vehicle (vehicle 2) travels on the right lane of the main highway, at the position of $x_2 = -30$ m and initial speed of 30 m/s. We examine how different lane change strategies influence the individual and total costs of the two controlled vehicles. In Experiment 4, we make the situation more demanding for both controlled vehicles to explore the benefits of cooperation. The scenario is shown in Figure 1(d). All other conditions are the same as in Experiment 3, only the initial gap between the two controlled vehicles is reduced to 10 meters.

The temporal evolution of running costs, speeds, desired lane sequences, and accelerations will be used to examine the behavior of the controlled vehicles. The predicted running costs and total cost are used to compare the performance of the controlled vehicles. The predicted (total) cost is the optimal cost-to-go, i.e. J^*. The predicted running costs are the integral of different running cost terms in the running cost function (11) from the current time t_0 to the terminal time t_f.

Delayed lane change (Experiment 1)

In Experiment 1, the changed initial conditions still provide incentives for the controlled vehicle to change lane but make it difficult to change lane immediately. The predicted running cost terms and total cost of different decisions are shown in TABLE 1, while the temporal evolution of running cost $\mathcal{L}$, desired lane sequence, speed and acceleration are shown in Figure 2(a)(b)(c)(d).

The leader on the right lane has a lower speed, hence the controlled vehicle has more desire

TABLE 1 Predicted running cost terms and total cost under different lane change sequences for Experiment 1.

Experiment 2	$\tau_{1,1} = 0, \delta_{1,1} = -1$	$\tau_{1,1} = 1, \delta_{1,1} = -1$	$\tau_{1,1} = 2, \delta_{1,1} = -1$	$\tau_{1,1} = 3, \delta_{1,1} = -1$	No LC
Safety cost	0.00	0.58	0.82	1.17	4.27
Equilibrium cost	18.99	11.12	7.98	7.02	2.53
Efficiency cost	20.00	27.50	35.00	42.50	80.00
Lane preference cost	8.00	7.00	6.00	5.00	0.00
Lane switch cost	0.20	0.20	0.20	0.20	0.20
Control cost	5.98	5.98	4.71	3.21	1.03
Total cost J^*	53.17	**52.39***	54.72	59.12	87.84

[1]: No lane change.

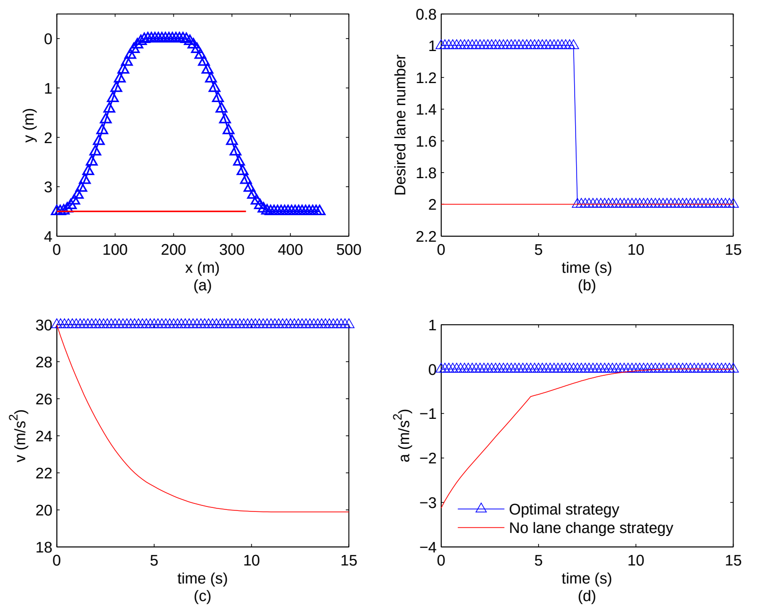

FIGURE 3 Experiment 2: (a) vehicle path in (x, y) plane, (b) desired lane sequence, (c) speed and (d) acceleration of optimal strategy with $\tau_{1,1} = 0, \delta_{1,1} = -1, \tau_{1,2} = 7, \delta_{1,2} = 1$ and no lane changes strategy.

to change lane to reduce efficiency cost as well as safety cost. However, the initial gap to the leader on the left lane is only 5 meter. This corresponds to an equilibrium speed of 2.5 m/s according to Eq. (14), much lower than the initial speed of 25 m/s. Changing lane immediately will result in a higher equilibrium cost and the controlled vehicle would have to lower its speed substantially to match the available gap to the leader on the left lane. TABLE 1 clearly shows that staying longer in the right lane results in higher safety cost and efficiency cost, while changing lane immediately leads to higher equilibrium cost and lane preference cost. The trade-off among the cost terms points out that the optimal strategy is to wait one second and then change left rather than to change lane immediately. Note that most utility-based lane change decision models fail to predict this delayed lane change.

TABLE 2 Predicted (total) cost under different lane change sequences of Experiment 2.

Experiment 2	$\tau_{1,2}=7, \delta_{1,2}=1$	$\tau_{1,2}=8, \delta_{1,2}=1$	$\tau_{1,2}=9, \delta_{1,2}=1$	$\tau_{1,2}=10, \delta_{1,2}=1$	$\delta_{1,2}=0$ [2]
$\tau_{1,1}=0, \delta_{1,1}=-1$	**7.4***	8.4	9.4	10.4	15.2
$\tau_{1,1}=1, \delta_{1,1}=-1$	infeasible[3]	21.76	22.76	23.76	28.56
$\tau_{1,1}=2, \delta_{1,1}=-1$	infeasible	infeasible	36.55	37.54	42.34
$\tau_{1,1}=3, \delta_{1,1}=-1$	infeasible	infeasible	infeasible	51.45	56.24
$\tau_{1,1}=4, \delta_{1,1}=-1$	infeasible	infeasible	infeasible	infeasible	69.51
$\tau_{1,1}=5, \delta_{1,1}=-1$	infeasible	infeasible	infeasible	infeasible	81.65
$\tau_{1,1}=6, \delta_{1,1}=-1$	infeasible	infeasible	infeasible	infeasible	92.83
$\tau_{1,1}=7, \delta_{1,1}=-1$	infeasible	infeasible	infeasible	infeasible	103.3
$\tau_{1,1}=8, \delta_{1,1}=-1$	infeasible	infeasible	infeasible	infeasible	113.1
$\tau_{1,1}=9, \delta_{1,1}=-1$	infeasible	infeasible	infeasible	infeasible	122.4
$\tau_{1,1}=10, \delta_{1,1}=-1$	infeasible	infeasible	infeasible	infeasible	131.3
$\delta_{1,1}=0$ [1]	-	-	-	-	166.95

[1]: no lane change. [2]: no secondary lane change. [3]: all infeasible combinations are due to violation of constraint (3).

Overtaking (Experiment 2)

Now we increase the prediction horizon to 15 seconds so that multiple lane change decisions within the prediction horizon are possible and test whether the controller generates the expected behavior in a simple overtaking scenario. The leader on the right lane travels with a speed of 20 m/s, while the controlled vehicle starts with the desired speed of 30 m/s, 50 meters behind the leader. No vehicle is traveling on the left lane. We evaluate different lane change strategies (staying on the same lane, only changing left, and changing left first and then right) at different decision time instants. The resulting cost of different strategies are summarized in TABLE 5.2. The infeasible combinations are situations that violate constraints of (3, 4), i.e. either the controller cannot predict a full lane change process within the prediction horizon or a subsequent lane change occurs before the initial one is completed.

When the controlled vehicle stays in the right lane, it incurs safety cost, equilibrium cost and efficiency cost. These terms demand the vehicle to decelerate to match the speed of the leader on the right lane. The total cost of this strategy is 166.95. The vehicle path in the (x, y) plane, desired lane sequence, speed and acceleration profiles of the follow-the-leader or no-lane-change strategy are shown in red lines in Figure 3.

It is clear from TABLE 5.2 that the longer the controlled vehicle stays behind the leader on the right lane, the higher cost it gets. Changing lane leads to lower cost compared to the keep-the-same-lane strategy. In addition, overtaking the leader results in lower cost compared to change-to-left-lane strategies. This is mainly due to the higher lane preference cost for staying in the left lane.

The optimal strategy is to change to the left lane immediately to avoid high safety and equilibrium cost and then change to the right lane after 7 seconds, i.e. immediately after satisfying constraint (3), to avoid high lane preference cost. This results in a cost of 7.4, significantly lower than the follow-the-leader strategy. The resulting optimal path, desired lane number sequence, speed and the acceleration are depicted with blue lines in Figure 3. As depicted in Figure 3(a), the optimal lane change strategy results in a much higher speed and longer distance traveled compared to those of the follow-the-leader strategy, which is a clear benefit for traffic efficiency.

Non-cooperative and cooperative merging (Experiments 3 and 4)

In Experiment 3, we consider the interaction and cooperation of two controlled vehicles in a merging scenario at a highway on-ramp. The possible strategies and corresponding predicted cost are shown in TABLE 3.

If vehicle 1 and 2 are *non-cooperative* controllers, it is evident that the best situation for vehicle 1 is to change left immediately. Given this situation, the best for vehicle 2 is to change left immediately. The running cost, speed and acceleration of the two vehicles in this situation are shown in Figure 4(a)(b)(c). In this case, vehicle 2 incurs a lane switch cost and higher lane preference cost, while vehicle 1 incurs a cost due to equilibrium speed, which is higher than the current speed. Thus vehicle 1 accelerates to match the equilibrium speed. This leads to a cost of 10.92 for vehicle 1 and 16.20 for vehicle 2, resulting in a total cost of 27.12. This defines a Nash equilibrium solution, i.e. neither vehicle can improve its individual situation unilaterally.

The best situation for *non-cooperative* vehicle 2 is keeping the same lane while vehicle 1 stays in the on-ramp. In this case, vehicle 2 only incurs a cost of 8.03 due to the lane preference cost, but vehicle 1 incurs a much higher cost of 34.76 due to the equilibrium cost, route cost and travel efficiency cost. The total cost of this strategy amounts to 42.79, significantly higher than the best situation for vehicle 1. This situation is worse for the collective cost of two vehicles. This is not a Nash equilibrium situation, since vehicle 1 can improve its situation by changing lane earlier.

When both vehicles are *cooperative*, they seek a coordinated strategy that minimizes the joint cost, i.e. *collective optimum solution*. Hence, the mainline vehicle will change left immediately to create space for the merging vehicle, although this implies a higher cost for himself compared to his best situation when vehicle 1 does not change lane. The merging vehicle will also change lane immediately. In doing so, the joint cost reaches the minimum. Note that in this experiment, the *collective optimum* solution coincides with the Nash equilibrium solution.

The gaming facet becomes more pronounced when the situation is more demanding for the merging and mainline vehicle in Experiment 4. The merging vehicle starts at the on-ramp at the same location, but with a lower initial speed of 20 m/s. The mainline vehicle (vehicle 2) travels on the right lane of the main highway, at the position of x_2 = -10 m and initial speed of 30 m/s. The predicted costs of different lane change strategies of vehicle 1 and 2 are also summarized in TABLE 3.

If both vehicles are controlled by *non-cooperative* or *autonomous* controllers, the best strategy for vehicle 1 is still to change lane immediately and given this situation the best for vehicle 2 is to change lane immediately. The situation is shown in Figure 4(d)(e)(f). This leads to a cost of 18.61 for vehicle 1. Although the immediate lane change will not lead to route cost for vehicle 1, the lower initial speed leads to higher equilibrium cost for vehicle 1 and hence it has to accelerate harder to match the equilibrium speed on the main highway compared to Experiment 3. Given this situation, the best for vehicle 2 is to change lane immediately, resulting in a cost of 16.20 for vehicle 2. However, the total cost for this situation is 34.36, not the optimum from the collective perspective. This solution is a Nash equilibrium, but not the collective optimum.

The best for *non-cooperative* vehicle 2 is keeping the same lane while vehicle 1 does not change lane or delay its lane change maneuvers to 2 seconds later. In this case, vehicle 2 is able to pass vehicle 1 to be the leader of the pair, resulting in a minimum cost of 8.00 for vehicle 2. However, none of them is a Nash equilibrium solution since the *non-cooperative* vehicle 1 would be better-off by changing lane earlier.

When both vehicles are *cooperative*, they seek the *collective optimum*. Vehicle 1 will delay its lane change after 3 seconds while vehicle 2 does not change lane. This will allow vehicle 2 to pass vehicle 1 before vehicle 1 merges and gives sufficient time for the gap between the new leader (vehicle 2) and the new follower (vehicle 1) on the main highway to grow so that the equilibrium cost of the merging vehicle will not increase substantially due to the lane change. This results in

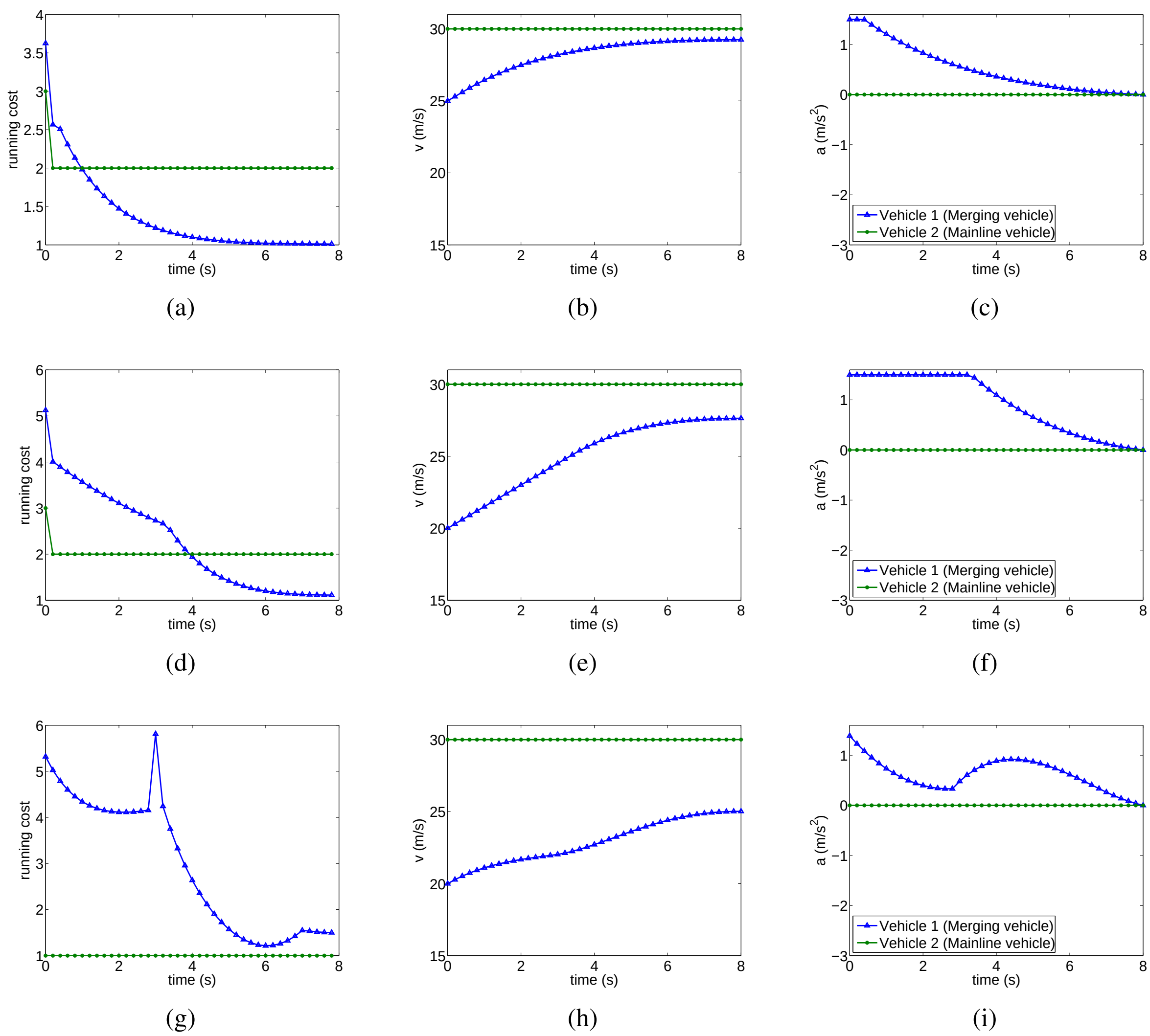

FIGURE 4 Experiment 3: (a) Running cost, (b) speed and (c) acceleration of the merging vehicle and the mainline vehicle under coinciding Nash equilibrium and collective optimum solutions Experiment 4: (d) Running cost, (e) speed and (f) acceleration of the merging vehicle and the mainline vehicle under Nash equilibrium solution Experiment 4: (g) Running cost, (h) speed and (i) acceleration of the merging vehicle and the mainline vehicle under collective optimum solution.

TABLE 3 Predicted cost J^* (cost of vehicle 1, cost of vehicle 2) in Experiments 3 and 4

Experiment 3	$\tau_{2,1}=0, \delta_{2,1}=-1$	$\tau_{2,1}=1, \delta_{2,1}=-1$	$\tau_{2,1}=2, \delta_{2,1}=-1$	$\tau_{2,1}=3, \delta_{2,1}=-1$	$\delta_{2,1}=0^{[2]}$
$\tau_{1,1}=0, \delta_{1,1}=-1$	**27.12**$^{*+}$ (**10.92**,16.20)	32.35 (11.00,21.36)	36.24 (11.03,25.20)	38.59 (11.04,27.54)	34.47 (11.04,23.42)
$\tau_{1,1}=1, \delta_{1,1}=-1$	28.73 (12.46,16.26)	27.73 (12.46,15.26)	32.07 (12.46,19.60)	34.74 (12.46,22.28)	30.92 (12.46,18.46)
$\tau_{1,1}=2, \delta_{1,1}=-1$	30.33 (14.07,16.26)	29.33 (14.07,15.26)	28.33 (14.07,14.26)	32.42 (14.07,18.35)	29.68 (14.07,15.61)
$\tau_{1,1}=3, \delta_{1,1}=-1$	32.18 (15.92,16.26)	31.17 (15.92,15.25)	30.17 (15.92,14.25)	29.17 (15.92,13.25)	29.61 (15.92,13.69)
$\delta_{1,1}=0^{[1]}$	51.00 (34.76,16.23)	50.00 (34.76,15.23)	48.99 (34.76,14.23)	47.99 (34.76,13.23)	42.79 (34.76,**8.03**)
Experiment 4	$\tau_{2,1}=0, \delta_{2,1}=-1$	$\tau_{2,1}=1, \delta_{2,1}=-1$	$\tau_{2,1}=2, \delta_{2,1}=-1$	$\tau_{2,1}=3, \delta_{2,1}=-1$	$\delta_{2,1}=0^{[2]}$
$\tau_{1,1}=0, \delta_{1,1}=-1$	34.36^{+} (**18.16**,16.20)	106.63 (18.18,88.45)	130.03 (18.18,111.85)	143.14 (18.18,124.96)	145.62 (18.18,127.44)
$\tau_{1,1}=1, \delta_{1,1}=-1$	35.83 (19.58,16.25)	34.83 (19.58,15.25)	83.95 (19.58,64.36)	97.06 (19.58,77.48)	99.83 (19.58,80.24)
$\tau_{1,1}=2, \delta_{1,1}=-1$	37.39 (21.14,16.26)	36.39 (21.14,15.26)	35.39 (21.14,14.25)	42.04 (28.84,13.20)	32.62 (24.62,**8.00**)
$\tau_{1,1}=3, \delta_{1,1}=-1$	39.07 (22.87,16.20)	38.07 (22.87,15.20)	37.07 (22.87,14.20)	36.07 (22.87,13.20)	**31.64*** (23.64,**8.00**)
$\delta_{1,1}=0^{[1]}$	54.57 (38.37,16.20)	53.57 (38.37,15.20)	52.57 (38.37,14.20)	51.57 (38.37,13.20)	46.37 (38.37,**8.00**)

*: Collective optimum solution. +: Nash equilibrium solution. [1]: no lane change for vehicle 1. [2]: no lane change for vehicle 2.

the minimum *joint* cost of 31.64, which is lower than that in the Nash equilibrium solution. The situation is shown in Figure 4(g)(h)(i).

Experiments 3 and 4 demonstrate the applicability of the proposed approach in computing optimal strategies for connected vehicle systems. The last experiment shows the clear benefit of cooperative controller compared to their non-cooperative counterpart. Unlike other work using game theoretical approaches in modeling human merging behavior (*14, 18*), the cost of different strategies in our approach is based on the *predicted* path of the controlled vehicle in the future, taking into account the *anticipated* behavior of surrounding vehicles. The optimal lane change strategy does not necessarily occur at the current time, but may happen in the future. In addition, each strategy corresponds to an optimal solution for the sub-problem and defines a *unique* and *continuous* vehicle path that can be sent to the lower-level vehicle actuators to track.

DISCUSSION ON EXTENSION TO HUMAN DRIVER MODEL

The proposed mathematical approach generates lane change sequences and accelerations in the future and is applicable for both *autonomous* and *connected* vehicle systems. The framework can be extended to an integrated model for driving behavior, since it has several distinctive features compared to the state-of-the-art lane change decision and car-following models. The approach can anticipate the lane changes in the future and can capture the interaction and cooperation of multiple vehicles in conflicting situations. The specification of objective function include human drivers' desires and traffic rules and the model can produce plausible lane-changing maneuvers while obeying safety and comfort requirements. The approach can lead to explicit target lane choices in the anticipated future, which is a favorable feature when modeling strategic overtaking, lane change decisions in multiple lanes highways (more than 3 lanes), dedicated lane operations, and mandatory lane changes. It has potential for selecting safe gap in high density conditions (*26*).

CONCLUSIONS

We put forward a modeling approach for optimal lane change times and accelerations of autonomous and cooperative vehicles based on receding horizon optimal control and dynamic game theory and demonstrated the workings of the modeling framework with numerical examples. The proposed approach generates predicted lane change sequences and accelerations and is applicable for both *autonomous* and *connected* vehicle systems. The resulting lane change decisions and car-following control inputs determine a *unique* and *continuous* path that can be used by the automated vehicle actuators to track. Numerical examples show that the controller produces plausible lane-changing and car-following maneuvers at the microscopic level in highway conditions and cooperative controller can lead to lower joint cost in conflicting situations such as merging scenarios.

The presented work focuses on the mathematical framework and illustration of the fundamental workings of the proposed approach. Extensions of the framework to model human driver behavior by including anticipation of traffic conditions on target lanes and adaptive model parameters remains interesting for future research. Future research is also directed to the systematic examination of the impact of the optimal lane change time and acceleration model on dynamic traffic flow features under realistic bottlenecks.

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