An algorithm for calculating the cycletime and greentimes for a signalized intersection

Taale, Henk

Publication date
1995

Document Version
Accepted author manuscript

Published in
Studies of the Faculty of Management Science

Citation (APA)
Taale, H. (1995). An algorithm for calculating the cycletime and greentimes for a signalized intersection. *Studies of the Faculty of Management Science*, *4*, 87-94.

AN ALGORITHM FOR CALCULATING THE CYCLETIME AND GREENTIMES FOR A SIGNALIZED INTERSECTION

Henk Taale

1. Introduction

For a signalized intersection with a fixed-time control strategy the cycletime and greentimes are the variables that influence the delay of vehicles. In the past a lot of effort has been put in finding a method to calculate an optimal cycletime and optimal greentimes that minimize delay. A famous example is the formula of Webster (1958), which is derived from a formula describing the average delay per vehicle.
In this article a method is described to calculate an optimal cycletime and greentimes, based on a generalized Webster formula and taking into account the minimum greentime and maximum degree of saturation for each movement.

2. Basic principles and definitions

For a movement of a signalized intersection certain characteristics can be defined. If we call G_r the greentime for movement r, than the effective greentime g_r can be defined as G_r-$t_{s,r}$+$t_{e,r}$, where t_s is the start lag and t_e the end gain. Another useful parameter is the *flow ratio*. If q_r is the flow and s_r is the saturation flow for movement r, than the flow ratio y_r is defined as q_r/s_r. The flow ratio can also be considered as the fraction of the time that is needed for movement r to handle all traffic. The parameter which relates the flow ratio to the greentime ratio (g_r/C, where C is the cycletime) is called the *degree of saturation*. This degree of saturation x_r is defined as $(y_r*C)/g_r$ (Akçelik, 1986).
For an intersection certain movements have a conflict with each other, which means that they are not allowed to receive the green light at the same time. A set of movements conflicting with each other, but where no other movement, which has a conflict with all other movements in the set, can be added to, is called a *maximum set of movements*. Within each maximum set of movements an *internal lost time* can be determined for a certain sequence of movements. The internal lost time is the sum of the intergreen times plus the sum of the start lags and minus the sum of the end gains of all movements in the set. The intergreen time is dependent of the sequence of movements and therefore another sequence of movements can give another internal lost time. For the calculation of the optimal cycletime the sequence of movements, within a maximum set of movements, with minimum internal lost time is used.

3. Conditions for a control strategy

In designing a (fixed-time) control strategy three conditions are important: the available time must be larger or equal than the time needed [1], the greentime of every movement must be larger or equal than the minimum greentime [2] and the degree of saturation of every movement must be less or equal than the maximum degree of saturation [3].
We consider the collection K_i of maximum sets of movements of all movements of an intersection. For every maximum set of movements K_i the internal lost time L_i and the load Y_i is known, where Y_i is the sum of the flow ratio's of all movements within K_i. According to condition [1] the following must hold for the cycletime C_i:

$$C_i \geq \frac{L_i}{1 - Y_i} \tag{A}$$

Condition [2] leads to:

$$C_i \geq \max\{ L_i + m_r \frac{Y_i}{y_r} | r \in K_i \} \tag{B}$$

where m_r is the minimum <u>effective</u> greentime (minimum greentime plus end gain minus start lag).
Finally, condition [3] gives:

$$C_i \geq \max\{\frac{x_{\max,r} L_i}{x_{\max,r} - Y_i} | r \in K_i\} \qquad \text{(C)}$$

where $x_{max,r}$ is the maximum degree of saturation for movement r, which is also known. This condition holds under the assumption that $x_{max,r} > Y_i$ for all movements r, otherwise negative cycletimes would be required. Because condition (3) is the same as condition (1), but more strict if $x_{max,r}<1$ for all $r \in K_i$, the combination of all three conditions leads to the following formula for the minimum cycletime:

$$C_i = \max\{\max\{\frac{x_{\max,r} L_i}{x_{\max,r} - Y_i} | r \in K_i\}, \max\{L_i + m_r \frac{Y_i}{y_r} | r \in K_i\}\} \qquad \text{(D)}$$

4. Calculations taking into account the minimum greentime

The famous formula of Webster for the optimal cycletime is:

$$C_i = \frac{1.5\, L_i + 5}{1 - Y_i} \qquad \text{(E)}$$

This formula can be generalized into:

$$C_i = \frac{F_1 L_i + F_2}{1 - \frac{Y_i}{F_3}} \qquad \text{(F)}$$

where F_1, F_2 and F_3 are called the Webster coefficients. The matching effective greentimes for all movements $r \in K_i$ are calculated with:

$$g_r = \frac{y_r}{Y_i}(C_i - L_i) \qquad \text{(G)}$$

Due to condition (4) it is possible that the cycletime becomes very large which is caused by movements with a very low flow ratio y_r. To correct this problem, the flow ratio of these movements is increased artificially to a point that the minimum greentime condition is satisfied. First the set N_i of movements $r \in K_i$ is determined for which the following condition holds:

$$y_r \leq m_r \frac{(1 - \frac{Y_i}{F_3}) Y_i}{(F_1 - 1 + \frac{Y_i}{F_3}) L_i + F_2} \qquad \text{(H)}$$

Than an arbitrary movement $p \in N_i$ is taken for which the "artificial" flow ratio y_p^* is calculated with:

$$a_1 = \frac{L_i + \sum_{r\in N_i} m_r}{F_3} \sum_{r\in N_i} \frac{m_r}{m_p} \qquad \textbf{(I)}$$

$$b_1 = \left(F_1 - 1 + \frac{Y_i - \sum_{r\in N_i} y_r}{F_3}\right) L_i + F_2 - \left(1 - 2\frac{Y_i - \sum_{r\in N_i} y_r}{F_3}\right) \sum_{r\in N_i} m_r \qquad \textbf{(J)}$$

$$c_1 = m_p\left(\sum_{r\in N_i} y_r - Y_i\right)\left(1 - \frac{Y_i - \sum_{r\in N_i} y_r}{F_3}\right) \qquad \textbf{(K)}$$

$$y_p^* = \frac{-b_1 + \sqrt{b_1^2 - 4a_1c_1}}{2a_1} \qquad \textbf{(L)}$$

The other "artificial" flow ratio's y_r^* for $r\in N_i\backslash\{p\}$ and the new sum of flow ratio's Y_i^* are calculated with the following formula's:

$$y_r^* = y_p^* \frac{m_r}{m_p} \qquad \textbf{(M)}$$

$$Y_i^* = Y_i - \sum_{r\in N_i} y_r + \sum_{r\in N_i} y_r^* \qquad \textbf{(N)}$$

5. Calculations taking into account the maximum saturation flow

After correcting for the minimum greentime, also the condition for the maximum saturation flow has to be satisfied. The movements $r\in K_i$ for which this condition does not hold, can be determined with (if $r\in K_i\backslash N_i$ than $y_r^*=y_r$):

$$\frac{y_r}{y_r^*} \geq x_{\max,r} \frac{\left(F_1 - 1 + \frac{Y_i^*}{F_3}\right) L_i + F_2}{(F_1 L_i + F_2) Y_i^*} \qquad \textbf{(O)}$$

This set of movements is named M_i and the cross-section of the sets N_i and M_i is named P_i. The next step is to take an arbitrary movement $q\in M_i$ and to calculate:

$$a_2 = x_{\max,q} \frac{L_i}{F_3} \sum_{r\in M_i} \frac{y_r}{y_q} \frac{x_{\max,q}}{x_{\max,r}} \qquad \textbf{(P)}$$

$$b_2 = x_{\max,q}\left(\left(F_1 - 1 + \frac{Y_i^* - \sum_{r\in P_i} y_r^* - \sum_{r\in M_i - P_i} y_r}{F_3}\right) L_i + F_2\right) - (F_1 L_i + F_2) \sum_{r\in M_i} y_r \frac{x_{\max,q}}{x_{\max,r}} \qquad \textbf{(Q)}$$

$$c_2 = -y_q\left(Y_i^* - \sum_{r\in P_i} y_r^* - \sum_{r\in M_i\backslash P_i} y_r\right)(F_1 L_i + F_2) \qquad \textbf{(R)}$$

$$\bar{y}_q = \frac{-b_2 + \sqrt{b_2^2 - 4 a_2 c_2}}{2 a_2} \tag{S}$$

Now, the other flow ratio's for the movements $r \in M_i \backslash \{q\}$ and the new sum of flow ratio's are calculated with:

$$\bar{y}_r = \bar{y}_q \frac{y_r}{y_q} \frac{x_{\max,q}}{x_{\max,r}} \tag{T}$$

$$\bar{Y}_i = Y_i^* - \sum_{r \in P_i} y_r^* - \sum_{r \in M_i \_ P_i} y_r + \sum_{r \in M_i} \bar{y}_r \tag{U}$$

Because for the movements $r \in M_i$ the flow ratio is increased, the sum of flow ratio's is also increased, leading to a greentime larger than the minimum greentime for the movements $r \in N_i \backslash M_i$. This greentime can become very large for a movement with a very low flow ratio and a low maximum saturation flow. To overcome this problem, the flow ratio for these movements has to be adjusted again. Therefor it is necessary to determine the set of movements $r \in N_i \backslash M_i$ which satify the following conditions:

$$y_r^* > m_r \frac{(1 - \frac{\bar{Y}_i}{F_3})\bar{Y}_i}{(F_1 - 1 + \frac{\bar{Y}_i}{F_3})L_i + F_2} \tag{V}$$

$$y_r^* > \frac{y_r}{x_{\max,r}} \frac{(F_1 L_i + F_2)\bar{Y}_i}{(F_1 - 1 + \frac{\bar{Y}_i}{F_3})L_i + F_2} \tag{W}$$

An arbitrary movement $t \in N_i \backslash M_i$ is taken and a new flow ratio is calculated with:

$$a_3 = \frac{L_i + \sum_{r \in N_i \backslash M_i} m_r}{F_3} \sum_{r \in N_i \backslash M_i} \frac{m_r}{m_t} \tag{X}$$

$$b_3 = (F_1 - 1 + \frac{\bar{Y}_i - \sum_{r \in N_i \backslash M_i} y_r^*}{F_3})L_i + F_2 - \left(1 - 2\frac{\bar{Y}_i - \sum_{r \in N_i \backslash M_i} y_r^*}{F_3}\right) \sum_{r \in N_i \backslash M_i} m_r \tag{Y}$$

$$c_3 = m_t\left(\sum_{r \in N_i \backslash M_i} y_r^* - \bar{Y}_i\right)\left(1 - \frac{\bar{Y}_i - \sum_{r \in N_i \backslash M_i} y_r^*}{F_3}\right) \tag{Z}$$

$$\tilde{y}_t = \frac{-b_3 + \sqrt{b_3^2 - 4 a_3 c_3}}{2 a_3} \tag{AA}$$

The other flow ratio's for the movements $r \in N_i \backslash (M_i \cup \{t\})$ and the new sum of flow ratio's are calculated with:

$$\tilde{y}_r = \tilde{y}_t \frac{m_r}{m_t} \qquad \text{(BB)}$$

$$\tilde{Y}_i = \bar{Y}_i - \sum_{r \in N_i - M_i} y_r^* + \sum_{r \in N_i - M_i} \tilde{y}_r \qquad \text{(CC)}$$

Finally, the cycletime and effective greentimes are calculated with

$$\tilde{C}_i = \frac{F_1 L_i + F_2}{1 - \frac{\tilde{Y}_i}{F_3}} \qquad \text{(DD)}$$

$$\tilde{g}_r = \frac{y_r}{\tilde{Y}_i}(\tilde{C}_i - L_i) \qquad \text{(EE)}$$

for $r \in K_i \backslash (N_i \cup M_i)$ and

$$\tilde{g}_r = \frac{\tilde{y}_r}{\tilde{Y}_i}(\tilde{C}_i - L_i) \qquad \text{(FF)}$$

for $r \in N_i \backslash M_i$ and

$$\tilde{g}_r = \frac{\bar{y}_r}{\tilde{Y}_i}(\tilde{C}_i - L_i) \qquad \text{(GG)}$$

for $r \in M_i$.

After this, again the set of movements N_i is determined and all steps are taken again until both the sets N_i and M_i are empty. This algorithm is carried out for all maximum sets of movements K_i. If the cycletime has a maximum for the maximum set of movements K_h, so that:

$$C_h \geq \tilde{C}_i \qquad (i = 1, \ldots, n) \qquad \text{(HH)}$$

than, for the generalized Webster formula, C_h is the cycletime that should be used and the accompanying greentimes G_r can be calculated with $g_r + t_{s,r} - t_{e,r}$.

6. Final remark and literature

The algorithm described in this article has been implemented in the computer program KRAAN, owned by DTV consultants the Breda, The Netherlands.

1. Webster, F.V., *Traffic Signal Settings*, Road Research Technical Paper No. 39, Road Research Laboratory, 1958
2. Akçelik, R., *Traffic Signals: Capacity and Timing Analysis*, Research Report ARR 123, Australian Road Research Board, 1986

Author: ir. Henk Taale
Ministry of Transport, Public Works and Water Management
Transport Research Centre (AVV)
P.O. Box 1031
3000 BA Rotterdam
The Netherlands