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STATE SPACE MODELING OF FLUID FLOW FOR THRUST CONTROL IN MEMS-BASED MICROPROPULSION

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Abstract: This paper presents a dynamic system approach for the modeling of fluid flow in microchannels to be used in thrust control applications. A micro-resistojet fabricated using MEMS (Microelectromechanical Systems) technology has been selected for the analysis. The device operates by vaporizing a liquid propellant, in this case water, and expelling it as gas that is accelerated by a micro-nozzle. The pressure variation due to boiling in the chamber might lead to unwanted behavior of the feed system and the frequency analysis in this case can indicate whether or not instabilities will be present. To handle this complex problem, the incompressible Navier-Stokes equations are linearized in the steady-state flow regime and then formulated in state space form to provide the necessary means for control analysis. Controllability and observability of the system are investigated considering low values of Reynolds numbers present in micro fluidics applications. Results from the analytical treatment are compared with CFD (Computational Fluid Dynamics) simulations of the microchannel to demonstrate the validity of the approach investigated.
Keywords: micropropulsion, thrust-control, resistojet, MEMS

1. Introduction

Micropropulsion is one of the key developments for the next generation of CubeSats. It will extend the range of applications for this class of satellites to include missions where, e.g., formation flying or station keeping maneuvers are necessary [1]. These maneuvers need the ability to control the thrust in magnitude and direction [2]. Usually, the thrust direction is fixed and the spacecraft needs to correct its attitude to compensate for unwanted torques generated by the misalignment between the thruster and the spacecraft center of mass. By controlling the thrust one can circumvent this issue and have an optimized velocity increment.

Many micropropulsion devices have been developed in the past two decades in an attempt to provide CubeSats with the mentioned capabilities. Among these systems, we can highlight some of the most promising ones due to aspects such as reliability and simplicity: pulsed-plasma thruster (PPT) [3–6], cold-gas thruster [7,8], vaporizing liquid microresistojet (VLM) [9–11], and free molecular microresistojet (FMMR) [12,13]. The PPT is advantageous in terms of the type of propellant (usually solid) and ease of operation, while its disadvantage is in the lack of control in the thrust levels and in the possibility of electro-magnetic compatibility issues. The cold-gas is more suitable for controlling the thrust but the high pressure needed to store the propellant in order to achieve reasonable performance sets a great challenge for its use, especially in small satellites and CubeSats. The VLM has similar problems with the storage pressure but might be less depending on the propellant choice. Finally the last one operates with very low pressure that might reduce the leakage and storage problems but it is still in an early phase of development needing more experimental tests.

In the perspective of thrust control, cold-gas and VLM are more suitable for magnitude control; however, for direction control there is no state-of-the-art micropropulsion system and only a few analysis have been done on this aspect [2].

In this paper, we consider the VLM as the baseline design. In this case, the magnitude control can be achieved by controlling the propellant mass flow in the feed system and, to this purpose, a proper mathematical model of the flow is necessary for system modeling and analysis. In this context, the state space formulation is very useful mainly to deal with multivariable systems as in the case of fluid flow. It also allows the assessment of zeros and poles locations in the complex plane that is crucial for stability analysis of the system.

The remainder of this paper is organized as follows: section 2 presents some general background, section 3 presents the equation development and modeling, section 4 presents the results of simulation and section 5 draws the conclusions.

2. Background

2.1 – Propulsion

The performance of micropropulsion systems can generally be analyzed using the same formulation as in normal sized systems. In this case, two parameters are of major interest when analyzing the performance of the thruster: specific impulse and thrust [14]. The thrust (F in equation [1]) is the force generated by the gas accelerated and expelled through the nozzle.

$$F = \dot{m}V_e + (P_e - P_a)A_e \qquad [1]$$

where $\dot{m}$ is the mass flow rate, V_e is the exhaust velocity, P_e and P_a the exit and ambient pressures, and A_e is the exit area. The specific impulse (I_{sp} in equation [2]) is a measure of efficiency regarding the consumption of propellant. Although the unit is given in seconds, it does not represent a measure of time but a measure of thrust per unit weight of propellant and it should be as high as possible for best propellant consumption efficiency.

$$I_{sp} = \frac{F}{\dot{m}g} \qquad [2]$$

where $g = 9.80665\,[\mathrm{m/s^2}]$ is the gravitational acceleration on Earth at sea level.

2.2 – Fluid dynamics

The Navier-Stokes equations have been used in the simulations to model the behavior of the flow [15,16]. These equations (equations [3-5]) are used to describe any flow in the continuum regime.

$$\frac{\partial \rho}{\partial t} + \nabla(\rho\vec{u}) = 0 \qquad [3]$$

$$\rho\left(\frac{\partial \vec{u}}{\partial t} + \vec{u}\cdot\nabla\vec{u}\right) = -\nabla p + \nabla\cdot\tau + f \qquad [4]$$

$$\rho C_v\left(\frac{\partial T}{\partial t} + T\cdot\nabla T\right) = -p\nabla\cdot\vec{u} + \nabla\cdot(k\nabla T) + \tau\cdot\nabla\vec{u} \qquad [5]$$

where ρ is the density, t is the time, $\boldsymbol{v}$ is the fluid velocity, p is the pressure τ is the viscous stress tensor, $\boldsymbol{f}$ is an external force acting on the control volume, R is the specific gas constant, T is the temperature, and C_v is the specific heat at constant volume.

In this paper, as we will see in next section, we consider the incompressible isothermal assumptions, i.e. the density is constant over time and space as well as the temperature.

3. Equations and modeling

The system considered in this paper is a Vaporizing Liquid Micro-resistojet (VLM) (). This system is being developed at the Space System Engineering (SSE) chair of Delft University of Technology (TU Delft) [12]. It is composed by a tank where the propellant is stored, a feeding section with a proportional valve to control the mass flow rate, and a thruster with a heating chamber to vaporize the propellant and accelerate it through a convergent-divergent nozzle. The eventual goal of this concept is to control the thrust levels the system is able to provide, which might be done by controlling the proportional valve before the chamber.

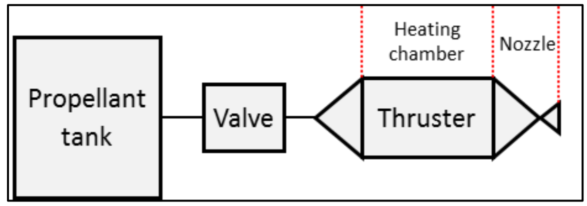

Figure 1 - Diagram of the Vaporizing Liquid Micro-resistojet.

In this context, as an initial step towards thrust control, we consider a single cylindrical channel connecting the valve to the chamber through which the liquid propellant is fed. The channel is modeled using Navier-Stokes equations considering the flow as incompressible and unsteady as seen in equation [7], where $\vec{u}$ is the velocity, ρ is the density of the fluid, p is the pressure, and η is the dynamic viscosity of the fluid.

$$\rho\frac{\partial \vec{u}}{\partial t} = -\rho\vec{u}\cdot\nabla\vec{u} - \nabla p + \eta\nabla^2\vec{u} \qquad [6]$$

Assuming the flow unidirectional and the density constant over time, then $\vec{u}\cdot\nabla\vec{u} = 0$ and equation [7] becomes:

$$\rho\frac{\partial \vec{u}}{\partial t} = -\nabla p + \eta\nabla^2\vec{u} \qquad [7]$$

Integrating over the length L of the channel in cylindrical coordinates one might re-arrange it to:

$$\rho\frac{\partial u}{\partial t} = \frac{\Delta p}{L} + \eta\left(\frac{\partial^2 u}{\partial r^2} + \frac{\partial u}{r\partial r}\right) \qquad [8]$$

and integrating it twice over r :

$$\frac{r^2\rho}{4\eta}\frac{\partial u}{\partial t} = \frac{r^2\Delta p}{4\eta L} + u + c_0 ln(r) + c_1 \qquad [9]$$

Applying the boundary conditions to calculate the constants:

$$\begin{cases} \dfrac{\partial u}{\partial r}\Big|_{r=0} = 0 \\ u\,|_{r=R} = 0 \end{cases} \qquad [10]$$

$$\begin{cases} c_0 = 0 \\ c_1 = -\dfrac{R^2}{4\eta L}\Delta p + \dfrac{R^2\rho}{4\eta}\dfrac{\partial u}{\partial t} \end{cases} \qquad [11]$$

where R is the radius of the channel.

Then integrating over the radius to get the average values we finally get the equation:

$$\frac{\pi R^4 \rho}{8\eta}\frac{\partial u}{\partial t} = -u\pi R^2 - \frac{\pi R^4 \Delta p}{8\eta L} \qquad [12]$$

If we define the velocity as the state of the system we can re-write it in the state space form $\dot{x} = Ax + B\bar{u}$ considering the pressure drop as the input $\bar{u} = \Delta p$:

$$\frac{\partial u}{\partial t} = -\frac{8\eta}{R^2\rho}u - \frac{\Delta p}{\rho L} \qquad [13]$$

The boiling process inside the chamber is a source of disturbances in the pressure drop, mainly caused by the explosion of bubbles or their collapse when they hit solid surfaces. This effect might be reduced with the inclusion of a controller in the closed loop to act on the inlet pressure and control the velocity avoiding possible backflows, for example. This controller, in practice, would act on the proportional valve controlling the mass flow. Figure 2 depicts the block diagram used to simulate the behavior of the system together with a PID (proportional-integral-derivative) controller. In this case, the disturbances (pressure drops caused by boiling) are included in the input of the plant model (microchannel). The reference input is the velocity of the flow which might be viewed as the mass flow rate as the density and the cross sectional area are constant. The PID controller is modelled as:

$$\frac{U(s)}{E(s)} = K_p + K_i\frac{1}{s} + K_d\frac{s}{Ns+1} \qquad [14]$$

where K_p, K_i, and K_d are the gains of the controller and N is the filter coefficient calculated to reduce high frequency responses of the derivative term.

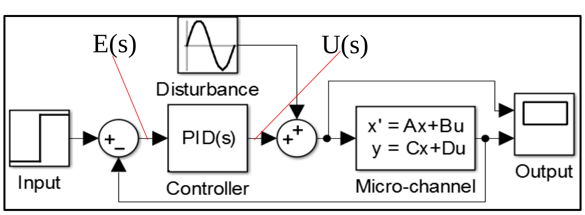

Figure 2 - Control loop for the microchannel.

4. Results

Simulations were carried out considering the parameters given in Table 1.

The system contains only one real pole located at $-8\eta / R^2\rho = -2.7322e+03$ which means that the system is stable in open-loop. The frequency response for the system is depicted in Figure 3 where we see an initial phase shift of 180° due to the nature of the signals. A positive pressure drop, where the pressure at the exit is higher than that at the inlet, causes the flow to go from the exit to the inlet therefore with negative velocity.

Table 1 - Parameters used in the simulations.

Parameter	Value [unit]
R	50 [μm]
L	1000 [μm]
ρ	1000 [kg/m^3]
η	8.5383e–4 [Pa s]

In Figure 4, the response of the system to a step input and a variable input are shown. The third graph is showing the response for the same variable input but using the conventional model for the microchannel assuming the flow steady [17], i.e. not including the time dependent velocity derivative. With these two graphs we can see a slight difference in the phase of the signals modeled by the inclusion of the time derivative.

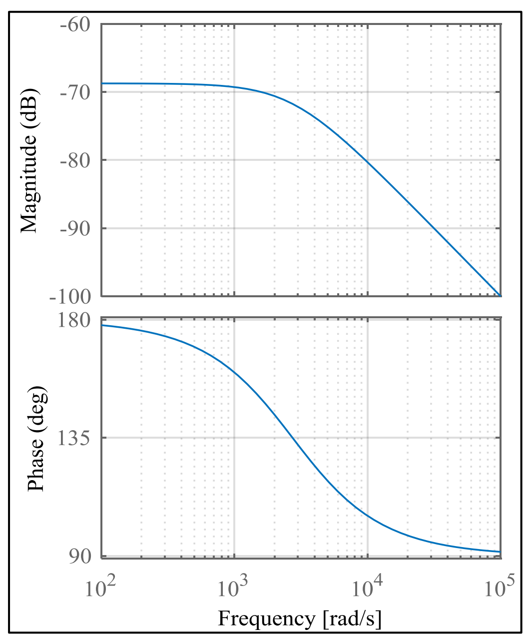

Figure 3 - Bode diagram of the system.

Figure 5 shows the response of the system to a step input in closed-loop with a controller and in open-loop only with a proportional gain in the input. The controller parameters were empirically set to $K_p = -4500$, $K_i = -1.5\text{e}{+}07$, and no derivative term, i.e. $K_d = 0$. Also, the output of the controller is subject to a saturation to correctly model the thruster with its pressurized tank. As we can see in the figures this saturation point is not achieved during the simulations.

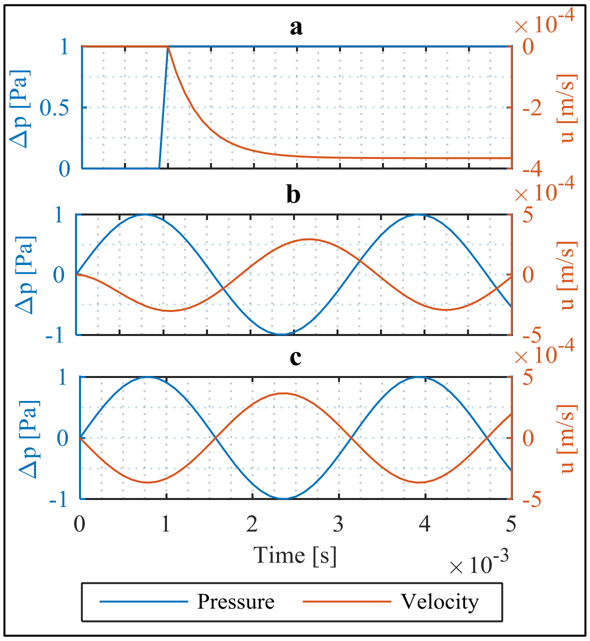

Figure 4 - Response to (a) step input, (b) sinusoidal input, (c) sinusoidal input with conventional circuit analogy.

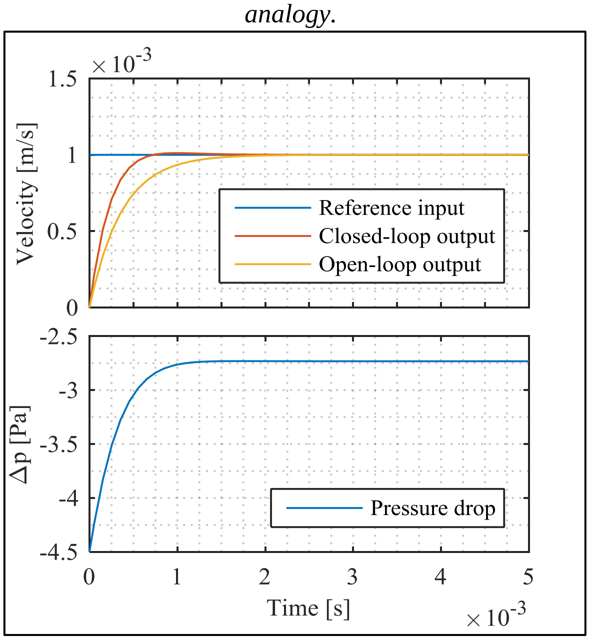

Figure 5 – (Top) Comparison between the response with and without control. (Bottom) Control signal for the pressure drop.

In the case of the open-loop response the gain was set to the pole of the system. With the controller there is a small overshoot of around 1% that can be corrected with an optimization of the gains which is not in the scope of this paper.

Figure 6 presents the response of the system now considering a disturbance in the input following a sinusoidal shape with amplitude of $0.5\,[\text{Pa}]$ and frequency of $1e+04\,[\text{rad/s}]$. In this case, the same controller was used and it was able to achieve the desired output.

5. Conclusions

This paper presented the modeling and linear system analysis of a micro-fluidic channel partially representing the feeding section of a micropropulsion system. The cylindrical channel was modeled based on the well-known Navier-Stokes set of equations and then formulated as a state-space system to provide a more intuitive view of the system for control analysis.

The frequency response was analyzed showing that the system has a very fast response which is expected due to the nature of the problem. This aspect have to be considered when designing the micropropulsion system since the usual assumption of steadiness in the flow might hide some characteristics of the dynamics as shown by Figure 4.

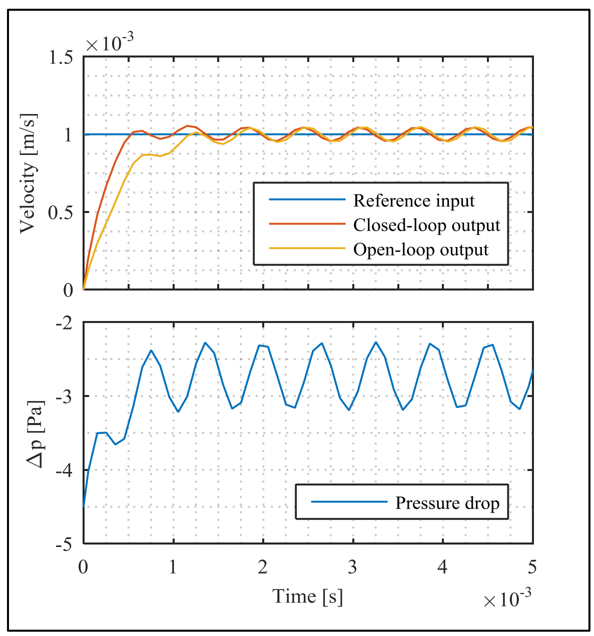

Figure 6 - Comparison considering a disturbance in the pressure drop.

Future work will be focused on the further development of this formulation to include other aspects in the simulation loop as well as other components such as a proportional valve to control the flow. Also, a more detailed model is interesting to evaluate the high frequency dynamics that might be present during operation and with the current approach are not being modeled since the time derivative of pressure is not included.

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