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# Motion Simulator 2-axes Input Design for Angular Accelerometer Calibration

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The calibration of Angular Accelerometers requires a controlled test with a known acceleration profile. However, current turn-tables have been designed primarily for generating a constant rotational velocity. To generate a profile with varying angular acceleration we propose using constant rotational velocity on the two axes of a 2-axes motion simulator. The proposed sequence was developed to obtain the required rotational acceleration signal quality using only rotational velocity input. The identified pattern is applied to an envelope of test conditions, resulting in test matrix. This method provides an alternative means to generate inputs that can be used to calibrate angular accelerometers using calibration hardware that is not primarily designed to provide accurate acceleration inputs.

## Nomenclature

AA Angular accelerometer  
DAS Data Acquisition System  
IMU Inertial Measurement Units  
PSD Power Spectral Density

### *Subscripts*

*i* Inner-axis  
*o* Outer-axis

### *Symbols*

$\alpha$  Angular acceleration,  $deg/s^2$   
 $\mathbf{i}$  Unit vector in X-axis  
 $\mathbf{j}$  Unit vector in Y-axis  
 $\mathbf{k}$  Unit vector in Z-axis  
 $\omega$  Angular velocity,  $deg/s$   
 $\psi$  Inner-axis rotation angle,  $deg$   
 $\vec{\omega}$  Associated angular velocity,  $deg/s$   
 $r$  Radius of a circle, m  
SXYZ Coordinate System  
 $t$  Time,  $s$

## I. Introduction

GENERATING precise input motions is crucial in inertial sensor calibration and system identification. Generally, a motion simulator is used for testing and calibrating such sensors. A motion simulator is a

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high precision device which through a multi-axis rotation system can simulate any type of rotational position, velocity, or acceleration inputs. Different motion simulators are available in various specification and serve many purposes related to inertial sensor performance evaluation.

In the case of angular accelerometers (AA), providing a sufficiently accurate angular acceleration reference is not trivial and has become one of the main concerns in dynamic angular sensor calibration, where the dynamic input is required in order to generate the proper excitation. Part of it is due to the present emphasis on gyroscopic sensors, in which static accuracy is the primary factor.<sup>1</sup> The available test equipment consequently follows this pattern. Motion simulators are usually more accurate during static (i.e. position) operations than during dynamic operations. Furthermore, the motion simulator bases bring unwanted dynamics, non-linearity, transport delays, etc. making it difficult to separate sensor issues from test equipment problems.<sup>2,3</sup>

AA are novel inertial sensors that are likely to play a significant role in future fault-tolerant flight control systems. They provides angular acceleration feedback in the design of state-of-the art fault tolerant flight control systems such as the Incremental Nonlinear Dynamic Inversion (INDI) technique<sup>4</sup> and Sensor-based Backstepping (SBB) approach.<sup>5,6</sup> A calibration is prerequisite before incorporating the AA into the fault-tolerant flight control systems. For this, a calibration procedure for the force-balance AA has been developed in a previous study.<sup>7</sup> The motion simulator of interest is a 2-axes angular velocity and position table suitable for simultaneous testing of several medium-sized Inertial Measurement Units (IMU) or Micro Electro Mechanical Systems (MEMS) sensors. Nevertheless, the previous method utilizes only one axis excitation input about the motion simulator inner axis, that is parallel to the sensor sensitive axis. The calibration table is designed to generate a smooth position reference. While the angular velocity reference is still acceptable, there are significant real discontinuities in the angular acceleration reference which significantly effect the calibration procedure.

The shortcoming in generating a viable angular acceleration reference is one of the current motion simulator limitations. A particular effect is apparent in the AA signal, where an additional frequency component is present at twice the original fundamental frequency, in constant angular velocity input motion.<sup>8</sup> Considering that the required input for angular accelerometers is a fairly rapid variation of angular velocity in the form of sinusoidal motion, the additional component will accumulate throughout the frequency spectrum.

In this paper we propose a new approach by rotating both axes simultaneously, each with a constant angular velocity. The coupling of inner and outer axis rotation generates the anticipated angular acceleration. This way, the generated input motion is independent from motion simulator's angular acceleration function. This study develops the design and implementation of a 2-axes, uniform circular input motion, for use in AA calibration tests. The initial step in this scheme involves the definition the equation of motions. Subsequently, analysis of the motion is extended by analysing the effective angular acceleration-like effect. Actual tests will comprise of an investigation of the motion simulator 1-axis output compared to the proposed sequence in terms of to what degree the quality of the acceleration reference has been improved.

The current study is structured in five sections. Section II introduces the overview of the motion simulator system. Section III describes the frame of reference, the proposed sequence and the equation of motion derivation for 2-axes motion simulator. The designed input based on the test envelope is realized into test matrix and a preliminary measurement are presented in Section IV. Then the conclusions are drawn in Section V.

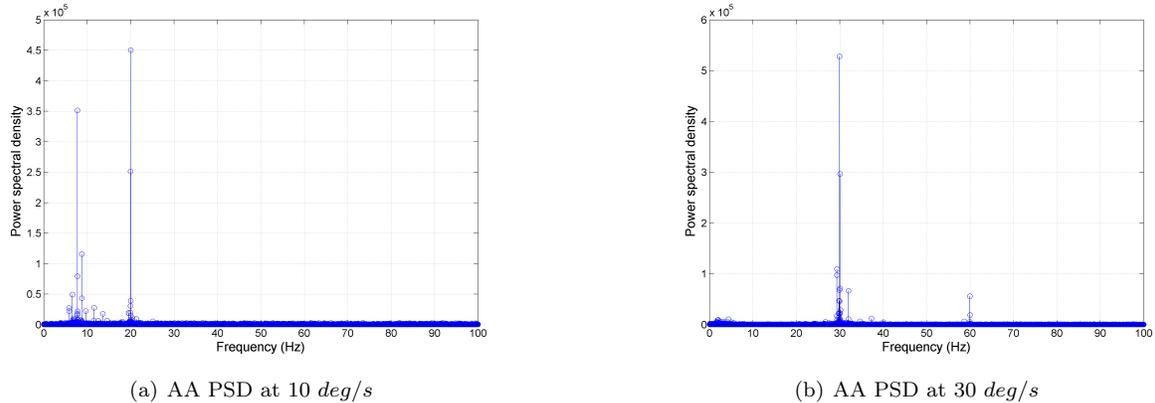
## II. Overview of the Motion Simulator

Two methods for the determination of a mechanical motion quantity are possible, in principle. With the first method it is measured directly, and with the second it is determined by differentiation and integration of the known mechanical motion quantity. In this study for example, the rotational motion of the turn-table is measured using an *angular position* transducer, and then its angular velocity and angular acceleration are estimated by means of differentiation. The integration or differentiation operations in this case (for the Acutronic AC2266L) are performed directly in the motion simulator computer.

Even though a modern motion simulator has a high precision, imperfections exist during the production process, or simply due to the interaction between its mechanical joint. One of the recently discovered effects are small oscillations in the angular velocity motion. These oscillations could be caused by either a slight flexibility in the table inner and outer axes coupling, or natural shaft resonances; both are likely to remain

a problem in our investigation as these are extremely difficult to repair or mitigate.<sup>9</sup>

A position-based, 2-axes motion simulator system comprising a turn-table and a control center, is utilized for the AA calibration in this study. It was observed that the turn-table slightly oscillates during its constant angular velocity motion, which manifests itself in the additional peak in the AA PSD analysis, see Figure 1. The additional peak appears in the spectral analysis at twice the fundamental frequency. This potentially complicates the dynamic test measurement, as the input may cover a range of frequencies.



**Figure 1. AA PSD at constant angular velocity motion of 10 deg/s and 30 deg/s which shows an additional frequency component at twice the fundamental frequency.**

Isolating the proper angular acceleration excitation at the required frequency means avoiding the use of sinusoidal input. Angular rates vary fairly rapid during sinusoidal motion, thus it is likely that the extra frequency components mix with fundamental components and higher harmonics of the intended test signal. Since the AA will measure the equivalent oscillation, it will reduce the modelling accuracy. Consequently, different methods to excite the AA should be developed.

A possibility to acquire angular acceleration excitation is to employ the pendulum method, which harnesses the potential energy of a swinging rod which is pivoted at one fixed end. In general, this method is implemented in IMUs, whether for the combination of gyroscopes and accelerometers,<sup>10</sup> with the addition of magnetometer,<sup>11</sup> or gyroscope-free IMUs.<sup>12</sup> However, this method requires a special pendulum apparatus, and in most cases, one that is customized for a specific type of IMU or sensor.

Similar to the pendulum method is the turning platform method,<sup>13</sup> which consists of turning the tested instrument's axis of sensitivity through a precisely known angle with respect to the vertical. Then the acceleration along this axis is determined by the turning angle and the acceleration due to gravity. Various precise turning devices (for instance, optical dividing heads, etc.) are widely used as a means for producing acceleration. Thus, the measurement of acceleration in this range is based on the measurement of the angle and the knowledge of gravitational acceleration at a particular point on the Earth's surface. Nevertheless, this method is particularly applicable for linear accelerometers and for accelerations below 1  $g$ .

Another known alternative is the centrifugal method, which serves to produce accelerations higher than that due to gravity. The centrifugal acceleration thus produced is determined by the platform's angular velocity and the distance from the rotation axis to the application point of the acceleration. For producing considerable acceleration amplitudes in the range of 0.01 – 30  $Hz$  the most preferable method consists of using a double centrifuge. The installation consists of two rotating platforms with parallel axes of rotation and one of the platforms being mounted on the other. The amplitude of the produced acceleration is then determined by the angular velocity of one platform and the distance between the platform axes, whereas the frequency is determined by the angular velocity of the second platform. Some work on this concept includes the biaxial test installation<sup>14</sup> and a testbed for calibrating angular-acceleration transducers.<sup>15</sup>

### III. 2-axes Turn-table Motion

In order to obtain adequate amplitudes in producing low-frequency angular accelerations, it is possible to use a system consisting of two platforms which rotate at a constant velocity about mutually-perpendicular axes. The following section will discuss the 2-axes platform frame of reference and the equation of motion for constant angular velocity application.

#### A. Motion Frame of Reference Definition

The motion of a solid body rotation about a fixed point is characterized by the motion of its center of mass. The mechanical motion quantities of a solid body are linear and angular displacements of its center of mass and all their time derivatives (displacement, velocity, acceleration, sharpness, and so on). The mechanical motion quantities are vector quantities, so they can be measured by one of two methods:<sup>16</sup>

- determination of the corresponding vector modulus and the angles characterizing the position of these vectors in the chosen reference system
- determination of the vector components by the axes of the chosen reference system

The methods for measuring mechanical motion quantities in this paper is the absolute (inertial) methods which is based on the law of inertia. This method is valid for classical mechanics, where mechanical motion quantities change in inertial space, where Newton's laws are valid.

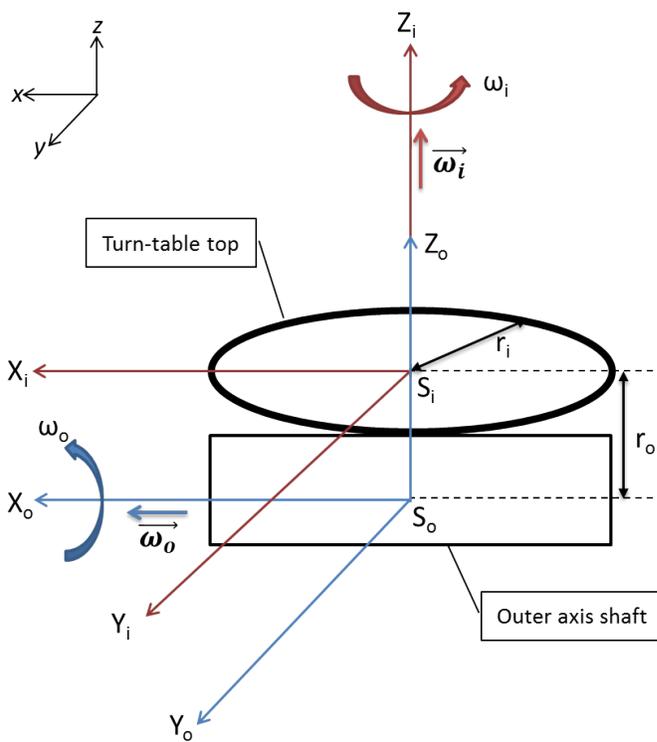


Figure 2. 2-axes motion simulator reference frame.

Figure 2 shows the 2-axes motion simulator frame of reference system in Cartesian coordinates. The  $S_i X_i Y_i Z_i$  coordinate system represent the inner-axis which carries the motion simulator top, where the sensor is mounted, and rotates around its  $Z$  axis. Whereas the  $S_o X_o Y_o Z_o$  coordinate system serves as the outer-axis, thus its axis of rotation  $X_o$  is parallel to the AA sensitive axis. The two systems are within  $r_o$  distance from each other's coordinate origin. Nevertheless, the rotational motion quantities are independent of the distance  $r_o$  between the table top and the axis of rotation for  $\omega_o$ .

The outer-axis is rigidly mounted on the table support, perpendicular to the inner-axis. Both axes meet at the center of rotation; the inner-axis is rotating without slipping and undergoes two simultaneous rotations. The table top is spinning about its axis with an angular speed  $\omega_i$  and associated angular velocity  $\vec{\omega}_i$ . While the outer-axis shaft is spinning about its axis with an angular speed  $\omega_o$  and associated angular velocity  $\vec{\omega}_o$ . Since the outer-axis is fixed, the rotation  $\omega_i$  do not affect its motion. Therefore its angular acceleration is due solely to  $\dot{\omega}_o$ . Meanwhile for the inner-axis, the angular velocity vector will change with time both due to  $\dot{\omega}_i$  as well as  $\omega_o \times \omega_i$ .

#### B. Combined 2-axes Constant Angular Rate Application

The analysis of the turn-table 2-axes kinematics is using the principle of constrained motion. It is assumed that mechanical friction and aerodynamic are very small and can be neglected. For simplicity at this stage, the motion analysis is limited only for the 2-axes turn-table.

Consider three set of axes shown in the Figure 2. Examine the problem by first defining the fixed coordinate system,  $xyz$ . Axes  $X_oY_oZ_o$  rotate with angular velocity  $\omega_o\mathbf{i}_o$  with respect to  $xyz$ . Axes  $X_iY_iZ_i$  rotate with angular velocity  $\omega_i\mathbf{k}_i$  with respect to  $X_oY_oZ_o$ . The total angular velocity with respect to the fixed axes is

$$\mathbf{\Omega} = \omega_o\mathbf{i}_o + \omega_i\mathbf{k}_i \quad (1)$$

where  $\mathbf{i}_o$  is the unit vector of axis  $X_o$  and  $\mathbf{k}_i$  is the unit vector of axis  $Z_i$ .

The measured angular acceleration however, depends on the orientation of the AA sensitive axis. In the normal 1-axis setup, the AA is mounted on the table top with its sensitive axis parallel to the inner-axis rotation, as the motion diagram in Figure 4(a). Another possible setup is by positioning the AA on its side, so that its sensitive axis is parallel with the table top or perpendicular to the inner-axis rotation, with the motion diagram shown in Figure 4(b).

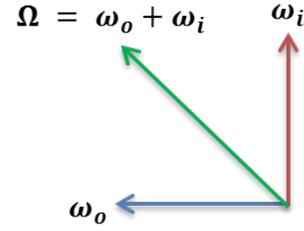
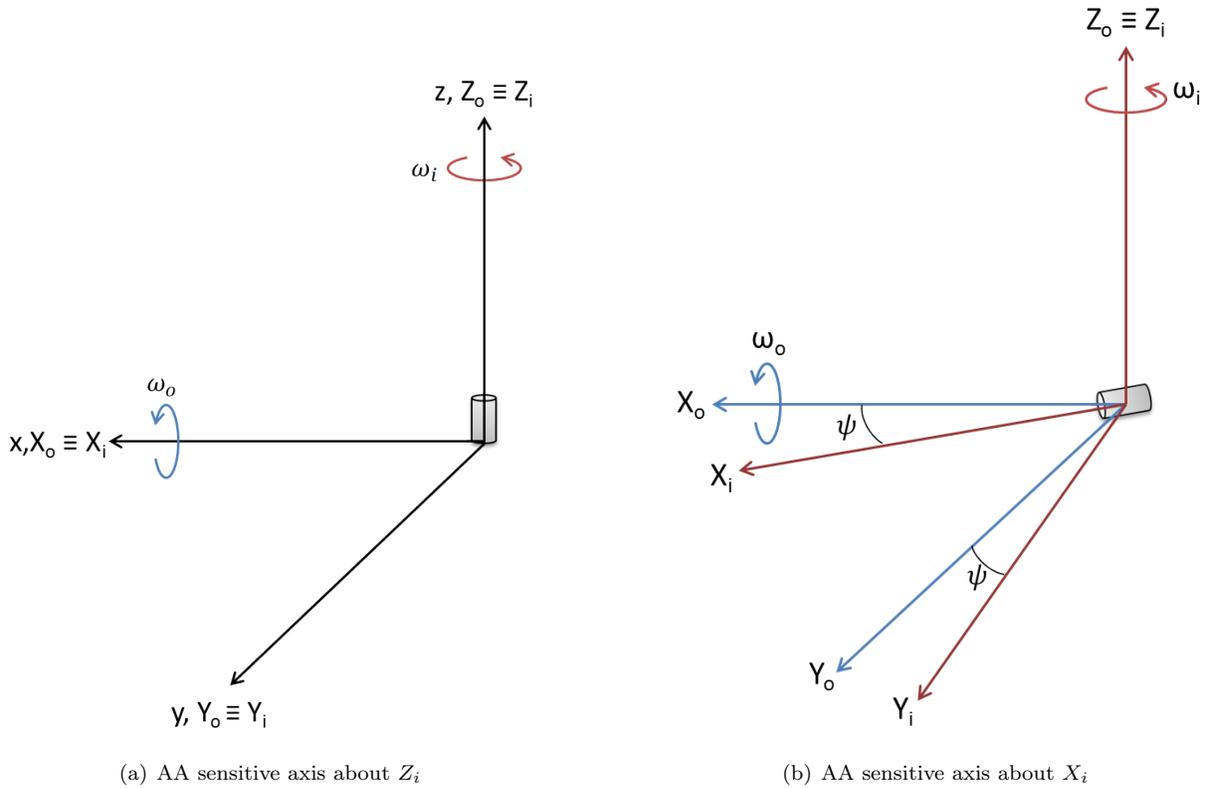


Figure 3. Total angular velocity vector.



(a) AA sensitive axis about  $Z_i$

(b) AA sensitive axis about  $X_i$

Figure 4. Motion diagram of AA sensitive axis orientations, about  $Z_i$  and about  $X_i$ .

### 1. Angular Accelerometer Sensitive Axis About the Motion Simulator Inner-Axis

In this first setup, the AA is installed at the center of  $S_iX_iY_iZ_i$  coordinate system, with its sensitive axis along the  $Z_i$  axis. Hence, it immediately detects  $\omega_i$  and any other rotation that affect the inner-axis. In order to calculate angular acceleration of the AA with respect to the fixed axes  $xyz$ , consider the derivative of Equation (1),

$$\begin{aligned} \left(\frac{d\mathbf{\Omega}}{dt}\right)_{xyz} &= \left(\frac{d}{dt}(\omega_o\mathbf{i}_o + \omega_i\mathbf{k}_i)\right)_{xyz} \\ &= \left(\frac{d}{dt}(\omega_i\mathbf{k}_i)\right)_{xyz} + \dot{\omega}_o\mathbf{i} \end{aligned} \quad (2)$$

at the instant,  $\mathbf{i}_o = \mathbf{i}$ . Here,  $\mathbf{i}$  does not change with respect to the axes  $xyz$  and therefore only the magnitude of  $\omega_o$  changes.

Apply Coriolis Theorem to calculate the time derivative of  $\omega_i \mathbf{k}_i$  with respect to the fixed frame. Take  $\omega_i$  constant in magnitude, but not direction, and  $\omega_o$  as constant. The rotation  $\omega_o$  will rotate the vector  $\omega_i$ , changing its direction.

$$\left( \frac{d}{dt} (\omega_i \mathbf{k}_i) \right)_{xyz} = \left( \frac{d}{dt} (\omega_i \mathbf{k}_i) \right)_{X_o Y_o Z_o} + \omega_o \mathbf{i} \times \omega_i \mathbf{k}_i \quad (3)$$

with  $\mathbf{k}_i = \mathbf{k}_o$ . In the  $X_o Y_o Z_o$  frame,  $\mathbf{k}_o$  does not change direction, then

$$\left( \frac{d}{dt} (\omega_i \mathbf{k}_i) \right)_{xyz} = \dot{\omega}_i \mathbf{k}_o + \omega_o \mathbf{i} \times \omega_i \mathbf{k}_o \quad (4)$$

Combination of Equation (2) and Equation (4) gives

$$\begin{aligned} \left( \frac{d\Omega}{dt} \right)_{xyz} &= \dot{\omega}_i \mathbf{k}_o + \omega_o \mathbf{i} \times \omega_i \mathbf{k}_o + \dot{\omega}_o \mathbf{i} \\ &= \dot{\omega}_o \mathbf{i} - \omega_o \omega_i \mathbf{j}_o + \dot{\omega}_i \mathbf{k}_o \end{aligned} \quad (5)$$

Since  $\omega_o$  and  $\omega_i$  are constant, then  $\dot{\omega}_o = 0$  and  $\dot{\omega}_i = 0$ . Equation (5) becomes

$$\left( \frac{d\Omega}{dt} \right)_{xyz} = -\omega_o \omega_i \mathbf{j}_o \quad (6)$$

The generated angular acceleration vector displayed in Figure 5 has a direction along  $-\mathbf{j}_o$ . This vector is perpendicular to the AA sensitive axis about  $k_o$  and consequently, does not affect the measured angular acceleration by the AA.  $\omega_o$  that is perpendicular to the AA sensitive axis will serve as the angular frequency, but furthermore, this setup will not produce the expected angular acceleration for the AA.

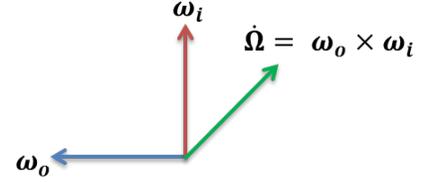


Figure 5. Angular acceleration as the cross product of the inner and outer-axis angular velocity vector.

## 2. Angular Accelerometer Sensitive Axis Perpendicular to the Motion Simulator Inner-Axis

In the second setup as shown in Figure 4(b), the AA is installed at the center of  $S_i X_i Y_i Z_i$  coordinate system, with its sensitive axis along the  $X_i$  axis which is parallel with the outer-axis of rotation,  $X_o$ . Since the outer-axis is fixed, only rotation  $\omega_o$  is affecting measurement in this direction. However, from the AA point-of-view, its sensitive axis will follow axis  $X_i$  which rotates with angular velocity  $\omega_i$  and rotational angle  $\psi$  towards axis  $X_o$ . This means that the angular velocity which influences the AA sensitive axis is equal to the projection of  $\omega_o$  on the  $X_i$  axis. Moreover, the AA will measure a time-varying term due to its change of orientation.

The angular velocity works on the AA then can be define as

$$\omega_{AA} = \omega_o \cos \psi \mathbf{i}_i \quad (7)$$

with  $\psi = \psi_0 + \omega_i t$ . Consider the initial inner-axis rotational angle  $\psi_0 = 0$ , gives

$$\omega_{AA} = \omega_o \cos(\omega_i t) \mathbf{i}_i \quad (8)$$

In order to calculate angular acceleration of the AA with respect to the fixed axes  $xyz$ , consider the derivative of Equation (8),

$$\begin{aligned} \alpha_{AA} &= \frac{d}{dt} \omega_{AA} = \frac{d}{dt} (\omega_o \cos(\omega_i t) \mathbf{i}_i) \\ &= -\omega_i \omega_o \sin(\omega_i t) \mathbf{i}_i \end{aligned} \quad (9)$$

The above expression shows that the combined 2-axes, constant angular velocity motions produce an alternating angular acceleration response for the AA. The angular acceleration vector is in the direction of unit vector  $\mathbf{i}_i$  of axis  $X_i$ , i.e. similar to the AA sensitive axis. The response is in the form of periodic sinusoidal output with an amplitude of  $\omega_i \omega_o$  and an angular frequency of  $\omega_i$ .

## IV. Designing Angular Accelerometer Test Input

Developing the test plan is crucial in order to characterize the sensor response. The first step involves recognizing the limit of the motion simulator, as well as the required operational condition of the AA. To be in-line with the applicable motion simulator input, the angular variables are described in degrees. The following subsections describes the test envelope identification and theoretical calculation of the generated angular acceleration.

### A. Determining the Performance Envelope

In operating the motion simulator, the factory limits of the equipment determine the maximum operational capability of each motion mode. Although the maximum bandwidth by design could reach up to 30  $Hz$  for the inner-axis and 20  $Hz$  for the outer-axis, the actual setup requires the terminal frequency to be 11  $Hz$ .<sup>17</sup> The terminal frequency is subsequently reduced to 11  $Hz$ , with consideration of the customized sensor Data Acquisition System (DAS) weight installed on the table top. Two other restrictions are the angular velocity and angular acceleration limits, for which this equipment has different boundaries on its inner and outer-axis. The inner-axis has an acceleration limit of 1000  $deg/s^2$  and a angular velocity limit of 1200  $deg/s$ . Whereas the outer-axis have an acceleration limit of 200  $deg/s^2$  and a angular velocity limit of 1000  $deg/s$ . These limitations are represented by the black, red and blue lines in Figure 6.

More importantly, the AA performance specification should also be taken into account in specifying the boundaries. The AA has a maximum customized range of 10  $rad/s^2$ <sup>a</sup> or 572.958  $deg/s^2$ . A noise level corresponding to 0.8  $deg/s^2$  is used as the lowest extent of the measured angular acceleration. Lastly, the AA frequency threshold is set at 0.4  $Hz$  for the minimum and 8.5  $Hz$  maximum. The green lines in Figure 6 mark the AA range and limitations.

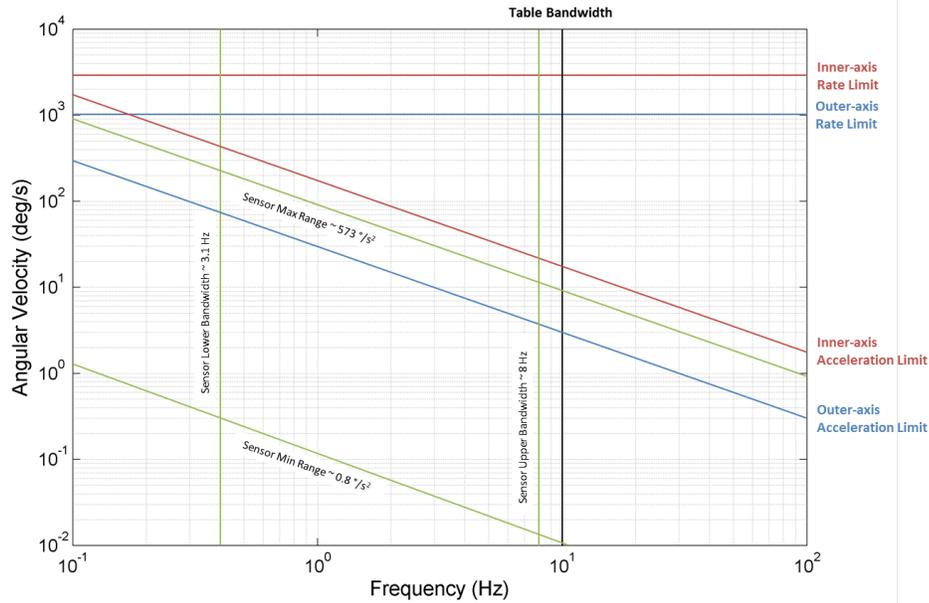


Figure 6. Motion simulator inner and outer-axis performance limitations and AA range.

Two important aspects arise based on the system's limitations related to Equation (9). The first implication is, that to achieve a 1  $Hz$  sine,  $\omega_i$  should be set to 360  $deg/s$ . With a angular velocity limit of 1200  $deg/s$ , the inner-axis could only accommodate angular frequency up to 3.006  $Hz$ <sup>b</sup> or equal to 1082.153  $deg/s$ . The second effect concerns the relation of  $\omega_i\omega_o$ , which defines the amplitude of the sine and represent the angular

<sup>a</sup>AA Specification, <http://www.crlsensors.com/>

<sup>b</sup>The test frequency points refer to the method developed in the previous study<sup>17</sup>

acceleration value. A maximum angular frequency of 3.006  $Hz$  for  $\alpha = 450 \text{ deg/s}^2$  or 78.6% of the maximum range for example, gives

$$\alpha = \omega_i \omega_o \Rightarrow \omega_o = \frac{\alpha}{\omega_i} = 0.416 \text{ deg/s}$$

## B. Test Matrix Design

The measurement series for AA calibration is designed to accommodate constant accelerations over the frequency range. The performance envelope described in Section A and pictured in Figure 6 gives the boundary in which the tests can be safely performed and valid for the sensor of interest.

**Table 1. 2-axes input test**

Item (#)	$\omega_i^1$		$\omega_o, (\text{deg/s})$		
	( $Hz$ )	( $\text{deg/s}$ )	400 $\text{deg/s}^2$	450 $\text{deg/s}^2$	500 $\text{deg/s}^2$
1	0.397	142.822	2.801	3.151	3.501
2	0.473	170.288	2.349	2.643	2.936
3	0.58	208.74	1.916	2.156	2.395
4	0.687	247.192	1.618	1.82	2.023
5	0.824	296.631	1.348	1.517	1.686
6	0.992	357.056	1.12	1.26	1.4
7	1.007	362.549	1.103	1.241	1.379
8	1.19	428.467	0.934	1.05	1.167
9	1.434	516.357	0.775	0.871	0.968
10	1.709	615.234	0.65	0.731	0.813
11	1.999	719.604	0.556	0.625	0.695
12	2.06	741.577	0.539	0.607	0.674
13	2.487	895.386	0.447	0.503	0.558
14	2.975	1071.167	0.373	0.42	0.467
15	3.006	1082.153	0.37	0.416	0.462

<sup>1</sup>  $\omega_i$  as angular frequency

To build the test matrix, the first step is defining the measurement frequency range. Using the lowest AA range of 0.4  $Hz$ , the frequency is raised 20% from the previous value. Additionally, the round frequency points are also included. These frequencies are adjusted to obtain an integer number of complete sine cycles for the spectral estimation purpose, and listed in the second column of Table 1. Note that due to the inner-axis angular velocity limit, the maximum angular frequency is restricted to 1082.153  $\text{deg/s}$  or 3.006  $Hz$ .

The second step is to convert the frequency in  $Hz$  into angular frequency in  $\text{deg/s}$ , shown in the third column of the same table. In this regard, 1  $Hz$  is equal to one full rotation per time unit, or 360  $\text{deg/s}$ . Based on the specified frequency range, the lowest angular frequency is 142.822  $\text{deg/s}$  and the highest is 1082.153  $\text{deg/s}$ .

Next, angular acceleration points are selected to calculate the required  $\omega_o$ . To obtain an adequate excitation as well as avoid the clipping of data, 80% of the maximum AA range is chosen. This gives an angular acceleration point of 458.367  $\text{deg/s}^2$ , which then simplified to 450  $\text{deg/s}^2$ . Two additional points are selected, with 50  $\text{deg/s}^2$  lower and higher difference. The associated  $\omega_o$  are listed in column 4 to 5 of Table 1.

Estimation of the generated angular acceleration sensed by the AA is presented in Figure 7. The fifteen plots represent the calculated angular acceleration for all the specified frequencies, at angular acceleration of 450  $\text{deg/s}^2$ . The lowest and the highest frequencies are featured in dashed-bold lines.

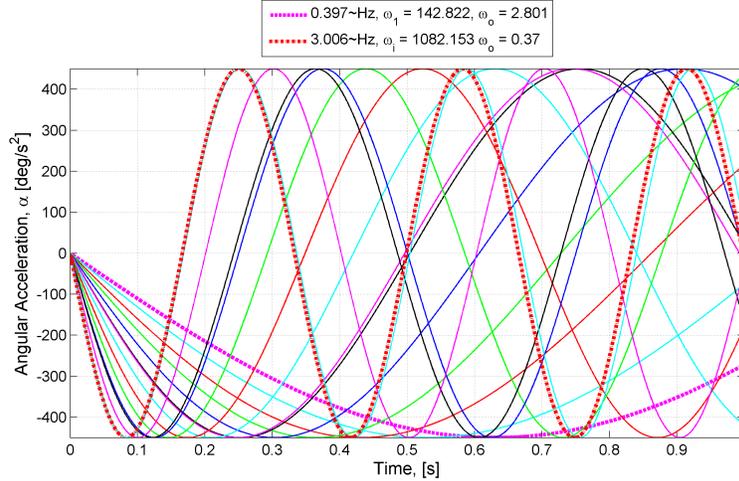


Figure 7. Theoretical angular acceleration as a function of two constant angular velocity  $\omega_i$  and  $\omega_o$ , for the specified frequency range.

To validate the proposed angular acceleration generation procedure, a series of measurements are planned on the motion simulator system based on the proposed test matrix in Table 1. However, due to the current setup limitation where the sensor DAS is required to be installed on top of the turntable top,<sup>18</sup> the preliminary measurement will only be performed for the lowest angular frequency and lowest angular acceleration. This due to the fact that the inner-axis will rotate rapidly with the increase of angular frequency, which is potentially hazardous to the bulk DAS setup.

Figure 8 presents the AA signal PSD for the angular acceleration point of  $450 \text{ deg/s}^2$  and a combination of  $\omega_i = 142.822 \text{ deg/s}$  and  $\omega_o = 2.801 \text{ deg/s}$ . The figure shows a single peak of frequency component, exactly at the specified frequency point of  $0.397 \text{ Hz}$ . Based on this, the proposed 2-axes sequence demonstrates the ability to eliminate the unwanted oscillation in 1-axis input motion. Furthermore, the AA data indicates that the combination of constant angular velocity motion between the two axes results in angular acceleration output.

Although the proposed method successfully generate the angular acceleration output motion for the AA, a prominent constraint emerges in terms of frequency bandwidth. The inner-axis maximum angular velocity which act as the angular frequency, limit the capability of the motion simulator to identify the full AA dynamics. The mechanical characteristic is unfortunately defines a fixed, narrow bandwidth for this type of motion simulator.

Observing the setup, some limitations might be attributed to the sensor DAS, AA alignment and 2-axes balance. The future exercise should investigates the relation and optimization among these elements, in order to apply the test matrix for the sensor calibration. Additionally, the outer axis rotation also means that Earth's rotational velocity, however small, needs to be taken into account.

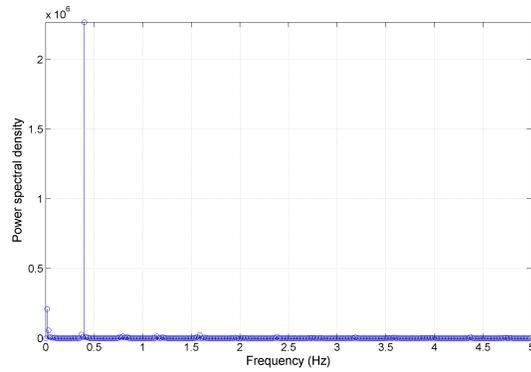


Figure 8. Theoretical angular acceleration as a function of two constant angular velocity  $\omega_i$  and  $\omega_o$ , for the specified frequency range.

## V. Conclusion

The 2-axes combined motion is intended to excite the AA in a calibration test, to determine its frequency response characteristics. In order to reproduce angular accelerations of a given amplitude and frequency, the inner-axis platform which carries the AA rotates at a constant angular velocity about the mutually perpendicular axes  $Z_i$  and  $X_o$ . The AA sensitive axis orientation plays an important part in order to measure the related resultant motion, which in this case is aligned with axis  $X_i$ .

Combination of the 2-axes, constant angular velocity motion generates an angular acceleration effect for the AA in the form of sinusoidal response, with an amplitude of  $\omega_i\omega_o$  and an angular frequency of  $\omega_i$ . The angular acceleration obtained by means of this method is affected by the mechanical limitations of the motion simulator, as well as the sensor specification. Therefore, it can be concluded that the resulted accelerating amplitudes is accurate over a narrow angular frequency range.

The principle used in exciting the AA using a combination of constant angular velocity enables one to angular acceleration response in its fundamental frequency. Thus the modelling of the AA will be independent from the extra frequency component when the angular velocity is vary in normal sinusoidal input. The proposed method provides an alternative means to generate inputs that can be used to calibrate angular accelerometers using calibration hardware that is not primarily designed to provide accurate acceleration inputs.

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