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Robust train trajectory optimization and train motion model calibration

Alejandro CUNILLERA PÉREZ

Robust train trajectory optimization and train motion model calibration

Dissertation

for the purpose of obtaining the degree of doctor
at Delft University of Technology
by the authority of the Rector Magnificus, Prof. dr. ir. H. Bijl,
chair of the Board for Doctorates
to be defended publicly on
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Meow!

飞弹, Rongata

Preface

And here I am, writing the last sentences of my thesis. Since locomotives and the railroad cars pulled by them centralize the content in the following pages of this manuscript, I will take the freedom to use these lines for a different purpose: thanking the people that have been important to me during my research.

To Nikola Bešinovic, thank you for your nice advice and support during the first year of my thesis. It is a pity that you had to leave the Netherlands, but at least we had the time to publish one of my favourite papers together. I wish you all the best in your career and hope that our paths will cross again soon.

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To my family, friends and colleagues, for your love, support and the good times spent together.

To Rongata (飞弹). She cannot read this, so I will tell her personally how thankful I am for her purrs, for waking me up with a big *Meow!*, for warmly welcoming me when arriving home and for her unconditional support.

I will continue now in Spanish.

A mis padres, por dar sentido a cada una de las acciones de mi vida.

A Yanyan, por hacer nuestros días más brillantes.

Y a Eva Xin, por enseñarme lo verdaderamente importante de la vida.

Alex Cunillera
Beijing, 2026

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Chapter 1

Introduction

1.1 Context and motivation

The railway industry faces a pressing need to cut down on energy usage due to society's environmental awareness, the increasing competition with other mobility modes and the current political sustainability goals (IEA, 2019). This challenge is driven by a mix of environmental responsibility, cost-effectiveness and advancements in technology. The steadily increasing energy costs are adding extra stress on the accounting of railway undertakings (DiDomenico & Dick, 2015), and part of this stress has already impacted travel fares. Reducing the amount of energy consumed is an incentive for railway companies, not only from an economical perspective, but also because it helps building a stronger and more sustainable public image. Energy consumption is directly linked to greenhouse gas emissions and other types of pollutants like soil contamination and microparticles. Rationalizing the energy usage should be at the forefront of railways' transformation into a more sustainable transport mode. Despite the fact that railways are already the most sustainable among mass-transport modes (IEA & UIC, 2017), there is still potential for reducing its environmental footprint.

Energy-efficient train driving is one of the most effective ways of saving energy in railway operation (Scheepmaker et al., 2017). It could be defined as any measure that can be implemented to drive a train from its departure to its destination with reduced energy consumption. Several techniques and measures are encompassed under the energy-efficient driving label. Train Trajectory Optimization, Driver Advisory Systems, Automatic Train Operation, energy-efficient timetabling and training drivers on energy-efficient driving techniques are some examples of these measures. Still, not any energy-efficient driving measure can be implemented in practice. For instance, the energy consumption is inversely proportional to running times, which may increase the operating costs in some cases. Passengers generally perceive long in-vehicle travel times negatively, so such a measure might lead to an undesired modal shift away from rail, lowering railway companies' revenues (Cheng & Tsai, 2014). Instead, railways have to be efficient, aiming to run on time with short journey times while saving as much energy as possible. In this regard, punctuality is the main priority. If a train runs late, it must try to recover as much delay as possible at the cost of consuming more energy. Timetables include running time supplements for accommodating potential delays and buffers over the minimum operational dwell times to prevent delays from stops that last longer than expected (Scheepmaker &

Goverde, 2015). These running time supplements and buffer times can be used to save energy if the train is running on time.

Train Trajectory Optimization is arguably the core of energy-efficient train driving. It is a nonlinear optimal control problem that aims to calculate the most energy-efficient control strategy for driving a train timely to its destination (Albrecht et al., 2016b). Such a strategy must comply with the departure and arrival times, and passing times and time windows given by the timetable. The train dynamics, namely the traction and brake characteristics, the mechanical and aerodynamic running resistances, and the resistance to motion due to the track geometry also play a fundamental role when calculating the optimal driving strategy. The infrastructure is also included in the Train Trajectory Optimization problem by means of its gradients, curves and speed limits. Several types of optimization methods have been devised for calculating energy-efficient train trajectories. Among these, indirect methods transform the Train Trajectory Optimization problem into a boundary value problem and solve the latter, while direct methods discretize the problem at carefully-selected points, turning it into a nonlinear programming problem which solution is the optimal trajectory (Scheepmaker et al., 2017). Both methods can be fast enough for real-time implementations, but in this thesis we focus on a direct method because it offers more flexibility in the modelling and it is easier to be implemented.

A mathematical analysis of the Train Trajectory Optimization problem shows that four driving regimes are possible in a trajectory that minimizes the energy consumption given running time constraints: applying traction at the maximum capability of the train, holding a constant speed, coasting and applying maximum brake (Albrecht et al., 2016b). This is a very valuable insight, since drivers can be instructed to coast, and to accelerate and brake hard to save energy (Luijt et al., 2017). Still, sudden acceleration changes and high acceleration or deceleration rates may affect the comfort of passengers (De Winkel et al., 2023). Timetables that incorporate energy-efficient trajectories allocate the available running time supplements in a way that maximizes the potential for saving energy (Scheepmaker et al., 2020a). Furthermore, it is important to adapt the timetable to the actual way of driving trains. The timetable provides trains a set of timing constraints called Train Path Envelope, which establish target times, or time windows at specific locations of the infrastructure (Wang & Goverde, 2016). Driver Advisory Systems and Automatic Train Operation that incorporate real-time Train Trajectory Optimization can adapt to the current operating conditions and to any updates in the timetable included in the Train Path Envelope. These two energy-efficient driving technologies also smooth railway operation by reducing the dispersion in the arrival times, leading to higher punctuality rates and lower infrastructure capacity usage (Franke et al., 2000).

Train Trajectory Optimization relies on a physical description of the dynamics of the train: the train motion model (Hansen & Pachl, 2014). It is a simple, yet accurate representation of the train dynamics which contains a handful of parameters that need to be estimated from data so that the train motion model can reproduce and predict the dynamics of the train accurately (Cunillera et al., 2023a). In this thesis, this parameter estimation process is referred to as train motion model calibration. The same model may reproduce the dynamics of two different train units. The difference is the values of the train motion model parameters, which are tailored to each train. The performance of energy-efficient driving technologies and Train Trajectory Optimization in particular depends

heavily on the accuracy of the model calibration. For example, freight trains usually have different compositions and geometries on different days. Historical data of freight trains with specific compositions may not be available for calibrating the train dynamics, so real time calibration is essential for guaranteeing good performance of energy-efficient driving techniques for such trains. Metro trains typically run through tunnels which add extra aerodynamic drag on the rolling stock. This translates into additional resistance to motion, which depends on the cross-section of the tunnels and the roughness of the walls, among other factors. This example evidences that the calibration of the train motion model does not only depend on the rolling stock, but also on the specific line that the train is servicing.

Despite the importance of train motion model calibration in the railway industry and academia, it has not received much recognition in the literature. Several ways of calibrating the train motion model exist: from computational analyses and closed-track tests, to data analysis from measurements recorded during real operation. We can distinguish offline calibration that analyzes large quantities of data in one go, producing one set of estimates of the train motion model parameters, and online calibration that analyzes data as soon as it is generated (Cunillera et al., 2023a). Among these, only the latter may capture any temporal parameter variations due to rolling stock ageing, different weather conditions or the impact of tunnels on the train dynamics.

Train Trajectory Optimization has been tackled mainly from a deterministic perspective, although railway operation is essentially stochastic and uncertain. The number of passengers onboard a train and the weather conditions are some examples of uncertain operating conditions that vary every day. Three types of sources of stochasticity and uncertainty are usually described in the literature: noise, parameter variations and unmodelled dynamics (Boltanski & Poznyak, 2011). Noise can be defined as random fluctuations in one or more variables. For instance, the wind direction is not constant over time, as it randomly varies making the aerodynamic drag of the train fluctuate. Parameter variations are significant fluctuations of the train characteristics included in the train motion model. For example, the resistance to motion is higher under headwinds than under tailwinds. In turn, all the phenomena that are not explicitly included in the model are unmodelled dynamics. For instance, the impact of the track cant or superelevation on the dynamics of the train is rarely included in the train motion model. The typical driving variability present in manually-driven trains is also generally not included in the train motion model and thus is technically an unmodelled dynamic, although it deserves special attention in railway operation. Reaction times and the applied acceleration and brake rates are simple examples of manual driving characteristics that vary among drivers, and from day to day for a single driver. The variability of manual driving characteristics has an impact on the punctuality rates and on the capacity of the railway network. For passengers comfort purposes, drivers accelerate and brake more gently than Automatic Train Operation, which can control the rate of change of the acceleration more precisely.

Some of the sources of stochasticity can be quantified and included in the train motion model in a different way, at the expense of complicating the train motion model and making the Train Trajectory Optimization problem harder to solve. The train motion model can be transformed into a stochastic differential equation that includes noise terms. However, characterizing the exact properties of the noise is difficult and requires analyzing

large amounts of data. Furthermore, determining which are the noisy components of the train motion model that are worth being considered is also nontrivial. The optimal control theory of stochastic differential equations is a well established theory that could allow to solve the resulting problem in some cases. Still, the solutions of stochastic optimal control problems depend on expectations of physical quantities and may not capture outliers or scenarios rarely observed in railway operation.

Parameter variations in different trajectories can be quantified by means of parameter estimation and the values obtained may be introduced in the train motion model (Cunillera et al., 2023a). This requires analyzing data from several realized train trajectories. If this data is measured in different operating conditions, the observed parameter variations can be associated to the conditions observed and be given physical meaning. For instance, headwinds increase the aerodynamic drag, while tailwinds decrease it. Analyzing data measured under both conditions allows to quantify their respective impact on the train dynamics. The Train Trajectory Optimization problem can then be solved for a specific set of parameter estimates, in the same way as in classical, deterministic Train Trajectory Optimization. For example, if headwinds are expected, the train trajectory to be used as a reference should be computed using the train motion model parameters estimated using data measured under headwind conditions. This gives more realism to the calculations and allows to have a better understanding of the physical meaning of the train characteristics to be used in the calculations. The parameter estimates used for calculating train trajectories must represent faithfully the expected operating conditions, otherwise the accuracy of the calculated trajectories may be jeopardized. The use of more relevant data in train trajectory calculations can help improving the performance of the energy-efficient driving measures and railway operation. In case of parameter variability during a trajectory, online estimation can capture this variability at each point of the track. For instance, the available power at the catenary depends, among other factors, on the number of trains obtaining energy from the substation feeding the catenary. The amount of tractive power that a train can apply can be lower than expected if several neighbouring trains are also consuming electricity from the same substation. This information can be incorporated for instance in a Train Trajectory Optimizer based on direct optimization, which modelling flexibility allows to introduce differences in the train dynamics at different locations of the track.

In turn, unmodelled dynamics require both mathematical modelling and field testing efforts to assess the impact of any phenomena. For example, the impact of curves on the dynamics of trains is still actively discussed (Wu et al., 2022b). Several formulas describing how curves add extra running resistance to motion exist, but there is no consensus on which one performs best. The effect of curves in the train motion model can be neglected at the cost of lower model accuracy.

Alternatively, Train Trajectory Optimization can be performed in a loop as the train performs its journey, receiving feedback on the performance of the train with respect to the optimal trajectories previously generated. This feedback loop allows to adapt the control strategy to the real performance of the train. The faster the loop is, the lower the potential impact of errors in the train motion model calibration and unmodelled dynamics will be. Regarding the impact of human driving, there are several ways of introducing it into the train motion model. A model of how human drivers apply traction and brake

could substitute the usual traction and brake effort term in the train motion model. The reaction times and the acceleration and deceleration rates can be considered as random variables. This new model would require to be calibrated from real data. Different drivers might show some differences in their driving style, and such variability could also be estimated from the analysis of data and introduced into the model.

This analysis of the possible ways of incorporating the stochasticity inherent in railway operation into the Train Trajectory Optimization process highlights a key question: How should the railway industry expect a train to be driven energy-efficiently if the operating conditions are uncertain? Can it be done in an effective, yet intuitive way for practitioners? How can train trajectories be calculated, so that they are robust against uncertain operating conditions? Since characterizing noise is difficult and adding terms to the train motion model that are not usually considered would require modelling and quantifying the impact of each operating condition separately, stochastic optimal control may not be suitable for this purpose. Adapting the control strategy to the actual performance of the train is not useful in planning stages, since no *a priori* information on the performance of the train is available before it runs. Parameter estimation is a more adequate candidate to characterize and quantify different operating conditions than the other methods that have just been described. The estimates obtained are intuitive from a physical point of view, in the sense that they can be given physical meaning based on the operating conditions under which the data used for their estimation was measured. It also allows to gauge in a simplified way the impact of human driving on the dynamics of a train. For instance, if a train driver applies traction gently, it is more important to introduce this in the train motion model than the actual maximum tractive power that the rolling stock can provide. Furthermore, a wide variety of parameter estimation techniques are available in the literature. An energy-efficient driving algorithm should be adapted to the style of each driver and human factors should be taken into account. Regarding Driver Advisory Systems, an advice that is far from the usual driving style or that asks the driver to perform uncommon actions is likely to be rejected. Driver's instruction should establish the basic driving notions and algorithms should also support drivers in line with the guidelines learned during the instruction.

Still, in the timetabling process it is not possible to know in advance which are the exact operating conditions that will take place. Moreover, the operating conditions can change over time, hindering adhering to the existing timetable. Even if the timetable is not modified based on the expected operating conditions, any available information on the potential operating conditions can be incorporated in the Train Trajectory Optimization process. For instance, if the weather forecast predicts rain for tomorrow, train motion model parameters estimated during wet operating conditions should be incorporated in the Train Trajectory Optimization problem instead of those corresponding to dry conditions. Furthermore, the analysis of the characteristics of manual driving recorded in rainy days can also be taken into account.

Train motion model calibration is therefore an adequate candidate to gauge the impact of the stochasticity of railway operation in the train dynamics and to produce trajectories that are robust against this stochasticity. Given the need of the railway industry for robust methods for optimizing train trajectories, in the sense that the uncertainties present in railway operation do not affect the calculated trajectories calculated, the main purpose of

this thesis is to bridge this gap and design effective ways of capturing the stochasticity of railway operation and incorporating this information in the Train Trajectory Optimization problem.

1.2 Research objective and research questions

Train Trajectory Optimization has been mainly addressed from a deterministic perspective, neglecting the effect of the uncertainty and the stochasticity inherent in railway operation on optimal train trajectories.

The main objective of this thesis is to develop models and algorithms to calculate robust train trajectories that optimize the running time and energy consumption while considering uncertainties and stochastic variations in the model input parameters.

We address this from a train-centric perspective where robust trajectories are calculated on-board or for timetabling purposes, and no automated conflict detection and resolution are available. The algorithms designed must be computationally efficient, as we also assume that all calculations can be performed by on-board processing units. This research objective is divided in the following research questions:

1. How can the main uncertainties and variations in the characteristics of train operations be quantified? (Chapters 2-5)
2. What is the influence of these variations on optimized train trajectories? (Chapter 6)
3. How can Driver Advisory Systems incorporate dynamic, inaccurate and limited real-time information? (Chapter 7)
4. How can the uncertainties present in train operation be integrated in a real-time train trajectory optimizer? (Chapter 8)
5. How can train path envelopes be computed, so that they are robust against these variations and drivable train trajectories are guaranteed? (Chapter 8)

1.3 Thesis contributions

1.3.1 Scientific contributions

The scientific impact of this thesis is abridged in the following points:

1. A literature review on train motion model calibration (Chapter 2)

The existing train motion model calibration techniques are classified in two main types depending on the purpose and speed of the estimation: offline and online techniques. Chapter 2 provides a deep overview and discussion of these train motion model calibration techniques. A research agenda for the following years and practical recommendations for the accurate calibration of the train motion model are proposed.

2. A robust train motion model calibration method (Chapter 3)

One of the main research gaps identified in the previous point is the lack of train motion model calibration techniques that are robust against noise, measurement outliers or wrong measurements. This gap is addressed in chapter 3, where a robust train motion model calibration method based on M-estimation is proposed, tested and analyzed.

3. Calibrating the train motion model parameters using online estimation (Chapters 4 and 5)

Data from trains in operation is analyzed in real time for calibrating the train motion model in chapter 4 and statistics on the volatility of the train motion model parameters are analyzed. Assessing whether the train motion model has been calibrated accurately is difficult when using real data due to the lack of ground truth data on the values of the train motion model parameters. In chapter 5 we address this issue by testing the performance of an online calibration method designed specifically for calibrating the impact of variations of the aerodynamic resistance on the train dynamics in several numerical scenarios.

4. An improvement to one of the state of the art algorithms for Train Trajectory Optimization (Chapter 6)

We observed that a modified version of the real time train trajectory optimizer proposed by Jibrin et al. (2021b) produces trajectories that are sensitive to the number of collocation points used to discretize the track. We solved this issue in chapter 6 by introducing Chebyshev collocation points in the track discretization process, making the algorithm less sensitive to the number of collocation points used.

5. The identification of the most critical train motion model parameters for optimizing train trajectories (Chapter 6)

We studied the volatility of optimal train trajectories with respect to variations of the train motion model parameters by means of a novel sensitivity analysis. The results revealed in chapter 6 spotlight the importance of calibrating the running resistance of a train for energy-efficient driving, of the appropriate instruction of train drivers, and some benefits of the implementation of Automatic Train Operation.

6. The analytical solutions of the train motion model (Chapter 7)

The train motion model is a differential equation that describes the dynamics of a train. We derive its analytical solutions in chapter 7, which can be used in a wide variety of scenarios that require reproducing or predicting the dynamics of a train: from timetable design and evaluation, to simulation, Driver Advisory Systems and Automatic Train Operation.

7. A coasting advice algorithm used by the train drivers working at NS in the Netherlands (Chapter 7)

The analytical formulas mentioned in the previous point have been used to improve the accuracy of the coasting advice algorithm developed by the Dutch railway undertaking NS. The resulting algorithm is computationally efficient and allows to predict arrival times in real time on tracks with complex gradient profiles.

8. Train Trajectory Optimization under uncertainty (Chapter 8)

A new Robust Train Trajectory Optimization algorithm is proposed in chapter 8. It utilizes as input several sets of estimates of the train motion model parameters that represent different operating conditions of interest. For example, if rain is expected, sets of train motion model parameters that were estimated under different wet rail conditions can be introduced in the robust Train Trajectory Optimization problem to calculate a robust-optimal control strategy to drive the train to its destination. Including the uncertainties inherent in railway operation in the Train Trajectory Optimization process allows to reduce the impact of these uncertainties in the trajectories described by trains. In practice, this is useful for mitigating the impact of weather on the punctuality rates, on the capacity of the railway network and on the amount of energy consumed, which will ultimately lead to providing a better service.

9. The proposal of a concept for Train Path Envelopes that are robust against uncertain operating conditions (Chapter 8)

Deterministic Train Path Envelopes define time windows at the timing points for each train to avoid train path conflicts. In chapter 8 the definition of Train Path Envelopes is upgraded by incorporating into it the different passing times obtained by means of Robust Train Trajectory Optimization.

1.3.2 Societal relevance and contributions

This research was performed in collaboration with NS. All the theories, methods and algorithms have been developed with the aim of producing practical outputs that not only can reach society through scientific publications and academical presentations, but also through useful algorithms that may be used at NS. The societal impact of this thesis can be described as follows:

- **Assisting train drivers at NS to arrive on time and energy-efficiently** (Chapter 7)

The coasting advice generator presented in chapter 7 was implemented in the tablets used by NS' train drivers in July 2022, and is contributing to the sustainable mobility of approximately 1.3 million passengers every day. The algorithm assists thousands of train drivers at NS on a daily basis to arrive timely at their destinations while saving energy by coasting. The reduced computation time with respect to the earlier version of the algorithm will allow NS to implement more advanced driving advice in the future and to increase the chance that the tablet battery will last an entire shift.

- **Giving train motion model calibration its due recognition in the railway industry and academia** (Chapters 2-6)

Despite that the effectiveness of timetabling tools, simulators and energy-efficient driving algorithms relies on an accurate calibration of the train motion model, the relevance of this step did not use to receive its due recognition in the railway industry and academia. The research presented in chapters 2-6 highlights the importance of this topic. These chapters will help practitioners and scholars to choose the right train motion model calibration technique to be used taking into account their needs and purpose, increasing the accuracy of the tools calibrated and smoothing their implementation.

- **Sustainability and punctuality of railways** (Chapters 6-8)

The research on Train Trajectory Optimization presented in chapters 6-8 aims to generate train trajectories that guarantee the punctuality of trains while minimizing the energy consumption. This reduction of the amount of energy consumed may also imply a decrease in the amount of greenhouse gas emissions, leading to a more sustainable freight and passenger mobility, to an overall better performance of railways and to improving the public image of railway companies.

1.4 Research outputs

Seven scientific papers form the core of this thesis. Most of the work presented in this thesis has been performed independently by the author. In addition to the supervisory team, one of the papers has been written in collaboration with colleagues from NS and their contributions are listed below. The chapters of this thesis are based on the following papers:

- Chapter 2: Cunillera, A., Bešinović, N., Lentink, R.M., van Oort, N., Goverde R.M.P. (2023). A literature review on train motion model calibration. *IEEE Transactions on Intelligent Transportation Systems*, 24(4), pp. 3660–3677.
- Chapter 3: Cunillera, A., Bešinović, N., Lentink, R.M., van Oort, N., Goverde R.M.P. (2023). Robust train motion model calibration based on M-estimation. (*Unpublished*).
- Chapter 4: Cunillera, A., Bešinović, N., van Oort, N., Goverde R.M.P. (2023). Real-time train motion parameter estimation using an Unscented Kalman Filter, *Transportation Research Part C: Emerging Technologies*, 143, p. 103794.
- Chapter 5: Cunillera, A., Lentink, R.M., van Oort, N., Goverde R.M.P. (2023). Adaptive estimation of train running resistance in tunnels: A Recursive Least Squares approach with multiple forgetting factors. (*Unpublished*).
- Chapter 6: Cunillera, A., Lentink, R.M., van Oort, N., Goverde R.M.P. (2023). Volatility of optimal train trajectories under parameter variations. (*Unpublished*).
- Chapter 7: Cunillera, A., Jonker, H.H., Scheepmaker, G.M., Bogers, W.H.T.J., Goverde R.M.P. (2023). Coasting advice based on the analytical solutions of the train motion model. *Journal of Rail Transport Planning & Management*, 28, 100412. The second, third and fourth co-authors defined the research question and contributed to the writing of the paper and to some numerical simulations.
- Chapter 8: Cunillera, A., Lentink, R.M., van Oort, N., Goverde R.M.P. (2026). Robust Train Trajectory Optimization. (*Unpublished*).

1.5 Outline of the thesis

This section describes the outline of this thesis, which is graphically shown in Figure 1.1. The research is divided in three topics: train motion model calibration, Train Trajectory Optimization and Robust Train Trajectory Optimization.

The train motion model calibration problem constitutes the main focus of chapters 2, 3, 4 and 5. These chapters address the first research question, which aims to quantify the main uncertainties and variations inherent in railway operation. Train motion model calibration is used to assess the impact of the mentioned uncertainties on a train’s dynamics. The state of the art on this topic is presented in chapter 2, which analyzes and classifies the existing train motion model calibration techniques. A research agenda and practical recommendations are proposed based on the findings of the literature review. Chapter 2 further elaborates on contextualizing and motivating the research presented in this thesis. Particularly, train motion model calibration is classified into two main types: offline and online calibration. Among the research gaps identified in chapter 2, two are addressed in the next chapters: robust calibration against measurement outliers and anomalous input data that can bias the estimation, and the online calibration of all the train motion model parameters required as input for Train Trajectory Optimization. Chapter 3 focuses on robust offline calibration. Operational data from multiple train trajectories are analyzed

to calculate robust estimates of the train motion model parameters. A set of estimates is computed for each of the trajectories considered, capturing the impact on the train dynamics of the operating conditions observed during each trajectory. A statistical analysis is performed to study the observed variability of the dynamics of the trains considered in this chapter. In chapter 4, an online calibration framework based on the Unscented Kalman Filter is used to estimate all the train motion model parameters used as input for Train Trajectory Optimization from train speed and location measurements. Robust online calibration is addressed in chapter 5, where Recursive Least Squares with multiple forgetting factors is proposed to capture the different volatilities of the running resistance parameters. The robustness of this method against different input data sampling times and additive noise intensities is evaluated in several scenarios.

Classical Train Trajectory Optimization, in the sense that it does not incorporate the uncertainties inherent in railway operation, is the main topic of chapters 6 and 7. Chapter 6 addresses the research question that aims to evaluate the influence of the variations present in railway operation on optimized train trajectories. A sensitivity analysis of the impact of individual variations of the train motion model parameters on the volatility of different shape indicators of optimal train trajectories is performed in that chapter. Furthermore, the influence of observed parameter variations on calculated optimal trajectories is determined by using the parameters estimated in chapter 3 as input. Chapter 7 addresses the research question on how to incorporate dynamic, inaccurate and limited real-time information in Driver Advisory Systems. It presents a new way of providing driving advice based on the analytical solutions of the train motion model, which are used to calculate the expected arrival time assuming that the train starts coasting immediately. The driving advice is provided to the train driver as a countdown to start coasting. This advice is recalculated and updated dynamically once per second, mitigating the impact of inaccurate or limited real-time information. The algorithm proposed in chapter 7 is used by the train drivers working for NS since July 2022.

Train Trajectory Optimization under uncertain operating conditions is the central subject of chapter 8, which addresses the research question on integrating the uncertainties inherent in railway operation into a real-time trajectory optimizer. As we observed in chapter 6, train motion model parameter variations have an impact on the shape of optimal trajectories. A new Robust Train Trajectory Optimization algorithm is proposed in chapter 8, taking as input several sets of train motion model parameter estimates characterizing different operating conditions of interest. This algorithm calculates the most energy-efficient control policy feasible for all the considered sets of parameters. Again, the parameters estimated in chapter 3 are used as input for the developed Robust Train Trajectory Optimization algorithm to assess its performance in different scenarios. The last research question concerns calculating train path envelopes robust against uncertain operating conditions. It is addressed in chapter 8 through the proposal of a new concept: Robust Train Path Envelopes. They consist of timing points, and time and speed windows allocated at specific locations along the track to provide a real-time traffic plan under train motion model parametric uncertainty. Robust Train Path Envelopes are calculated from the earliest and latest passing times of the trajectories calculated by the Robust Train Trajectory Optimization algorithm. In chapter 6 we observe that Train Path Envelopes must account for parameter variations in the train motion model and driving variability

in the passing times. The proposed definition of Robust Train Path Envelopes specifically addresses this issue.

Finally, the main conclusions drawn from this thesis are presented in chapter 9 along with recommendations for future research.

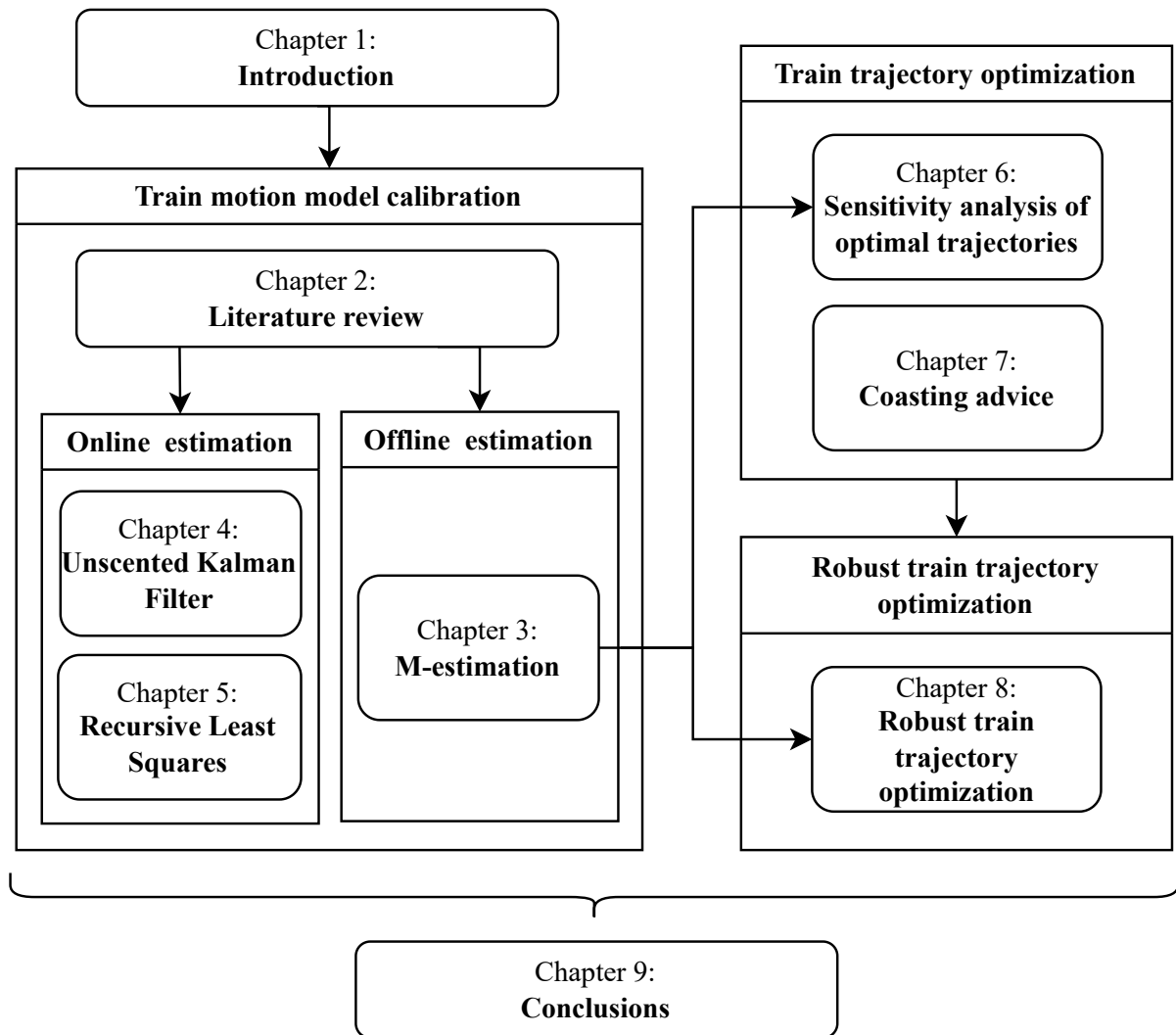


Figure 1.1: Overview of the structure of this thesis

Chapter 2

A literature review on train motion model calibration

Apart from minor changes, this chapter has been published as Cunillera, A., Bešinović, N., Lentink, R.M., van Oort, N., Goverde R.M.P. (2023). A literature review on train motion model calibration. *IEEE Transactions on Intelligent Transportation Systems*, 24(4), 3660–3677.

2.1 Introduction

Most railway simulators and several railway applications like timetabling, energy-efficient driving tools, and train trajectory generation algorithms in Driver Advisory Systems (DAS) and Automatic Train Operation (ATO) rely on a dynamical system that models the train dynamics: the train motion model (Jaekel & Albrecht, 2014). This model is obtained from Newton’s second law and requires as input several train and track characteristics, namely the maximum tractive and brake effort and power, the running resistances that affect its motion, the track geometry, the maximum acceleration rates, the brake rates to be used, the mass and the rotating mass factor (Hansen & Pachl, 2014). The model can reproduce and predict a train’s movement along a track accurately if the parameters that describe the mentioned train and track characteristics are carefully determined (Wang et al., 2020).

An accurate train motion model calibration is essential to guarantee the performance and effectiveness of the railway applications that include this model (De Martinis & Corman, 2019). Simulators utilize the motion model to reproduce a train’s dynamics in both real and hypothetical scenarios. For instance, they are used to perform capacity assessments of both existing and future railway lines (Landex et al., 2008), to calculate track section occupancy and arrival times that are used as input for timetabling tools (Medeossi et al., 2011), to estimate energy consumption (Kapetanović et al., 2021), to support infrastructure planning (Čamaj et al., 2021), to test new signalling systems (Quaglietta, 2014) and to train drivers (Tichon, 2007). However, a wrong calibration of the train motion model embedded in a simulator usually leads to erroneous calculations. An inaccurate

capacity assessment can produce an increase of the number of disturbances and an overall decrease in the service reliability. An inaccurate calculation of track section occupation times may lead to more trains approaching yellow and red signals and to infeasible timetables. New infrastructure and signalling usually require large investments (Solmes, 2009), and errors in the planning stage lower the effectiveness of the mentioned investments.

In train trajectory optimizers, the train motion model is utilized along the timetable and the allocated path in the track to generate energy-efficient train trajectories that fulfill the arrival and passing times described in the timetable. To this end, they describe the reference speed to be followed at every point of the path. Train trajectory optimizers require a precise model calibration, which, in the event of not fully capturing the train's behaviour, could lower the drivability of the generated trajectories and cause deviations from the expected arrival times, reducing the energy savings. In turn, DAS and ATO generate respectively driving advice and the right amount of tractive and brake effort to be applied to drive the train automatically. The train motion model is used in these applications to generate the reference trajectories to be followed, usually by means of an embedded train trajectory optimizer or a set of precalculated trajectories (Wang et al., 2019). The accuracy of these applications may be affected by a wrong calibration. An imprecise driving advice is unlikely to be accepted by drivers, reducing their trust on DAS. Similarly, this impacts ATO by causing deviations from the expected arrival time and the expected stopping locations, and by increasing the probability of overspeeding, which triggers the intervention of Automatic Train Protection systems and drivers in the control process. As a consequence, DAS and ATO require a highly precise calibration, which usually hinders their accurate implementation.

Train motion model calibration is usually a difficult problem in the railway industry and academia due to the spatial and temporal parameter variability shown by some of the mentioned characteristics and the existence of several external factors that influence the train dynamics (Cunillera et al., 2022). For instance, wear may alter the tractive effort performance of engines over time. Strong winds may increase the aerodynamic drag of the train, augmenting its running resistance and energy consumption. Computational Fluid Dynamics (CFD) and scaled wind tests are usually performed to estimate the running resistance parameters, but the results obtained are usually inaccurate (Somaschini et al., 2018). Full-scale tests can also be used to calibrate train dynamics, although these tests are usually resource and capacity demanding (Rochard & Schmid, 2000). The main issue regarding the mentioned procedures is that the calibration may not always be representative of the current operation conditions, which lowers the accuracy of the calibrated model and the applications that incorporate it. Consequently, the train motion model calibration has to be updated often or performed continuously. To address this, the available operational data recorded on board represents a resourceful alternative for calibrating the train motion model and to boost the performance of railway applications, as it allows to capture the current operation conditions and train characteristics.

Such calibration may be performed in two ways when using operational data as input: online, by processing on-board collected data on-the-go, and offline, by analyzing historical data. These online and offline model calibration procedures have complementary scopes that determine the kind of applications in which they are embedded. Online model calibration can be used to detect the variability of the parameters of the train

motion model in real time and enhance its accuracy by accommodating these variations. Furthermore, it is highly suitable for freight trains with variable rolling stock composition, for which there is no historical data available (De Martinis & Corman, 2019). However, the algorithms used need to be fast and computationally efficient. In this regard, the effects of varying electric power, weather, wind, adhesion, track, brakes and engine condition, and driving style can be assessed in real time and utilized to increase the accuracy of real-time applications such as online energy-efficient driving algorithms, DAS and ATO (Wang & Goverde, 2016). In turn, offline model calibration can process a larger amount of data coming from several sources and may be easier to implement. Thus, more powerful but slow and computationally demanding techniques can be applied offline to support decision makers by enhancing the accuracy of simulation and timetabling applications (Bešinović et al., 2017; Goverde et al., 2016). Usually, offline approaches produce a single estimate for each target parameter that constitutes the best fit to the considered dataset, as opposed to online calibration, that produces time-varying estimates.

In this chapter we review the current state of the art in online and offline train motion model calibration, addressing particularly those methods that utilize on-board measurements. This chapter is of special relevance in the railway industry and academia, since no recent literature review paper on this topic is available. Rochard & Schmid (2000) is the latest review on running resistance calculation for high speed trains, focusing on collecting and comparing several running resistance equations that have been developed by scholars and railway undertakings from different countries. However, most of these formulas constitute empirical or semi-empirical approximations that usually overestimate the running resistance of current trains. Despite its limited scope, it is one of the most influential articles on train motion model calibration. In turn, one of the most extensive lists of empirical running resistance formulas can be found in Iwnicki (2006, Chapter 13.2.2.2). However, we do not recommend using such formulas if either calibration is possible or, alternatively, if the manufacturer formulas are available. A recent review (Shrestha et al., 2019b) focuses on a close topic: the estimation of adhesion, namely the ratio of the tangential force between wheel and rail, and the wheel load. Most of the techniques covered there require modelling the tractive wheels dynamics and measuring the wheels rotating speed with an encoder, which is compared to the train speed to gauge the wheel slip. Although adhesion estimation is essential for determining the applied tractive effort and brake, we do not review adhesion estimation techniques in this chapter, as we do not cover techniques that require wheels angular speed or traction torque measurements. Covering railway applications that benefit from train motion model calibration is also out of the scope of this chapter. This chapter and Shrestha et al. (2019b) constitute a complete overview of train motion model estimation techniques. Moreover, model calibration is closely related to monitoring algorithms, which are reviewed in Li et al. (2017a), focusing on rolling stock health monitoring and fault detection. Regarding other fields of transport, motion model calibration is also covered in the case of wheeled vehicles (Andersen et al., 2015), flapping wing micro-aerial vehicles (Khan & Akmeiliawati, 2021), and aircrafts (Klein & Morelli, 2006).

Considering that no literature review on train motion model calibration based on on-board measurements can be found in the existing literature, we review articles published from January 2000 to December 2021. We selected 42 articles and the techniques found

are classified in two main categories: online and offline algorithms. Then, the existing techniques are clustered according to the type of method used for calibration. Last, each technique is analyzed individually, outlining their main advantages and disadvantages.

The main contributions of this literature review chapter are:

- A broad overview, description and critical discussion of the existing train motion model calibration methods.
- Highlighting the relevance of train motion model calibration in many areas of railway research and industry, which may ultimately lead to a broader and more effective calibration of railway applications and to boost their performance.
- A research agenda based on the current state of the art and research gaps.
- Practical recommendations for practitioners and scholars.

The remainder of this chapter is structured as follows. We illustrate in chapter 2.2 the train motion model calibration problem. We present the methodology used for this literature review in chapter 2.3. We describe and discuss the train motion model calibration methods found in the existing literature in chapter 2.4 and outline the main findings of the overview in chapter 2.5. Next, we propose a research agenda for the coming years and practical recommendations in chapter 2.6. Last, we outline the main conclusions of this literature review in chapter 2.7.

2.2 The train motion model calibration problem

In this chapter we introduce the train motion model calibration problem and outline the input parameters to be estimated. Furthermore, we describe the main sources of variation of each parameter, which highlight the difficulty of the calibration problem. We refer to Cole et al. (2017); Wu et al. (2016) for a deeper discussion on train dynamics modelling.

The dynamics of a train along its movement on a track can be described according to Newton's second law (Hansen & Pachl, 2014), as

$$m\rho\frac{dv}{dt} = f(v) - r(v) - w(s), \quad (2.1)$$

where t represents the time, s the train location, v the train speed, m the train mass, ρ the rotating mass factor accounting for the train's rotating parts inertia, $f(v)$ the tractive and brake effort, $w(s)$ the influence on the track geometry on the train dynamics and $r(v)$ the running resistance, which is represented as a quadratic function of speed called the Davis equation (Davis, 1926),

$$r(v) = r_0 + r_1v + T_f r_2 v^2, \quad (2.2)$$

where r_0 , r_1 and r_2 are nonnegative running resistance parameters that characterize the resistance to motion of a train. T_f corresponds to the extra running resistance observed when running through tunnels. T_f takes unit value when the train is running in the open air and a value larger than one when running through a tunnel due to the extra

aerodynamic drag. This tunnel factor depends on several factors, like the train head geometry, the rolling stock composition, the length of the train and the tunnel, the relative cross section of the train with respect to the tunnel and the surface of the tunnel's walls (Niu et al., 2020).

The resistance due to the track geometry, $w(s)$, accounts for the resistance due to the gradients and curves in the first and second term of Eq. (2.3), respectively,

$$w(s) = mg \sin(\alpha(s)) + m \frac{k}{R(s)}, \quad (2.3)$$

where g is the gravity acceleration, $\alpha(s)$ is the angle of the slope of the track at the location s . Factors like the curve radius and superelevation, the train speed and length, the condition and design of the train wheels, and the rail maintenance condition and lubrication are acknowledged to influence the curve resistance (Iwnicki, 2006), although the influence of each factor on the resistance to motion is still not well understood (Wu et al., 2021). The curve resistance is mainly modelled as the equivalent gradient that would produce the same resistance to motion, with a hyperbolic dependence on the radius of the curves $R(s)$, being k the associated proportional constant (Hay, 1991). This means that higher resistances would be experienced on tighter curves. This equivalent-gradient description is usually called the Schmidt formula (Schmidt, 1927). Although it is the most widely used for modelling the resistance due to the curves, it is still a simplification of the actual resistance and might not always be accurate, particularly on tight curves and uphill sections, where the Schmidt formula largely underestimates the curve resistance, leading sometimes to freight trains stalling on those sections (Szanto, 2012, 2018). Moreover, the influence of the train on the curve resistance (Wu et al., 2021, 2022b) highlights the impact of curves on the dynamics of high speed trains, despite the typical large radii of the curves found in the infrastructure used by high speed trains. More sophisticated formulas have been proposed (Sapronova et al., 2017), although they are not used as widely as the Schmidt formula (Wu et al., 2022b). In spite of the relevance of the curve resistance on a train's dynamics, in this chapter we do not cover the current state of the art on curve resistance estimation, since an exhaustive theoretical and experimental study is required first to understand it better and model it accurately.

Moreover, the maximum effort that can be applied is limited by adhesion, and the engine and brake characteristics and wear,

$$-f_{\min}(v) \leq f(v) \leq f_{\max}(v), \quad (2.4)$$

where $f_{\max}(v)$ and $f_{\min}(v)$ are the maximum tractive and brake effort.

Furthermore, the train acceleration is usually bounded due to operational constraints like the predefined braking curves that describe a train's deceleration, and comfort constraints to guarantee the integrity of freight rolling stock compositions and the comfort of passengers. Thus,

$$-a_{\min}(v) \leq \frac{dv}{dt} \leq a_{\max}(v), \quad (2.5)$$

where $a_{\min}(v)$ and $a_{\max}(v)$ are the maximum and minimum deceleration and acceleration, respectively. The jerk, namely the rate of change of the acceleration, is also another

important factor that is usually taken into account to guarantee the comfort of passengers, since abrupt acceleration changes may be felt uneasy by passengers,

$$\left| \frac{d^2v}{dt^2} \right| \leq \gamma_{\max}. \quad (2.6)$$

with $\gamma_{\max}$ the maximum jerk that can be applied along a journey.

Moreover, besides the explicit parameter dependencies that are shown in Eqs. (2.1)-(2.6), several of the mentioned parameters may also show spatial and temporal variations. Some physical sources of train motion model parameter variation are the wind force and direction, the weather and temperature, the available power at the catenary, the wheel and rail adhesion condition, and the wear of wheels, rail, mechanical parts and engine. In the running resistance r_0 and r_1 describe the mechanical resistance to motion, while r_2 describes the aerodynamic drag resistance. r_0 accounts for the internal frictions and the contact ellipses between wheels and rails, while r_1 stands for the effect of the air momentum on the train dynamics and the flange friction between wheels and rails. The value of r_2 is determined by the train head geometry and cross section, and its value may increase when running through a tunnel due to the extra aerodynamic drag. In addition, adhesion may cap the maximum applicable tractive and brake effort. For instance, bad rail and wheel maintenance may lead to a lower adhesion condition. Moreover, water, snow and fallen leaves on rails may also lower the available adhesion, so these variations are also season-dependent. At high speeds, the maximum tractive effort depends on the tractive power, which in turn depends on the engine wear and temperature and the available catenary power.

Braking in railways is a complex process that is characterized by long braking distances, low friction and delays in the response times. Friction determines the effective maximum brake effort that can be applied, since wheels slip if the brake effort exceeds the friction limit. Friction is an uncertain factor limited by the mass of the train, adhesion, the current moisture and weather, the applied sanding and the wheels, brake and rail maintenance condition. In undisturbed normal operation conditions, Automatic Train Protection (ATP) systems and predefined braking curves prevent from braking at the maximum capacity. Therefore, adhesion and manual driving variability constitute relevant sources of variations when braking, so the performed braking curves may be the target of calibration procedures in normal operation conditions instead of the maximum brake effort, while keeping the braking model simple. Furthermore, some braking procedures also consider reduced adhesion conditions, like the European Train Control System (ETCS). Among braking systems, those based on air brakes show a particularly complex behaviour. When the air brake is triggered, a difference in the pressure is produced by a compressor in the leading locomotive, propagating in the pneumatic system along the following cars at finite speed lower than the speed of sound (Hay, 1991). This produces a delay in the response time of brakes of each car or wagon, which, in the extreme case of old and very long freight trains, may lead to differences of the order of a minute between the brake triggering at the locomotive and at the last wagon (Günay et al., 2020). Thus, in these types of trains the brake has to be applied gently in order to reduce the risk of derailment. Electro-pneumatic brake systems can homogenize and reduce the response time along the train length by transmitting the brake command electronically to all cars and wagons (Günay et al., 2020). Dynamic braking systems brake a train by turning

its kinetic energy into electric energy that can be reused, returned into the catenary or simply dissipated as heat (Günay et al., 2020). These systems help reducing the mechanical wear on wheels and brake shoes produced by air brake systems. However, dynamic brakes usually have a lower braking capacity, therefore, a smart brake blending strategy is needed to combine both braking systems (Wu et al., 2022a). Typically, dynamic brakes are applied first, and if the achieved brake rate is not sufficient, then air brake is applied. Still, the uncertainties on the air brake system must be considered in this scenario.

What is more, mass-specific parameters can be obtained by introducing the train mass in the remaining model parameters, which reduces the number of parameters to be estimated by one. The mass can be estimated from the rolling stock operational empty weight and the passengers or freight load, and can be considered as constant between consecutive stops. Alternatively, weigh-in-motion systems can be used to estimate the mass of a train by measuring the load of each axle, thus also detecting uneven load distributions. These systems have been predominantly used by infrastructure managers to detect overloaded trains, to control uneven axle loads that may produce extra wear on rails and boost the risk of derailment, and to assess the load capacity on bridges (Pintão et al., 2022). To this end, the mass can be estimated by placing piezoelectric sensors, strain gauges or accelerometers on the rails, and measuring the movement and bending of the rails under the weight of the wheels when passing above the sensors. Weigh-in-motion can also be performed using on-board equipment, for instance, by placing accelerometers on axles, although this approach is scarcely used (Mosleh et al., 2020). On-board mass measurements can be used along the estimated mass-specific parameters and the rotating mass factor in ATO to determine the exact amount of tractive and brake effort to be applied for driving the train.

In manually-driven trains parameter variations may arise due to variations in the driving process. These driving-induced parameter variations have to be carefully determined in order to improve the realism of railway simulators and energy-efficient driving algorithms. We can distinguish up to four driving regimes in a train trajectory between two stops (Scheepmaker et al., 2017; Goverde et al., 2021).

1. Acceleration to reach a target speed or the track speed limits.
2. Cruising at the given target or track maximum speed, which can be performed in several ways. A constant cruise speed can be kept by holding the right amount of tractive effort to compensate running and track geometry resistances, by braking to avoid overspeeding on downhill track sections, or approximated by performing traction and coast cycles around the target cruise speed.
3. Coasting, which means applying no traction or brake.
4. Braking, which is usually performed by following predefined braking curves or a constant brake rate.

In train motion model calibration, coasting takes a special role due to the absence of effort applied, so the running resistance can be determined from speed measurements provided that the track geometry description is accurate enough. Acceleration is the second most important driving regime for model calibration, since train drivers can be instructed to

apply maximum traction to accelerate, which simplifies the calculations needed for the calibration. Then, several model parameters and bounds can be calibrated if the maximum tractive effort or the traction engine's efficiency and the consumed energy are known or measured. Moreover, estimating the train motion model parameters is more difficult for the cruising and braking regimes, since cruising is a complex regime that can be performed in several ways, and since the combination of running resistance and applied brake may lead to overestimating the running resistance when braking (Cunillera et al., 2022).

Consequently, the running resistance parameters, the maximum tractive effort and power, the maximum brake effort, the train mass and rotating mass factor and the performed braking curves have to be carefully calibrated to minimize the impact of physical and driving-induced sources of parameter uncertainty on railway applications.

2.3 Methodology

We searched for articles in two well-known databases, Scopus and IEEE Xplore, to review the existing literature on train motion model calibration that utilizes on-board measured operational data as input and those techniques that can be adapted for utilizing such data. We performed a boolean search using combinations of the keywords "model", "calibr*", "rail*", "monitor*", "parameter*", "estimat*", "determin*" in title, abstract and keywords, filtering the topics not related to railways. We restricted the search scope to academic articles in English, including journal articles and conference proceedings published from January 2000 to December 2021 that are available online.

To refine the selection we checked the articles' titles, keywords and abstracts manually. This refinement, however, was found to be inefficient, since some of the articles that used perform train motion model calibration for enhancing the accuracy of another method did not state it in the mentioned parts of the article. Due to this secondary role, the title, abstract and keywords do not usually reflect the fact that a new model calibration technique is introduced in these articles, so this manual inspection was mainly used to remove those articles that do not perform model calibration on the train motion equation parameters. This highlights the need to emphasize the relevance of train motion model calibration within the scientific community, who should give more visibility to train motion model calibration by making its usage explicit in the abstract and keywords of their articles.

Then, forward and backward snowball research became crucial in order to find most of the articles that perform train motion model calibration. We checked the references and citations of the articles found and we also explored other articles published by the same authors. Last, we performed a final refinement of the selected articles, restricting to those that present train motion model calibration approaches that do not require the usage of extra measurement equipment. In total, 42 articles were finally selected for this literature review: 21 conference and 21 journal articles. Figure 2.1 shows the number of journal and conference articles reviewed in this chapter per year of publication. We observe an increasing publication trend that might be due to the growing amount of data available and the increasing need for achieving higher accuracy levels by calibrating the algorithms that embed the train motion model.

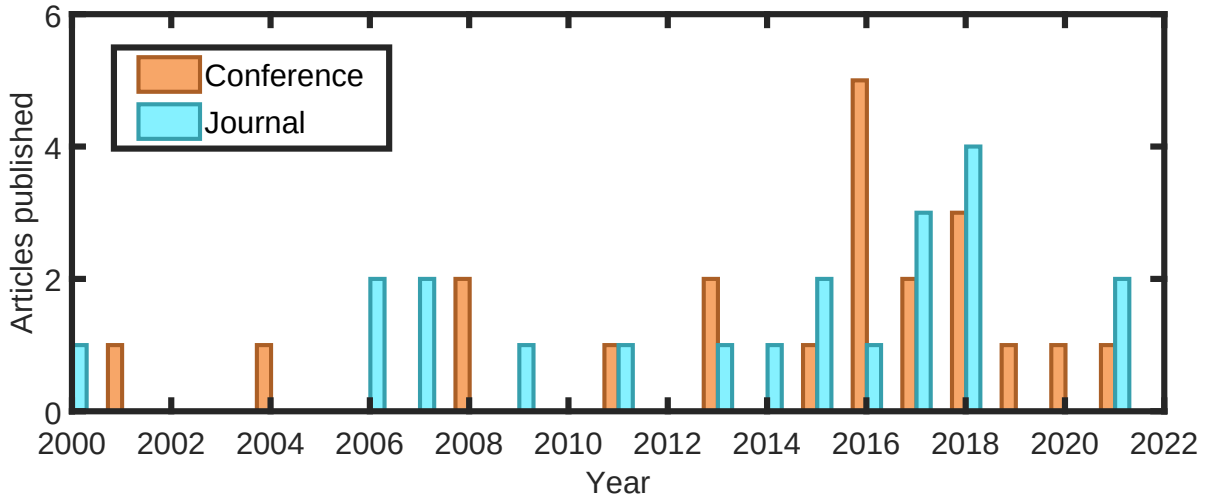


Figure 2.1: Distribution of journal and conference per year of publication

2.4 Literature review

Train motion model calibration is a key step towards an accurate implementation and performance of railway applications. Train motion model calibration approaches can be encapsulated in three main groups depending on the stage in which the calibration is performed.

1. In early rolling stock design phases, scaled models, wind tests, Computational Fluid Dynamics (CFD) and simulations are performed to estimate the train motion model parameters, particularly the running resistance (Baker et al., 2019).
2. Full scale tests are performed in controlled scenarios like test tracks or in level and straight track sections to estimate the input parameters of the motion model.
3. For trains in service, operational data is analyzed to perform model calibration.

In this literature review we cover train motion model calibration techniques that use operational data or can be minimally adapted to use it, since using operational data shows many advantages with respect to CFD and tests. For instance, they require less resources, benefit from a larger amount of data available and may lead to estimates that represent the current operating conditions more faithfully. Furthermore, test runs may not be possible for trains of variable composition, thus model calibration based on operational data may be crucial for such railway systems (Michálek, 2018). The archetypical example are freight wagons, since flat, open and closed wagons show radically different dynamics.

In full-scale tests, most efforts have been dedicated to estimating the running resistance. The European Committee for Standardization outlines the main full scale tests for determining the resistance parameters in a European standard (CEN European Standard, 2018). One of these tests consists of pulling the train at a constant speed by means of a windlass and using a dynamometer to measure the resistance to motion. However, this test can only be performed at low speeds and requires the train mass as input. Coasting tests are widely used to explore the train dynamics in a broader speed interval. The rolling

stock is accelerated until a target speed is reached and then it decelerates by coasting. This test requires to be repeated several times for statistical purposes. A third type of test can be performed by driving the train at a constant low speed while measuring the consumed tractive power (Aradi et al., 2015). This method requires an accurate knowledge of the traction efficiency and its accuracy is easily affected by unexpected accelerations. The main problems regarding the mentioned tests are that they are very demanding resource-wise and that they may not lead to accurate results. These tests generally require to be performed in level and straight track sections and are very sensitive to errors in the track geometry description. New full-scale tests that allow for running resistance and rotating mass factor estimation without requiring an accurate track geometry description have been proposed in Somaschini et al. (2016, 2018). Parameters r_1 and r_2 from Eq. (2.2) are obtained from three or more standard coasting tests at the same track section with different starting speeds and using Eq. (2.1) on the speed difference to triangulate the mentioned parameters. Meanwhile, r_0 and the rotating mass factor are obtained from swinging tests at low speeds, where the train is let to coast and move back in a steep uphill track section.

The tractive effort can be determined by setting load cells at the coupling between the locomotive and the next car (Rangelov, 2012). To this end, the load cells measure the pulling force needed to move all the cars that succeed the measuring coupling. Alternatively, the applied tractive effort can also be determined by measuring the pulling force at the drawbar between the locomotive and a dynamometer car (Radosavljevic, 2006), which is a special measurement car that is coupled for measuring the mentioned pulling force, among other magnitudes. These methods underestimate the tractive effort, since the measurements do not include the tractive effort needed to accelerate the locomotive. Nevertheless, these measurements can be combined with power consumption measurements to enhance the accuracy of the tractive effort measurements. Furthermore, dynamometer cars are typically not suitable for measuring the tractive effort during real operation, so they are mostly used by manufacturers in full-scale tests to determine the characteristics of locomotives' engines.

In this literature review we classify operational data-based calibration techniques based on their way of analyzing data: online techniques that perform the calibration on-the-go and offline ones that analyze data from one or more trajectories at once. These two main types of calibration techniques show substantial differences and complement each other with respect to their applicability. Online model calibration techniques perform adaptive parameter estimation that allows to capture parameter variability along the train run, as opposed to offline techniques that usually produce static parameters that constitute the best fit for the entire analyzed trajectory. Online techniques may therefore be able to capture varying environmental conditions, like the typical increment of the running resistance due to the extra aerodynamic drag in tunnels, the effect of varying wind strength and direction on the resistance along a train's trajectory and the impact of varying adhesion conditions on the applied maximum tractive effort at different parts of the track. In turn, offline techniques capture the average behaviour of a train given the operating condition and their estimates. Offline techniques can analyze larger quantities of data in one go, benefitting from a larger amount of historical data available and more computational power. In contrast, online ones have to be suitable for on-board implementations, which

requires them to be faster and less computationally-demanding. However, the smaller amount of data used by online techniques to generate the estimates makes them more sensitive to data anomalies that can affect their accuracy. Railway applications like simulators, timetabling tools and train trajectory optimizers use static train motion model parameters as input, which makes offline techniques more suitable for calibrating such applications than online ones that produce time-varying estimates. Nevertheless, online techniques are of special relevance for monitoring freight trains and trains with varying rolling stock composition in real time due to the lack of historical data available, and for assessing the impact of varying weather conditions.

Tables 2.1 and 2.2 show a classification of the train motion model calibration approaches covered in this chapter, clustered in terms of method and estimation technique used. Table 2.1 gathers offline model calibration techniques and Table 2.2, the online ones. They include the data used as input, whether the techniques use data measured in real operation, the estimated parameters, whether the estimation is also performed in tunnels and the type of railway system targetted in each article. In Tables 2.3 and 2.4 the main characteristics of each method are outlined, including the computation speed, the implementation difficulty, whether the method is robust against measurement outliers and anomalous data, the driving regimes in which are usually applied and some additional comments. Figure 2.2 shows a schematic representation of the mentioned techniques, including classifications with respect to their real-time applicability and different technique-type clustering.

2.4.1 Offline train motion model calibration

Regression

Least Squares Regression (LSR) is the most widely used method to estimate the running resistance parameters and, particularly, the main method used for analyzing data from coasting tests. Speed and location measurements are used along with the track geometry description and the applied effort, if available, to calculate the value of the running resistance needed to match each individual pair of speed and location observations. In the case of a coasting train in a flat and straight track, the train decelerates only due to the running resistance. Then, this method estimates the optimal parameters by minimizing the sum of squared errors between the measurements and the model predictions.

This technique has been mainly used to estimate the running resistance parameters from coasting tests data. Particularly, low speed coasting and cruising tests data is analyzed in Hoffrichter et al. (2014), where the drive-train efficiency is also estimated. r_2 is estimated in Kim et al. (2006); Kwon (2018), and the latter included the relative train speed with respect to the wind speed in the quadratic term of the resistance. r_2 in tunnels is estimated in Hansen et al. (2017); Li et al. (2017b) to gauge the impact of the extra aerodynamic drag on the train dynamics. The speed measurements are weighted in Hansen et al. (2017) to compensate the uneven speed measurements distribution and to avoid underfitting r_2 in the least-visited speed intervals. In Lukaszewicz (2007a), the author concludes that the impact of the wind can be mitigated by coasting in both directions and averaging among all tests, that an inaccurate value of the mass and the rotating factor may bias the estimation, and that r_1 is the most uncertain and difficult

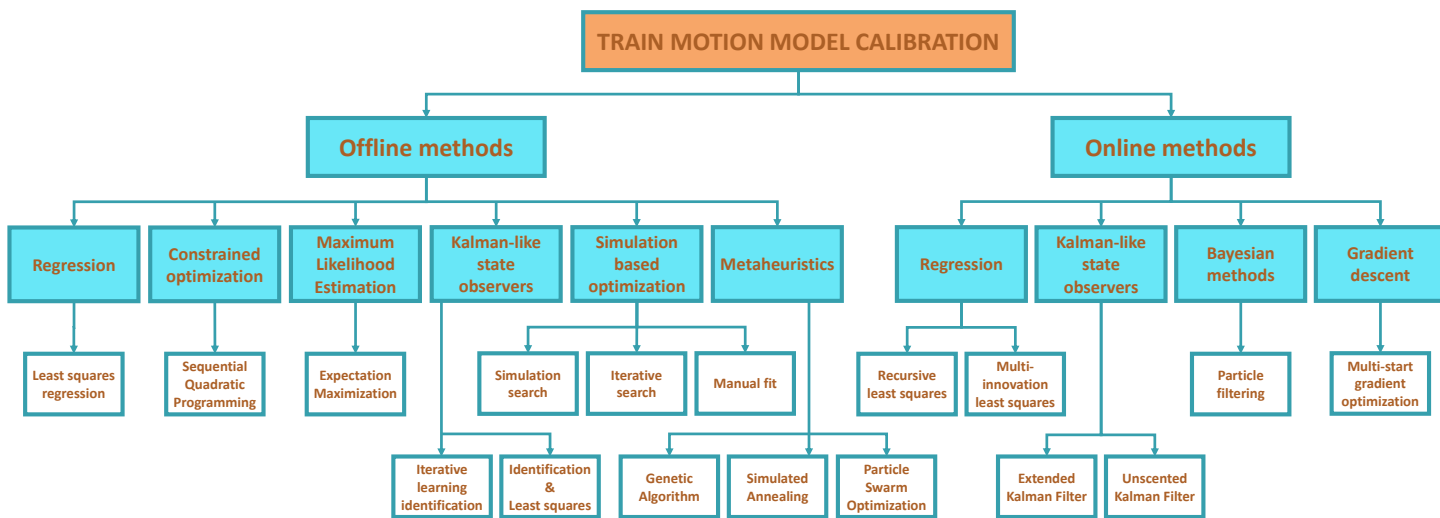


Figure 2.2: Schematic representation of the train motion model calibration techniques included in this literature review

Method	Technique	Articles	Input data	Operational data	Parameters estimated	Tunnels	Railway system
Regression	Least squares	Hansen et al. (2017)	Location, speed	Yes	r_2 in tunnels	Yes	Main line
		Hoffrichter et al. (2014)	Location, speed, energy consumption	No	r_0, r_1 , traction efficiency	No	Hybrid hydrogen locomotive
		Kim et al. (2006)	Location, speed	No	r_0, r_1, r_2	Yes	High speed
		Kwon (2018)	Location, speed	No	r_0, r_1, r_2	No	High speed
		Li et al. (2017b)	Location, speed	No	r_2 in tunnels	Yes	High speed
		Lukaszewicz (2007a,b, 2009)	Location, speed	No	r_0, r_1, r_2	No	Freight trains
		Manabe et al. (2015)	Location, speed, traction and brake notches, weather data	Yes	r_0, r_1, r_2, m	Yes	Main line
		Mancini et al. (2001)	Location, speed, wind speed	No	r_0, r_1, r_2	No	High speed
		Michálek (2018)	Location, speed	No	r_0, r_1, r_2	No	Freight trains
		Miyauchi et al. (2018)	Location, speed, electric power, traction and brake effort command	No	r_0, r_1, r_2	No	Monorail
		Ogawa et al. (2017)	Location, speed, notch, voltage, brake effort	Yes	r_0, r_1, r_2	Yes	Main line
		Radosavljevic (2006)	Location, speed, tractive effort	No	$r_0, r_1, r_2, f_{\max}$	No	Diesel-electric locomotive

		Rochard & Schmid (2000)	Location, speed	No	r_0, r_1, r_2	No	High speed trains
Constrained optimization	Sequential Quadratic Programming	Aradi et al. (2015)	Location, speed, energy consumption	Yes	r_0, r_1, r_2, m , mechanical efficiency	No	Electric train
Maximum Likelihood Estimation	Expectation Maximization	Jaoua et al. (2018)	Location, speed, voltage, current	No	m	No	Not stated
State observers	Iterative learning identification	Wang & Qi (2008)	Location, speed	No	r_0, r_1, r_2	No	Main line
	State observers & least squares	Vandanjon et al. (2016)	Location, speed, energy consumption, wind speed	No	r_0, r_1, r_2 , electric power parameters	No	High speed
Simulation-based optimization	Search	De Martinis et al. (2013)	Location, speed, energy consumption	Yes	r_0, r_1, r_2	No	Main line
		Do et al. (2020)	Location, acceleration	Yes	r_0, r_1 , brake distance	No	Tram
		Hansen et al. (2017)	Location, speed, energy consumption	Yes	r_2 in tunnels	Yes	Main line
		Nespoulous et al. (2021)	Location, speed, energy consumption	No	r_0, r_1, r_2 , traction and brake efficiencies, auxiliary power	No	High speed
		Szanto (2016)	Location, speed, tractive effort	Yes	r_0, r_1, r_2	No	Freight trains
	Iteration-based search	Powell & Palacín (2015)	Location, speed	Yes	brake rate, cruise speed	Yes	Metro

	Manual fit	De Fabris et al. (2008)	Location, speed	Yes	traction and brake performance, cruise speed	No	Main line
Metaheuristics	Genetic Algorithm	Bešinović et al. (2013a,b)	Track occupation data	Yes	r_0 , r_1 , r_2 , train length, $f_{max}(v)$, brake rates, cruise speed	No	Main line
		De Martinis & Weidmann (2016a,b)	Location, speed, energy consumed and recovered	Yes	r_0 , r_1 , r_2 , traction and brake performance, cruise speed	Yes	Freight trains
		De Martinis & Corman (2018)	Location, speed, energy consumption	Yes	r_0 , r_1 , r_2	No	Main line
		Sánchez et al. (2021)	Average fuel consumption	Yes	r_0 , r_1 , r_2	No	Diesel trains
	Particle Swarm Optimization	Graber et al. (2016)	Energy consumption	No	r_0 , r_2 , line resistivity, traction efficiency, rotating mass coefficient	Yes	Metro
		De Fabris et al. (2011); Medeossi et al. (2011)	Location, speed	Yes	traction and brake performance, cruise speed	No	Main line

Table 2.1: Details from selected articles on offline calibration, sorted by method and technique.

Method	Technique	Articles	Input data	Operational data	Parameters estimated	Tunnels	Railway system
Regression	Recursive least squares	Moroz et al. (2017); Gorbachev et al. (2017)	Location, speed, applied effort	No	r_0, r_1, r_2, m	No	Monorail
		Liu et al. (2018)	Location, speed	Yes	r_0, r_1, r_2	Yes	Metro
	Multi-innovation least squares	Liu et al. (2018)	Location, speed	Yes	r_0, r_1, r_2	Yes	Metro
Kalman-like state observers	Unscented Kalman Filter	Cunillera et al. (2021)	Location, speed	Yes	r_0, r_1, r_2 , train length, $f_{\max}(v)$, brake rates, cruise speed	No	Main line
		Howlett et al. (2004)	Location, speed	No	r_0, r_2	No	Freight trains
	Extended Kalman Filter	Jin et al. (2018a)	Location, speed, applied effort	No	r_0, r_1, r_2	No	High speed
Bayesian methods	Particle filtering	Xie et al. (2016)	Location, speed	No	r_0, r_1, r_2	No	High speed
Gradient descent	Multi-start gradient optimization	De Martinis & Corman (2019)	Location, speed, power consumed and generated	Yes	r_0, r_1, r_2	Yes	Main line

Table 2.2: Details from selected articles on online calibration, sorted by method and technique.

Offline method	Articles	Computation time	Implementation difficulty	Robust method	Driving regimes	Additional comments
Regression	Hansen et al. (2017); Hofrichter et al. (2014); Kim et al. (2006); Kwon (2018); Li et al. (2017b); Lukaszewicz (2007a,b, 2009); Manabe et al. (2015); Mancini et al. (2001); Michálek (2018); Miyauchi et al. (2018); Ogawa et al. (2017); Radosavljevic (2006); Rochard & Schmid (2000)	Fast	Low	No	Coasting	A constrained regression method should be used to avoid obtaining wrong estimates of r_1
Constrained optimization	Aradi et al. (2015)	Medium	Medium	Yes	All	Accuracy sensitive to initialization values
Maximum likelihood estimation	Jaoua et al. (2018)	Slow	High	Yes	All	Accuracy sensitive to initialization values
State observers	Wang & Qi (2008); Vandanjon et al. (2016)	Fast	High	No	All	Requires training. Calibration may fail if an input trajectory differs significantly from the ones used for training
Simulation-based optimization	De Martinis et al. (2013); Do et al. (2020); Hansen et al. (2017); Nespoulous et al. (2021); Szanto (2016); Powell & Palacín (2015); De Fabris et al. (2008)	Medium	Low	No	All	Finds local optima. The simulator used may bias the estimates

Metaheuristics	Medeossi et al. (2011); Bešinović et al. (2013a,b); De Martinis & Weidmann (2016a,b); De Martinis & Corman (2018); Sánchez et al. (2021); Graber et al. (2016); De Fabris et al. (2011)	Slow	Low	No	All	Finds local optima. Explores the parameter space extensively
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Table 2.3: List of characteristics of each offline estimation method.

Online method	Articles	Computation time	Implementation difficulty	Robust method	Driving regimes	Additional comments
Regression	Moroz et al. (2017); Gorbachev et al. (2017); Liu et al. (2018)	Fast	Low	Yes*	All	*: robust if a measurement window is considered in each iteration
Kalman-like state observers	Howlett et al. (2004); Cunillera et al. (2021); Jin et al. (2018a)	Fast	High	No	All	Accuracy highly sensitive to initialization values
Bayesian methods	Xie et al. (2016)	Medium	Medium	Yes	All	Accuracy dependent on set of parameter candidates
Gradient descent	De Martinis & Corman (2019)	Slow	Low	No	All	Computation time too slow for real-time calibration if the measurement window is large

Table 2.4: List of characteristics of each online estimation method.

parameter to be calibrated due to the predominance of the other two parameters in low and high speeds. Several empirical formulas that model individual running resistance parameters dependencies on train characteristics like number of cars, number of axles, train length or axle load are calibrated in Lukaszewicz (2007b, 2009). r_0 depends linearly on axle load and number of axles and might be affected significantly by the state of the track. r_1 depends on the train length, but not on the train mass, and no effect from air momentum drag is distinguished, contradicting the general knowledge about r_1 . In turn, r_2 has two parts: a constant one depending only on the front and rear of the train and a component that increases linearly with train length. This is supported by Mancini et al. (2001), where the impact of bogie fairings on the running resistance is evaluated, observing that they may reduce up to 10% of the total aerodynamic drag, including some impact on r_0 . In Michálek (2018), new running resistance formulas are estimated with the aim to update the phenomenological formulas used in the Czech Republic. The existing methodological formulas are found to be too conservative, being most of them not correct physically. The results show that r_0 has two components: one mass-dependent and a constant one, while r_2 depends linearly on the number of new fronts of a train, namely loaded wagons behind empty ones. The running resistance parameters of a monorail are estimated in Miyauchi et al. (2018). The estimates obtained from coasting tests are compared with well-known formulas in Rochard & Schmid (2000), finding that most of these formulas overestimate the running resistance. Among those, Armstrong and Swift's formula (Armstrong & Swift, 1990) is the most accurate. In Radosavljevic (2006), the maximum tractive effort and running resistance parameters are estimated from pulling and coasting tests, respectively. Regarding operational data, running resistance estimation under various circumstances is explored in Manabe et al. (2015), including tunnels and different weather conditions using monitoring data. The results obtained from operational data are found to be consistent with coasting tests in Ogawa et al. (2017).

Some of the advantages of LSR are that it allows to analyze large amounts of data, it is fast and easy to be implemented. This made LSR the most frequently used offline calibration method, particularly for analyzing coasting data. However, errors in the track description may lower its accuracy. LSR often leads to negative values of r_1 , which is not physically correct as the running resistance parameters should be nonnegative and standard LSR is not suited for constrained regressions. Moreover, speed measurement records usually show an uneven distribution over speed intervals, leading to overestimating parameters in the most frequently-visited speed intervals and underestimating them in the least-visited ones. Furthermore, ordinary least squares algorithms are not robust under the presence of outliers in the measurements.

Constrained optimization methods

Parameter estimation can be formulated as a constrained optimization problem by choosing an appropriate objective function to be minimized with respect to the target parameters, and by adding constraints to the optimization model like upper and lower bounds to the target parameters. Several techniques can be used to solve the resulting constrained optimization problem. For instance, Sequential Quadratic Programming (SQP) (Nocedal & Wright, 2006) models the constrained optimization problem as a quadratic

programming subproblem that is solved iteratively until the set of estimates converges to a solution.

SQP is utilized in Aradi et al. (2015) to estimate the mass, mechanical efficiency and running resistance parameters from speed, location and energy consumption data, distinguishing the individual contributions of the locomotive and cars to the total running resistance. The error in the accumulated energy consumption is used as the objective function. SQP is found to be robust, although the rotating mass factor was omitted and the running resistance was overfitted in the most visited speed interval.

Formulating the parameter estimation problem as a constrained optimization one allows for more flexibility in the model and the techniques to be applied for solving it than LSR. SQP can be used to avoid obtaining negative values of r_1 and shows more robustness against measurement noise than LSR. However, the computation time may be larger and may depend on parameter initialization.

Maximum likelihood estimation

Maximum Likelihood Estimation (MLE) aims to find the set of parameter values that maximizes the probability of the values of the target parameters with respect to the available observations. This maximum can be obtained by means of numerical methods like the Expectation-Maximization algorithm, where the maximum is calculated iteratively. At each iteration, the algorithm computes the expected value of the log-likelihood function of the estimates of the previous iteration conditioned to the speed measurements, and then a new set of estimates that maximize the mentioned log-likelihood is obtained. To this end, a particle filter has to be used to approximate the probability density functions required for the computation of the log-likelihood.

MLE is used in Jaoua et al. (2018) to estimate the train mass, showing that estimating the mass may lead to a more accurate speed profile reconstruction. However, the algorithm is validated using only simulated data.

MLE may produce estimates robust under measurement noise and missing data, provided that a sufficiently large set of trajectories is used as input. However, MLE is computationally expensive and may be sensitive to the parameter initialization. Furthermore, understanding and deriving the likelihood equations may be a complex task depending on the mathematical background of the user.

Kalman-like state observers

In control theory, state observers are used to estimate the internal state of a system from observations of a certain set of variables. These state observers may perform parameter estimation by introducing the target parameters as state variables.

An Iterative Learning Identification (ILI) that exploits the repeating characteristics of train trajectories is proposed in Wang & Qi (2008). ILI is able to improve the estimation iteration after iteration. To this end, at each iteration a state observer is utilized to produce a new estimation of the target parameters based on the error between the predicted and measured variables. Several state observers are combined with LSR in Vandanjon et al. (2016) to estimate the running resistance and some electric power parameters. However, this method failed to calibrate r_0 .

While ILI is guaranteed to converge if the dynamical system under consideration fulfills a certain set of requirements, this approach needs to be explored further to assess whether it is able to produce accurate estimates when using real data, particularly in the case of trajectories that deviate significantly from the ones used for training the model.

Simulation-based optimization

Simulators can also take a relevant role in parameter estimation. Given a set of input parameters, simulators produce data that can be compared with real measurements to assess the performance of the mentioned parameters. Search algorithms, gradient descent, iteration-based search and manual fit are among the different approaches that can be used to find the set of parameters that reproduces the real measurements more accurately.

Simulation-based optimization is used to estimate the running resistance parameters in the following articles. A commercial optimization toolbox is used in De Martinis et al. (2013) to calculate the estimates that minimize the squared difference between the observed and the calculated energy consumption, provided that the speed profiles match. The braking distance of trams is also estimated in Do et al. (2020) using coasting operational data. Traction and brake efficiencies and auxiliary power are also estimated in Nespoulous et al. (2021), targeting to match energy consumption records, finding that the parameters that are most related to energy consumption are r_2 and the traction efficiency. Tractive effort measurements are used in Szanto (2016) to generate speed profiles and calibrate r_0 first at low speeds, and then r_2 at high speeds. In Hansen et al. (2017), r_2 is estimated in tunnels by minimizing the squared deviation between the measured and the calculated speed time series subject to the measured energy consumption and speed. There, the tractive effort is obtained from the consumed power, assuming constant traction efficiency. This method is found to be more robust than LSR and the authors observed that theoretical values of r_2 clearly overestimate the running resistance. Furthermore, the mass factor is shown to influence the value of r_2 , and the inclusion of energy consumption data may add an extra layer of precision to the estimates.

In Powell & Palacín (2015), an iterative search is used to estimate deceleration rates and the cruise speed, targeting to match the observed running time and minimize the root mean squared error between the measured and calculated speed profiles. Performance parameters representing the actual value relative to the maximum of the traction, cruise speed and brake rates are calibrated manually in De Fabris et al. (2008). Although the simplified kinematic models used in the last two articles constitute a good first order calibration, they may not be able to capture complex driving behaviour.

Simulation-based optimization methods offer more modelling flexibility than other approaches and are generally easy to be implemented. However, these methods usually lead to local optima and the quality of the estimates is correlated to the level of realism of the simulator, which internal configuration may constitute a source of bias with respect to the model calibration.

Metaheuristics

Metaheuristic algorithms are also a common way of solving simulation-based optimization problems in industry and academia due to their ease of implementation and practicality.

In these algorithms, a set of parameters whose performance is evaluated is called a candidate, and the set of candidates is called population. The initial population is randomly generated, and each algorithm proposes a different way of evolving the population.

In a Genetic Algorithm (GA), at each iteration the candidates that show the worst performance are eliminated, and new ones are generated by combining and modifying randomly the remaining candidates, so that the population is kept constant at each iteration. The running resistance is estimated in several articles using GA (Bešinović et al., 2013a,b; De Martinis & Weidmann, 2016a,b; De Martinis & Corman, 2018; Sánchez et al., 2021). Particularly, GA is combined in Sánchez et al. (2021) with a SQP-based speed profile calculator, finding results consistent with coasting tests. A quadratic interpolation routine was used to assess whether the newly generated candidates are promising or not, allowing to save computation time from the SQP calculation. Furthermore, train lengths, maximum tractive effort and power, brake rates and the cruise speed are estimated in Bešinović et al. (2013a,b) from track occupation data and statistical distributions of those parameters are calculated. Acceleration and deceleration rates and the traction performance rate are also estimated in De Martinis & Weidmann (2016a,b).

In Particle Swarm Optimization (PSO) the candidates in the population move over the parameter space while sharing information between them, so that they are influenced by their individual and the population best-fit locations. PSO is used in Graber et al. (2016) to calibrate the dynamics of a metro vehicle for a simulator. The track was divided in several segments, and the running resistance parameters, the rotating mass factor, the overhead wire resistivity and the power efficiency were calculated in each segment.

In Simulated Annealing (SA), the candidates move randomly in the parameter space, introducing a temperature parameter that controls the mobility of the particles through the parameter space. This temperature is gradually lowered through the iterations, reducing the candidates' speed and guaranteeing the algorithm convergence close to a local minimum. Two articles De Fabris et al. (2011); Medeossi et al. (2011) build on De Fabris et al. (2008) by substituting the manual fit by SA, which simplifies the calibration procedure. However, the calibration performance is limited again by the simplified model used.

As simulation-based optimization methods, metaheuristic algorithms show similar pros and cons. However, they may cover the parameter space more extensively at the cost of showing the highest computation times of the considered methods.

2.4.2 Online train motion model calibration

Regression

LSR can be adapted to perform online estimation by adding recursively in each iteration the last set of measurements to the least squares procedure and calculating the resulting parameter update. The resulting method, Recursive Least Squares (RLS), is enhanced as Multi-Innovation Least Squares (MILS) by considering not only the error between the model predictions and the last set of measurements, but the one with respect to the last few observations, which constitute the observation window. MILS can also be implemented with a varying window size (V-MILS), which helps producing more robust estimates in the presence of measurement outliers or missing data.

RLS is used in Moroz et al. (2017); Gorbachev et al. (2017) to estimate the train weight and the running resistance parameters r_0 and r_1 . In both articles RLS is validated using simulation data with added noise representing the effect of wind. Several adaptive parameter estimation algorithms including RLS, a regularized version of RLS that guarantees the positivity of r_1 , MILS and V-MILS are tested in Liu et al. (2018). These methods are verified using real data from the ATO system of a metro train and estimate the running resistance accurately. Both MILS and V-MILS outperform RLS.

RLS shows the same advantages and disadvantages as LSR. It is simple, easy to be implemented and computationally very efficient. The inherited sensitivity to measurement outliers may be particularly problematic for RLS due to the iterative data processing procedure. However, MILS and V-MILS may solve this issue. Moreover, V-MILS is robust under the presence of missing or anomalous measurements. Furthermore, RLS allows to weight less the oldest measurements by introducing a forgetting factor.

Kalman-like state observers

Several nonlinear versions of the Kalman Filter have been used for estimating train motion model parameters in real time. In the Unscented Kalman Filter (UKF) the state and noise statistics are sampled deterministically by means of a technique called Unscented Transformation and propagated through the nonlinear train motion model in order to determine the state estimates at each iteration. Another nonlinear version of the Kalman Filter is the Extended Kalman Filter (EKF). The main difference with respect to UKF is that EKF considers the state distribution a Gaussian random variable, which is propagated through a linearization of the dynamical system.

In Howlett et al. (2004), a simulated case study shows that UKF can estimate the running resistance parameters r_0 and r_2 when a train is coasting on a level track. The results obtained are sensitive to the initialization value of the covariance matrix and the authors conclude that a rigorous filter initialization procedure is required for the successful implementation of the algorithm. Similar conclusions are drawn in Cunillera et al. (2021), where UKF is applied in the four driving regimes mentioned in chapter 2.2. There, UKF is combined with a driving regime identifier that determines in real time the current driving regime. Then, the remaining train motion model parameters and their maximum bounds are estimated by means of a post-processing module that analyzes the outputs of the UKF and the driving regime identifier. The authors conclude that tractive and brake effort measurements are an asset for enhancing the estimation accuracy. EKF is combined in Jin et al. (2018a) with Gaussian sum theory to model any non-Gaussian state and noise statistics as a sum of Gaussian random variables that can be processed by a set of EKFs to estimate the running resistance. There, the number of Gaussian terms is reduced after each iteration of the algorithm by selecting those with the largest weights to guarantee real-time performance.

Kalman-based filters are widely spread in the industry, as they can produce very accurate estimates, deal with non-Gaussian noise and are computationally efficient. However, their main drawback is that they are very difficult to be implemented and tuned. Furthermore, their performance depends critically on the covariance initialization value.

Bayesian methods

Particle Filtering is a different online calibration technique that can deal with non-Gaussian dynamics, where the state statistics is evaluated by means of a random sampling.

In Xie et al. (2016), the running resistance parameters are described as a weighted sum of a given set of predefined candidates. This procedure assumes that the real value of the parameters will lay within the mentioned set of predefined parameters. The estimation is performed by means of a Particle Filter that calculates the posterior probability of each parameter candidate subject to the most recent set of measurements and chooses the most likely parameter among the candidates as output if its probability is more than 90%, and it is computed as the weighted sum of all the probabilities associated to the parameter candidates otherwise.

Bayesian methods are conceptually simple, yet powerful and may deal with non-Gaussian noise. However, the major drawback of the implementation proposed in Xie et al. (2016) is that it may be inaccurate if the set of predefined parameter candidate does not represent the current real value of the target parameters. Compared to the previous two online calibration methods, Particle Filtering is not deterministic and is more expensive computationally.

Gradient descent

In optimization, gradient descent is a technique that aims to minimize a function iteratively, by evaluating the function in the direction where it decreases faster with respect to the current estimates.

In De Martinis & Corman (2019), a multi-start gradient optimization procedure is used to estimate the running resistance parameters from a finite window containing the last few measurements. The driving regime is determined first from the acceleration and net power consumed for every set of measurements, while the predicted tractive effort is used as the measure of performance of the proposed framework. Two different objective functions are explored: the root mean square error and the sum of absolute errors between the predicted and the measured tractive effort, being the latter faster, less sensitive to outliers and the one used in the latter parts of that paper.

However, this method can only perform online estimation for small measurement windows due to its high computation time.

2.5 Main findings

In total, we have reviewed 42 articles. 34 on offline calibration techniques and 8 on online approaches. The reviewed techniques have been applied to most types of rolling stock: from monorail to high-speed trains. Most articles focus on determining a few parameters, being the running resistance parameters those that have received most attention due to their impact on the energy consumption.

Least Squares Regression (LSR) is the most frequently-used calibration method due to its simplicity and performance. The acceleration is usually calculated from speed measurements and the force balance in Eq. (2.1) is used to find the running resistance parameters that best fit the measurements. However, in Hansen et al. (2017), this procedure was

found to be less consistent than fitting the squared error between the measured speed and the predicted speed calculated from the previous speed measurement. All the regression methods were explored using a squared error-based objective function. The sum of absolute errors has been explored in De Martinis & Corman (2019), showing that in some cases it can be more computationally efficient than a squared error objective while still producing similar results. Other offline techniques can be used to estimate more parameters at once, at the expense of longer computation times. Regarding online calibration techniques, further research is needed to reach a reliable on-board implementation.

Several factors that limit the calibration accuracy are systematically overlooked:

- The nonnegativity of the running resistance parameter constraint is usually introduced manually in LSR by either fixing r_0 and r_1 or by setting $r_1 = 0$. This latter parameter, r_1 , is generally considered to be the most difficult running resistance parameter to be calibrated. Running resistance parameters estimation is a constrained optimization problem that should be addressed by using an appropriate constrained optimization method. In the case of LSR, this can be solved by using a modified version of LSR that allows for constrained optimization.
- Speed measurements are often distributed unevenly along a train's speed range, leading to overfitting the parameters in the most-visited speed intervals.
- So far, only a handful of authors have developed calibration techniques that are robust against anomalous data or outliers (Aradi et al., 2015; Liu et al., 2018; De Martinis & Corman, 2019). However, these articles mainly focus on estimating the running resistance parameters, and only Aradi et al. (2015) estimated the train mass and mechanical efficiency.
- All the reviewed methods require either an accurate track geometry description or to be used on a level and straight track section. This is not a hard constraint, since a test for determining running resistance and the rotating mass factor without requiring track geometry information has been designed in Somaschini et al. (2016, 2018). However, the train speed has to be measured at the exact same locations along different runs, so extending this method for using operational data may require further research.
- We have observed that validating and comparing the existing calibration techniques is generally difficult due to the lack of target parameters' ground-truth data and the wide variety of real and simulated case studies used. Some of the techniques are validated by means of data generated in simulations, although this does not guarantee their performance or accuracy in a real scenario.

Despite offline calibration has received more attention in the literature, some articles highlight the need for on-board calibration (Lukaszewicz, 2007b, 2009), particularly for rolling stock of variable composition and freight trains. Some of the discussed articles challenge the general knowledge on the physical sources of train motion model parameter variation (Lukaszewicz, 2009). Furthermore, just a few articles have addressed determining the relevance of individual sources of variation (Hansen et al., 2017; Lukaszewicz, 2007b, 2009) or driving-induced variations (Bešinović et al., 2013a,b; Cunillera et al.,

2021; De Martinis & Corman, 2019). Remarkably, only Hansen et al. (2017); Li et al. (2017b) focus on estimating the extra running resistance in tunnels, despite its impact on the train dynamics and energy consumption.

Online calibration techniques show a wide variety of ways of analyzing the available measurements. Some of them use only the last measurement available to generate the estimates at each iteration of the algorithm (Cunillera et al., 2021; Liu et al., 2018; Howlett et al., 2004; Moroz et al., 2017; Gorbachev et al., 2017; Jin et al., 2018a), which guarantees a faster computation time at the cost of more sensitivity of the estimates to anomalous data and noise. In contrast, some techniques consider all the available measurements at each iteration, and add the new ones as soon as they are generated (Xie et al., 2016). This produces an increasingly larger computation time that might become an obstacle in the event of an onboard implementation. Other techniques include a measurement window consisting of a finite number of the most recently recorded measurements (Liu et al., 2018; De Martinis & Corman, 2019). The size of the window has to be fine-tuned for each technique to optimize the balance between computation time and robustness against measurement outliers and missing data. In Liu et al. (2018), measurement windows including from 2 to 6 measurements are explored (with a sampling rate of 1s), showing that larger windows produce more accurate estimates than the standard Recursive Least Squares that only uses the latest measurement available. Moreover, a varying window size has also been explored (Liu et al., 2018). In turn, in De Martinis & Corman (2019) very large windows that include 30 to 60 measurements were found not suitable for real-time calibration, so the measurement window had to be reduced to 10 measurements (sampling rate, 1s). However, the size of the window and the computation time depend on each technique, the efficiency of the implementation and the computational power available. Surprisingly, the addition of a forgetting factor that contributes to gradually give less weight to old measurements than to the newly generated ones is still to be explored in train motion model calibration.

The analysis of the literature shows another advantage of using operational data for train motion model calibration over full-scale tests: the applicable range of speed of the calibration techniques. On the one hand, pulling and cruising full-scale tests, including the new ones proposed in Somaschini et al. (2016, 2018), have to be performed at low speeds. On the other hand, most techniques that analyze operational data could be applicable in any speed range. For instance, RLS has been historically used to analyze coasting data, which usually spans the upper speed interval of the train, however, this method can also be used to analyze data below 10km/h (Hoffrichter et al., 2014). Similarly, the Unscented Kalman Filter has been used to analyze coast curves in Howlett et al. (2004) and has been extended to cover the whole trajectory in Cunillera et al. (2021).

2.6 Research agenda and practical recommendations

Based on the train motion model calibration literature review presented in chapter 2.4 and the findings outlined in chapter 2.5, we hereby outline open research questions and practical recommendations.

2.6.1 Research agenda

Online calibration for on-board energy-efficient driving algorithms

An adequate implementation and usage of on-board energy-efficient driving algorithms like real-time train trajectory optimizers, DAS and ATO requires performing several iterative rounds of tests and measurements to calibrate such algorithms and achieve their target levels of precision. This is a difficult and resource-consuming task that is usually performed in early implementation stages and fine-tuned at the beginning of their operational life. However, as mentioned in chapter 2.2, the wear generated in the mechanical parts of the train and its engines, varying environmental and seasonal conditions may have an impact on the train dynamics. This can be modelled as parameter variability, which may lower in time the accuracy of the calibrations. Furthermore, in the case of freight trains with variable rolling stock composition, the usual calibration tests cannot be performed (Lukaszewicz, 2007b, 2009). This fact highlights the need for an online train motion model calibration framework for on-board energy-efficient applications, which is still missing in the current literature. Such framework could benefit from the large amount of data that is measured and stored on-board. It would have to calibrate accurately in real time the usual input parameters and bounds of energy-efficient applications mentioned in chapter 2.2, from mass-specific running resistance parameters to bounds on the tractive effort and power and, in the case of manually driven trains, the performed brake curves. Moreover, it should be computationally efficient in order to allow for real-time on-board calculations and easy to be configured to perform the estimations automatically. This latter characteristic would reduce the need for tests, lowering partially the economic, capacity and resource costs associated to the calibration procedures during the implementation phase. Chapter 4 constitutes an initial step towards addressing this research gap.

Calibration frameworks robust against anomalous data and varying conditions

Nowadays, railways are fitted with a wide variety of sensors that produce the input data required by algorithms like train motion model calibrators. However, all measurements are affected by noise. This may affect the accuracy of the calibration, especially when outliers or wrong measurements are introduced as input. Moreover, coverage issues may lead to missing data. An archetypal example is the missing Global Navigation Satellite System (GNSS) location data at tunnels where the satellite coverage is lost. These data issues affect particularly online calibration algorithms that analyze the measured data on-the-go, as these errors may stand out from the limited amount of measurements used at each calculation, biasing the estimates. Train motion model calibration frameworks need to be robust under the presence of outliers and missing data, particularly those that are designed for on-board applications. The accuracy of the estimates must not be affected by such anomalous data in order to guarantee the reliability of the estimates in most operational scenarios. As shown in chapter 2.4, online calibration algorithms add a layer of robustness to their estimations by analyzing several measurements at the same time, either by using the last few measurements (Liu et al., 2018; De Martinis & Corman, 2019), or by using all the available measurements (Xie et al., 2016). However, analyzing more than one set of measurements at each iteration is more demanding computation-

ally, which might become an obstacle in the event of an onboard implementation. Most online calibration algorithms require using an initial guess of the estimates and a robust framework should be robust against the mentioned initial guess, although this feature is still to be explored in the scientific literature. Regarding offline calibration procedures, Least Squares Regression is the most popular technique among them, however, it may be highly sensitive to measurement outliers. Least absolute deviations, which is only explored in De Martinis & Corman (2019), constitutes a usual alternative to least squares when aiming for robustness. Furthermore, a robust calibration framework covering most train motion model parameters is still to be developed. What is more, the track geometry description is a simplification of the actual geometry of the track, constituting a significant source of uncertainty and error for the train motion model. Most calibration procedures either consider the track as flat, or assume that the track geometry description is perfectly accurate. Therefore, a calibration procedure that uses operational data as input and does not require knowing the track geometry description, in line with Somaschini et al. (2016, 2018), would also be of practical interest. In conclusion, more research on robust train motion model calibration is needed to enhance the reliability of the estimates and to boost the performance of railway applications. In this thesis, a robust offline train motion model calibration method based on M-estimation is proposed, tested and analyzed in chapter 3. In turn, the robust online estimation of the running resistance parameters is addressed in chapter 5 by means of Recursive Least Squares with multiple forgetting factors.

Quantifying sources of parameter variation

A systematic study of the individual physical sources of parameter variability based on operational data is still missing in the scientific literature. In chapter 2.2 we described several sources of parameter variation for the train motion model. Passing through tunnels and running under headwind may increase the running resistance and the energy consumption. The train mass may vary depending on the load, especially in the case of freight trains. Fluctuations in the catenary power may vary the maximum tractive effort that can be applied. These are just a few examples of the existing sources of variation affecting a train's dynamics. As we outlined in chapter 2.5, some individual sources of parameter variations have been studied in a small portion of the articles covered, but more research is required to quantify the individual contributions. A study of the impact of rolling stock characteristics and the main variations present in a fleet of trains with the same composition may emphasize the importance of individual train calibrations and train-tailored solutions. Particularly, energy-efficiency research, like energy-efficient driving, could benefit from more studies on the impact of running through tunnels on the running resistance, building on Hansen et al. (2017); Li et al. (2017b). Analogously, online calibration may contribute to understanding further the impact of curve resistance on the dynamics of a train. Moreover, weather may also influence train motion model parameter variations. Public weather data sources could be used for analyzing seasonal and weather-induced variations (Manabe et al., 2015). The impact of wind speed and direction could also be assessed, as it may also influence the train dynamics. For example, the Sauthoff formula is a widely used variation of Davis equation that accounts for the wind speed in its quadratic term, however, it assumes a wind speed of 15km/h (Rochard & Schmid, 2000). This research could also validate running resistance formulas that aim to be more

accurate, like the wind-dependent one proposed in Trivella et al. (2021). The knowledge obtained could be of interest for rolling stock manufacturers, who could optimize their designs taking into account the observed variability. Moreover, the relevance of on-board calibration could be further highlighted by this research, as well as the need for algorithms robust against the initial guess of the estimates, so that their accuracy is not affected by uncertainties and variations of the operating conditions. Alternatively, driving-induced parameter variability could also be researched. The maximum applied tractive effort and power, the applied brake rates and switching points between driving strategies are affected by driving style variations, particularly in the case of manually-driving trains (Bešinović et al., 2013a; Cunillera et al., 2021; Medeossi et al., 2011). Comparing the results of manually and automatically-driven trains could also be of interest to further gauge the advantages and disadvantages of railway automation. Furthermore, driving-induced variations could be assessed for different schedule deviation scenarios, that is to say, whether trains run on-time, delayed or early.

2.6.2 Practical recommendations

A public dataset for model validation and benchmarking

We suggest the creation of a publicly available dataset for validating and benchmarking the performance of train motion model calibration frameworks. Data availability is an issue in railways research. Some of the techniques analyzed in this literature review are validated by means of simulations, although this does not guarantee the performance or accuracy of the proposed approaches in real life. Moreover, comparing the performance and results obtained by those techniques that are tested using real measurements is difficult due to the large variety of case studies found in the literature, spanning from monorail to high speed trains. Furthermore, usually no ground-truth data for the real value of the estimated parameters exists. These three facts constitute a remarkable hurdle for the validation and comparison of train motion model calibration frameworks. We recommend a minimum number of 50 runs of a train unit running on a line to be included in such dataset, including an accurate track description of the considered line, GNSS location, speed and applied effort measurements. Train mass, tractive wheels rotation speed and energy consumption measurements could be a plus that would allow for more flexibility in new calibration techniques. Up to the authors knowledge, there are no such publicly available datasets for operating trains (Pappaterra et al., 2021).

Good practices for train motion model calibration

Train motion model calibration is generally a difficult problem and we have detected some common pitfalls in the articles covered, which are outlined in chapter 2.5. Input data should be weighted to avoid underfitting the target parameters in the least-visited speed intervals, as speed measurements show an uneven distribution along a train's speed range. Moreover, r_1 is generally considered the most difficult running resistance parameter to be estimated. Calibration procedures usually lead to a negative value of r_1 , which is not physically correct. This can be solved by applying a constrained optimization procedure like Constrained Least Squares, SQP, or by adding a regularization term to the cost

function. Furthermore, the value of the rotating mass factor may affect the running resistance estimation, thus it must be taken into account in order to produce accurate estimates (Hansen et al., 2017).

Regarding the data sampling rate for calibrating the train motion model, we suggest a sampling rate from 1Hz to 10Hz for all the mentioned variables except for the energy consumption, as higher sampling rates require more filtering, storage and subsampling, while lower sampling rates may not be suitable for most applications or may lead to inaccurate estimates. The European Committee for Standardization recommends a minimum sampling rate of 10Hz for train measurements (CEN European Standard, 2018), however, the mentioned inconveniences of higher sampling rates should be taken into account.

We do not recommend using accelerometers data for train motion model calibration, as they may produce noisy data, leading to estimates that are less accurate than those inferred from other data sources (Kim et al., 2006).

Train motion model calibration should be at least an essential preliminary step in the implementation of any railway application to guarantee its accuracy and effectiveness. We strongly suggest to use online techniques for calibrating continuously any on-board application that utilizes the train motion model. Particularly, train control algorithms, Driver Advisory Systems and Automatic Train Operation usually have very high precision requirements, so model calibration is vital for such applications. The performance and realism of simulators and train trajectory optimizers also require an exhaustive calibration, which may be performed offline using large amounts of data from historical measurement records.

Train motion railway calibration and the existing railway systems

Any of the presented online and offline calibration techniques are suitable for main line, freight and high speed trains. However, regression algorithms that only analyze coasting data are not suitable for monorail, urban/light and metro trains due to the absence of coasting regimes or their typical short duration in such railway systems. Instead, any of the other calibration techniques presented in this chapter could be used for calibrating the dynamics of these latter systems.

The role of infrastructure managers

In chapter 2.6.1 we mentioned that the track geometry description is a source of error in the train dynamics due to the fact that it is a simplification of the real geometry. Moreover, no train motion model calibration framework that utilizes operational data and that is robust against errors in the track geometry description exists yet. Therefore, we request infrastructure managers to describe the track geometry as accurately as possible, as this can enhance the train motion model performance and benefit other railway applications, like ETCS continuous brake curves calculation.

Giving visibility to train motion model calibration in Academia

We would like to demand researchers to state clearly in the title or abstract that train motion model calibration is performed in this chapter, as this will contribute to drawing more attention to this topic. When gathering the articles that cover train motion model

calibration for preparing this literature review article we faced an unexpected challenge. Scholars tend not to mention explicitly that train motion model calibration is performed when it is used as a means to calibrate an algorithm that embeds the motion model. This hinders the task of learning new calibration techniques and good practices.

2.7 Conclusions

An accurate and effective implementation of railway applications like timetabling tools to Driver Advisory Systems and Automatic Train Operation requires calibrating a train motion model that describes the dynamics of a train running on a track. In order to highlight the importance of train motion model calibration in the railway industry and academia, we have presented a critical review of the train motion model calibration techniques that are found in the scientific literature. Particularly, we have focused on those techniques that use operational data, since such techniques benefit from larger data availability, do not require scheduling resource-demanding tests, and may lead to estimates that represent better ordinary operation conditions than tests and simulations. To this end, we have clustered the existing calibration techniques in two categories: online calibration that analyzes the measured data on-the-go, and offline calibration that is able to analyze data from several trajectories in one go. We have described and analyzed each technique, identifying its pros and cons, as well as the most common problems and pitfalls.

Based on this analysis, we have proposed a research agenda for the coming years. We have highlighted the need for an accurate online calibration framework for on-board energy-efficient driving applications, as well as for frameworks that are robust under measurement outliers and missing data that may decrease the calibration accuracy. We have described this calibration problem and listed the main sources of parameter variation, including physical sources like wind and weather, and manual driving that produces driving-induced variations. In this regard, we also proposed a quantitative research of the parameter variability due to each individual source.

Last, we have suggested also several practical recommendations regarding train motion model calibration. Particularly, the railway community would benefit from a publicly available dataset for validating and benchmarking newly developed calibration frameworks. In addition, we have outlined the main pitfalls to avoid when calibrating the train motion model, recommended a sampling rate for the measurements used as input to the calibration frameworks and highlighted the role of infrastructure managers in describing the track geometry accurately for improving the performance of such calibration frameworks and the role of researchers in giving visibility to this topic within academia.

Chapter 3

Robust train motion model calibration based on M-estimation

3.1 Introduction

The energy-efficiency of railways constitutes one of their main advantages over other transport systems (Baker et al., 2010). Despite having the upper hand on energy-efficiency, a large amount of research is being performed to improve railways' efficiency, punctuality and service reliability (Scheepmaker et al., 2020b). Particularly, energy efficient driving is a popular research topic that can further reduce the energy consumption of railways while contributing to a timely, safe and comfortable railway operation (De Martinis & Corman, 2018; Scheepmaker et al., 2017). Energy efficient driving may impact the railway industry through Train Trajectory Optimizers, Driver Advisory Systems, Automatic Train Operation, simulators and timetabling tools.

An effective implementation of any energy efficient driving algorithm requires a careful calibration using historical data. Indeed, most energy efficient driving algorithms embed an equation based on Newton's second law that models the dynamics of a train running on a track: the train motion model (Scheepmaker et al., 2017). These algorithms use this equation to predict and reproduce the dynamics of a train, requiring a precise calibration of its internal parameters in order to achieve the target accuracy and performance. The maximum tractive effort and power that can be applied, the brake rates and the running resistance parameters are among the input parameters of the train motion model that have to be calibrated.

Two main types of train motion model calibration techniques can be distinguished: online techniques that analyze on-boards measurements as they are generated and offline techniques that can analyze a larger amount of data at once but are not suitable for real-time data analysis. They serve different purposes: online calibration allows to explore spatiotemporal variations of the train dynamics, while offline techniques aim to calculate a single set of estimates that fits best the model. In this chapter we address offline calibration, which has been tackled by means of a wide variety of methods. Ordinary Least Squares (OLS) regression (Lukaszewicz, 2007a; Michálek, 2018; Rochard & Schmid, 2000) is the simplest and most widely used offline calibration method, although outliers and highly leveraged measurements tend to bias the estimates produced. Sequential Quadratic Programming (Aradi et al., 2015) formulates the calibration problem

as a constrained optimization problem, allowing to add constraints to the estimates (for instance, nonnegativity). Maximum Likelihood Estimation (Jaoua et al., 2018), utilizes a particle filter to approximate the probability density function of the estimates, which are computed iteratively. Its performance relies on an adequate initial guess of the estimates. An Iterative Learning Identification method has been explored (Wang & Qi, 2008), combining OLS with state observers to improve iteratively the accuracy of the estimates. Simulation-based optimization (De Martinis et al., 2013; Hansen et al., 2017; Szanto, 2016) and metaheuristics (Bešinović et al., 2013a; De Martinis & Corman, 2018; Sánchez et al., 2021) allow for more flexibility in the modelling at the expense of longer computation times. See chapter 2 for an extensive review and discussion on online and offline train motion model calibration.

One of the main challenges in train motion model calibration is its robustness against anomalous data (Cunillera et al., 2023a). Robustness here refers to the insensitivity of the calibration to input data outliers or wrong measurements (Rey, 1983). In practice, measurement outliers may bias the estimates, lowering their accuracy. Some sensors eventually lose coverage, creating gaps of missing data, which also compromises the estimation quality. Despite the importance of creating robust estimates, a robust train motion model calibration method covering most of the parameters of the models is still missing in the literature (Cunillera et al., 2023a).

In this chapter, we address this gap by proposing a computationally-efficient robust train motion model calibration method based on M-estimation. This method utilizes an objective function in which the influence of the residuals of the estimation model is lower than in OLS. This allows to reduce the impact of outliers on the calculated estimates, making the estimation robust. Unlike OLS, M-estimators do not usually have a closed analytical solution, so the estimates have to be calculated iteratively. M-estimation has been used in a wide variety of scenarios related to transport systems: vehicle state estimation (Xue et al., 2022), satellites flying in formation (Guo & Zhao, 2021; Sun et al., 2023), flight attitude estimation (Wang et al., 2011), location estimation (Ho et al., 2020), relative GPS positioning (Chang & Guo, 2005), and integration of Global Navigation Satellite Systems and Inertial Navigation System measurements (Jiang et al., 2022). M-estimation is used in this chapter to estimate the running resistance parameters.

Furthermore, we also calibrate the maximum tractive effort and power applied, the maximum applied brake effort and the deceleration rates used when braking by means of a sliding median filter. While the running resistance parameters are explicitly included in the dynamical system that models the dynamics of the train, these other parameters appear as bounds of some of the variables of the model. M-estimation is not an appropriate method for estimating these bounds. Instead, we calculate them from the applied traction and brake effort and the deceleration rates observed. We remove potential outliers in the effort, tractive power and deceleration time series by means of a sliding median procedure and take the maximum over all the observations in each trajectory.

The main contributions of this chapter can be summarized as follows:

- The proposal of a robust train motion model calibration method that estimates the running resistance parameters, the maximum tractive and brake efforts, the maximum tractive power and the most restrictive speed-dependent deceleration rates observed.
- A comparison of several M-estimation error measures for the robust estimation of the running resistance parameters.
- A demonstration of the performance of the method in a case study with real data from trains running in the Dutch railway network, including a statistical analysis of the estimated train motion model parameters. From these statistics we infer an analysis of the behavioural factors of train drivers that are relevant in practice for modelling train dynamics.

The method proposed in this chapter will be of significance for railway practitioners and scholars. A robust train motion model calibration will enable to ease model-related inaccuracies that arise when implementing automation algorithms. This method mitigates the impact of measurement outliers in the calibration, enhancing the performance of timetabling and simulation tools that rely on the train motion model.

The remainder of the chapter is organized as follows: Next, we describe the train motion model calibration problem in chapter 3.2. We introduce the proposed robust parameter estimation method in chapter 3.3. We test the performance of this method in a case study with real data from trains running on the Dutch railway network in chapter 3.4. Last, we gather the main conclusions of this research in chapter 3.5.

3.2 Train motion model calibration

We proceed now to describe the train motion model calibration problem by introducing the train motion model, explaining each of the parameters to be calibrated and formulating mathematically the calibration as an optimization problem that utilizes as input measurements of some of the variables of the model.

The train motion model is usually modelled by means of Newton's second law as follows (Douglas et al., 2017):

$$\frac{dv}{dt} = u(t) - r(v(t)) - w(s(t)), \quad (3.1)$$

where $s(t)$ is the train location in m and $v(t)$ the speed in m/s. t is the time in s, $u(t)$ is the tractive and brake effort in N/kg. All the terms in the right hand side term of Eq. (3.1) have been divided by the train mass and the dimensionless rotating mass factor ρ to make them mass-specific, which allows to keep the train motion model affine in its parameters. $w(s(t))$ is the resistance to motion due to the gradients and curves of the track, in N/kg.

$$w(s(t)) = \frac{g}{\rho} p(s(t)) + \frac{k}{\rho R(s(t))}, \quad (3.2)$$

with $g = 9.81 \text{ m/s}^2$ the acceleration of gravity, ρ the dimensionless rotating mass factor, $p(s(t))$ the gradient of the track at $s(t)$ in vertical meters per meter advanced, $R(s(t))$ the radius of the track at $s(t)$ in m and k is a constant that gauges the impact of curves on the train dynamics in Nm/kg. $r(v(t))$ is the running resistance in N/kg and it is characterized by the so-called running resistance parameters r_0 , r_1 and r_2 , which are nonnegative,

$$r(v(t)) = r_0 + r_1 v(t) + r_2 v^2(t). \quad (3.3)$$

The running resistance directly influences the energy consumption of a train and plays an important role in shaping the trajectory of a train when it is coasting, that is to say, when the train runs without applying traction or brake. Still, the running resistance parameters are often inaccurately determined. This uncertainty constitutes a source of error when reproducing or predicting the dynamics of a train, having an impact on the performance of any algorithm that relies on the train motion model. Train manufacturers perform field tests and computational experiments to determine these parameters, although these tests may not capture the behaviour of the train during daily operating conditions. Thus, data measured onboard during the working life of the rolling stock has more potential for estimating these parameters during relevant operation scenarios (Cunillera et al., 2023a). The existing literature shows that Ordinary Least Squares (OLS) regression is the most widely-used technique for estimating the running resistance parameters for its simplicity. However, it is not exempt of issues since its estimates have a tendency to violate the nonnegativity of the linear running resistance parameter r_1 . Furthermore, OLS is not a robust parameter estimation technique since it is significantly affected by measurement outliers.

The tractive and brake effort $u(t)$ in Eq. (3.1) is limited by adhesion and the engine and brake characteristics. These constraints can be modelled as

$$-u_{b,\max}(v) \leq u(t) \leq u_{f,\max}, \quad (3.4)$$

$$u(t)v(t) \leq p_{\max}, \quad (3.5)$$

with the bounds being $u_{f,\max}$ and $p_{\max}$ the maximum tractive effort and power, respectively, in N/kg and W/kg. $u_{b,\max}(v)$ is the maximum brake effort in N/kg, with $u_{b,\max}(v)$ positive. These bounds are uncertain as well, since adhesion and mechanical wear of the engine and the brakes may affect the traction and brake capacity of the train. When considering electric trains, the overhead power has also a direct influence on the tractive power. Estimating these bounds is also important in order to provide accurate inputs to simulators, timetabling and energy-efficient driving algorithms. In this chapter we consider the maximum observed brake effort in Eq. (3.4) instead of the maximum braking capability of the train, since trains do not brake at their maximum capacity during normal operation. For simplicity, we consider $u_{b,\max}$ constant for all speeds. In this regard, we model the deceleration rates of a train in more detail as

$$u(t) - r(v(t)) - w(s(t)) \geq -a_{\min}(v), \quad (3.6)$$

with $a_{\min}(v) > 0$ in m/s^2 . The Dutch Automatic Train Protection System ATB-EG requires decelerating with minimum rates at 5 predefined speed steps: between 0 and 40km/h, 40 and 60km/h, 60 and 80km/h, 80 and 130km/h and over 130km/h. We describe $a_{\min}(v)$ as a piecewise constant function of the speed:

$$a_{\min}(v) = \begin{cases} a_{40} & \text{if } v \leq 40\text{km/h}, \\ a_{60} & \text{if } 40\text{km/h} < v \leq 60\text{km/h}, \\ a_{80} & \text{if } 60\text{km/h} < v \leq 80\text{km/h}, \\ a_{130} & \text{if } 80\text{km/h} < v \leq 130\text{km/h}, \\ a_{140} & \text{if } 130\text{km/h} < v, \end{cases} \quad (3.7)$$

with $a_{40}, a_{60}, a_{80}, a_{130}, a_{140}$ in m/s^2 and strictly positive. We assume that the braking capability of the train is sufficiently large so that all these deceleration rates are achievable under normal operation conditions. Each train driver may brake slightly differently, so analyzing observed deceleration rates allows to obtain a deeper insight into the braking style of human drivers.

The set of parameters to be estimated are therefore $r_0, r_1, r_2, u_{f,\max}, p_{\max}, u_{b,\max}, a_{40}, a_{60}, a_{80}, a_{130}$ and a_{140} . In this chapter we do not estimate a_{140} since the maximum speed of the track is 140km/h and in our case study trains do not usually brake in the speed step between 130 and 140km/h. Therefore, not enough braking data is available to estimate a_{140} accurately. The speed step of a_{130} is then extended to model the braking behaviour at higher speeds, since in a previous work on online parameter estimation we did not observe significant differences between a_{130} and a_{140} (Cunillera et al., 2022).

Train motion model calibration requires measurements of some of its variables. We define the measurement times $t_k, k = 1, \dots, M$. Although measurements are usually sampled at a constant measurement rate, missing or wrong measurements are also common. We assume that the measurement times t_k are sampled at a constant rate $\Delta t = t_{k+1} - t_k$. At each measurement time we define train location, speed and tractive/brake effort measurements, $s_k = s(t_k), v_k = v(t_k)$ and $u_k = u(t_k), k = 1, \dots, M$. We can approximate the acceleration of the train from two consecutive measurements and assume that it is constant between measurements, $a_k = (v(t_{k+1}) - v(t_k)) / \Delta t, k = 1, \dots, M - 1$. This allows to discretize Eq. (3.1) at the measurement times,

$$a_k = u_k - r_0 - r_1 v_k - r_2 v_k^2 - w(s_k), \quad k = 1, \dots, M - 1. \quad (3.8)$$

This equation can be rearranged as an identification model of the running resistance parameters. Denoting $\theta = (r_0, r_1, r_2)^\top$, we find

$$y_k = x_k^\top \theta, \quad k = 1, \dots, M - 1, \quad (3.9)$$

with $y_k = u_k - w(s_k) - a_k$ and $x_k = (1, v_k, v_k^2)^\top$.

The goal of train motion model calibration is obtaining a faithful description of the train dynamics that allows to have a deeper insight into the influence on the physics of a train, and to boost the accuracy, performance and reliability of any algorithm that embeds the train motion model. In this regard, the estimates must be able to explain the measurements obtained, save for any measurement error or bias. We define the residuals of the identification model

$$z_k = y_k - x_k^\top \theta, \quad k = 1, \dots, M - 1. \quad (3.10)$$

The train motion model calibration problem can be modelled as an optimization problem that seeks to minimize the sum of the estimation errors at each measurement time, given a certain convex measure $q(z)$:

$$\min_{\theta} J = \min_{\theta} \sum_{k=1}^{M-1} q(z_k). \quad (3.11)$$

In OLS the error measure is quadratic, which is not robust since outliers are likely to bias the estimation. M-estimation aims to mitigate the impact of such outliers by choosing appropriate error measures. We elaborate further on this in chapter 3.3.

The bounds on the tractive effort in Eqs. (3.4)-(3.5) and the maximum deceleration rates in Eq. (3.6) can also be discretized at the measurement times,

$$-u_{b,\max} \leq u_k \leq u_{f,\max}, \quad (3.12)$$

$$u_k v_k \leq p_{\max}, \quad (3.13)$$

$$-a_k \leq a_{\min}(v_k). \quad (3.14)$$

The estimation of these bounds requires filtering the tractive and brake effort and the deceleration measurements, since bounds are particularly sensitive to measurement outliers of those variables.

3.3 Methodology

3.3.1 M-estimation

The main drawback of OLS is its sensitivity to measurement outliers. The objective function of OLS depends quadratically on the residuals of the identification model in Eq. (3.10). This means that the largest residuals have more influence on the value of the objective function. For example, a residual that is 10 times larger than the other residuals of the objective function would have 100 times more influence on its value. OLS tends to mitigate the influence of such large residuals, as this is usually the most efficient way of reducing the value of the objective function. If measurement outliers are introduced in the identification model, they will bias the estimates due to OLS' tendency to adapt the estimation to minimize the influence of large residuals, reducing the accuracy of the estimation. Moreover, data anomalies and missing measurements sometimes occur, adding extra sources of error in the estimation procedure. In train motion model calibration, GPS location measurements might be noisy and unreliable for estimation purposes. Thus, robust methods are essential to guarantee an accurate and non-biased train motion model calibration.

M-estimation solves these issues by replacing the quadratic objective function of OLS by a more convenient measure of the error between the measurements and the estimates that reduces the influence of outliers. M-estimation can be considered as a generalization of maximum likelihood estimation (Huber, 1964). The error measure must be convex, symmetric, have a minimum centered at zero and grow less rapidly than the square of the residuals. The resulting regression is more robust with respect to outliers or wrong

measurements than OLS. The lax requirements on the properties of the error measure allow to design a wide variety of measures $q(z)$, see De Menezes et al. (2021) for a comprehensive discussion of 48 different robust error measures for M-estimation. Table 3.1 gathers the error measures and weight functions tested in this chapter.

M-estimation problems are usually solved iteratively since the minimization problem in Eq. (3.11) does not usually have an analytical closed solution, unlike in OLS. The solution of Eq. (3.11) must verify

$$0 = \frac{\partial J}{\partial r_j} = \sum_{k=1}^{M-1} \frac{dq(z_k)}{dz_k} \frac{\partial z_k}{\partial r_j}, \quad j = 0, 1, 2. \quad (3.15)$$

We can rewrite this equation by defining the weights $\gamma(z)$,

$$\gamma(z_k) = \frac{1}{z_k} \frac{dq(z_k)}{dz_k}, \quad k = 1, \dots, M-1. \quad (3.16)$$

Then, Eq. (3.15) turns into

$$0 = \sum_{k=1}^{M-1} z_k \gamma(z_k) \frac{\partial z_k}{\partial r_j}, \quad j = 0, 1, 2, \quad (3.17)$$

which are the same equations that would result in a Reweighted Least Squares problem:

$$\min_{\theta} \frac{1}{2} \sum_{k=1}^{M-1} \gamma(z_k^{(l-1)}) z_k^2, \quad (3.18)$$

with $z_k^{(l-1)}$ the estimates obtained in the previous $(l-1)$ -th iteration. Therefore, the M-estimation problem can be solved iteratively as a Reweighted Least Squares problem. The error measure is rewritten as a quadratic function of the error multiplied by the remaining non-quadratic term. In each iteration this latter term is considered as constant by evaluating in it the latest estimates or an initial guess in the first iteration. The resulting optimization problem is a Weighted Least Squares regression that can be solved easily. This procedure is repeated until the estimates converge to a value, which is the solution of the original M-estimation problem.

Still, the objective function of M-estimators presented in Eq. (3.11) is sensitive to the scale of the measurements or of the identification problem (Goshtasby, 2012). Unlike measurement errors, not all the residuals have the same variance and M-estimation needs to compensate for this by estimating their scale before performing each iteration. We use studentized residuals in the weights to address this issue:

$$\tilde{z}_k^{(l-1)} = \frac{z_k^{(l-1)}}{S \sqrt{1 - h_k}}, \quad k = 1, \dots, M-1 \quad (3.19)$$

where $\tilde{z}_k^{(l-1)}$ is the studentized residuals based on the estimates of the $(l-1)$ -th iteration, h_k is the leverage of the residual $z_k^{(l-1)}$ and S is an estimate of the standard deviation of the residuals equal to the median absolute deviation of the residuals from their median divided by 0.6745 (Rousseeuw & Croux, 1993). These studentized residuals are introduced in Eq. (3.18) to give less weight to high-leverage residuals. This helps to identify any potential outliers, since high-leverage residuals are usually candidates to be outliers.

M-estimator	Error measure $q(z)$	Weight $\gamma(z)$	Tuning constant c
Andrews	$q(z) = \begin{cases} c^2 \left(1 - \cos \frac{z}{c}\right) & \text{if } z \leq c\pi \\ 2c^2 & \text{if } z > c\pi \end{cases}$	$\gamma(z) = \begin{cases} \frac{c}{z} \sin \frac{z}{c} & \text{if } z \leq c\pi \\ 0 & \text{if } z > c\pi \end{cases}$	1.339
Cauchy	$q(z) = \frac{c^2}{2} \log \left(1 + \left(\frac{z}{c}\right)^2\right)$	$\gamma(z) = \left(1 + \left(\frac{z}{c}\right)^2\right)^{-1}$	2.385
Fair	$q(z) = c^2 \left(\frac{ z }{c} - \log \left(1 + \frac{ z }{c}\right)\right)$	$\gamma(z) = \left(1 + \frac{ z }{c}\right)^{-1}$	1.3998
Huber	$q(z) = \begin{cases} \frac{z^2}{2} & \text{if } z \leq c \\ c \left(z - \frac{c}{2}\right) & \text{if } z > c \end{cases}$	$\gamma(z) = \left(\max \left(1, \frac{ z }{c}\right)\right)^{-1}$	1.345
L_2 (OLS)	$q(z) = \frac{z^2}{2}$	$\gamma(z) = 1$	-
L_p	$q(z) = \frac{ z ^c}{c}$	$\gamma(z) = z ^{c-2}$	1.2
Logistic	$q(z) = c^2 \ln \cosh \left(\frac{z}{c}\right)$	$\gamma(z) = \frac{c}{z} \tanh \frac{z}{c}$	1.205
Talwar	$q(z) = \begin{cases} z^2/2 & \text{if } z \leq c \\ c^2/2 & \text{if } z > c \end{cases}$	$\gamma(z) = \begin{cases} 1 & \text{if } z \leq c \\ 0 & \text{if } z > c \end{cases}$	2.795
Tukey	$q(z) = \begin{cases} \frac{c^2}{6} \left(1 - \left(1 - \left(\frac{z}{c}\right)^2\right)^3\right) & \text{if } z \leq c \\ \frac{c^2}{6} & \text{if } z > c \end{cases}$	$\gamma(z) = \begin{cases} \left(1 - \left(\frac{z}{c}\right)^2\right)^2 & \text{if } z \leq c \\ 0 & \text{if } z > c \end{cases}$	4.685
Welsch	$q(z) = \frac{c^2}{2} \left(1 - e^{-(z/c)^2}\right)$	$\gamma(z) = e^{-(z/c)^2}$	2.985

Table 3.1: M-estimation error measures and weights tested in this chapter (De Menezes et al., 2021; Zhang, 1997).

3.3.2 Robust estimation of the bounds of the train motion model variables by means of a sliding median filter

Besides the running resistance parameters, the maximum tractive effort and power $u_{f,\max}$ and $p_{\max}$, the maximum observed brake effort $u_{b,\max}$, and the deceleration rates a_{40} , a_{60} , a_{80} and a_{130} , also have to be estimated. We utilize a sliding median filter on the tractive and brake effort u_k , on the observed decelerations a_k , and on the applied tractive power, that is, the product $u_k v_k$, provided that u_k is strictly positive. The sliding median filter is centered at each measurement and takes the left and right neighbour whenever any of those correspond to a traction regime for estimating $u_{f,\max}$ or $p_{\max}$, to a brake regime for estimating $u_{b,\max}$, or to a deceleration regime for estimating the deceleration rates. This allows to eliminate measurement outliers. For the estimation of the deceleration rates we only consider those observed decelerations that take place while non-zero brake effort is applied. Then, we obtain $u_{f,\max}$ and $p_{\max}$ as the maximum of the filtered positive tractive effort measurements and applied power, respectively, and $u_{b,\max}$ as the maximum of the filtered observed brake effort measurements. Last, we classify the filtered observed deceleration rates according to the speed steps in Eq. (3.7) and take the maximum in each step to estimate a_{40} , a_{60} , a_{80} and a_{130} . As we mentioned in chapter 3.3.1, the speed step corresponding to a_{130} also includes speeds larger than 130 km/h due to the lack of braking data at speeds higher than 130 km/h.

3.4 Results and discussion

3.4.1 Case study

We show the performance of the proposed train motion model M-estimator in a case study with data from trains running from Arnhem Centraal to Nijmegen, two stations located in the Eastern part of the Netherlands. This line is 18.542 km long, does not include tunnels and is mostly flat. In this regard, the track gradients and speed limits are used as input of the proposed method. We consider 6 double-deck Intercity trains with 4 cars (VIRM-IV) operated by NS and driven manually. We analyze 191 trajectories realized in 2021 consisting of time, GPS location, speed and engine torque measurements sampled every 5 seconds, which are used as input to the parameter estimator. For each rolling stock unit we have 46, 25, 24, 27, 37 and 32 trajectories, respectively. Figure 3.1 shows a speed-distance diagram with all the trajectories used in this chapter.

The engine torque measurements at times t_k , $T_k = T(t_k)$, are transformed to the mass-specific effort applied at the wheels u_k by means of the gearbox efficiency η_{ag} , the constant gear ratio i_{ag} , the diameter of the powered wheel d_w , the mass m and the rotating mass factor ρ as follows (Kapetanović et al., 2022):

$$u_k = \begin{cases} \frac{2T_k i_{ag} \eta_{ag}}{d_w m \rho} & \text{if } T_k \geq 0, \\ \frac{2T_k i_{ag}}{\eta_{ag} d_w m \rho} & \text{if } T_k < 0. \end{cases} \quad (3.20)$$

with $\eta_{ag} = 97\%$, $i_{ag} = 73/17$, $d_w = 0.88$, $m\rho = 273328$ kg.

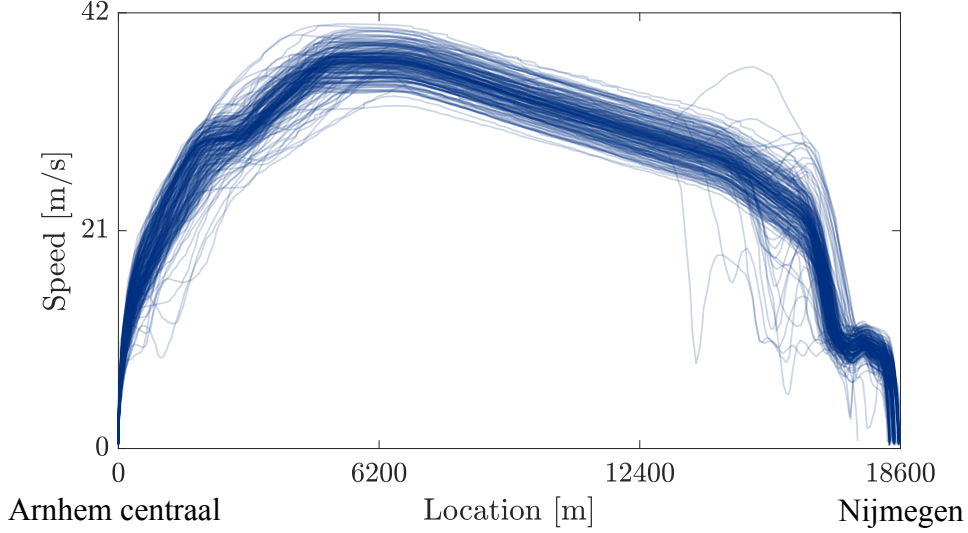


Figure 3.1: Speed-distance diagram of the 191 trajectories used in this case study.

This step constitutes a source of bias in any estimation that utilizes the efforts u_k as input. Wheels have a conical shape, so their diameter is not constant along a train's trajectory (Bernsteen et al., 1983). For example, wheelsets tend to be pushed towards the outer part of curves where the outer wheel's diameter over the rail is larger than the inner's. This produces a slight oscillation in the bogies after curves, although the swaying is dampened quickly. The diameter of the wheels of VIRM-IV trains have a minimum value of 0.84 m and a maximum value of 0.92 m according to the rolling stock manufacturer, so we take the mean in the calculation of the effort at the wheel. The considered track has some curves, however, that data was not available in this study. Although the train mass can be estimated from the rolling stock operational empty weight and the load or occupation data, in this research only the operational empty weight and the maximum passenger load is available. In line with NS' usual mass estimation, we assume that trains are always loaded at 35% of their maximum passenger capacity. Likewise, the gearbox efficiency and the rotating mass factor may also differ from the value provided by the manufacturer. Taking this into account, we utilize the measurements during the coasting regime to estimate the running resistance parameters using M-estimation. These measurements are independent of any bias in the calculation of the tractive and brake effort from the available measurements, since at the coasting regime the effort u_k is zero.

Data quality is another important source of bias and error in this chapter. Speed measurements are rounded to the nearest km/h during the measurement or storage procedure, which affects the accuracy of the estimation. The infrastructure geometry data used as input to Eq. (3.2) does not include curves despite the real track not being straight. Moreover, the gradients data may also be inaccurate. Given these data issues, the added value of this chapter is the proposal of a robust method for estimating the running resistance parameters and the bounds of the variables of the train motion model, rather than producing accurate estimates of the running resistance parameters.

In this case study, we transform the engine torque measurements to the effort applied at the wheel by means of Eq. (3.20). We filter those measurements corresponding to the coast regimes observed during the trajectories, that is to say, when the train applies zero tractive

and brake effort. The running resistance parameters are calculated by means of 10 M-estimation error measures, and their performance is tested and compared. The estimates of the running resistance produced by the error measure that showed the best performance are statistically analyzed and used as input of the sliding median filter to calibrate the maximum observed deceleration rates and the traction and brake characteristics. Last, we fit the probability density functions of the obtained estimates and discuss the results obtained.

3.4.2 Comparing different error measures

First, we test M-estimation for the 10 error measures that are shown in Table 3.1, aiming to quantify which one is the most suitable for train motion model calibration. This is by no means a trivial question due to the lack of ground truth data of the running resistance parameters. We use a robust coefficient of determination to assess the goodness of fit of the estimation (Engelen et al., 2004):

$$R^2 = 1 - \frac{\sum_{k \in G} \text{var}(z_k(\hat{\theta}))}{\sum_{k \in G} \text{var}(y_k)}, \quad (3.21)$$

where G is the set of indices of the measurements that are not identified as outliers, $\hat{\theta}$ is the estimate of the running resistance parameters and $\text{var}(\cdot)$ is the variance operator. This robust coefficient of determination leaves out of the calculation the outliers, which are identified as those measurements with residuals z_k larger than the estimate of the standard deviation of the residuals S that was used to studentize the residuals in the final iteration of the estimation. We take the mean of the robust coefficients of determination over all the trajectories and compare the performance of the error measures.

The results obtained show that Fair is the error measure that has the largest mean robust coefficient of determination ($R^2 = 0.9438$), suggesting that it is the most suitable error measure for estimating the running resistance, at least for this dataset. It is closely followed by L_p and the Logistic error measures. OLS shows the second worst performance, outperforming only its truncated version: Talwar's measure. The good performance of Fair and L_p measures is in line with the existing literature (Zhang, 1997). Huber's error measure is usually among the ones that performs consistently best, however, it ranked 6th. From now on, in this chapter we use Fair as the error measure in all the calculations.

The robustness of M-estimation methods depends on the dataset to be fitted and on the geometrical properties of the error measures used. In practice, we recommend comparing the statistics of the residuals with the error and weight functions to select the most appropriate M-estimator and refine the value of its tuning constant. See De Menezes et al. (2021); Zhang (1997) for graphical representations of the error and weight functions of the M-estimators shown in Table 3.1. Regarding the geometrical properties of the error measures, the influence of large residuals on the error measure is limited in the Fair, Huber, L_2 , L_p and Logistic measures, while for the other error measures in Table 3.1 their influence is asymptotically reduced or simply zero. Considering this geometric difference, to optimize the robustness of the estimation we recommend selecting the Fair or the Cauchy M-estimator depending on the statistics of the residuals, and refine the value of the tuning constants to reduce the influence of any highly leveraged observations that could bias the estimates.

3.4.3 Estimating the train motion model parameters

We estimate the train motion model parameters on each trajectory, obtaining 191 sets of estimates. The running resistance parameters are estimated by means of M-estimation, while a sliding median filter is used to estimate the remaining train motion model parameters. Table 3.2 gathers the statistics of the parameter estimates along with a comparison with the values provided by the rolling stock manufacturer whenever appropriate.

Parameter [units]	Mean	Variance	Manufacturer value	Difference (%)
r_0 [N/kg]	0.0067	4.7e-06	0.0067	0.34
r_1 [N/(kg m/s)]	3.1e-04	4.1e-08	1.5e-04	100
r_2 [N/(kg(m/s) ²)]	2.2e-05	2.7e-11	2.8e-05	-23
$u_{f,\max}$ [N/kg]	0.34	0.0076	0.52	-34
$p_{\max}$ [W/kg]	5.21	0.37	5.26	-0.92
$u_{b,\max}$ [N/kg]	0.17	6.4e-04	0.50	-66
a_{40} [m/s ²]	0.29	0.0032	-	-
a_{60} [m/s ²]	0.36	0.0038	-	-
a_{80} [m/s ²]	0.38	0.0046	-	-
a_{130} [m/s ²]	0.35	0.0077	-	-

Table 3.2: Statistics of the parameter estimates.

We observe that the mean of the estimate of the running resistance parameter r_0 is almost identical to the value provided by the manufacturer as they differ by less than 1%. The difference of r_1 with respect to the manufacturer value is notably larger, as the mean of the estimates doubles the theoretical value. This parameter is generally considered to be uncertain and hard to estimate (Lukaszewicz, 2007a). It is known to depend on the train length and on the smoothness of the surfaces of the rolling stock. For instance, HVAC air sources influence the value of r_1 , including whether the train windows are open or not. The other two terms of the running resistance have usually a larger weight in the total resistance. The linear term is much smaller than r_0 and the quadratic term for lower and higher speeds, respectively. Still, r_1 can be estimated more accurately when using data from low speeds. This is an issue in this research since low speed intervals are scarcely visited when coasting, so r_1 may have been fitted with less accuracy than the other two running resistance parameters. More coasting measurements at low speeds would be required to assess whether r_1 can be estimated more accurately. Regarding r_2 , we notice that the average of the estimates is 23% lower than the manufacturer value. This indicates that the manufacturer may have overestimated this parameter. Manufacturers are often conservative when it comes to calculating the energy consumption of the rolling stock since railway operators are sometimes entitled to compensation if the rolling stock actual energy consumption is higher than expected. Moreover, theoretical values typically overestimate the quadratic running resistance parameter (Hansen et al., 2017) and they rarely model the effect of wind on its value (Rochard & Schmid, 2000; Trivella et al., 2021).

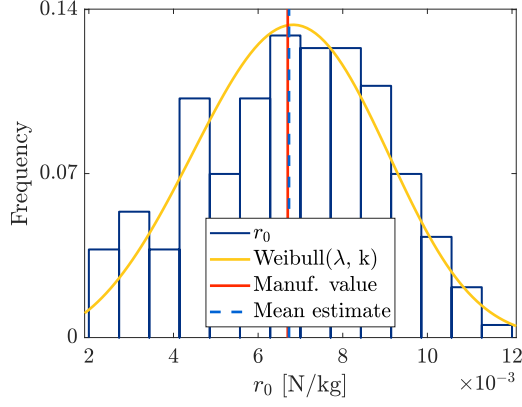
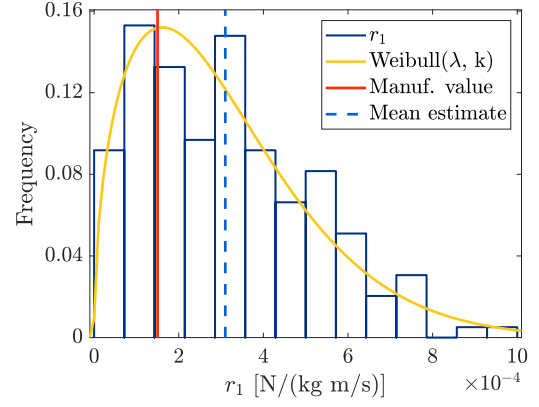
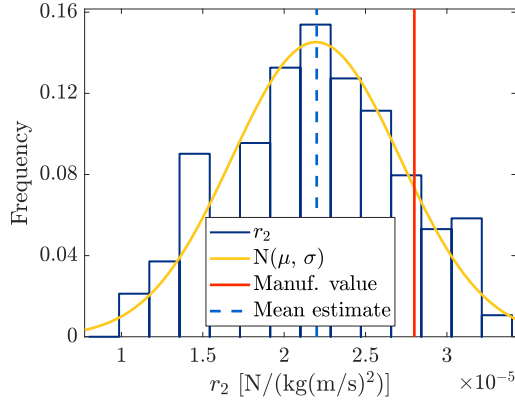
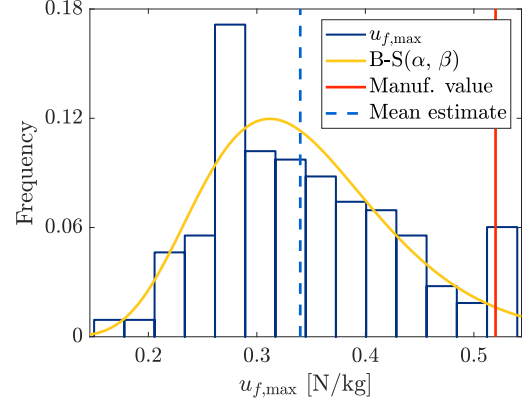
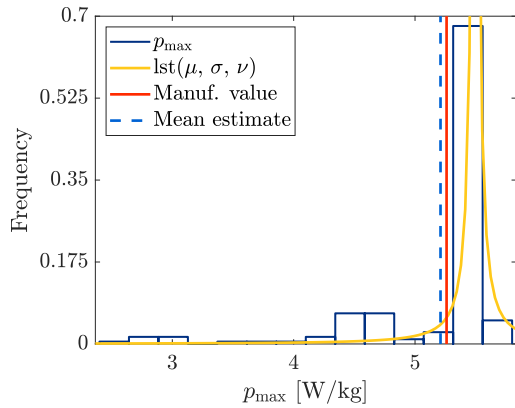
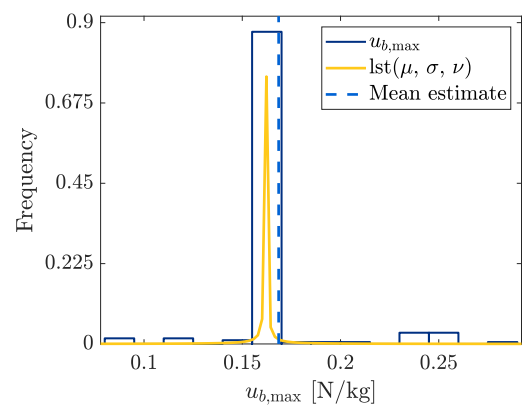
The mean of the estimates of $u_{f,\max}$ is found to be more conservative than the manufacturer estimate for the maximum capability of the train. Train drivers do not start

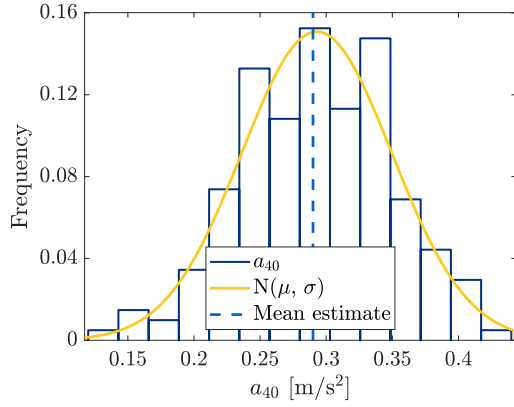
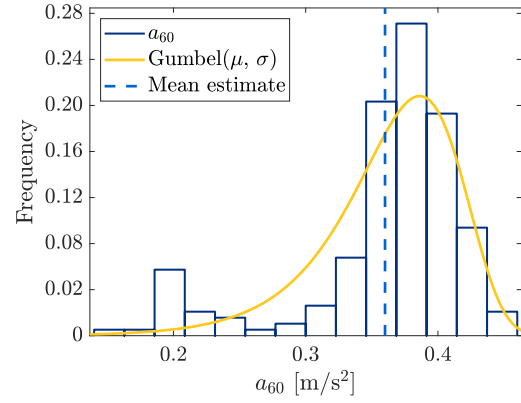
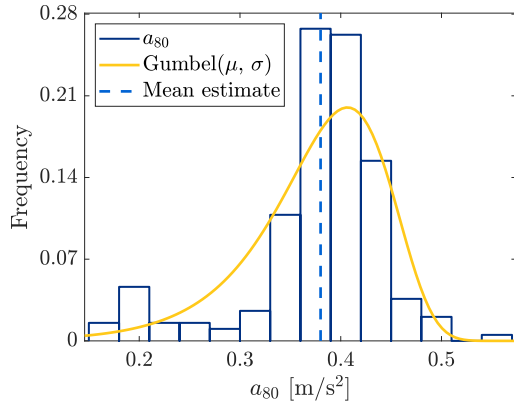
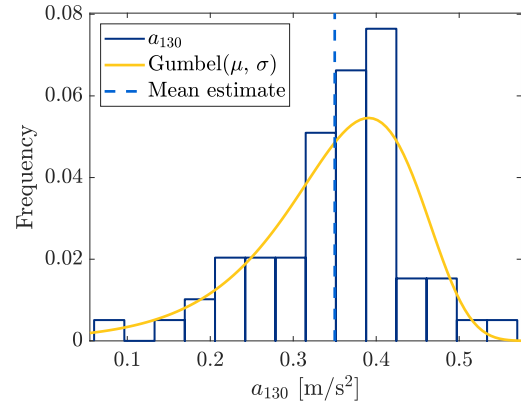
accelerating at the maximum rate for passenger comfort purposes, so the observed difference may be due to how trains are driven manually. The average of $u_{f,\max}$ is approximately equal to two thirds of the manufacturer value, which could be a usual position of the driving command at low speeds. When the speed is higher, the maximum tractive effort that can be applied is limited by $p_{\max}$. According to the manufacturer values, this transition takes place around 10 m/s, or 36 km/h. This speed is reached shortly, so by then train drivers might not have applied full traction yet. This would explain the observed value of the average of $u_{f,\max}$. $p_{\max}$ is remarkably close to the manufacturer value besides the effect of the mass uncertainty and the bias potentially introduced in the torque-to-wheel transformation. In line with what we have just mentioned regarding $u_{f,\max}$, in most cases the train driver must have placed the driving command in the full traction position.

Like $u_{f,\max}$, the average maximum brake effort observed, $u_{b,\max}$, shows a similar trend. It is again approximately one third of the manufacturer value. Actually, when passing a yellow signal, the Dutch Automatic Train Protection system ATB-EG requires train drivers to apply at least one third of the maximum brake rate. Otherwise, ATB-EG triggers the emergency brake. Train drivers are encouraged to brake gently and smoothly for passenger comfort purposes, so the maximum observed brake effort is likely to be lower than the maximum capability of the rolling stock. Still, placing the brake command to one third of its maximum rate could also be commonly used for approaching a lower speed limit, like the 40 km/h limit at the entrance of Nijmegen station, or when braking to a standstill. This may be supported by the difference between a_{40} and the other three deceleration rates, a_{60} , a_{80} and a_{130} . The latter three show similar mean deceleration rates, which is probably linked to the statistics of $u_{b,\max}$. a_{40} is lower than the other three deceleration rates since trains already run at 40 km/h or less when drivers have to stop at a station. ATB-EG does not require to brake at a minimum brake rate at speeds lower than 40 km/h, so drivers are not forced to reach the mentioned one-third brake rate and may brake more gently. Moreover, drivers also have to aim to stop at specific locations along the platform. They might need to brake harder during the last seconds before stopping, but the sampling rate of the data used in this research does not allow to assess this. a_{130} shows a higher variance than the other three deceleration rates. We observed braking at speeds higher than 80 km/h only in one third of the considered trajectories, which translates in less observations and estimates of a_{130} .

3.4.4 Fitting the estimates to probability distributions

We used MATLAB's distributionFitter app to fit the estimates of each parameter to a probability distribution. Up to 18 different distributions are tested in each case. The distributions are fitted by means of a maximum likelihood estimation procedure, so we utilized the log-likelihood of each fit to select the most adequate one. Distributions with a larger amount of statistical parameters are penalized by means of the Akaike information criterion to avoid overfitting. Figures 3.2-3.11 show histograms of the likelihood of each parameter along with the corresponding fitted distribution represented by a yellow line. When compared to Table 3.2, these histograms offer a broader understanding of the statistics of the estimates. We performed a Kolmogorov-Smirnov test to assess whether the estimates are not likely to be drawn from the selected distribution. The details of each distribution are gathered in Table 3.3.

Figure 3.2: Fitted distribution for r_0 Figure 3.3: Fitted distribution for r_1 Figure 3.4: Fitted distribution for r_2 Figure 3.5: Fitted distribution for $u_{f,max}$ Figure 3.6: Fitted distribution for p_{max} Figure 3.7: Fitted distribution for $u_{b,max}$

Figure 3.8: Fitted distribution for a_{40} Figure 3.9: Fitted distribution for a_{60} Figure 3.10: Fitted distribution for a_{80} Figure 3.11: Fitted distribution for a_{130}

Variable	Distribution	Distribution parameters	K-S test statistic	K-S hypothesis rejected?
r_0	Weibull	$\lambda = 0.0075, k = 3.55$	0.0424	No
r_1	Weibull	$\lambda = 3.4\text{e-}04, k = 1.49$	0.0545	No
r_2	Normal	$\mu = 2.2\text{e-}05, \sigma = 5.2\text{e-}06$	0.0439	No
$u_{f,\max}$	Birnbaum-Saunders	$\alpha = 0.25, \beta = 0.33$	0.0642	No
$p_{\max}$	Location-Scale t-dist.	$\mu = 5.49, \sigma = 0.032, \nu = 0.60$	0.1675	Yes
$u_{b,\max}$	Location-Scale t-dist.	$\mu = 0.16, \sigma = 1\text{e-}06, \nu = 0.35$	0.4215	Yes
a_{40}	Normal	$\mu = 0.29, \sigma = 0.057$	0.0469	No
a_{60}	Gumbel	$\mu = 0.39, \sigma = 0.040$	0.1376	Yes
a_{80}	Gumbel	$\mu = 0.41, \sigma = 0.054$	0.1310	Yes
a_{130}	Gumbel	$\mu = 0.39, \sigma = 0.078$	0.0916	No

Table 3.3: Fitted distributions of train motion model parameters.

The histogram of r_0 is shown in Figure 3.2. It is fitted to a Weibull distribution that is approximately symmetrical. Some estimates are slightly less than the double of the manufacturer value and some three times lower. Considering the effect of the train load on the manufacturer value, r_0 would fall between $6.3\text{e-}03$ and $7.0\text{e-}03$ N/kg, which does not fully explain the large variations observed around the mean. The dispersion observed is captured neither by the manufacturer value nor by the mean of the estimates shown in Table 3.2 but still these two values are almost identical. This is probably due to the fact that manufacturers obtain r_0 as the mean of the values obtained over several tests.

Figure 3.3 shows the distribution of r_1 . It is also fitted to a Weibull distribution with positive skew. The large tail towards high values of the parameter may indicate that the estimation of r_1 might not have been very accurate. Still, the peak of the distribution is centered close to the manufacturer value. The mean is higher than expected due to the skewness of the distribution. Again, the train load does not explain the observed distribution, as r_1 would fall between $1.4\text{e-}04$ and $1.6\text{e-}04$ N/(kg m/s).

Figure 3.4 shows the distribution of r_2 . A normal distribution was found to be the best fit for the estimates of r_2 . This is not correct from a physical point of view, since r_2 is nonnegative while the normal distribution allows for negative values of this parameter. The train load would cause r_2 to vary between $2.65\text{e-}05$ and $2.96\text{e-}05$ N/(kg (m/s)²), but the observed bell-shaped distribution goes beyond these two values. Instead, factors like the wind may have a major role on producing the volatility of r_2 .

Figure 3.5 presents the distribution of $u_{f,\max}$, which is fitted to a Birnbaum-Saunders distribution. Two peaks arise over the yellow line that depicts the fitted distribution: one around 0.27 N/kg next to the mean and another one at the right hand side of the plot, around 0.54 N/kg. These are roughly 50% and 100% of the maximum tractive effort. In this sense, the right peak corresponds to the full traction position of the driving command and the left one to the 50% power position. The distribution mean is found then around the 66% of the maximum effort. This complements the analysis of Table 3.2, giving a more accurate insight on the usual characteristics of the driving style.

In Figure 3.6 the probability distribution of $p_{\max}$ is fitted to a Location-Scale t-distribution centered at 5.49 W/kg, slightly more than the mean calculated in Table 3.2. This peak is determined by the maximum power that the engine can deliver since we did not observe many estimates at the right of the peak. These rare higher outliers may be due to extra power in the catenary. Lower outliers might either correspond to low catenary power, to unfavourable environmental conditions affecting the engine to a lower position of the driving command or to any other non-critical defect. The latter is more likely taking into account the impact of the driving behaviour on $u_{f,\max}$.

Figure 3.7 gathers the histogram of $u_{b,\max}$, which is also fitted to a Location-Scale t-distribution. Both this and the distribution of $p_{\max}$ show large kurtosis, that is to say, these distributions show a large peak at the mean. The distribution of $u_{b,\max}$ is centered around a third of 0.5m/s^2 , which is the minimum brake rate required by ATB-EG for braking at speeds higher than 40 km/h in case of a yellow signal. Maybe this position of the brake command is also used in service braking. Higher maximum brake rates are also observed, however, all of them are well below the theoretical maximum brake rate, which supports the fact that train drivers brake gently.

Figure 3.8 presents the probability distribution of a_{40} , which is fitted to a Normal distribution. Like we mentioned in the case of r_2 , a_{40} cannot be drawn formally from such distribution due to the nonnegativity constraint, which would require using a truncated version.

Last, Figures 3.9, 3.10 and 3.11 show the distributions of a_{60} , a_{80} and a_{130} , respectively. All these deceleration rates are fitted to Gumbel distributions, which usually model the distribution of the maximum among the samples of several distributions. The deceleration rates at each trajectory are realizations of a random variable and we are estimating their maximum over each trajectory, so Gumbel distributions constitute an adequate candidate for the model of the distributions of these rates. The distribution of a_{40} shows a clearer bell-shape with respect to the other three deceleration rates. This may be produced by the absence of a required minimum brake rate and the need to brake to a standstill at a specific location along the platform. The distributions of the deceleration rates are remarkably different from the distribution of the maximum observed brake effort $u_{b,\max}$. This is due to the fact that the amount of brake applied is not constant along the whole brake curve, so the observed maximum brake effort may have been applied at different speeds in different trajectories, leading to a higher dispersion in the statistics of the deceleration rates.

The Kolmogorov-Smirnov test indicates that we can reject the hypothesis that $p_{\max}$, $u_{b,\max}$, a_{60} and a_{80} are drawn from their respective fitted distributions. The distributions obtained differ significantly from the ones presented in Bešinović et al. (2013a). The differences observed may be explained by the fact that in that chapter the parameters were estimated using track occupation data instead of GPS location and speed and a completely different estimation method was utilized.

3.4.5 Discussion

We found r_1 more difficult to be estimated than r_0 and r_2 because the data available is not uniformly sampled over the whole speed interval visited by the train. In fact, higher speeds are more frequently measured while coasting, which adds some uncertainty in the estimation of r_1 . The mean of the estimates of r_0 is remarkably close to the value provided by the manufacturer, while r_2 seems to have been overestimated by the manufacturer. This latter fact is in line with the typical conservative approach taken to calculate the energy consumption of a train.

The trains considered in this case study are driven manually, which has a major impact on the observations of $u_{f,\max}$ and $u_{b,\max}$. Regarding these two parameters, the statistics show that we did not observe the maximum capability of the rolling stock, but rather how train drivers apply traction and brake. This is particularly relevant, as it allows us to gauge the impact of manual driving on the driving characteristics and to include this data in energy efficient driving algorithms, making them more realistic and adapted to the actual manual driving style. $u_{f,\max}$ shows a large dispersion due to the mentioned characteristics of manual driving. Our results show that train drivers start applying traction smoothly for passenger comfort purposes. Before reaching 40 km/h, most drivers place the traction command at 50% of its maximum capability. The command was placed at full traction only in 6% of the trajectories, although typically it was placed at most between 50% and 90%. The results show that Dutch driver's braking style seems to be strongly influenced by ATB-EG. Train drivers tend to brake gently, mostly placing the

brake command right at one third of its range. This value is linked to ATB-EG, which in the event of a yellow signal requires applying a minimum brake rate equal to 33% of the maximum brake rate. Braking harder is usually either not necessary, or perceived as uncomfortable by passengers. When it comes to the maximum tractive power, $p_{\max}$, we did not observe many variations. The maximum power distribution was centered at a power slightly higher than the manufacturer value. The lack of variations observed show that $p_{\max}$ is determined essentially by the engine characteristics and not by external conditions. Data from a longer time period would be required to assess whether wear influences $p_{\max}$ over time.

The observed deceleration rates a_{60} , a_{80} and a_{130} show very similar statistics. They rarely exceed 0.5 m/s^2 , a usual comfort threshold in mainline railways. The statistics of a_{40} are slightly different due to the need to stop accurately at a specific location along the platform. The distribution of a_{40} is skewed more towards lower deceleration rates since a 40 km/h speed limit is placed before entering Nijmegen station. For speeds higher than 40 km/h the speed limit ahead forces drivers to brake with higher rates to avoid overspeeding, while once below 40 km/h being accurate to stop at the right location is more important. These two different behaviours at speeds higher and lower than 40 km/h are in line with what we will observe in chapter 4.

Field tests under controlled experimental conditions would be of special relevance for validating the calibration and for selecting the most appropriate error measure and its tuning constant given the accuracy of the sensor measurements.

3.5 Conclusion

The performance of energy efficient driving algorithms is essentially linked to a careful calibration of the parameters of the train motion model. This calibration is usually done by performing Ordinary Least Squares (OLS) regressions on speed data measured during parts of trains' trajectories where they coast. Measurement outliers are likely to bias the parameter estimates in OLS, affecting in some cases the performance of energy efficient driving algorithms. Moreover, the existing literature focuses on estimating the running resistance parameters and pays less attention to determining the limits on the traction and brake effort and power.

In this chapter we have presented a robust parameter estimation method for calibrating the train motion model. We used a robust regression method, M-estimation, to estimate the running resistance parameters. A sliding mean filter was used to remove measurement outliers in the applied effort and the observed deceleration rates. The maximum value over each trajectory is taken to estimate the maximum observed tractive and brake effort, the maximum tractive power and the deceleration rates at the speed intervals supervised by the Dutch Automatic Train Protection system ATB-EG. We tested the proposed method in a case study with measurements from 6 trains running in the Dutch railway network in 2021. Using data measured during normal operation allows to obtain a more relevant overview of the dynamics of a train than when tests are performed, since we can capture the influence of the existing operating conditions and the driver's style on the train motion model parameters.

Our results demonstrate that M-estimation can produce robust estimates that are not biased by measurement outliers, outperforming in this sense most of the existing train motion model calibration algorithms. We compared 10 M-estimation error measures including OLS, finding that Fair measure is the most adequate one for estimating the running resistance parameters.

The quality of the input data is usually an issue in train motion model calibration, and in this chapter we also faced some difficulties in this regard. The accuracy of the estimates could be higher if the sampling rate of the measurements was higher (around 1 s). The location and speed measurements were found to be noisy. Regarding the infrastructure data, the information about the curves was missing and the height profile data was inaccurate. The calculation of the applied tractive and brake effort at the wheel from engine torque measurements constitutes a source of bias in the input data. We had to use data measured when trains coast in order to avoid this bias. This issue could be relieved by introducing an estimation or measurement of the rolling stock mass and load and other relevant traction characteristics. Then, the data obtained during the regimes where the train is applying traction and brake could be introduced without compromising the accuracy of the estimates. Furthermore, accurate train mass measurements could be used along with the rotating mass factor to avoid having to use mass-specific parameters. Still, the method proposed and the insights obtained in this chapter are of relevance to rolling stock manufacturers, railway operators and researchers. The observed role of behavioural aspects of manual driving is of particular relevance due to its impact on punctuality, energy-efficiency and capacity. Practitioners may take these findings into consideration in railway operation.

Several potential further research topics of interest remain open. More accurate data with a higher sampling rate could be used to assess the impact of the weather on the train dynamics and the characteristics of human driving. A comparison of the parameters estimated from automated and manually-driven trains could identify potential parameter correlations that arise when trains are driven manually. A deeper analysis of human braking style would require to consider the observed reaction times, the locations where the train driver starts to brake, the rolling stock brake response times and other psychological and human-machine interaction factors. Data measured during several years could be used to quantify up to which extent rolling stock ageing affects a train's dynamics. Moreover, data from different sensors and sources could be combined by means of sensor fusion to boost the accuracy of the train motion model calibration. Last, a comparison between different train motion model calibration algorithms in the same study would be of interest, as it would help to determine quantitatively the advantages and disadvantages of each technique. However, the lack of an open-access data set of train trajectories measurements is hindering any comparison (Cunillera et al., 2023a).

Chapter 4

Real-time train motion model calibration using an Unscented Kalman Filter

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4.1 Introduction

Railway digitisation is boosting the sensorization of trains, tracks and signalling systems. Thousands of variables are continuously monitored, from passing times at a certain signal to the temperature in each car. The amount of information that is measured, generated and stored is increasing dramatically and the possibility of exploiting the produced data opens new horizons for calibrating and improving current mathematical and expert rule-based methods and for devising new ones. As a consequence, data-driven algorithms are becoming increasingly popular in the railway science and industry (Ghofrani et al., 2018). Recent surveys of data-driven and big data analytics applications in railway systems can be found in De Martinis & Corman (2018) and Ghofrani et al. (2018). However, the combination of data-driven techniques and classical approaches show promising results in boosting the performance and accuracy of current methods.

On-board applications like Driver Advisory Systems (DAS) and Automatic Train Operation (ATO) are among the solutions that may benefit most from data-driven and hybrid approaches by obtaining more accuracy and even by allowing the implementation of real-time and custom-train based solutions. DAS and ATO generally use a mathematical model based on Newton's second law in order to generate the reference speed profile to be followed, which includes the speed that the train should have at every point of the track along its movement. However, this train motion model has several input parameters that have to be carefully calibrated in order to reproduce train dynamics accurately.

Real-time train motion model calibration allows to obtain a more faithful representation of the current train characteristics for on-board applications. The calibration is usually performed offline by means of the engine characteristics and the train running resistance parameters provided by rolling stock manufacturers or by analyzing historical data registered during tests and standard operation. However, railway operation is fundamentally stochastic, which lowers the effectiveness of the mentioned offline calibration approaches with respect to real-time calibration. This is due to the fact that train dynamics and its model parameters show spatial and time-dependent variations due to factors like the engine and train wear and faults, the adhesion condition, the weather and the driving style. Therefore, individual train tailored real-time calibration becomes a key element for accounting for the current operation conditions and the parameter variations that may jeopardize the accuracy of applications like DAS and ATO. For instance, an ATO that is not well calibrated could lead to altered punctuality rates, to trains stopping far from the allocated locations in the platform, to overspeeding with possible brake intervention from the Automatic Train Protection system, to reducing trust in the system by the driver, and to a decrease of the passenger comfort. Furthermore, such a real-time calibration must be performed on-board, which exerts extra pressure on the available on-board computational resources.

In this chapter, a real-time parameter estimation algorithm is used for determining the train motion model parameters in real time, aiming to develop a framework that can calibrate the input parameters of real-time Train Trajectory Optimization algorithms that are embedded in energy-efficient on-board applications like DAS or ATO. This algorithm, the Unscented Kalman Filter (UKF), is a well-known mathematical parameter estimator for nonlinear systems. It uses on-board location and speed measurements of a train running under normal, unperturbed operation conditions, as input data to estimate the train running resistance parameters. This algorithm also requires the applied tractive and brake efforts as input, which are calculated from the last estimations of the running resistance parameters and the gradient profile, so that the efforts match the train acceleration. Moreover, the algorithm is complemented by two modules. The first one is a driving regime recognition module that identifies the current driving regime, that is, whether the train is applying traction to accelerate, cruising, coasting or braking. The second one is a post-processing module that determines parameter bounds like the maximum tractive effort and power and driving style features like the applied brake rates, the cruise speed and the switching points between driving strategies. The mentioned train characteristics constitute most of the input parameters of on-board energy-efficient applications, where they are used to compute reference energy-efficient driving trajectories and the associated driving advice or control policy. Consequently, the proposed real-time calibration framework may boost the performance of such energy-efficient applications and facilitate their implementation by providing accurate estimates of their input parameters based on the current state of the train and the operation conditions. In particular, the train running resistance parameters estimated by the UKF are directly related to the train energy consumption, since they model the main losses of kinetic energy of a train. Therefore, the proposed framework can be used to determine the energy consumption more precisely, which, combined with energy-efficient DAS or ATO, may lead to improved and more efficient railway operations.

The UKF is a filter that can be used for state and parameter estimation of nonlinear systems devised by Julier & Uhlmann (1997). In this technique, the covariance is propagated by means of a deterministic sampling procedure called Unscented Transformation. The state distribution is approximated by a Gaussian random variable, although it is represented by several carefully chosen sample points representing the mean and covariance. In each iteration, these points are propagated through the nonlinear system in order to calculate a new mean and covariance estimate. In this chapter, a UKF is used for estimating the train running resistance parameters, while the input data used in this filter are the train location and speed measurements. UKF has also been used in other transport modes, like vehicles (Antonov et al., 2011; Liu et al., 2021), aircraft (Khadilkar & Balakrishnan, 2011) and ships (Peng et al., 2019).

In railway systems, several approaches have already been considered for online train motion model parameter estimation, such as multi-innovation theory (Liu et al., 2018), multi-start gradient-based search (De Martinis & Corman, 2019), expectation maximization based on sliding windows (Jin et al., 2018b), UKF (Howlett et al., 2004) and Gaussian sum theory combined with Extended Kalman Filters (Jin et al., 2018a). However, most of them focus on determining a limited number of model parameters, such as the running resistance parameters (Liu et al., 2018; De Martinis & Corman, 2019; Howlett et al., 2004; Jin et al., 2018a) and the adhesion coefficient (Jin et al., 2018b). In contrast, the proposed framework, besides determining the three running resistance parameters and calculating the applied tractive and brake effort in real time, also aims to provide estimations of the maximum tractive power applied and to model driving style features like the applied brake rates and the switching points between driving regimes. These parameters are usually used as input of Train Trajectory Optimization algorithms. Therefore, the proposed framework can improve the accuracy of such algorithms and the applications that usually embed them, like DAS and ATO. Some of the mentioned techniques are applied for a specific driving regime, like coasting (Liu et al., 2018; Howlett et al., 2004) and braking (Jin et al., 2018b), while only De Martinis & Corman (2019), Jin et al. (2018a) and the proposed framework are operative in any driving regime. UKF is also faster than expectation maximization (De Martinis & Corman, 2019), since that technique takes more than 6 seconds per iteration, while UKF can process a whole trajectory in less than a second. When compared to EKF (Jin et al., 2018a), UKF propagates the statistics of random variables nonlinearly, while EKF linearizes the process function that describes the evolution of such random variables and thus, in some cases, UKF can be more accurate than EKF.

Reflecting on the existing online train motion model parameter estimation literature, UKF was first used estimating two of the three running resistance parameters in Howlett et al. (2004). The algorithm was validated from simulated speed and location data of a train coasting on a level and straight track. In this chapter we extend the UKF algorithm by estimating all the running resistance parameters and we validate the results obtained using real location and speed measurements. The addition of a driving regime recognizer and a module to obtain parameter bounds and statistics gives this framework an added value with respect to the existing online parameter estimation algorithms, since the proposed framework constitutes the online parameter estimation algorithm with the broadest and most complete scope in the current scientific literature.

The contributions of this chapter are:

- Proposing an accurate real-time train motion model parameter estimation framework.
- Estimating in real time the input parameters of on-board energy-efficient railway applications like DAS and ATO from train location and speed measurements.
- Showing in a case study with real data that the proposed framework can be used effectively for model calibration in railways.
- Calculating bounds and obtaining statistics of driving style parameters and applied maximum tractive power and brake rates.

The remainder of the chapter is organized as follows. In chapter 4.2 the train motion model is introduced and some of the main sources of variability and uncertainty of the model input parameters are described. The proposed online parameter estimation algorithm is presented in chapter 4.3. Its performance is assessed in a case study, and the results obtained are discussed in chapter 4.4. Last, final remarks are outlined in chapter 4.5.

4.2 The train motion model

The train motion model that aims to reproduce the dynamics of a train can be described by means of Newton's second law,

$$\frac{dv}{dt} = f_t(v) - f_b(v) - r(v) - g(s), \quad (4.1)$$

where v stands for the train speed (in m/s), s for its location (in m) and t for the time (in s), which in this case is the independent variable. Moreover, f_t is the mass-specific applied tractive effort (in N/kg), $f_t = F_t/(\rho m)$, where F_t is the tractive effort (in N), ρ is the rotating mass factor (dimensionless) and m the train mass (in kg), f_b is the mass-specific applied brake effort (in N/kg), $f_b = F_b/(\rho m)$, where F_b is the brake effort (in N), and $g(s)$ is the mass-specific term (in N/kg) that accounts for the effects of the track geometry on the train dynamics. $r(v)$ is the mass-specific running resistance, which can be described by means of the Davis equation (Davis, 1926):

$$r(v) = r_0 + r_1 v + r_2 v^2, \quad (4.2)$$

where r_0 , r_1 and r_2 are nonnegative parameters (in N/kg, N/(kg m/s) and N/(kg(m/s)²), respectively).

Most of the mentioned parameters show time and spatial variations and they are influenced by wear and external factors. The applied tractive effort f_t is often limited by two variables that also show variations with respect to the manufacturer specifications: the maximum tractive effort and power. The maximum tractive effort that a train engine can apply is limited by factors like adhesion, which is the friction between the drive wheels and the rail, and engine wear. In turn, adhesion depends on factors like wheel and rail

wear and weather. Yet a train cannot apply this maximum effort at any speed. At higher speeds, the tractive effort is limited by the electric power available and the maximum power of the engine, which in turn may show variations due to the engine fault record and wear.

In manually-driven trains, the driving style also produces variations in the running speed, the acceleration and deceleration rates and the coasting and braking points, which are the locations where the train neither applies traction nor brake and where it starts braking, respectively. Moreover, under normal operation conditions, the brake effort f_b is limited by weather conditions and wear, by Automatic Train Protection systems, by comfort constraints and by the use of predetermined brake curves and rates. The train driver may also apply a brake rate lower than the maximum one that is limited by the train characteristics, and some variations might be observed when comparing the brake rate applied in different runs. Therefore, these driving-induced variations constitute an extra source of parameter uncertainty that differs from the physical sources of variation mentioned earlier. Driving-induced variations may also be the target of online parameter estimation frameworks, as they are of special relevance for on-board energy-efficient applications like DAS. The observed driving-induced variations could be introduced as input to a DAS in order to produce a driving advice adapted to the particularities and preferences of the current driver, maximizing the probability of compliance in following the driving advice, guaranteeing the efficacy of the energy-efficient algorithm and increasing the overall satisfaction and confidence of train drivers with respect to the DAS.

Running resistance parameters r_0 , r_1 and r_2 are usually computed by manufacturers by means of phenomenological equations, computational fluid dynamics (Rochard & Schmid, 2000) or by performing run-down tests (Lukaszewicz, 2007a). However, their values are significantly influenced by external factors like wind (Trivella et al., 2021), the presence of tunnels, and the train and rails wear. In particular, r_2 has a strong dependence on the train head geometry. Since energy consumption is strongly related to these parameters, most efforts on train motion model calibration have been devoted to monitoring them.

Even though train mass is not made explicit in Eq. (4.1), since the parameters in the mentioned equation are considered mass-specific, it is also a source of parameter uncertainty in the train motion model. A train tare is usually well-determined, while train load varies at stations due to passengers boarding and disembarking and freight loading and unloading, although it is easier to calculate the load mass of freight trains in comparison to passenger trains. Moreover, in the case of diesel-powered locomotives the fuel mass decreases during the journey. Using mass-specific parameters allows to reduce in one the number of parameters to be estimated and to keep the linearity of the model with respect to the parameters while considering the effects of the mass variability in the dynamics.

Infrastructure managers publish periodically a simplified description of the geometry of tracks and platforms, consisting of the gradients, curves and cant. Depending on the level of detail of the track geometry description, it usually shows some uncertainties that may affect the performance of the train motion model. For instance, the track is usually divided in intervals that contain several different gradients. Nevertheless, the track geometry description only includes the most representative or restrictive gradients in each track interval, as this description usually focuses on safety and operation targets.

Furthermore, the gradients are depicted as piecewise constant, and they are sometimes made continuous by means of parabolic links. Curves are described in a similar way, in terms of the radius of curvature. Overall, this constitutes a simplified description of the existing track geometry. The difference between the real track geometry and the description provided by infrastructure managers constitutes a source of error that may lead to inaccuracies in the term $g(s)$ of the train motion model and the reproduced train dynamics. In the proposed monitoring algorithm we assume that the track geometry description used is perfectly accurate, as this helps to keep the model simple and the effect of any mismatch between the real track geometry and its description might be up to some extent mitigated by overfitting the running resistance parameters.

4.3 Methodology

A parameter estimation algorithm has been developed to monitor the variability of the train motion model parameters in real time. The proposed algorithm is formed by three modules. First, a UKF module estimates the running resistance parameters. On-board location and speed measurements are used as input into this module along with the calculated tractive or brake effort needed to match the observed train acceleration. Eq. (4.1) is used to calculate the mentioned effort using the last running resistance parameters produced by the UKF. Next, the train acceleration is evaluated from the filtered speed and it is utilized along the calculated tractive and brake effort in a driving regime detection module for determining whether the train is applying traction to accelerate, holding a constant speed, coasting or braking. Last, the results obtained are post-processed on-line in a feature extraction module in order to determine parameters like the maximum tractive effort and power, brake rates, switching points between driving strategies and cruise speeds. Figure 4.1 shows a flowchart of the proposed algorithm. Although the proposed methodology is validated using real data from a mainline train in chapter 4.4, it could be adapted to any other railway system, provided that the internal parameters of the three modules are fine-tuned to adapt the framework to the particularities of each railway system. The remainder of this chapter is organized as follows. The UKF module is described in chapter 4.3.1, the driving regime recognition module is outlined in chapter 4.3.2, and chapter 4.3.3 describes the post-processing module.

4.3.1 Unscented Kalman Filter (UKF)

The UKF is a state observer that can perform parameter estimation of nonlinear systems by including the mentioned parameters in the state vector. This technique considers the system dynamics and the measurement procedure to be stochastic and models the corresponding noises as Gaussian white noise. The UKF evaluates the nonlinear evolution of the state statistics by means of the Unscented Transformation (UT), which is a method for approximating the statistics of a random variable that is transformed nonlinearly. In UT, the state statistics are thus expressed by means of a special set of points, the sigma points, that are chosen so that their mean and covariance are equal to the state respective values. Then, to calculate the evolution of the state statistics these sigma points are propagated nonlinearly through the dynamical system and the measurement function that

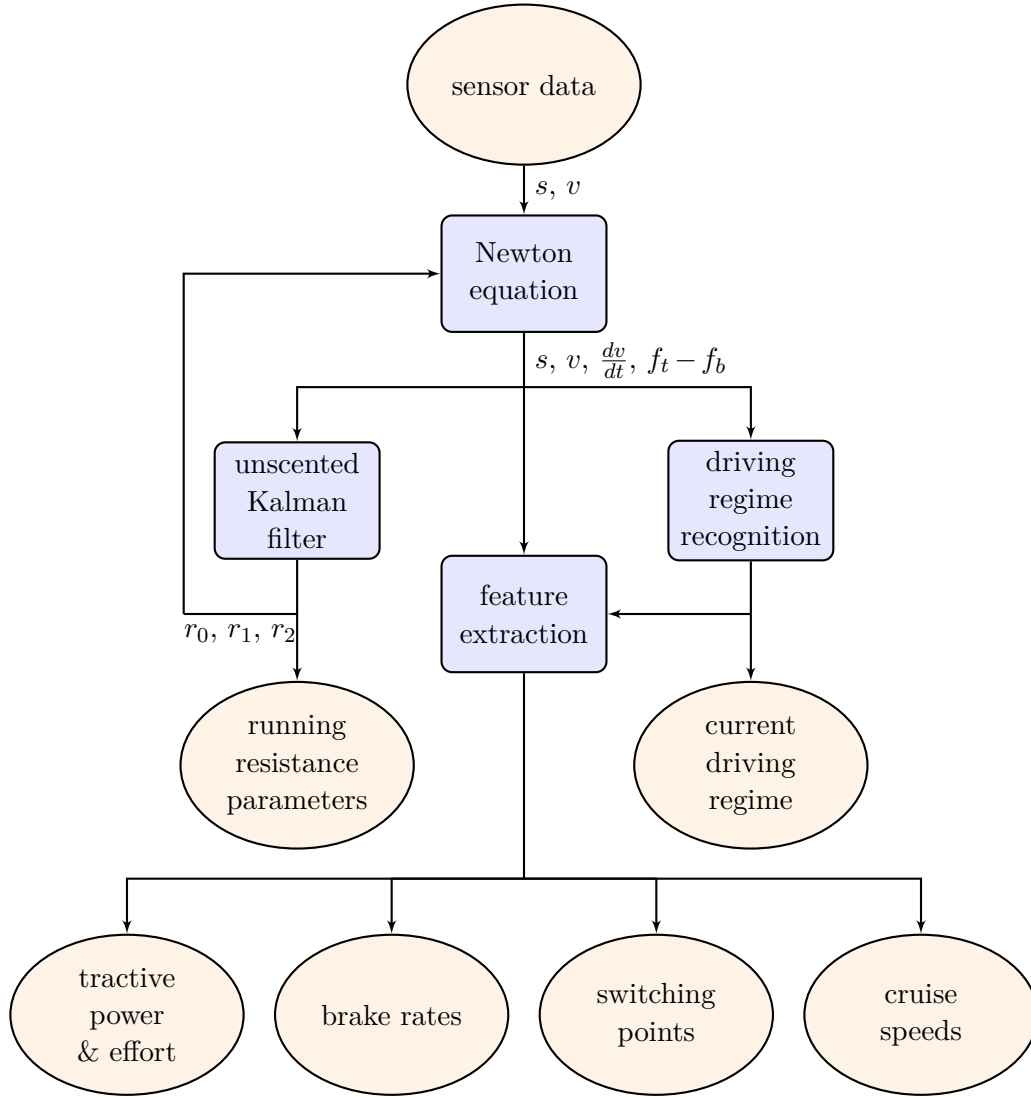


Figure 4.1: Flowchart of the proposed real time estimation algorithm.

maps the state vector into the measured variables. After that, the mean and covariance of the state at the next time step are calculated as weighted sums of the statistics of the propagated sigma points. The main advantages of UT is that it offers a computationally efficient way of calculating the state statistics of nonlinear systems, with an accuracy up to the third moment when the state statistics is symmetric. Furthermore, the sampling is deterministic, so it offers more consistent results than Monte Carlo sampling methods.

The UKF procedure can be described as follows. First, at every iteration, sigma points of the previous state are calculated and propagated through the system dynamics model, and the mean and covariance of the state at the current time are computed by means of a weighted sum of the propagated sigma points. Second, the propagated sigma points are mapped through the measurement function into the sigma points in the measurement space, which are used to obtain the expected value of the measured variables and the associated covariance. Last, the cross-covariance of the state and the measurements is calculated to obtain the Kalman gain, which is used to perform the estimation and to compute its covariance. This chapter aims to give an insight into the UKF.

Let $x^\top = (s, v, r_0, r_1, r_2)^\top$ be the train state vector of size L , including the running resistance parameters to be estimated and Δt the sampling rate of the location and speed measurements. Since measuring is a time-discrete process, the system evolution can also be considered discrete and evaluated in time steps equal to the measurement sampling rate Δt , so that $x_k \equiv x(k\Delta t)$, for $k = 0, 1, 2, \dots$. Considering that the evolution of the location and speed is determined by Eq. (4.1) and neglecting the fact that the running resistance parameters can evolve due to unmodelled dynamics, the discretization of the evolution of x can be described as follows:

$$x_k = f(x_{k-1}, u_{k-1}) + \nu_{k-1} \quad (4.3)$$

where $f(x_{k-1}, u_{k-1})$ is the process function, $u_k = f_{t,k} - f_{b,k}$ is the control variable, which corresponds in this case to the tractive and brake efforts and ν_{k-1} accounts for the noise in the process at the $(k-1)$ -th time step, modelled as a Gaussian random variable of zero mean and covariance matrix Q .

$$f(x_k, u_k) = x_k + (v_k \Delta t + 0.5 a_k (\Delta t)^2, a_k \Delta t, 0, 0, 0)^\top \quad (4.4)$$

where a_k is the acceleration term

$$a_k = u_k - r_{0,k} - r_{1,k} v_k - r_{2,k} v_k^2 - g(s_k). \quad (4.5)$$

Let z_k be the measurements at the time step k . The measurement function h is in this case linear, since measurements for the train location and speed are available in this case,

$$z_k = (s_k, v_k)^\top = h(x_k) + \omega_k, \quad (4.6)$$

where ω_k is the measurement noise, which is again considered to be Gaussian with zero mean and covariance matrix R .

As mentioned earlier, in the UKF the state distribution is represented by a vector x consisting of L Gaussian random variables. At time step $k-1$ it has mean $\bar{x}_{k-1}$ and covariance P_{k-1} . In order to describe its statistics, UT requires to calculate a minimum of $2L+1$ sigma points $X_{i,k-1}$, $i = 0, \dots, 2L$ to be able to capture the state statistics, with L the number of variables of the state. In this chapter, van der Merwe's scaled sigma point allocation is used (van der Merwe, 2004), which is one of the existing ways of calculating sigma points (Menegaz et al., 2015),

$$\begin{aligned} X_{0,k-1} &= \bar{x}_{k-1}, \\ X_{i,k-1} &= \bar{x}_{k-1} + (\sqrt{(L+\lambda)P_{k-1}})_i, \quad i = 1, \dots, L, \\ X_{i,k-1} &= \bar{x}_{k-1} - (\sqrt{(L+\lambda)P_{k-1}})_{i-L}, \quad i = L+1, \dots, 2L, \end{aligned} \quad (4.7)$$

where $\lambda = \alpha^2(L + \kappa) - L$ is a scaling parameter, α and κ are parameters that are used to regulate the spread of the sigma points around the mean and the weights of each sigma point when calculating the statistics of the observed variables. The square root of a matrix is in this case performed as a Cholesky factorization and $(\sqrt{(L+\lambda)P_{k-1}})_i$ corresponds to the i -th column of matrix $\sqrt{(L+\lambda)P_{k-1}}$. In order to obtain the prior distribution of the

state at time step k , the corresponding sigma points $Y_{i,k}$ are calculated by propagating the sigma points $X_{i,k-1}$ through the system dynamics function in Eq. (4.8),

$$Y_{i,k} = f(X_{i,k-1}, u_{k-1}), \quad i = 0, \dots, 2L. \quad (4.8)$$

Then, the mean and covariance of the prior distribution of the state x at time step k can be calculated by means of the UT as a weighted sum of the propagated sigma points, being w_i^m and w_i^c be the sigma weights associated to each sigma point for the calculation of the mean and covariance, respectively,

$$\begin{aligned} w_0^m &= \frac{\lambda}{L+\lambda}, \\ w_0^c &= \frac{\lambda}{L+\lambda} + 1 - \alpha^2 + \beta, \\ w_i^m &= w_i^c = \frac{1}{2(L+\lambda)}, \quad i = 1, \dots, 2L, \end{aligned} \quad (4.9)$$

where β is a parameter usually used to incorporate prior knowledge of the state distribution. Therefore,

$$\bar{x}_k \approx \sum_{i=0}^{2L} w_i^m Y_{i,k}, \quad (4.10)$$

$$P_{x,k} \approx \sum_{i=0}^{2L} w_i^c (Y_{i,k} - \bar{x}_k)(Y_{i,k} - \bar{x}_k)^\top + Q. \quad (4.11)$$

The UKF corrects the prior statistics of the state by means of the information obtained from the measured variables. To this end, the propagated sigma points $Y_{i,k}$ are mapped into the measurement space through the measurement function h .

$$Z_{i,k} = h(Y_{i,k}), \quad (4.12)$$

Analogously, the expected value of the measured variables and the associated covariance can be calculated through the UT,

$$\bar{z}_k \approx \sum_{i=0}^{2L} w_i^m Z_{i,k}, \quad (4.13)$$

$$P_{z,k} \approx \sum_{i=0}^{2L} w_i^c (Z_{i,k} - \bar{z}_k)(Z_{i,k} - \bar{z}_k)^\top + R. \quad (4.14)$$

Last, the Kalman gain K_k is calculated from the cross-covariance function $P_{xz,k}$,

$$P_{xz,k} \approx \sum_{i=0}^{2L} w_i^c (Y_{i,k} - \bar{x}_k)(Z_{i,k} - \bar{z}_k)^\top, \quad (4.15)$$

$$K_k = P_{xz,k} P_{x,k}^{-1}. \quad (4.16)$$

Therefore, the new state estimate $\tilde{x}_k$ and its covariance P_k can be calculated from the prior covariances, the Kalman gain and the expected value of the measured variables,

$$\tilde{x}_k = \bar{x}_k + K_k(z_k - \bar{z}_k), \quad (4.17)$$

$$P_k = P_{x,k} - K_k P_{z,k} K_k^\top. \quad (4.18)$$

Even though the accuracy of the estimations does not depend significantly on the initial state of the system, it is specially sensitive to its initial covariance (Wan & Van Der Merwe, 2000). Moreover, parameters α , β and κ have to be carefully chosen in order to maximize the accuracy of the UT. The methodology used for selecting these parameters will be covered in chapter 4.4.

4.3.2 Driving regime recognition module

In this chapter, we determine the driving regime in real time according to two criteria: the calculated applied tractive effort and brake, taking into account the current location and speed, and the train acceleration (De Martinis & Corman, 2019). First, we calculate the applied effort from the speed measurements. To cope with the noise present in those measurements, we define a tolerance of ± 0.025 N/kg in the applied effort in order to determine the coasting regime. Then, if the applied effort is higher than the upper bound, the train is applying traction, and if it is lower than the lower bound, it is braking. This bound constitutes approximately 5% of the maximum applicable tractive effort, according to the engine characteristics provided by the manufacturer. The lower bound for zero traction has been determined using the same value for simplicity, and both bounds have proven to determine correctly the driving regime on a predominantly flat track, as will be shown in the next chapter. Furthermore, the proposed framework should be able to distinguish whether a train is applying traction, brake or coasting even in steep uphill or downhill track section by means of this procedure, provided that the track description is sufficiently accurate. Moreover, if a train is exerting tractive or brake effort, the observed acceleration is considered: if it is between $\pm 0.06 \text{ m/s}^2$, the train is cruising, otherwise it is applying traction to accelerate or braking, respectively. This bound neglects small speed variations up to 2 km/h between consecutive speed measurements, mitigating thus the impact on the cruising regime recognition of measurement errors, small driving variations and inaccuracies in the track description.

4.3.3 Feature extraction module

The results obtained from the UKF are post-processed in order to determine parameters like the maximum tractive effort and power that the engine can apply, the applied brake rates and information regarding the different driving regimes. To this end, the driving regime recognition module takes an essential role in extracting some data from the speed profiles. The extracted features are specially relevant for calibrating algorithms like train trajectory optimizers and for assessing the variations in manual driving styles.

Maximum tractive power and effort can mainly be observed when applying traction to accelerate. At low speeds the maximum tractive effort is assumed to decrease linearly with the speed or to be approximately constant. Here, it is calculated by fitting a linear model to the filtered tractive effort time series when the train is continuously accelerating between 10 km/h and 30 km/h. The maximum tractive power is computed by taking the mean of the filtered tractive effort divided by the train speed when the train is continuously accelerating between 50 km/h and 130 km/h. However, in the case of diesel-powered locomotives, the maximum tractive power is not unique, since each engine notch can apply a different maximum tractive power. This leads to jumps in the hyperbolic part of the tractive effort curves. This could be addressed, for instance, by monitoring the notch changes. Nevertheless, this multiple notch approach will not be considered in this chapter since here only data from electric-powered trains is used. The maximum tractive power and effort calculated by means of this procedure might not correspond to the real maximum capability of the engine. A lower adhesion condition may lead to wheel slip, lowering the effective applicable tractive effort below the engine capability. The overhead

power may also influence the applied maximum tractive power, for example, when several trains depart from a station at the same time, the available catenary power is lower, reducing the acceleration rate. Besides physical sources of variation, driving-induced maximum tractive power and effort can also be observed, for instance, when a train driver accelerates gently to avoid disturbing the passengers' comfort or compromising the integrity of the composition of freight trains. Therefore, although the mentioned observations may not represent the real traction capability of the train, they can still be considered as input of energy-efficient on-board applications like DAS to reduce their impact on the expected arrival time.

To calculate the brake rates, the fact that a train might not decelerate with a constant brake rate is taken into consideration. *Automatische TreinBeïnvloeding Eerste Generatie* (ATB-EG, in English, Automatic Train Protection - First Generation) is the most-widely installed Automatic Train Protection system in the Dutch railway network. Besides high speed lines, the maximum speed limit in the Dutch railway network is 140 km/h. If there is no hazard ahead and the permanent speed limit of the track allows so, ATB-EG may show a Green signal, allowing the train to run with a maximum speed of 140 km/h. In the event of a yellow signal, ATB-EG may show 4 different signals, Yellow13, Yellow8, Yellow6 and Yellow, requiring the train to decelerate until reaching a speed below 130 km/h, 80 km/h, 60 km/h and 40 km/h, respectively. Moreover, the train driver is required to decelerate with a minimum deceleration rate until reaching the target speed, although the brake rate might be different for each type of yellow signal. If the driver fails to do so, ATB-EG intervenes and brakes the train. Therefore, in this chapter five speed steps are defined to build a deceleration model: over 130 km/h, 130 - 80 km/h, 80 - 60 km/h, 60 - 40 km/h and 40 - 0 km/h, and the mean observed deceleration rate in each speed step is calculated. Therefore, the proposed brake rate model may also be suitable for calculating mean deceleration rates in each ATB-EG speed step in the event of a yellow signal.

The values of the observation window of the maximum tractive effort and power and the speed steps related to ATB-EG are selected to match the particularities of the Dutch railway network and the rolling stock considered in the case study presented in the next chapter. These values should be updated in any application to adjust the feature extraction module to the expected braking behaviour and the tractive performance of the considered train.

Last, this module also calculates statistics of the cruise speed and the length of the final brake and of the previous coasting regime. To this end, the information obtained in the driving regime recognition module is utilized in the feature extraction module to calculate the final coasting and braking times. Moreover, the cruise speed during a cruising regime is easily determined by calculating the median of the filtered speeds along the mentioned regime.

4.4 Results and discussion

4.4.1 Case study

In this chapter, 67 runs of the same rolling stock unit in the Dutch railway network, departing from Eindhoven Centraal to 's-Hertogenbosch between March 2020 and December 2020, are considered. The mentioned speed profiles have been filtered, discarding the perturbed ones, that is, eliminating those that show an unexpected deceleration or stop due to a yellow or red signal. The considered unit is a manually-driven double-deck Intercity train with six carriages (VIRM-VI) operated by NS, the main railway operator in the Netherlands.

The available data for each run consists of GPS location and speed measurements, both of them sampled every 10 s. The GPS measurements are mapped into the track in order to calculate the mean gradient affecting the train at that time, taking into account its length. The track is 32103m long and is predominantly flat, with small, but nonsteep uphill and downhill gradients. The speed is measured in km/h, with an accuracy of 1 km/h. Figure 4.2 shows the speed profile of all the considered trajectories. Overall, a pattern can be distinguished in most of them consisting of an initial acceleration regime, cruising at a certain speed and coasting before braking. The overall maximum speed is 140 km/h save near the departure and arrival locations, where the speed limit is equal to 40 km/h. Moreover, variability is observed in the speed profiles due to the manual driving, not only in the running time between the two stations, but also in the cruising speed, the coasting and braking points and in the acceleration and deceleration rates when braking. However, the deceleration rates when coasting are significantly similar for all the speed profiles. Even though the maximum speed of the track is 140 km/h, in some cases the train runs with higher speed. The variability of the speed profiles and the driving style will be studied later in this chapter.

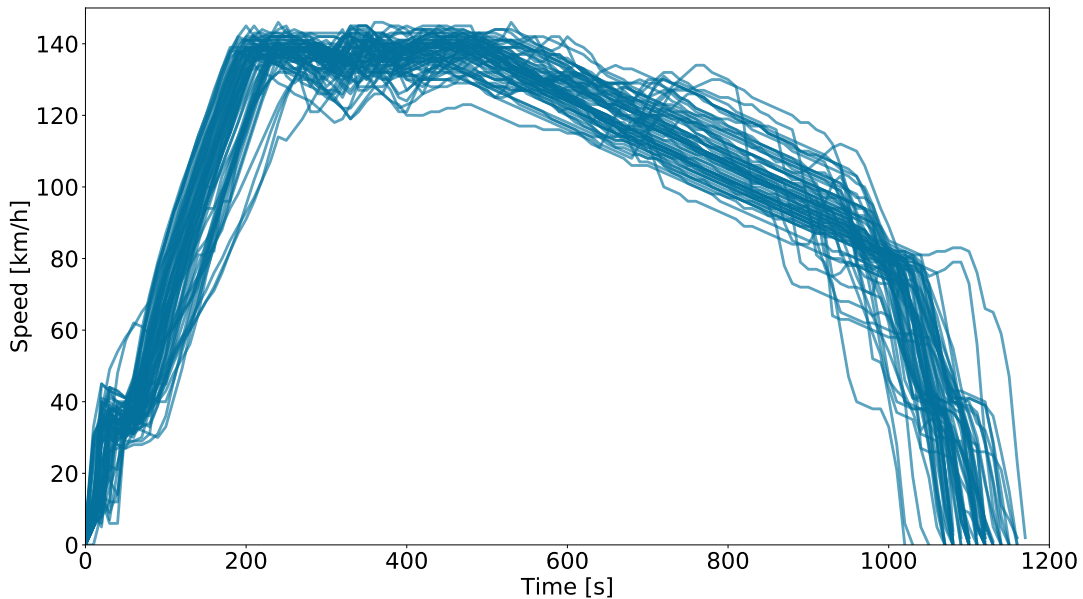


Figure 4.2: The 67 considered speed profiles in the case study from Eindhoven Centraal to 's Hertogenbosch.

4.4.2 Unscented Kalman Filter

The tractive and brake effort needed to match the observed acceleration or deceleration for each location and speed measurement is calculated by means of Eq. (4.1), thus generating an effort time series for each speed profile. It is used along the location and speed measurements and the track height profile as input for the UKF algorithm. The train is assumed to be loaded at 35% of its seat capacity and the rotatory mass coefficient is considered constant and equal to 6% of the empty mass of the train, $\rho = 1.06$, which corresponds to the value provided by the manufacturer. In order to estimate the running resistance parameters, they are considered as state variables along with the train location and speed, as described in chapter 4.3.1. The evolution of the state variables at each time step is therefore described by Eqs. (4.3) and (4.4), where $\Delta t = 10\text{s}$, matching the measurement sampling rate. Taking into account the short computation time of the algorithm, the considered UKF implementation could be able to cope with smaller sampling rates, which might lead to more accurate estimates. However, subsampling the available data led to less accurate results, particularly in the driving regime recognition and the feature extraction modules. Considering the data available, we used the smallest sampling rate in this case study. No process noise is considered due to the difficulty of quantifying it without using ground data of the running resistance parameters and the train location and speed, which is unavailable in this case study, so the term ν_{k-1} is equal to zero at every time step. However, we have noticed that small additive Gaussian white noise in the acceleration may lead to slightly richer dynamics of the estimates. We have also observed that either stronger acceleration noise or small noise in the running resistance parameters' equations lead to wrong estimates, namely negative estimates of the resistance parameters and unrealistic values of the applied effort. The UKF calculates the prior distribution of the state variables at the next time step, which is updated by means of the information obtained from the location and speed measurements at that time step. The standard deviation of the measurement noise of the location is considered to be 4m and 1 km/h in the case of the speed.

Nevertheless, the UKF parameters have to be carefully selected in order to produce accurate estimations. These parameters are van der Merwe's scaled sigma point allocation parameters α , β and κ , and the initial covariance matrix of the state vector. This parameter selection has been performed according to two criteria: First, by maximizing the total sum of the logarithms of the conditional likelihood of the measurements with respect to the estimated location and speed, and their respective estimated variances, assuming that the distribution of these variables is Gaussian (Scardua & Da Cruz, 2017). Second, by fine-tuning the obtained parameters, considering that the running resistance parameters are nonnegative. UKF was found to be very sensitive to the initialization values of the covariances of the measurements and the running resistance parameters. For instance, an increase in two orders of magnitude in one of the initial covariances of any running resistance parameter led to wrong running resistance parameter estimates that violated their nonnegativity and to calculating the applied effort calculation erroneously. Table 4.1 shows the parameters used in the UKF.

Parameter	Value	Parameter	Value
α	0.001	β	2
κ	-2	$\sigma_{s,\text{ini}}$	1 m
$\sigma_{v,\text{ini}}$	0.0018 km/h	$\sigma_{r_0,\text{ini}}$	10^{-6} N/kg
$\sigma_{r_1,\text{ini}}$	10^{-8} N/(kgm/s)	$\sigma_{r_2,\text{ini}}$	10^{-9} N/(kg(m/s) ²)

Table 4.1: UKF initialization parameters. α , β and κ are van der Merwe's scaled sigma point allocation parameters and $\sigma_{s,\text{ini}}$, $\sigma_{v,\text{ini}}$, $\sigma_{r_0,\text{ini}}$, $\sigma_{r_1,\text{ini}}$ and $\sigma_{r_2,\text{ini}}$ are the initial standard deviation of the location, speed, r_0 , r_1 and r_2 , respectively.

Figure 4.3 shows the results obtained by the UKF for a single speed profile. The upper plot shows the measured speed profile as a blue dotted line and the estimated speed is shown as a red line. It can be observed that the speed profile is accurately reproduced by the UKF. Moreover, the four mentioned driving regimes, namely acceleration, cruising, coasting and braking, can be distinguished and they take place in this order, being coasting the longest regime. The second and third plots show in red the calculated acceleration and deceleration rates for every measurement and the calculated tractive and brake effort, respectively. The dashed lines represent the maximum deceleration rate, -0.66 m/s^2 , and the speed-dependent maximum applicable tractive effort. This boundary has been calculated by means of the manufacturer values of the maximum applicable tractive effort and power, and assuming that for low speeds the maximum tractive effort is constant. It is observed that the applied tractive effort exceeds the mentioned theoretical boundary at the end of the acceleration regime. This will be analyzed later when discussing the results obtained from the feature extraction module. Next, along with the constant speed regime a first interval where the acceleration is zero is observed, followed by a second interval in which the train reaccelerates briefly to gain speed and coasts when the train speed is higher than the target cruise speed. This exemplifies two of the most frequently used cruising techniques in manual driving: holding a constant speed and oscillating around a target speed by applying traction and coasting cycles. Then, the acceleration is slightly negative along the coasting regime, where the applied effort is approximately zero, save for small deviations due to noise in the measurements. Therefore, a small tolerance around zero traction and brake is required in the driving regime recognition module to determine the coasting regime. Last, during the final brake, it can be observed that in this case the train deceleration is lower than the 59% of the maximum deceleration rate, so the driver decelerates the train progressively. The running resistance parameters are shown in the last three plots. The UKF estimations are depicted in red, while the theoretical values are represented by a horizontal blue dashed line. It can be observed that the three parameters show similar dynamics. Moreover, the theoretical values are used as the initial state in the UKF, the running resistance parameters start decreasing at the beginning of the trajectory, which indicates that the theoretical values overestimate the real time-variant parameter values. For lower speeds, the parameters increase again, which might hint an inverse proportional relation between the resistance parameters and the speed. Although there is a tunnel at the entrance of an intermediate station, Best, we did not observe any influence on the estimated running resistance, probably due to the large cross-section of the tunnel. However, further studies would be required to assess the performance of

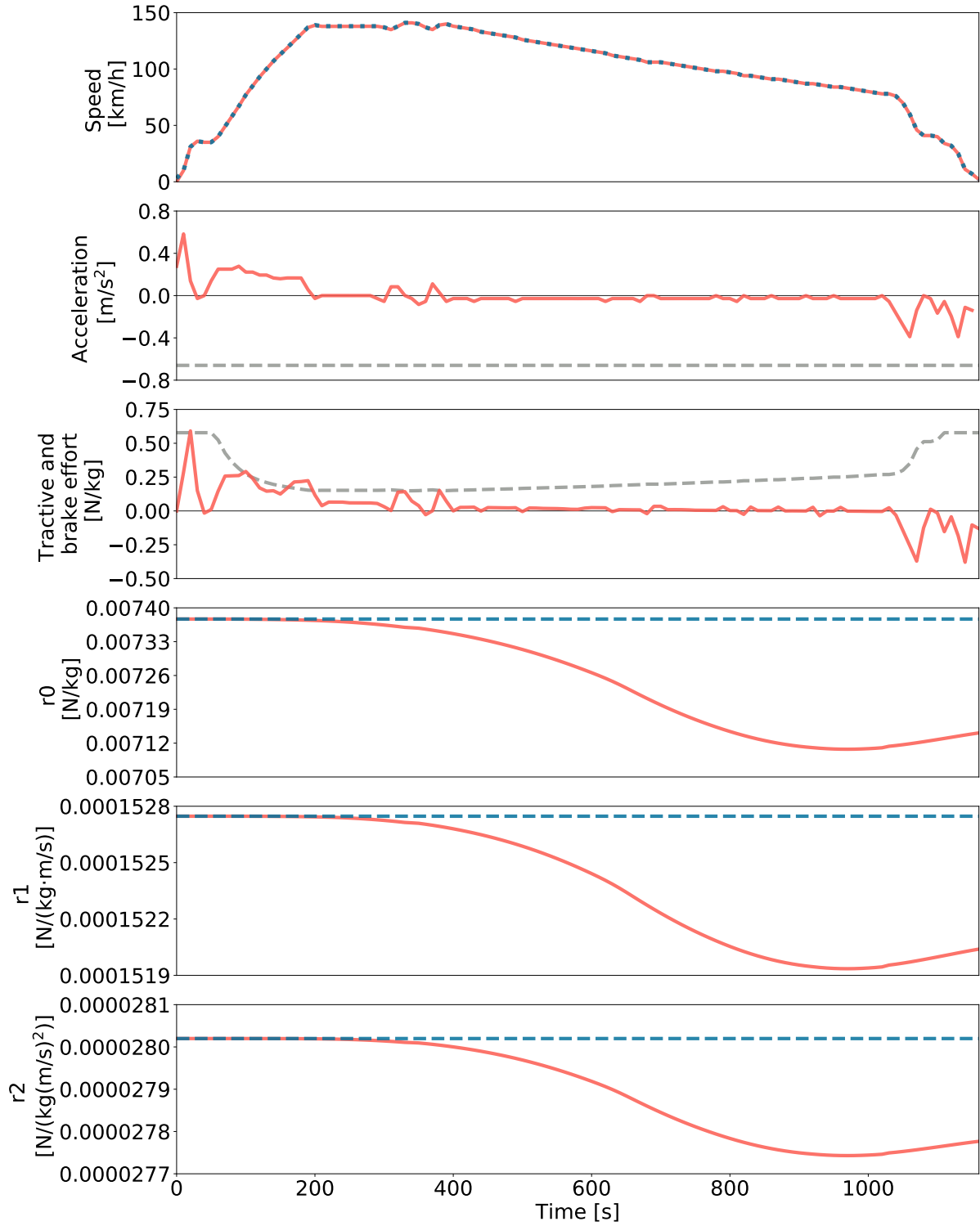


Figure 4.3: Example of the performance of the UKF on a single speed profile.

the proposed framework on detecting the extra aerodynamic drag when running through tunnels and its impact on the running resistance.

Figure 4.4 shows a histogram of the estimated values of r_0 , r_1 and r_2 from all the speed profiles. The manufacturer value is represented by a vertical dashed line. The negative

skewness of the distributions shows that the statistics here are biased due to the usage of the manufacturer value as the initial condition of the UKF algorithm. However, besides the outliers, parameter r_0 is bounded between 0.0072 N/Kg and 0.00738 N/Kg. The manufacturer value is near the mentioned upper bound and the distribution is biased due to its usage as the initial value of the UKF. Therefore, the manufacturer value overestimates the value of r_0 . The other two running resistance parameters, r_1 and r_2 , show similar statistics.

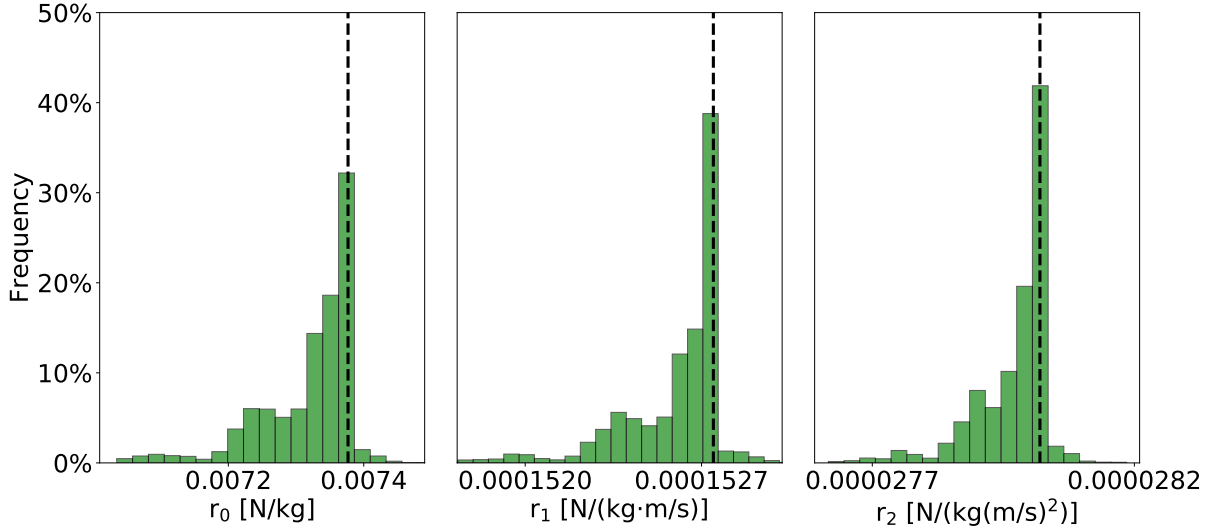


Figure 4.4: Histogram of the occurrence of parameters r_0 , r_1 and r_2 in the 67 speed profiles processed by the UKF. Their manufacturer values are represented by vertical black dashed lines.

Figure 4.5 shows a stacked area plot of each of the terms of the running resistance, where the parameters r_0 , r_1 and r_2 are those obtained by the UKF for the speed profile shown in Figure 4.3. In turn, the resistance due to the track geometry is depicted in magenta, in the same scale as the running resistance, to allow for comparison. The term r_0 is represented in dark blue, the term that depends linearly on the speed is represented in lilac and the quadratic term, in light blue. It may be observed that the variations of the constant term do not produce significant variations in the total running resistance, so this term may be considered constant in each interstation. The linear term accounts for up to the 11% of the total running resistance, only 2% below the constant contribution. However, Howlett et al. (2004) claim that this term contribution is small compared to the other two, and neglect this parameter and overestimate parameters r_0 and r_2 in order to overfit the total running resistance. Moreover, Bešinović et al. (2013b) do not consider the linear term since it produces a negligible variation in the running times (Łukaszewicz, 2001). It may be concluded then, that even though the linear term of the running resistance can be sometimes neglected, this cannot be done systematically for any type of rolling stock, at least when energy consumption and accuracy are the main focus of the parameter estimation. At higher speeds, the quadratic term accounts for up to the 76% of the total running resistance, being therefore the most relevant term for energy consumption. As a consequence, most efforts have to be dedicated to estimating r_2 .

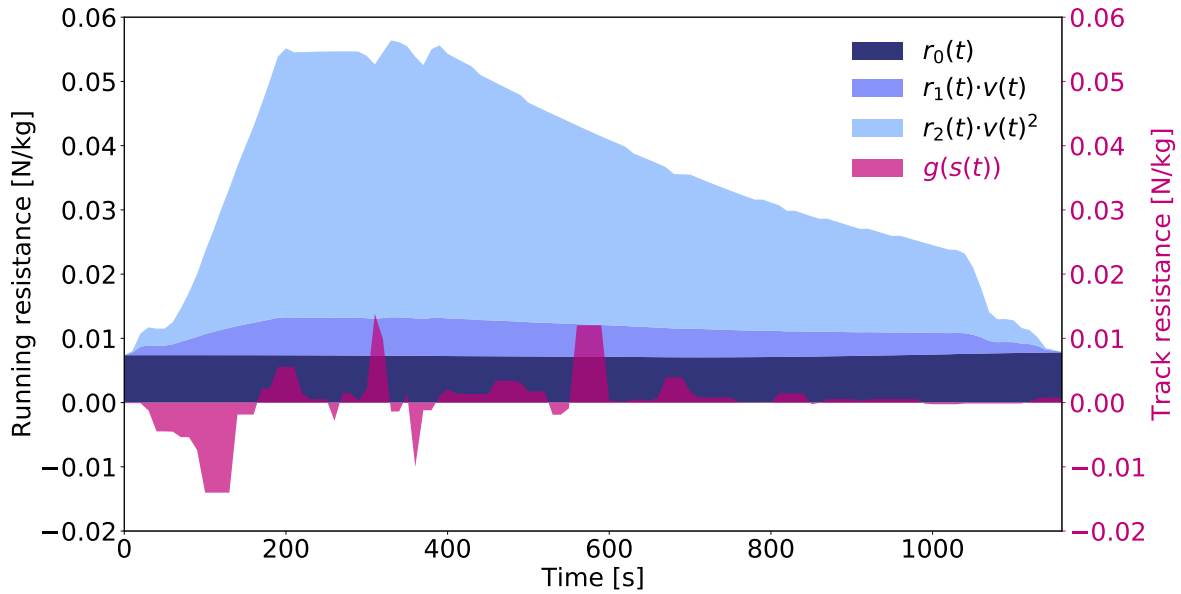


Figure 4.5: Stacked area plot of the contribution to the running resistance $r(v)$ of each of its terms, namely the constant term r_0 , the linear term, r_1v and the quadratic term r_2v^2 . The resistance due to the track geometry is represented in the right axis.

4.4.3 Driving recognition module

The driving regime recognition algorithm described in chapter 4.3.2 is used to assess the variability of the speed profiles with respect to the driving style. To this end, the cruising speed, and the coasting and braking times are calculated for each speed profile. The driving regime recognition module estimates the current driving regime for each measurement sample by means of the ruleset defined in chapter 4.3.2. Figure 4.6 shows the segmentation of the speed profile represented in Figure 4.3 in terms of the different driving regimes. It may be observed that the reaccelerations part of the cruise regime is not fully considered as cruising, but short traction, coasting and brake regimes are also determined along the mentioned cycles.

For this trajectory, the train applies traction to accelerate in 16.4% of the total running time and it cruises at a constant speed during 17% of the time. In this interstation, the train coasts during 57% of the trajectory, and it brakes during the remaining 9.3%.

This module allows for obtaining statistics on driving style features like the cruising speed and the length of the final coast and brake regimes. Figure 4.7 (left) shows a histogram of the observed cruise speeds in the 67 considered train runs. The cruise speed distribution is clearly skewed towards 140 km/h, which corresponds to the maximum speed of the track. This speed is exceeded in 12% of the speed profiles. Figure 4.7 (center) shows a histogram of the lengths of the coast regime before the final brake. The final coast regime length can vary significantly, from 2.5 to 10 minutes. Statistics on the final brake are shown in Figure 4.7 (right). Even though the length of the final brake regime is usually considerably shorter than the coast regime, they show comparable

variations, since the coefficient of variation of the brake times is 20%, while the coefficient of variation of the coast times is 30%.

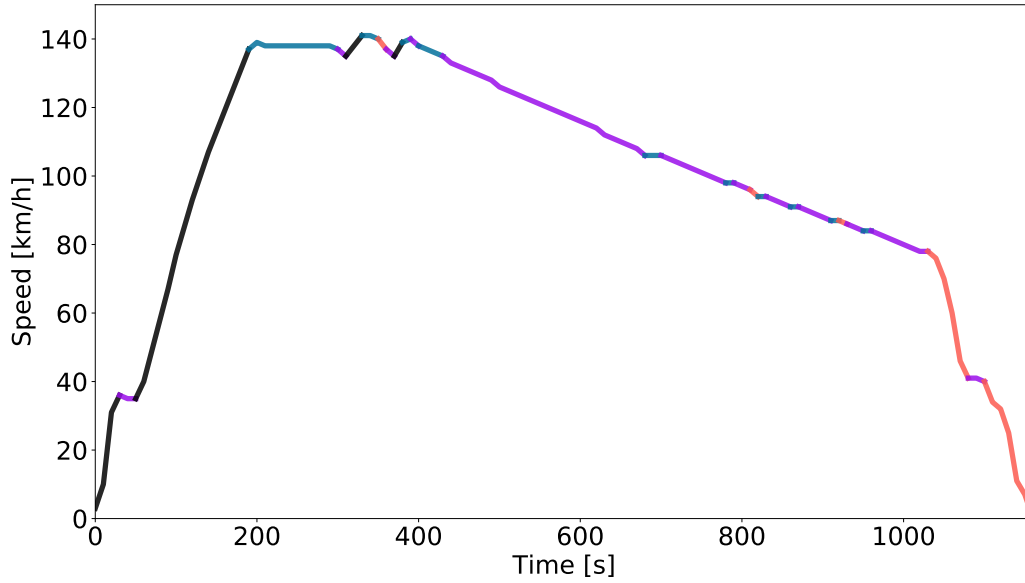


Figure 4.6: Driving regimes of the speed profile shown in Figure 4.3. The traction regime is depicted in black, cruising in blue, coasting in purple and braking in red.

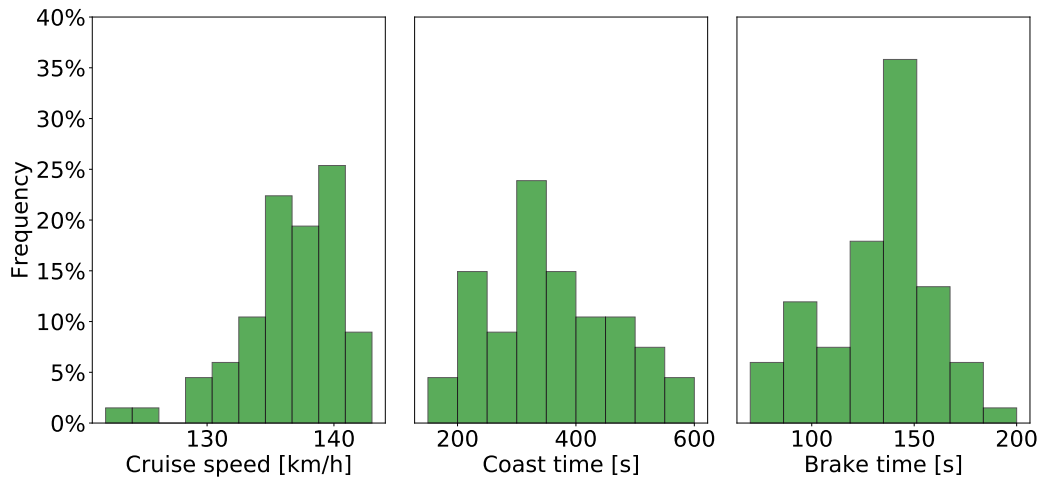


Figure 4.7: Left: Histogram of the observed cruise speeds. Center: Histogram of the observed length of the coast regime before the final brake. Right: Histogram of the observed length of the final brake regime.

4.4.4 Feature extraction

The feature extraction module is used to obtain statistics of the brake deceleration rates for each of the ATB-EG speed steps. Figure 4.8 shows the observed brake rates from all the considered speed profiles in blue, while the mean brake rates in each speed step are represented in red, and the associated standard deviations are depicted as a shadow red area. Each speed step is separated by a vertical black line. A large variability is observed in the brake rates produced by the manual driving, however, the lower the train speed, the higher the mean applied brake rate. This is linked to the fact that brake systems usually provide a larger brake effort at lower speeds and that drivers are encouraged to decelerate gradually, since braking gently increases passenger comfort. Moreover, the variability is larger for lower speeds. The wider variance of the brake decelerations at speeds lower than 40 km/h might be due to the fact that ATB-EG does not associate a target deceleration rate for this speed step and that the train has to prepare to stop to a standstill at the stopping point of the track, which requires a certain level of accuracy. At larger speeds, drivers also tend to start braking gradually, therefore the initial deceleration rates are usually lower than the final ones. It is noteworthy that the ATB-EG speed steps between 40 km/h and 80 km/h and those speed steps higher than 80 km/h respectively share a similar braking behaviour, including the mean observed brake rate.

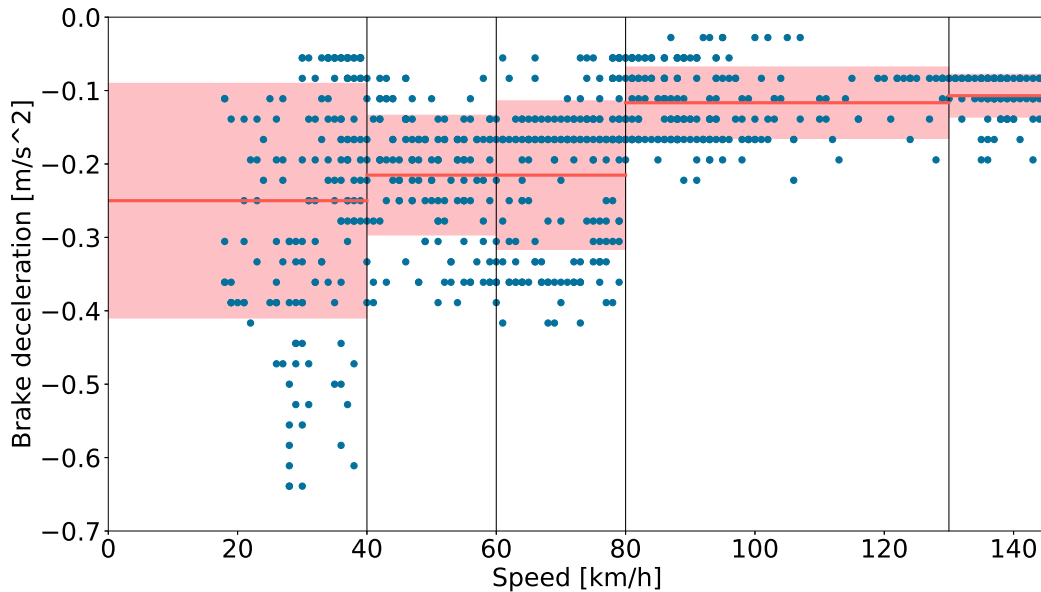


Figure 4.8: Observed brake rates and mean brake rate in each ATB-EG speed step.

The maximum applicable tractive effort and power take a relevant role in railway applications like Train Trajectory Optimization. An accurate estimation of these two quantities is essential for calculating accurately the running time and energy consumption of a train. However, estimating maximum bounds for the tractive effort and power from speed measurements can be complex. The information on whether the train driver is applying maximum traction or not is not available in this case study. Moreover, in the case of manual driving, the time in which the driver usually applies maximum traction

is limited. When a train is at a standstill, train drivers tend to apply traction gently in order to guarantee passenger comfort. Therefore, the first tractive effort measurements or calculations have to be discarded. Moreover, the maximum applicable tractive effort is usually modelled as constant for low speeds, and proportional to the inverse of the speed at higher speeds. The maximum applicable tractive effort in these two traction steps only coincides at a certain transition speed, which, according to the manufacturer values of the maximum tractive effort and power of the engine, in this case takes place around 38 km/h. However, the transition speed may vary depending on many factors such as the adhesion condition, the available electric power and the engine condition. Therefore, to guarantee that the tractive effort measurements and calculations used to calculate the mentioned upper bounds belong to the right tractive step, those tractive effort calculations that take place around the transition speed have to be discarded. In addition, in this case study there is a 40 km/h speed limit that affects the train trajectory when applying traction to accelerate. Train drivers tend to reach such a speed limit by reducing the applied traction. Once the train is running past the mentioned speed limit, train drivers usually reaccelerate gently, that is to say, they do not apply maximum traction immediately. Therefore, in the maximum tractive effort and power calculations the calculated tractive effort for speeds below 10 km/h and between 30 km/h to 50 km/h are discarded.

In the case of the maximum tractive effort, due to the small speed window and the small amount of data available due to the low measurement rate, few data are available. Moreover, the low accuracy of the speed measurements, which is determined within an error of 1 km/h, introduces extra noise in the calculation of the tractive effort, producing a large dispersion in the set of calculated efforts at low speeds. Therefore, there is not enough data available in order to calculate relevant statistics of the maximum applied tractive effort. Nevertheless, in the case of the maximum applied tractive power, there is enough data available. Again, the data close to the maximum speed of the track, 140 km/h, is discarded. Figure 4.9 shows the calculated tractive power for each speed measurement corresponding to the initial traction regime of each of the 67 considered train speed profiles. The calculated tractive power is represented by blue dots, and the value provided by the train manufacturer for the maximum tractive power is represented by a horizontal grey dashed line. Moreover, the observed maximum tractive power of each trajectory between 50 km/h and 90 km/h are represented by red crosses. Two clouds of points are observed. One in the lower left part of the figure, below the horizontal grey dashed line and for speeds below 115 km/h, and another in the upper right part of the figure, above the manufacturer value, for higher speeds. Furthermore, a line pattern in both clouds is observed due to the speed discretization with an accuracy of 1 km/h. However, all trajectories allocate tractive power points in both clouds. Therefore, a transition may take place in a certain point in the track. The difference in the calculated tractive power between both clouds is high enough so that it cannot be produced due to an inaccurate gradient profile of the track. Therefore, the observed transition may be due to the presence of several power stations, supplying energy with less power to the mentioned part of the track near 's Hertogenbosch station than to the next part of the track. Moreover, for the lower speeds, the train is departing from the station, where the available electric power can be lower than expected due to the fact that there are several trains using the available electric power. This explains the difference in tractive

power and the fact that the transition does not take place at a specific speed, since, as can be observed in Figure 4.2, there is a wide variability in the passing speeds in each point of the track. Moreover, there is a large variability in the observed maximum applied tractive power for lower speeds, although the manufacturer value constitutes in this case an accurate upper bound. The mentioned variability is probably due to the manual driving style, but nonhuman sources of maximum applied tractive power like variations in the available electric power at the station and train load may also be secondary sources of variability, which relevance could be assessed by analyzing data from ATO-driven trains.

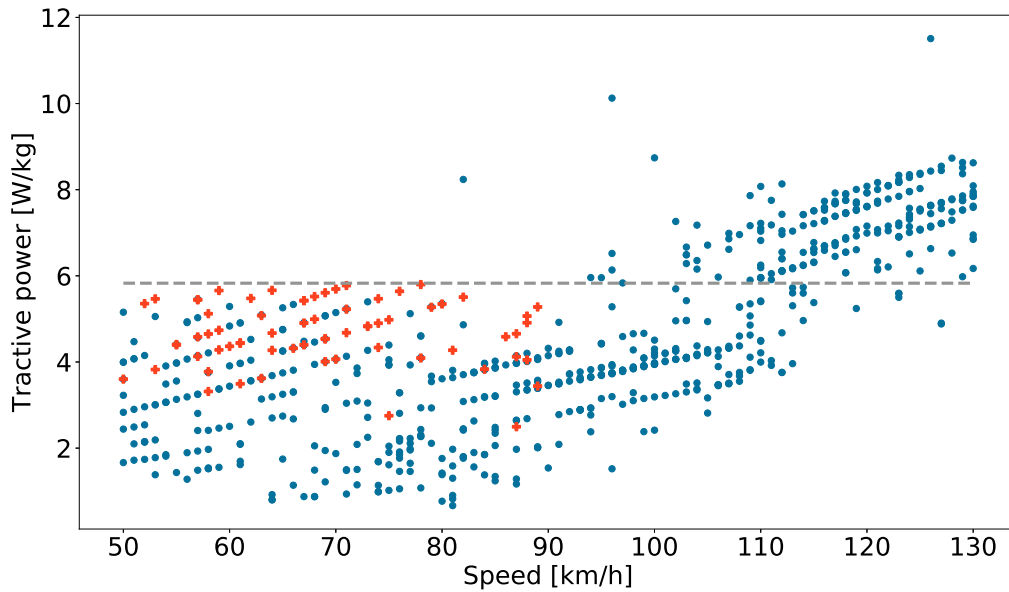


Figure 4.9: Tractive power diagram of the initial acceleration regime of the 67 considered speed profiles.

4.5 Conclusion

A train motion model based on Newton's second law is the core of railway applications like energy-efficient Train Trajectory Optimization, DAS and ATO. This model can reproduce train dynamics accurately when its internal parameters are carefully tuned. However, this parameter estimation is usually difficult, since they may vary stochastically in time due to a broad diversity of causes, from the current weather and adhesion condition to the mechanical wear of the train and the driving style of the driver. Therefore, an accurate real-time train motion model parameter estimation is essential to guarantee the performance and energy-efficiency of on-board railway applications and operations.

A real-time train motion model parameter estimation framework consisting of 3 modules has been devised. First, an Unscented Kalman Filter (UKF) module uses location and speed measurements as input data along the calculated traction and brake efforts needed to match the observed acceleration to estimate the train running resistance parameters. Second, the observed acceleration and the calculated tractive and brake efforts

are used in a driving regime recognition module to distinguish the driving regime at each measurement time. To this end, four driving regimes are defined: applying traction to accelerate, cruising, coasting and braking. Last, a feature extraction module is utilized to obtain statistics of the applied brake rates and the maximum applied power of the engine.

The proposed framework has been tested in a case study using real data, showing that the UKF is able to track the location and speed measurements and to provide an estimation of the running resistance parameters. The driving regime recognition module determines the current driving regime successfully. Some statistics have been computed on the observed cruise speeds, and the coast and brake lengths. The impact of manual driving on the applied brake rates and the maximum applied tractive power has been studied by means of the feature extraction module. In particular, the observed brake rates have been modelled in terms of the ATB-EG speed steps, showing that drivers tend to brake gently. From a statistical point of view, in this case three speed-dependent braking behaviours are observed: one for speeds higher than 80 km/h, another for speeds between 40 km/h and 80 km/h, and the last one for speeds lower than 40 km/h.

The proposed framework has a limitation regarding the availability of traction and brake efforts measurements, which are required to obtain an accurate estimation of the running resistance parameters. Even though calculating the applied tractive and brake efforts based on the observed acceleration allows to obtain approximate values of the running resistance parameters, it would smooth part of the running resistance dynamics, compromising then the performance of the UKF. Moreover, the driving regime recognition module could be updated in order to enhance its accuracy, and the information obtained could be used in real time to improve the performance of the parameter estimation module. Furthermore, a higher measurement sampling rate would improve the accuracy of the proposed framework.

This research opens new possibilities in railway operation data analysis. The case study included in this chapter could be extended to demonstrate that the proposed framework is also able to calibrate train dynamics of several rolling stock units and to detect the existing parameter variability within the different units. This would allow to demonstrate the importance of individual train calibration in the railway industry and academia. The performance of the proposed framework could be studied also for different types of trains: from high speed trains to light rail and metro. To address this, the internal parameter configuration of the framework should be retuned based on the characteristics of those railway systems. In addition, the spatial variability of parameters could be assessed by comparing the results obtained from trains running on different lines, which may lead to determining the impact of different track maintenance conditions on the dynamics of a train. The proposed framework could also allow to study temporal parameter variability. For example, data spanning a whole year could be used to study seasonal parameter variations, like the ones due to different temperature, weather, adhesion conditions and train wear. Moreover, data could be classified in an hourly basis and the impact of a tight timetable and delays on manual driving style could be assessed. What is more, if the data used is linked to the train driver, individual driving style statistics could be computed and the differences between them could be studied.

Chapter 5

Adaptive estimation of train running resistance: A Recursive Least Squares approach with multiple forgetting factors

5.1 Introduction

The increasing passenger demand and the current network congestion are pushing the railway industry towards automated driving. Automation can exploit marginal capacity gains while ensuring the sustainable, safe and efficient transport of passengers and freight. Railway automation relies on Train Trajectory Optimization to calculate the reference trajectories to be followed along the way to a train's destination. These trajectories are generated by means of a model of the train dynamics, which has to be calibrated to reproduce accurately the behaviour of each train. Train motion model calibration is required to maximize the performance of Train Trajectory Optimizers and to enable the successful implementation of automation solutions. In practice, monitoring and continuously calibrating the train motion model is essential since the train dynamics show significant variations along the train's journey.

The dynamics of a train are influenced along its journey by a wide variety of phenomena. The aerodynamic drag on the train increases when it runs through tunnels, decelerating the train (Iwnicki, 2006). When the windows are open, more air enters the cars (Baker et al., 2019). Trains passing other trains and stationary structures also add extra aerodynamic drag (Schetz, 2001). The mechanical resistances are determined by suspension losses, friction within bearings, wheels' geometry and complex interactions between wheel and rail (Bernsteen et al., 1983). All this increases the running resistance, which gathers the non-conservative mechanical and aerodynamic forces that result in kinetic energy losses, regardless of whether the train is applying traction or brake. Higher running resistance leads to increased energy consumption since the train will need to apply more traction along its journey to compensate for the extra loss of kinetic energy.

In railways, the running resistance is usually modelled as a quadratic function of the speed known as the Davis equation (Davis, 1926). It gathers the mechanical resistance to

motion in the speed-independent and the linear terms, and the aerodynamic resistance in the quadratic term (Bernsteen et al., 1983). The Davis equation is parameterized by three coefficients, one for each term, which require to be calibrated to guarantee its accuracy (Cunillera et al., 2023a). At high speeds, the quadratic term is the major contributor to the running resistance, so special consideration must be taken to estimate this parameter. The running resistance is usually calibrated by means of empirical formulas, or run down tests (Rochard & Schmid, 2000). These approaches assume the running resistance parameters as constant along a train’s trajectory. This simplified description of the running resistance fails to capture any potential spatiotemporal variations of the running resistance parameters. The aerodynamic resistance is known to vary more widely and frequently than the mechanical resistance along a train’s journey, making the quadratic parameter more volatile and its estimation more challenging. Narrow tunnels are an archetypical example of this phenomenon, as the quadratic running resistance parameter can even double its value when entering the tunnel and return to its original value when exiting it. Some efforts have been dedicated to gauge the effect of tunnels on the running resistance, see Hansen et al. (2017) and references therein. This is a complex problem due to the wide variety of factors that influence the dynamics of trains in tunnels (Niu et al., 2020). In railways, model calibration is applied in a wide variety of scenarios like assessing the stiffness of rails (Hoelzl et al., 2023), predicting train body vibrations (Peng et al., 2023), studying the influence of faulty axle-box bearings on the train dynamics (Guo et al., 2024), estimating the available adhesion (Shrestha et al., 2019a,b) or estimating wheel-rail contact forces in real time (Strano & Terzo, 2018). However, the current literature lacks a method that analyzes operational data to simultaneously estimate the running resistance parameters while dynamically capturing the broader volatility of the quadratic resistance parameter.

Variations of the train dynamics along a trajectory can be modelled as fluctuations of the train motion model parameters, and such fluctuations can be captured by means of online calibration. This type of calibration monitors the train dynamics in real time, processing data as soon as it is generated. It updates continuously the estimates of the train motion model parameters, constituting a perfect framework for gauging spatiotemporal parameter variations like the ones shown by the quadratic running resistance parameter. Several techniques have been explored in online train motion model calibration, as we reviewed and discussed in detail in chapter 2. Gorbachev et al. (2017) and Moroz et al. (2017) introduced standard Recursive Least Squares in railways for estimating the running resistance parameters. They assumed the quadratic running resistance parameter had been previously calculated from the rolling stock’s aerodynamic drag. The speed-independent and the linear running resistance parameter were estimated in a computational case study with additive noise emulating the impact of wind. Gorbachev et al. (2017) aimed to estimate the running resistance accurately to enable the implementation of an adaptive train control system that guarantees passengers comfort. In turn, Moroz et al. (2017) aimed to model the dynamics of passenger wagons accurately. Liu et al. (2018) compared several methods based on Recursive Least Squares to estimate the running resistance parameters of an automatically-driven metro train, aiming to improve the accuracy of automated driving strategies in metro lines. They showed that performing the recursive estimation over a window formed by a handful of the latest measurements

produces more accurate running resistance parameter estimates than standard Recursive Least Squares. A particle filter was used in Xie et al. (2016) to estimate the running resistance parameters among a predefined set of estimate candidates. The authors addressed the online running resistance estimation problem when the measurement noise is non-Gaussian, and managed to keep the computation time low enough for real-time calculations. Jin et al. (2018a) also considered non-Gaussian noise when estimating the running resistance parameters. They combined several Extended Kalman Filters with Gaussian sum theory. Non-Gaussian noise and state variables are represented as sums of Gaussian variables, and each of them is filtered by an Extended Kalman Filter with the aim of estimating the running resistance among a wide set of prespecified candidates for the values of the parameters. De Martinis & Corman (2019) estimated the running resistance parameters by means of a multi-start gradient descent method over sliding windows of measurements. The authors tested both a mean quadratic error and a least absolute error measure of performance, finding the latter more robust. They also considered losses of GPS coverage when running through tunnels and reconstructed the train trajectories using measurements of the power consumed, but their results do not show an increase in the running resistance inside the tunnels. The long computation time of their estimation method jeopardizes potential real-time implementations in the case of the widest sliding windows considered in their study. Several authors have explored using the Unscented Kalman Filter. It was used for the first time in Howlett et al. (2004), where the quadratic and speed-independent running resistance parameters were estimated while the linear one was considered equal to zero. The method was validated for coasting trains in a computational case study where a train coasts and Gaussian white noise is added to both the acceleration of the train and the resistance parameters, which are assumed to be constant along the whole journey. This method was also used in chapter 4 to analyze real measurements. The running resistance parameters were estimated along with the observed traction and brake characteristics. One of the main drawbacks of Kalman-like filters is that they contain several internal parameters that are difficult to initialize. The accuracy of the calibration is also sensitive to the initialization of the covariance. However, none of these papers demonstrate that the methods proposed can capture the variability inherent in the quadratic term of the running resistance.

In this chapter we propose an algorithm based on Recursive Least Squares with multiple forgetting factors for estimating the running resistance of a train in real time. It is an online parameter estimation algorithm that calibrates the running resistance parameters by minimizing the quadratic error of the observed and the estimated running resistance every time that a new set of measurements is produced. It utilizes location, speed and tractive and brake effort measurements as input. However, standard Recursive Least Squares tends to become insensitive to parameter variations over time since new and old measurements are equally weighted. In contrast, when a forgetting factor is introduced, the newer measurements are weighted more heavily in the estimation than the older ones. Still, if the value of the forgetting factor is the same for all the parameters to be estimated, a jump in a parameter may lead to wrong estimates of the other parameters. This is due to the fact that quadratic error objective functions usually lead to updates in more than one parameter at once (Sharma et al., 2016). This issue is called estimation windup (Shin & Lee, 2020) and can be solved by assigning different forgetting

factors to each parameters, depending on their relative intrinsic volatility (Vahidi et al., 2005). This is exactly the case of the running resistance parameters, where the quadratic one shows jumps and faster variations than the other two, especially when the train runs through tunnels. This method has been successfully applied in road vehicle mass and gradient estimation, where the gradients vary frequently along the way while the mass decreases slowly in time (Vahidi et al., 2005; Li et al., 2020). For the first time in online estimation of train running resistance, we explore adding multiple forgetting factors to Recursive Least Squares. Our main objective is to design a method that allows to estimate the three running resistance parameters in real time, taking into account their different volatilities along the journey. Our results show that the proposed method can be used to capture the variability of the quadratic running resistance parameter in real time while also estimating the mechanical resistance accurately. The algorithm is particularly suited for detecting such sudden changes in the running resistance, for instance, when a train enters or exits a tunnel.

The proposed method has several theoretical advantages over other existing train running resistance estimators. The most important one, and the one that adds value to the proposed method, is that it is the only existing method that can capture the different relative volatilities of the running resistance parameters. It can calculate simultaneously the three running resistance parameters, while some authors simplify the estimation by fixing the value of one of them (Gorbachev et al., 2017; Moroz et al., 2017; Howlett et al., 2004). Computationally, it is less expensive than gradient descent optimization methods (De Martinis & Corman, 2019), particle filters (Xie et al., 2016), and Extended (Jin et al., 2018a) and Unscented Kalman Filters (Howlett et al., 2004; Cunillera et al., 2022). The proposed method is conceptually simple, and easy to be implemented and initialized. It constitutes an upgrade to the Recursive Least Squares method presented in Gorbachev et al. (2017) and Moroz et al. (2017). Liu et al. (2018) studied the performance of different methods based on Recursive Least Squares, which in essence add robustness to the estimation by including several of the latest measurements in each iteration of the estimation. The extra layer of robustness is obtained at the cost of less sensitivity of the estimates with respect to jumps in the ground truth values of the parameters. The proposed method complements the findings of Liu et al. (2018) by capturing the different relative volatilities of the running resistance parameters.

In chapters 3 and 4 we observed that the lack of ground truth data on the values of the running resistance parameters hinders assessing whether the estimates are accurate or not. This is due to the fact that in normal operating conditions, the train motion model parameters may fluctuate and differ from the values provided by the rolling stock manufacturer. In this chapter, we test the proposed online algorithm in a simulated case study where we can exactly control the parameter variations and quantify accurately its performance. We generate train trajectories using a quadratic running resistance parameter that varies along the trajectory. Particularly, we mimic drifts, jumps and oscillations of this parameter while keeping the other two parameters constant, aiming to demonstrate that the proposed method can estimate the three parameters simultaneously and accurately. We obtain measurements of the location, speed and tractive and brake effort at a fixed sampling rate, and use them to calculate the value of the running resistance at every measurement time. First, we select the values of the forgetting factors in a numerical sce-

nario and show that the running resistance parameters are estimated accurately. Next, we analyze the performance of the proposed method for different values of the measurement sampling rate, observing that the accuracy of the method decreases for sampling rates larger than 5 s. We also test the robustness of the proposed method by adding Gaussian white noise to the observed value of the running resistance. Capturing the variability of the running resistance from real data allows to perform precise energy consumption and train trajectory calculations, so the proposed method will be relevant for railway practitioners and scholars with an interest in simulation, timetabling and energy-efficient driving. We also showcase an application of the proposed method, where we utilize the estimates obtained as input for a train trajectory optimizer to calculate energy-efficient train trajectories that can be used as reference in railway planning and operation.

The proposed method synergizes with several railway automation tools that require accurate estimates of the running resistance parameters as input. It is of particular relevance when deploying new types of rolling stock or new rolling stock compositions. For example, the impact of infrastructure elements like tunnels on the running resistance of new types of rolling stock can be quantified during real operation by means of the proposed method without the need to conduct expensive and capacity-consuming field tests. The performance of Train Trajectory Optimization algorithms is also enhanced when a more detailed description of the running resistance is available, as provided by the proposed method. Feedback-loop train control strategies, like Model Predictive Control, benefit from real time estimates of the running resistance parameters. The control strategy to be followed can be dynamically updated based on the current operating conditions if the parameter estimates show deviations from their a priori values. In practice, this allows Automatic Train Operation systems to be more precise and to adapt their performance to the existing operating conditions. In turn, Driving Advisory Systems can provide more accurate advice when using as input parameter estimates accurately generated in real time.

To sum up, the main contributions of this chapter are:

- We propose a new parameter estimation algorithm based on Recursive Least Squares with multiple forgetting factors that calibrates the running resistance parameters of a train in real time.
- We show that the proposed method is able to capture the different spatiotemporal variabilities of train running resistance parameters. None of the methods previously available in the railway literature consider the mentioned different parameter volatilities in the estimation. The method proposed in this chapter is therefore of practical relevance for potential applications where the running resistance should be estimated with high accuracy.
- We validate the proposed algorithm in simulated scenarios, which allows to quantify its performance more accurately than when only real measurements are analyzed.
- We illustrate a potential application of the proposed method for enhancing Train Trajectory Optimization, automation, and ultimately, railway planning and operation.

The remainder of this chapter is organized as follows. Chapter 5.2 introduces the proposed method. The results obtained are shown and discussed in chapter 5.3 and the final conclusions and remarks are summarized in chapter 5.4.

5.2 Methodology

5.2.1 Recursive Least Squares with multiple forgetting factors

The dynamics of a train are often described by Newton's second law (Hansen & Pachl, 2014):

$$m\rho\frac{dv}{dt} = F(t) - R(v(t), s(t)) - W(s(t)), \quad (5.1)$$

where m is the mass of the train, ρ is a dimensionless rotating mass factor, v is the speed, t the time, s the location of the head of the train, $F(t)$ the amount of tractive and brake effort applied by the engine at time t , $R(v(t), s(t))$ is the running resistance of the train, which is usually described as a quadratic function of the speed as follows:

$$R(v(t), s(t)) = R_0(s(t)) + R_1(s(t))v(t) + R_2(s(t))v^2(t), \quad (5.2)$$

with $R_0(s)$, $R_1(s)$ and $R_2(s)$ the running resistance parameters. $W(s(t)) = mgp(s(t))$ is the resistance to motion due to the track gradients. $g = 9.81 \text{ m/s}^2$ is the gravity acceleration and $p(s(t))$ is the mean gradient of the track along the train length, in permille. Unless stated, all the mentioned variables are in SI units.

We define the measurement sampling rate Δt as the time between two consecutive measurements. We consider it invariant along the train's journey. Then, the measurement times are defined as $t_{k+1} = t_k + \Delta t$, for $k \geq 0$. We assume that location, speed and tractive and brake effort measurements of the train are available at any time, so that for each time step t_k we have a new set of location $s_k = s(t_k)$, speed $v_k = v(t_k)$, and effort $F_k = F(t_k)$ measurements. We define the parameters to be estimated at time t_{k+1} , $\hat{R}_{0,k}, \hat{R}_{1,k}, \hat{R}_{2,k}$, where $\hat{R}_{m,k} = \hat{R}_m(s_k)$ for $m = \{0, 1, 2\}$. $R_{m,j} = R_m(s_j)$ for $m = \{0, 1, 2\}$ are the parameters that were estimated from earlier measurements. We use Eq. (5.1) to find a relation between the parameters $\hat{R}_{m,k}$ and the GPS location and speed measurements. We calculate the acceleration in the left hand side of the equation, $a_k = (v_{k+1} - v_k)/\Delta t$, and the resistance due to the track geometry, $W_k = W(s_k)$, from the available measurements. We assume that the measurement rate Δt is small with respect to the rate of change of the accelerations a_k and the variations of the track geometry resistance W_k , $k \geq 0$, so these two terms can be considered approximately constant between consecutive measurements. Defining the observed running resistance at time t_k , $y_k = F_k - W_k - m\rho a_k$, we rewrite Eq. (5.1) as an identification model:

$$y_k = \hat{R}_{0,k} + \hat{R}_{1,k}v_k + \hat{R}_{2,k}v_k^2 + \epsilon_k, \quad (5.3)$$

where ϵ_k gathers the effect of noise and any other train dynamics not included in the model at time t_k .

We define the objective function of the least squares problem with multiple forgetting factors by decomposing the usual least squares objective function in three terms, one for

each parameter, and weighting each of them with a different forgetting factor $\lambda_m \in (0, 1]$, for $m = \{0, 1, 2\}$.

$$J_k = \min_{\hat{R}_{0,k}, \hat{R}_{1,k}, \hat{R}_{2,k}} \frac{1}{2} \sum_{j=0}^k \lambda_0^{k-j} (y_j - \hat{R}_{0,k} - R_{1,j}v_j - R_{2,j}v_j^2)^2 + \frac{1}{2} \sum_{j=0}^k \lambda_1^{k-j} (y_j - R_{0,j} - \hat{R}_{1,k}v_j - R_{2,j}v_j^2)^2 + \frac{1}{2} \sum_{j=0}^k \lambda_2^{k-j} (y_j - R_{0,j} - R_{1,j}v_j - \hat{R}_{2,k}v_j^2)^2. \quad (5.4)$$

This decomposition allows the method to gauge which parameter is more likely to explain the changes observed in the running resistance (Vahidi et al., 2005). The forgetting factors weight each term and they are selected taking into account the expected variability of each parameter. If any of the forgetting factors is strictly less than one, the older observations are weighted exponentially less in the estimation than the newer observations. Notice that the variables $R_{m,j}$, $j = 0, \dots, k$ that appear in Eq. (5.4) will be conveniently substituted by the estimates $\hat{R}_{m,j-1}$ later in the derivation of the solutions of the Recursive Least Squares problem.

We can obtain the values of the estimates $\hat{R}_{m,k}$ that minimize Eq. (5.4). After a few manipulations that are analogous to those in the derivation of the solution of the standard Recursive Least Squares problem, we obtain

$$\hat{R}_{0,k} = \hat{R}_{0,k-1} + L_{0,k}(y_k - \hat{R}_{0,k-1} - R_{1,k}v_k - R_{2,k}v_k^2), \quad (5.5)$$

$$\hat{R}_{1,k} = \hat{R}_{1,k-1} + L_{1,k}(y_k - R_{0,k} - \hat{R}_{1,k-1}v_k - R_{2,k}v_k^2), \quad (5.6)$$

$$\hat{R}_{2,k} = \hat{R}_{2,k-1} + L_{2,k}(y_k - R_{0,k} - R_{1,k}v_k - \hat{R}_{2,k-1}v_k^2), \quad (5.7)$$

where $L_{m,k}$ are the gains of each estimate and $P_{m,k}$ stand for the covariance of the estimates at time t_k ,

$$L_{0,k} = P_{0,k-1}(\lambda_0 + P_{0,k-1})^{-1}, \quad (5.8)$$

$$L_{1,k} = P_{1,k-1}v_k(\lambda_1 + v_k^2 P_{1,k-1})^{-1}, \quad (5.9)$$

$$L_{2,k} = P_{2,k-1}v_k^2(\lambda_2 + v_k^4 P_{2,k-1})^{-1}, \quad (5.10)$$

$$P_{0,k} = (1 - L_{0,k})P_{0,k-1}\lambda_0^{-1}. \quad (5.11)$$

$$P_{1,k} = (1 - L_{1,k}v_k)P_{1,k-1}\lambda_1^{-1}. \quad (5.12)$$

$$P_{2,k} = (1 - L_{2,k}v_k^2)P_{2,k-1}\lambda_2^{-1}. \quad (5.13)$$

Substituting now $R_{m,k}$ by the estimates $\hat{R}_{m,k-1}$ (Georgiou & Lindquist, 2013), and rearranging the terms, we obtain the following recursive expression for the estimates of the running resistance parameters:

$$\begin{pmatrix} \hat{R}_{0,k} \\ \hat{R}_{1,k} \\ \hat{R}_{2,k} \end{pmatrix} = \begin{pmatrix} 1 & L_{0,k}v_k & L_{0,k}v_k^2 \\ L_{1,k} & 1 & L_{1,k}v_k^2 \\ L_{2,k} & L_{2,k}v_k & 1 \end{pmatrix}^{-1} \begin{pmatrix} \hat{R}_{0,k-1} + L_{0,k}(y_k - \hat{R}_{0,k-1}) \\ \hat{R}_{1,k-1} + L_{1,k}(y_k - v_k \hat{R}_{1,k-1}) \\ \hat{R}_{2,k-1} + L_{2,k}(y_k - v_k^2 \hat{R}_{2,k-1}) \end{pmatrix}. \quad (5.14)$$

When a new set of measurements is available, the observed running resistance y_k is calculated, the gains and covariances L_m and P_m are updated and the estimates $\hat{R}_m$ are calculated by means of Eq. (5.14). Still, this method requires an initialization of the estimates $\hat{R}_{m,0}$ and the corresponding covariances $P_{m,0}$. This initialization is executed by performing an Ordinary Least Squares regression over a handful of the first measurements.

5.2.2 Ensuring the non-negativity of the running resistance parameters

From a physical point of view, the running resistance parameters must be non-negative. However, the proposed method, as presented in chapter 5.2.1, does not theoretically guarantee their non-negativity. Although we did not observe negative estimates in our experiments, the proposed method can be modified to avoid producing negative estimates, following the methodology shown in Nascimento & Zakharov (2016). It results in a computationally-efficient procedure that can incorporate box constraints in the estimates. Non-negativity constrains are simply box constraints with arbitrarily high upper bounds. At each iteration of the algorithm, the non-negativity of all the parameter estimates produced by means of Eq. (5.14) is checked. Whenever at least one of them is negative, the Recursive Least Squares procedure must be solved using the box-constrained Recursive Least Squares algorithm based on dichotomous coordinate-descent presented in Nascimento & Zakharov (2016). The reader may refer to this paper and Arablouei & Dogançay (2012) for more details on constrained Recursive Least Squares estimation. Given the small scale of the estimation problem along with the small computational complexity of both the method proposed in this chapter and its constrained modification (Arablouei & Dogançay, 2012), finding negative estimates would represent no computational burden. The resulting method conserves its efficiency and practicality for real time running resistance parameter estimation. Although both methods provide the same level of accuracy, obtaining negative estimates with the proposed method may imply data issues like that the measurements analyzed are too noisy, highly leveraged, that wrong measurements have been introduced in the estimation loop, or that the measurements have not been properly filtered.

5.3 Results and discussion

5.3.1 Setting up the case study

We demonstrate the performance of the proposed method in three numerical scenarios. Several jumps, drifts and oscillations are introduced in the quadratic running resistance parameter in each scenario, aiming to replicate the impact of spatiotemporal phenomena on the train dynamics. We use the first scenario as a test to select the right combination of forgetting factors and to show that the proposed algorithm is able to estimate the running resistance parameters accurately. A deeper analysis is performed in the second scenario, where R_2 shows different variations. Particularly, we analyze the performance of the method for different measurement sampling rates and its robustness against additive noise. In turn, the third scenario is used to show the potential application of the esti-

mates produced by the proposed method for optimizing train trajectories. The running resistance parameters R_0 and R_1 are constant in each of the three scenarios. The first and third scenario share the same variations of R_2 , while the second one shows a different set of variations. All scenarios share the same gradient profile, where gradients are piecewise constant with different positive and negative values to test the performance of the proposed method over a wide variety of gradient values. We integrate the equations of motion numerically using as input the precalculated running resistance parameters and extract location, speed and tractive and brake effort measurements. The integration time step is set to 0.0001 s and the measurements are subsampled to match the measurement rate Δt . Throughout this chapter, we perform the simulations modelling the dynamics of an Intercity train VIRM-IV operated by the Dutch railways NS (Scheepmaker et al., 2020a). The train mass m is 257859 kg, the rotating mass factor ρ is 1.06 and the train is 200 m long. The train accelerates by applying a speed-dependent tractive effort $F(t) = \min(142600, 1438000/v)$ N. When it brakes, the brake effort is constant and equal to $0.5m\rho$ N. At each location along each journey, the gradient is averaged over the train length to obtain the mean gradient influencing the train dynamics. The speed and gradient profiles used in each scenario are shown in Figure 5.1. The speed profiles generated for estimating the running resistance parameters in the first, second and third scenario are shown in blue in the upper, middle and lower plots, respectively. The first two consist of an initial acceleration regime, a long coast and a final brake, while the third one also includes an intermediate constant speed regime followed by a short brake at high speed and a reacceleration. In each of the three plots, the track gradients are depicted in pink.

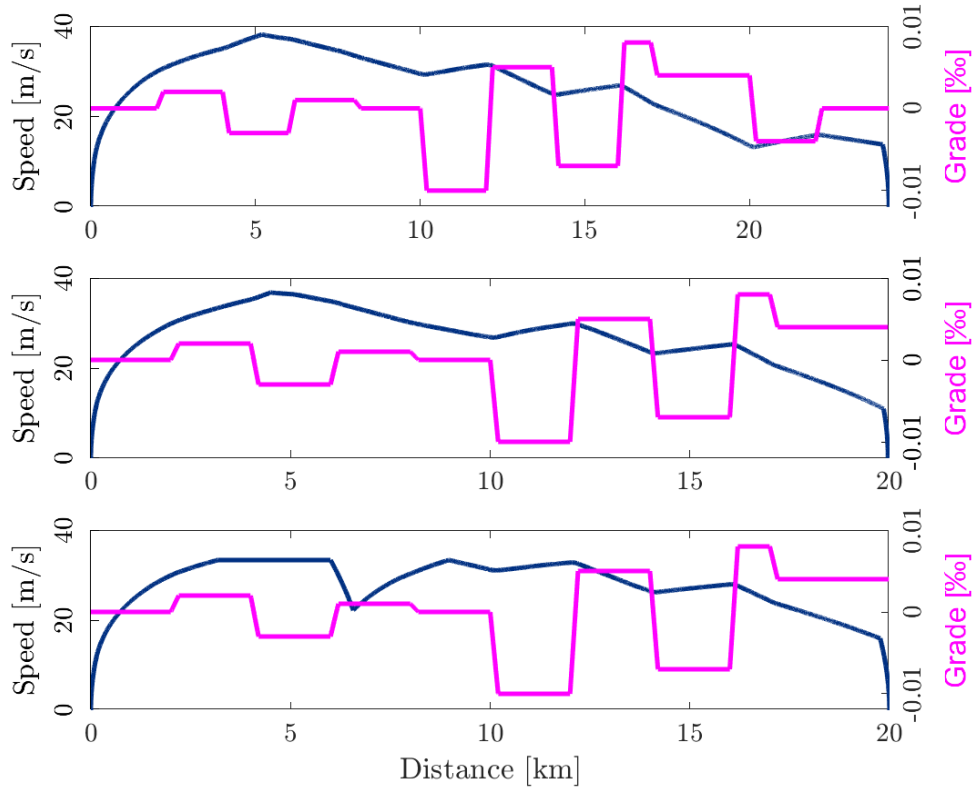


Figure 5.1: Speed and gradient profiles used in the first (upper) and second (middle) and third (lower) scenario.

5.3.2 Selecting the forgetting factors and testing the method

We proceed to select the forgetting factors in the first scenario, where R_0 and R_1 are constant through the whole trajectory and R_2 shows several jumps, drifts and oscillations of different frequencies. We model R_2 as a continuous function along the train trajectory. This means that any spatial phenomena leading to a jump in R_2 is interpolated over the length of the train as it passes through the location where the jump would take place. Intuitively, parameters that show a larger and more frequent variability are typically associated to lower forgetting factors. The lower the value of a forgetting factor, the more sensitive the estimation is to parameter variations. In contrast, forgetting factors close to 1 lead to estimates with more inertia, and the information included in the most recent measurements may not have enough weight to change the value of the estimates. Therefore, the forgetting factors of the error terms of R_0 and R_1 , λ_0 and λ_1 , should be close to 1, while the one associated to R_2 , λ_2 , should be lower than the other two. We fix the values of λ_0 and λ_1 , and gradually decrease the value of λ_2 until the three running resistance parameters are accurately estimated and decreasing further λ_2 does not further reduce the error in the estimate of R_2 . We measure the error in R_0 and R_1 in percentage of error. The error in R_2 is calculated as the Root Mean Square Error (RMSE). We repeat this for several values of λ_0 and λ_1 , finding that several combinations of the values of the forgetting factors lead to similar results. For simplicity, we choose the configuration $\lambda_0 = \lambda_1 = 1$ and $\lambda_2 = 0.8$, which leads to accurate estimates and is in line with the values used in similar studies (Vahidi et al., 2005). The measurement sampling rate is $\Delta t = 0.1$ s. Figure 5.2 shows that the running resistance parameters are accurately estimated in the first scenario.

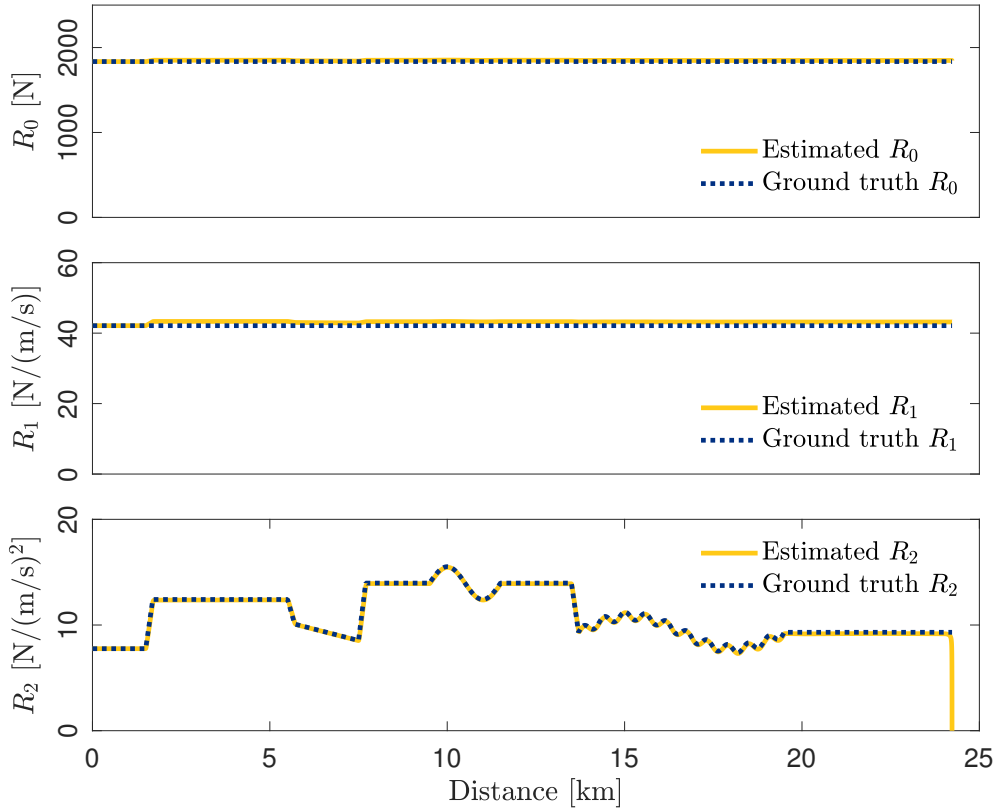


Figure 5.2: Estimation of the running resistance parameters in the first scenario.

The parameter estimates are represented by a yellow line, while the ground truth data is depicted as a blue dotted line. The estimates of both R_0 and R_1 are approximately constant, showing slight variations that resemble the dynamics of R_2 . Still, the maximum error in absolute value between R_2 and its ground value is 3.95%, while the mean errors in R_0 and R_1 are 0.68% and 2.33%, respectively. The maximum errors in R_2 take place close to the largest jumps shown by the values of the parameter. This is due to the inertia in the dynamics of the estimates, as the proposed method estimates the dynamics with a small delay despite the larger weight of the new measurements in the estimation. This phenomena will become more evident when we explore the effect of larger measurement sampling rates. The estimation of R_2 is wrong during the final brake. Braking impulsive disturbances were recently identified in Chor et al. (2023) as one of the main drawbacks of this parameter estimation method. Consequently, we ignore the final brake in the calculation of the errors throughout the chapter. Later in this chapter we will explore whether braking at higher speeds also leads to wrong estimates of R_2 . Still, trains do not brake frequently and the estimation could be reinitialized after the train stops braking. Alternatively, the dynamics of R_2 at the locations where the train brakes to a standstill could be estimated during a new trajectory where the train departs in the opposite direction and applies traction instead of brake.

Initializing the estimates and their covariance by performing a Least Squares Regression over the first 10 measurements led to accurate recursive estimates of the running resistance parameters, and enlarging this initial window did not enhance the accuracy. Smaller windows were found to initialize the estimates correctly, but not the covariances since during the first recursive iterations the estimates deviated briefly from their ground truth values.

5.3.3 Assessing the robustness of the method with respect to the measurement sampling rate and additive Gaussian white noise

In the second scenario we study the performance of the proposed method for different values of the measurement sampling rate and its robustness against Gaussian white noise added to the observations of the running resistance. R_0 and R_1 are again constant, but their values differ from the ones selected in the first scenario. In turn, R_2 shows fast and slow drifts, jumps and oscillations. Particularly, the dynamics of R_2 between km 6.5 and 9.5 is a sum of three oscillations with different frequencies. We considered 7 different measurement rates, 0.1, 0.2, 0.5, 1, 2, 5 and 10 s, subsampled the available measurements accordingly and estimated the parameters. We observed that the estimation failed to estimate the running resistance parameters correctly for the two largest measurement rates. The identification model in Eq. (5.3) is based on Euler's forward method for integrating the equations of motion. If the integration step, which in this case is equal to the measurement rate, is large, the numerical integration errors increase, biasing the estimation. Therefore, the results presented next describe the performance on the measurement sampling rates lower than or equal to 2 s.

Figure 5.3 exemplifies the performance of the proposed method in the second scenario for three values of Δt : 0.1 s, in yellow, 0.5 s, in pink, and 2 s, in grey. The true values

of the parameters are again represented by a dashed blue line. The estimation during the final brake is removed in all cases. We observe that the estimation is more accurate for the smaller measurement rates. The grey line shows more inertia: the estimation reacts slower to sudden variations of R_2 for the larger values of Δt . This inertia effect can be mitigated by lowering the forgetting factor λ_2 , at the cost of having estimates more sensitive to measurement outliers and noise. We also observe that the estimates of R_2 corresponding to the higher rates show some inaccuracies in the form of peaks. The initial peak is produced by the initialization of the covariance, as we mentioned earlier. However, the method is able to reduce this error along the way. We noticed that a larger initial estimation window may eliminate this phenomena. Several intermediate peaks also arise in intermediate points of the trajectory. Particularly, some stand out around kilometers 4, 10, 12, 14 and 16. These peaks are not related to the covariance of the estimates, but to the integration error. These are locations where the track gradients change, and the integration step is large enough to introduce errors that bias the estimation.

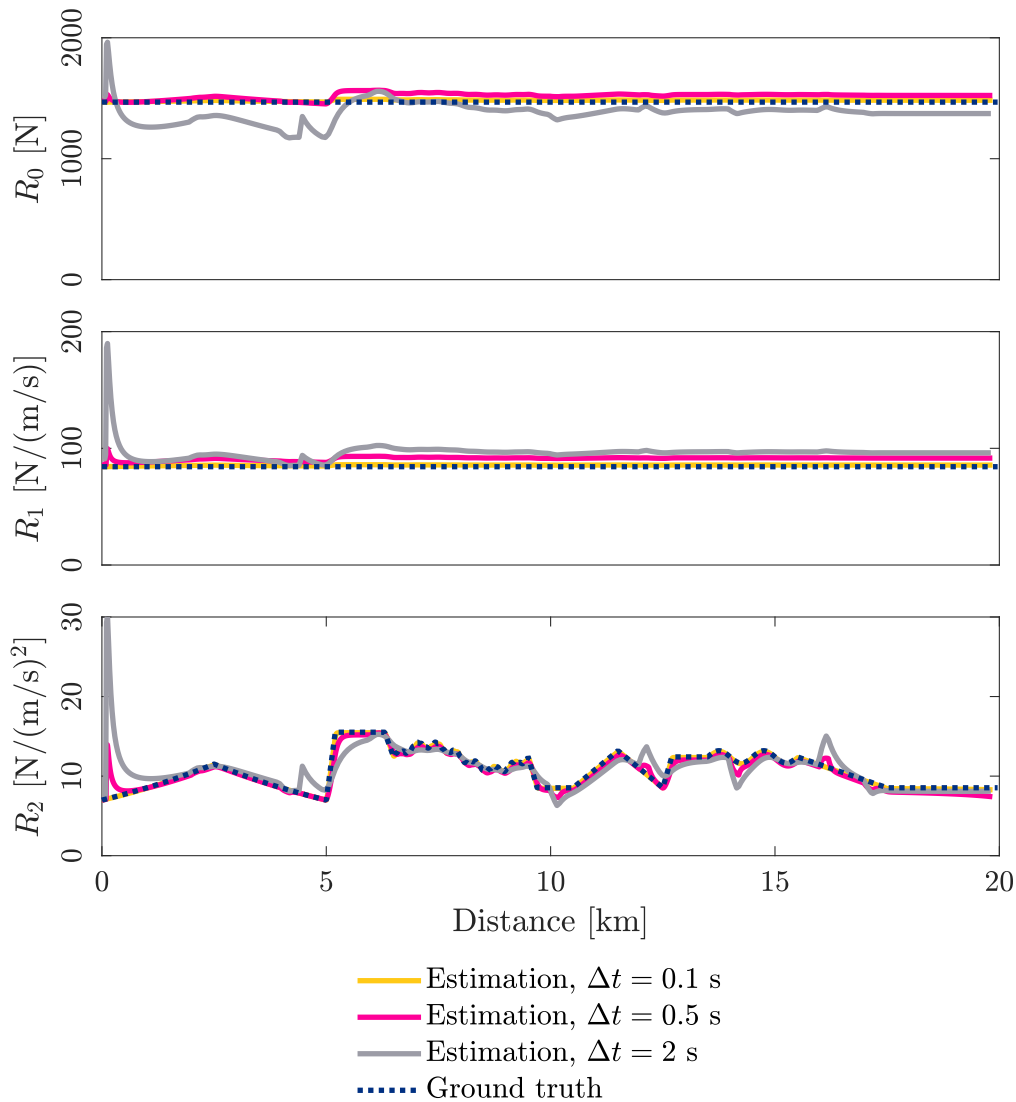


Figure 5.3: Estimation of the running resistance parameters in the second scenario for different values of the measurement sampling rate.

We quantify now the accuracy of the estimation for different values of the measurement rate. Figure 5.4 shows the Root Mean Square Error (RMSE) of the estimates of R_2 for different values of the measurement sampling rate. We did not include the initial and final part of the trajectory where the estimation was not accurate due to the covariance initialization and the final brake, respectively. As observed before, the RMSE increases with the sampling rate.

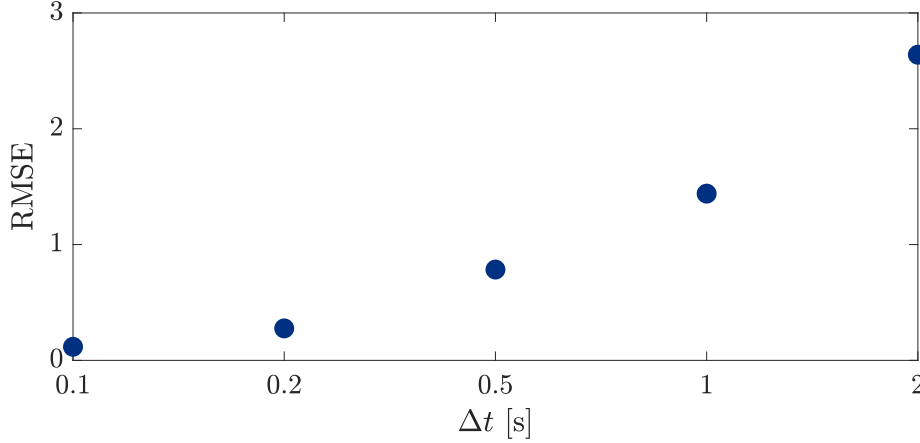


Figure 5.4: Root Mean Square Error (RMSE) of the estimates of R_2 for different values of the measurement sampling rate.

When it comes to R_0 and R_1 , their mean errors in absolute value also increase with the sampling rate. Particularly, the mean error of R_0 grows from 0.96% for $\Delta t = 0.1$ s to 3.0% for $\Delta t = 1$ s, although it decreases to -5.6% for $\Delta t = 2$ s. In turn, the mean error of R_1 rises from 1.5% to 14.9%. Considering that the estimates are drawn from errorless measurements, sampling rates lower than 1 s (for example 0.1, 0.2 or 0.5 s) should be used in practice to estimate the running resistance parameters accurately.

Figure 5.5 shows the distribution of the errors of the estimates of R_2 in % of the ground truth value at each measurement time. The error distributions obtained using measurement times $\Delta t = 0.1, 0.2, 0.5, 1$ and 2 s are depicted in yellow, dark blue, magenta, light blue and grey, respectively. Again, we discarded the initial errors due to the covariance initialization and those that arise during the final brake. We observe that R_2 is generally underestimated as a consequence of the slight overestimation of the other two running resistance parameters. Furthermore, large sampling rates tend to bias the estimation of R_2 more towards values lower than the real values of R_0 . Large sampling rates also lead to error distributions with fatter and longer tails due to the mentioned estimation inertia and the integration errors when the track gradients change. Still, for small sampling rates the estimates remain within a 5% distance with respect to the real values of R_2 .

We add Gaussian white noise to the observations of the running resistance y_k in Eq. (5.3) to assess the robustness of the method. The measurement sampling rate is fixed to $\Delta t = 0.1$ s. We vary the standard deviation of the noise from 0.001% to 1% of the maximum value of the observations y_k and calculate the RMSE of R_2 in 100 different noise realizations for each value of the standard deviation. Figure 5.6 shows the distributions of the RMSE calculated over 100 realizations of each value of the noise standard deviation.

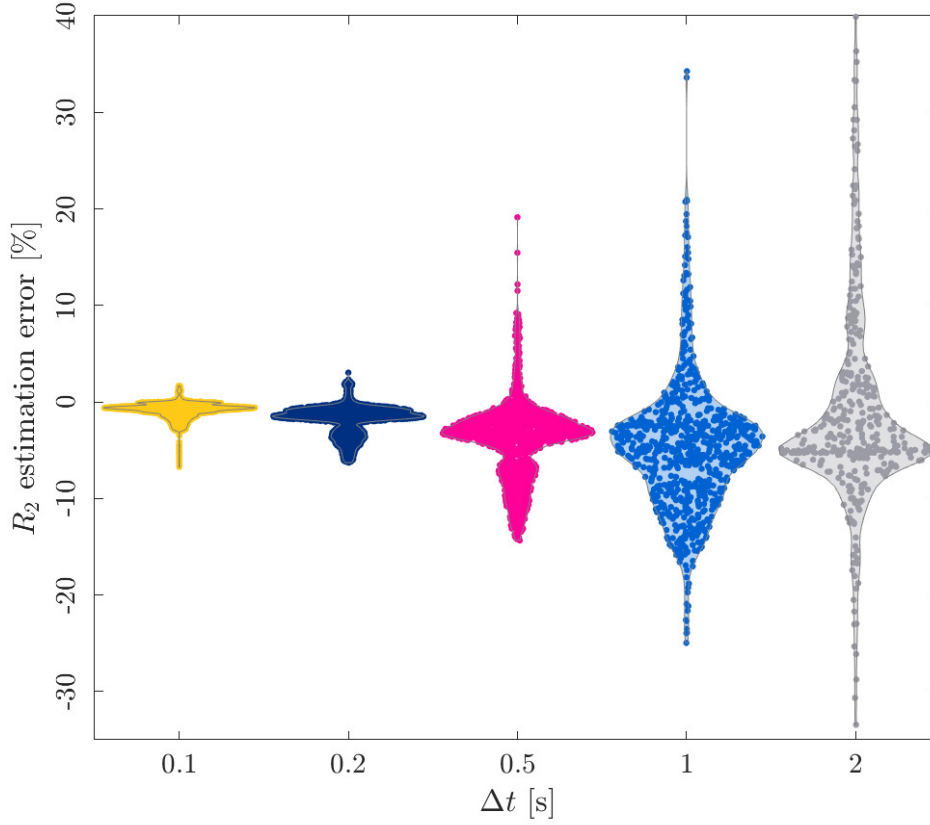


Figure 5.5: Distributions of the estimation errors of R_2 for different values of the measurement sampling rate.

The vertical axis represents the RMSE in logarithmic scale. We observe wide variations for the noisiest trajectories: in some cases R_2 is accurately estimated all over the trajectory, while sometimes it is estimated with a large bias. This highlights that the performance of the proposed method is sensitive to the accuracy of the least squares initialization of the parameters. If the parameter initialization is not correctly performed, any initial bias is dragged through the rest of the trajectory. The initial Least Squares Regression is widely known to be not robust against noisy or anomalous data, so when noise leads to a wrong initialization of the parameters R_0 and R_1 , the Recursive Least Squares estimation with multiple forgetting factors is not able to correct this bias. This phenomena is more likely to happen when analyzing very noisy data, which explains the results in Figure 5.6. We may conclude then that an accurate initialization of the running resistance parameters is essential to guarantee the performance of the proposed method. Regarding the least noisy observations, the results show that the proposed method is able to estimate accurately the quadratic running resistance parameter. In practice, the initialization window shall be sufficiently large, the measurement rate must be kept small, preferably below 1 s, and filtering the location, speed and effort measurements would also be beneficial for estimating the running resistance parameters accurately.

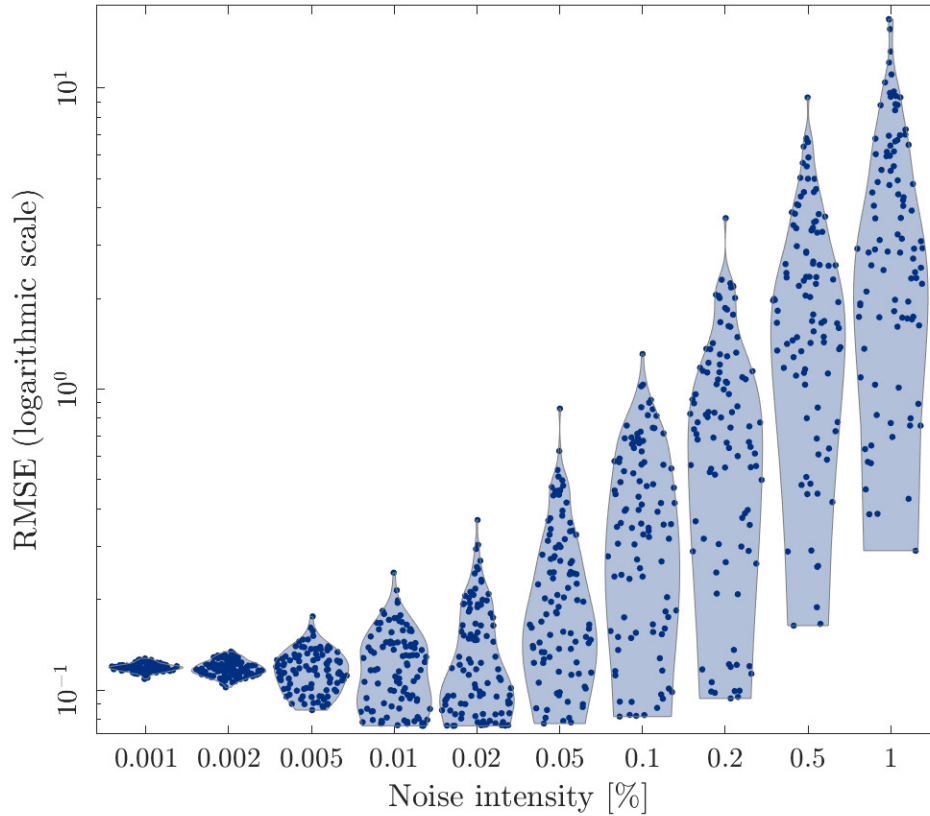


Figure 5.6: Distributions of the estimation errors of R_2 for different standard deviations of added Gaussian white noise. The sampling rate is $\Delta t = 0.1$ s.

5.3.4 Adaptive parameter estimation for Train Trajectory Optimization

Last, we study a potential application of the estimates obtained using Recursive Least Squares with multiple forgetting factors: Train Trajectory Optimization. This technique is used to calculate reference energy-efficient trajectories for timetabling and railway operation. Deviations between these reference trajectories and the ones actually performed are often observed due to the difference between the real and the estimated values of the running resistance parameters and the resistance to motion produced by the track geometry. These deviations become particularly evident when the train coasts, that is to say, when no traction or brake is applied and the train and the dynamics of the train are influenced only by the running resistance and the track geometry. During the cruise regime, the amount of traction applied should equal the running resistance to keep a constant speed. Therefore, an accurate calibration of the running resistance is essential to guarantee the accuracy and effectiveness of optimal trajectories.

The running resistance parameters are often considered constant through the whole trajectory, neglecting the impact of spatiotemporal phenomena on the train dynamics. The quadratic running resistance parameter is often modelled as a piecewise constant function of the train location, with jumps located when the train enters or exits tunnels

(Hansen et al., 2017). However, the dynamics of the running resistance parameters may be more complex than this. For instance, the cross-section of tunnels and the roughness of their surfaces are usually uneven throughout their whole length, producing small drifts and fluctuations in R_2 . Introducing a more complex model of the quadratic running resistance parameter in the Train Trajectory Optimization process would enhance the accuracy of the calculated trajectories and, ultimately, the accuracy of railway planning and operation.

In this subchapter we aim to motivate the need to model the running resistance of trains accurately for calculating optimal train trajectories. Three different models of the ground truth curve of R_2 introduced in Figure 5.2 have been estimated and used as input for calculating optimal train trajectories in the third scenario described in chapter 5.3.1. We utilize an algorithm that transforms the Train Trajectory Optimization problem into a convex quadratically-constrained linear programming problem. This algorithm will be introduced in chapter 6. This algorithm divides the distance to be covered by the train in small intervals. The state of the train, namely its speed and running time, is calculated at the points that separate each pair of intervals. The values of R_2 at each of these points are interpolated linearly from the time series of the estimates of the three models.

Figure 5.7 shows three optimal train trajectories calculated using estimates of R_2 that are modelled with different levels of detail. The running time in the three trajectories is 900 s and the track is 20km long. No speed limits are considered. The track has been discretized in 20000 intervals to guarantee the accuracy of the numerical integration later in this chapter, when we will track the optimal trajectories using the real values of the running resistance parameters. The upper plot shows the optimal speed-distance diagrams, the middle one the optimal amount of tractive and brake effort to be applied at each location, and in the lower one the values of the estimates of R_2 used at each location for each one of the models. The blue lines represent the model in which R_2 is estimated at regular time intervals using Recursive Least Squares with multiple forgetting factors. The values of R_0 and R_1 are constant along the whole trajectory, and their values are selected as the mean of the estimates obtained. The dotted yellow line represents the optimal trajectory calculated using an R_2 estimated as a piecewise constant function of the train location. In this model, the track is divided in 5 sections. The boundaries between consecutive sections are placed at the locations where the largest jumps in the real values of R_2 take place. These are the kilometric points 1.5, 5.5, 7.5 and 13.5. We used Ordinary Least Squares regression to estimate the running resistance parameters in each section, however, this method led to wrong estimates in every section except for the first one. This is due to the high leverage of the observations of the running resistance at those locations where R_2 is not constant. High-leverage observations influence significantly an Ordinary Least Squares regression, and in this case the estimates are biased to the extent that some of them are negative and all deviate from their ground truth values. To address this issue, we fixed R_0 and R_1 to their real value in every section, and used Ordinary Least Squares to estimate only R_2 . The resulting estimated model of R_2 is represented by a dotted yellow line in the lower plot of Figure 5.7. Last, the dashed pink line shows the trajectory calculated when R_2 is considered constant along the whole trajectory. Its value is equal to its open-air value, as specified by the rolling stock manufacturer. This model mimics disregarding the impact of spatiotemporal phenomena on R_2 .

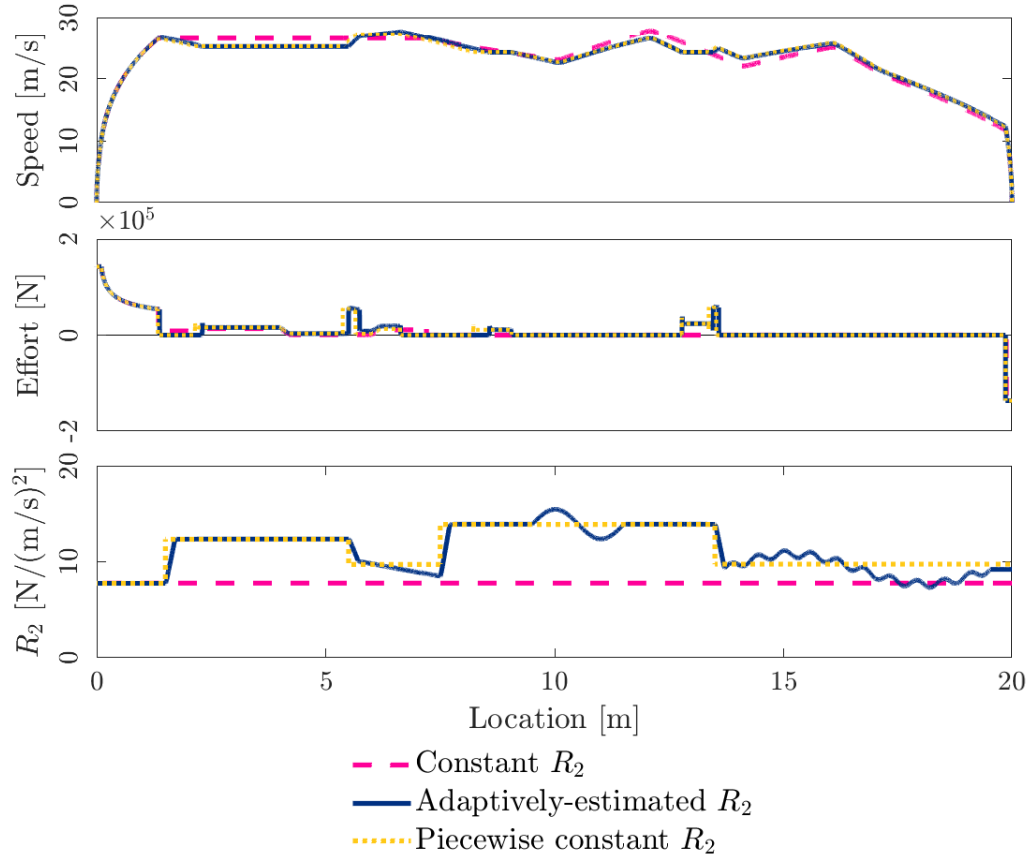


Figure 5.7: Optimal train trajectories calculated using estimates of R_2 modelled with different levels of detail.

As we can observe in the lower plot in Figure 5.1, the trajectory used to estimate the running resistance parameters in the third scenario has a cruise and a brake phase at high speeds between kilometric points 4 and 6.5. We compared the accuracy of the estimates of R_2 obtained using Recursive Least squares with multiple forgetting factors that are shown as a full blue line in the lower part of Figure 5.7 with respect to their ground truth values, which are represented as a dashed blue line in the lower plot of Figure 5.2. The comparison shows that the cruise and brake phase at high speeds did not jeopardize the accuracy of the estimates. The proposed method only struggles to keep the accuracy of the estimates when the train brakes at low speeds. Regarding the optimal trajectories in Figure 5.7, the trajectory produced using R_2 constant differs significantly from the other two, which are very similar. Comparing the estimates of R_2 produced by Recursive and Ordinary Least Squares in the lower plot of Figure 5.7, we observe that the main oscillations of R_2 are neglected by the Ordinary Least Squares regressions. Still, although the main trends of R_2 are captured in the piecewise constant description of R_2 , we have to take into account that Ordinary Least Squares failed to produce accurate estimates when estimating simultaneously the three running resistance parameters. This issue is solved by means of Recursive Least Squares with multiple forgetting factors.

In practice, the trajectories shown in Figure 5.7 are simply references that have to be tracked. The estimation errors in the running resistance may produce difference between the reference and the realized trajectories. We assess this by integrating the equations of motion numerically at each of the locations s_j , $j = 0, \dots, N$, where the train trajectory optimizer calculates the state of the train. We also use the optimal amount of tractive and brake effort to be applied at each location $F(s_j)$ shown in the middle plot in Figure 5.7, and the real values of the running resistance parameters R_0 , R_1 and $R_2(s_j)$, $j = 0, \dots, N - 1$. Furthermore, we observe that the gradient of the track $p(s)$ can be described as the derivative of the height of the track h with respect to the train location, $p(s) = dh(s)/ds$. Finally, we obtain the following recursive equations for calculating the tracked speed and the running time:

$$v_{j+1}^2 = v_j^2 + \frac{2}{m\rho}((F(s_j) - R_0 - R_1 v_j - R_2(s_j) v_j^2)(s_{j+1} - s_j) - mg(h(s_{j+1}) - h(s_j))), \forall j = 0, \dots, N - 1. \quad (5.15)$$

$$t_{j+1} = t_j + (s_{j+1} - s_j)/v_j, \forall j = 0, \dots, N - 1. \quad (5.16)$$

Particularly, we determine the stopping error at destination, the difference between the expected and the actual arrival times, and the amount of energy consumed. If the train arrives at the original stopping location with speed higher than zero, we assume that the train keeps on braking until stopping to a standstill. Tracking the trajectory calculated using the recursive estimates R_2 shows that the train would stop 0.35 m beyond the expected stopping point and would arrive 1.3 s earlier than expected. Tracking the trajectory calculated using piecewise constant values of R_2 also leads to stopping later than expected and to arriving slightly early. In this case, the train would stop 6.3 m further than expected, and arrive 2.2 s early. The results show that both trajectories can be tracked accurately and that the errors in the stopping location and the arrival time are small enough so that in practice they can be reduced by means of a more advanced tracking strategy. The tracking performance is slightly better when the most complex model of R_2 is used to calculate the reference trajectory, however, producing accurate train trajectories relies on an adequate performance of the estimation method used, and Ordinary Least Squares initially failed to produce reliable estimates of the three running resistance parameters. Regarding the expected energy consumption of both trajectories, the optimal trajectory calculated using the piecewise constant model of R_2 leads to 0.6% higher energy consumption. Tracking the trajectory calculated using constant R_2 leads to larger stopping and arrival time errors than the other two trajectories: the train would stop 117 m early. The train would be required to reaccelerate to stop at the expected point of the platform. The delays produced by the underestimation of the running resistance and the correction of the stopping point add up to 173 s. More advanced driving strategies might still drive the train to its destination, at the expense of a higher energy consumption than expected, and probably to reaccelerations and brakes that might jeopardize passengers' comfort. If the tracking strategy used in a Driver Advisory System or in Automatic Train Operation with Grade of Automation 2 is naive like the one used in this chapter, the train driver would have to actively intervene to prevent undesired stops or other disturbances produced by the errors introduced by wrong running resistance parameter estimates.

5.3.5 Challenges and limitations of a potential practical application

The practical implementation of the proposed method should be subject to validation in field tests that are beyond the scope of this chapter. Hereby we provide several practical recommendations to overcome the potential challenges that may arise in a real implementation of the method.

Determining the optimal size of the initialization window and the values of the forgetting factors is the first of these challenges. We observed that a wrong initialization of the estimates may jeopardize the performance of the method. Field tests would be required to determine the size of the initialization window. It should be large enough to allow to initialize the running resistance parameters accurately and include at least 10 measurements, but not excessively large to introduce large fluctuations of the running resistance parameters that may bias the initialization. We noticed that small initialization windows are not suitable for initializing the covariance of the estimates accurately. Its size will depend also on the measurement sampling rate and the quality of the measurements. Usually, the initialization of the estimates will take place during the first seconds of the train departure, so we recommend using the data measured during the initial constant tractive effort phase that typically takes place in energy-optimal train trajectories. When it comes to fine-tuning the forgetting factors, we recommend determining them by means of detailed simulations on digital twins of rolling stock with different compositions.

The quality of the input data, namely the available speed, location and tractive and brake effort measurements, also plays an important role in a practical implementation of the proposed method. It affects both the performance of the estimation and the size of the initialization window. Our experiments show a loss of accuracy when analyzing sparsely-sampled data. Our recommendation is to use data with sampling time lower than 1 s. In this regard, the quality of the measurements is subject to the sensors available onboard. Automation will boost the implementation of higher quality sensors for measuring the train speed, location and tractive and brake effort. Such sensors will produce less noisy measurements, although a filtering procedure is in all cases necessary to remove part of the noise before analyzing and storing the generated measurements. In our previous research on train motion model calibration we observed that tractive and brake effort measurements are critical to estimate the motion model parameters along the whole train trajectory, however, they are usually not available (Cunillera et al., 2022, 2023a). This is also a potential limitation of the applicability of the proposed method. When effort measurements are not available, only coasting regimes, that is to say, the parts of the trajectory where the train applies zero traction and brake, can be analyzed to determine the running resistance parameters accurately.

Our results show that the performance of the proposed method decreases when analyzing data measured while the train brakes at low speeds. On the one hand, these low speed brake regimes usually take place before stopping, so the estimation can be reinitialized when departing from the stop. On the other hand, the estimates have lower accuracy in those locations where the brake measurements were recorded, and are therefore not reliable for practical purposes. The implication of this fact on automation and Train Trajectory Optimization is minimal because this phenomenon takes place near the end of a trajectory or can be solved by reinitializing the estimates. We recommend analyzing

data recorded when the train runs in the opposite direction to estimate the parameters in the locations where the train applies brake at low speeds and to consider in practice the running resistance parameters approximately constant during these final brake regimes.

One of the main applications of the proposed method is for enabling the practical implementation of automation tools. In this chapter we have showcased how the proposed method can boost the accuracy of Train Trajectory Optimizers by providing a more detailed description of the running resistance. The continuous monitoring of the running resistance allows to implement real time control strategies based, for instance, on Model Predictive Control. We believe that this method will be also of practical relevance for Driver Advisory Systems. Still, the proposed method produces as outputs time series of parameter estimates, while not every algorithm used in automation utilizes as input such a high detailed description of the running resistance. For example, the driving advice algorithm introduced in chapter 7 utilizes as input parameters that are piecewise constant along the train trajectory. Parameter estimates that are sampled with high frequency will increase the computational burden of the driving advice algorithm and jeopardize its real time performance. In some cases, a post-processing tool may be required to subsample the estimates or to adapt them to the required input of each algorithm.

5.4 Conclusion

Estimating the running resistance parameters allows to perform accurate calculations of the energy consumed by trains and to optimize train trajectories effectively. Still, a simultaneous estimation of the running resistance parameters that captures the variability along the trajectory of the parameter that models the aerodynamic drag constitutes a challenge that has not been fully addressed yet. The lack of ground truth data regarding the exact values of the running resistance parameters hinders the assessment of the accuracy and performance of any estimates.

In this chapter we have addressed these issues by proposing a running resistance parameters estimation method based on Recursive Least Squares with multiple forgetting factors. It has been widely explored in road vehicle research, particularly in the simultaneous estimation of a vehicle mass and the road gradients. However, forgetting factors had not been used yet in estimation of a train's running resistance (Cunillera et al., 2023a). We have validated the proposed method in a computational case study, where we can select the variability of the running resistance parameters in several scenarios and quantify the accuracy of the estimation.

We have shown that the proposed method can estimate the quadratic running resistance accurately, provided that the estimates are carefully initialized. The forgetting factors are selected so that the method is more sensitive to the variations of the quadratic running resistance parameter than to those of the other two, which is in line with the expected physical variability of the parameters. The robustness of the method with respect to the measurement sampling rate and additive Gaussian noise in the observed running resistance has been demonstrated and analyzed statistically. Furthermore, we illustrated that the calculated estimates can be used as input for Train Trajectory Optimization. We calculated optimal train trajectories using as input estimates of the quadratic running resistance parameter that model it with different levels of detail. Among the models

considered, the estimates produced by the proposed method show the highest level of detail. We assessed how these optimal train trajectories can be tracked, observing that introducing a more detailed description of this parameter in the Train Trajectory Optimization problem shows several advantages with respect to using less-detailed models of the running resistance. Moreover, we have also analyzed the main challenges that would arise in a practical implementation of the method.

This method has several advantages over other existing running resistance estimators: it is computationally inexpensive, conceptually simple, and easy to be implemented and initialized. It provides reduced sensitivity to measurement outliers due to the introduction of multiple forgetting factors. It is worth noticing that we observed that the proposed method is sensitive to the initialization of the estimates and their covariances, that it struggles to keep the estimation accurate when the train brakes at low speeds, and that its accuracy is lower for high measurement sampling rates. Our results support the analysis in Chor et al. (2023), where the authors mention that large sampling rates and biases in the initialization of the estimates lower the accuracy of the proposed method. Also, the running resistance parameters should be nonnegative (Cunillera et al., 2023a). The proposed method can be adapted to ensure the nonnegativity of the estimates (Nascimento & Zakharov, 2016).

The method presented in this chapter could be explored further, as several research questions remain open. Although the method has been shown to be accurate in numerical scenarios, it is still to be validated using real data from field tests or from operational data. Such tests could particularly focus on analyzing the impact of tunnels on the train dynamics. A comparative study evaluating and benchmarking this and other existing on-line running resistance calibration methods would be of significant scientific and practical relevance. Furthermore, the Recursive Least Squares estimation routine could alternatively be substituted by a more robust estimation procedure, and the performance of both approaches could be compared and analyzed.

Chapter 6

Volatility of optimal train trajectories under parameter variations

6.1 Introduction

Energy-efficient driving is one of the most popular sets of techniques and strategies that are used in the railway industry to save energy (Scheepmaker et al., 2017). The current sustainability goals constitute a major challenge for the industry. For example, the European Commission aims to reduce by 90% the Greenhouse Gas (GHG) emissions of the transport sector by 2050 (European Commission, 2020). In this race, railways are departing from an advantageous position, as they emit considerably less GHG than other freight and mass-passenger transport modes (EEA, 2022). Still, energy-efficient driving can reduce 20% to 30% the energy consumed in railway operation (ON-TIME, 2014). Therefore, energy-efficient driving’s role for meeting the sustainability goals will be crucial. Beyond emissions and energy consumption, energy-efficient driving can also contribute to increasing the punctuality rates, to reducing the operational costs and to optimizing the capacity of the network. All this promotes the development of a stronger and eco-friendly corporate branding for railway operators.

A wide variety of techniques can be found under the energy-efficient driving label. Train drivers are instructed to save energy by coasting (Luijt et al., 2017). The expertise of drivers (van der Meulen, 2008) and historical data (Yin et al., 2016a) are used to develop energy-efficient speed control algorithms. Coasting advice is calculated from real-time location and speed information (Cunillera et al., 2023b). Regenerative braking transforms part of the kinetic energy lost while braking into electricity that can be pumped back into the catenary (Scheepmaker & Goverde, 2020). Train Trajectory Optimization (TTO) is used to calculate the reference speed of a train along its journey so that it consumes the minimum amount of energy while arriving on time (Albrecht et al., 2016b). TTO is usually a core module of Driver Advisory Systems and Automatic Train Operation (Howlett et al., 1994), which help improving the punctuality rates while contributing to saving energy (Yin et al., 2017).

Regarding TTO, mathematical control theory shows that an optimal trajectory, namely the one with minimum energy consumption for a given target running time, combines up to four different driving regimes: maximum traction, cruising at a constant speed, coasting and maximum braking. TTO can therefore be transformed into a new problem: determining the optimal switching points between driving regimes (Albrecht et al., 2016b). The mentioned optimal trajectory is determined by the train’s dynamics and by the characteristics of the infrastructure. TTO can incorporate these dynamics in the calculations through a train motion model drawn from Newton’s second law.

The train motion model is parametrized by several physical characteristics of the train: the mass, the maximum tractive and brake effort, the maximum tractive power, the resistance to motion and the percentage of kinetic energy lost in the movement of the rotating parts of the train. Two different trains will likely show different characteristics (Norris, 2008), so TTO requires an accurate calibration of the train motion model parameters for producing accurate reference trajectories. A survey of techniques for calibrating train motion model parameters can be found in chapter 2. Furthermore, when considering a single train, variations in the train motion model parameters are also observed. The exact values of the parameters depend on the operating conditions. For example, the wind strength and direction affect the running resistance, being larger under headwinds and lower under tailwinds (Trivella et al., 2021). Wet rails or fallen leaves on rails may reduce the adhesion between wheel and rail, lowering the effective maximum traction and brake that can be applied. The maximum tractive power is influenced by the engine wear and the available power in the catenary, which may vary in time. The mass of a train depends on the number of passengers on-board and on the load. Particularly, the passenger demand varies appreciably from peak to off-peak hours, leading to fluctuations in the consumed energy (Wang et al., 2017). Inaccuracies in the characterization of train motion model parameters may lead to deviations from the reference trajectory that the train is aiming to follow, requiring a recalculation of the reference trajectory if such deviations are large (Zhou et al., 2021).

The parametric variability of the train motion model has an impact on optimal trajectories, although the extent of this impact has not been quantified yet in the existing literature. Optimal trajectories can be characterized by the locations in which the driving regimes switch, the cruise speed, and the speed at which the train starts braking. Since the running time is constrained in the TTO problem, variations in the train motion model parameters will lead to differences in the optimal cruise and brake speeds, and in the energy consumption. The existing literature on TTO has mainly considered the train motion model parameters static and fixed (Scheepmaker et al., 2017). On a flat track, the optimal cruise and brake speeds are uniquely determined for each feasible value of the running time (Albrecht et al., 2016b). These two speeds are related by an equation that depends explicitly on the running resistance parameters. Still, other train motion model parameters can influence the value of these speeds through their impact on the running time. Most efforts have been dedicated to analyze variations in the consumed energy. Liu & Golovitcher (2003) observed that the cost-time curve, namely the rate of change of the energy consumption with respect to the running time, is equal the negative costate variable relative to the running time, which in turn is related to the optimal cruise speed through the running resistance parameters. This equation was derived and analyzed fur-

ther in Albrecht et al. (2016a) and extended in Howlett (2016) for non-flat tracks and in Albrecht et al. (2020) for intermediate points of the track in a two-train safe separation problem. The analytical solutions of the train motion model have been gathered in (Cunillera et al., 2023b), which may allow to explore analytically the impact of parameter variations on an optimal trajectory on a flat track after constraining the running time and the track length. However, the algebra could be tedious, requiring some approximations in terms of Taylor expansions, and resulting formulas might not be practical. When it comes to computational results, the sensitivity of optimal trajectories with respect to perturbations in the location of the switching points between driving strategies has been explored in Goodwin et al. (2020) to test the robustness of energy-efficient trajectories. Trivella et al. (2021) computationally studied the impact of the wind direction and strength on the energy consumption of optimal trajectories. Zhang et al. (2017) tested the robustness of optimized trajectories with respect to uncertainties in the traction and brake response time and transmission delay. However, the exact impact of train motion model parameter variations on the shape of optimal train trajectories is still to be quantified.

In this chapter, we aim to quantify the volatility of optimal train trajectories with respect to variations of the train motion model parameters. We fix the running time of the train and perform a sensitivity analysis by varying one by one the parameters and computing an optimal train trajectory for each value of the varying parameter. We study the impact on the energy consumption and on the cruise and brake speeds of energy-efficient trajectories. We also quantify how the running time and the energy consumption are affected by parameter variations in minimum time-optimal trajectories. A sensitivity analysis is performed in two scenarios: a long flat track where an optimal cruise regime can take place and a short flat track where no cruise regime is observed. Both scenarios consider the track flat, however, gradients and curves on the track also affect the shape of optimal trajectories, and most notably during coast regimes. A systematic assessment of the impact of variations of the track geometry on optimal trajectories cannot be performed due to the infinite combination of gradients, curves and their values that could be studied. Still, we assess the impact of a short gradient in the middle of the final coast regime in the previous long flat track scenario that allows for a cruise regime. With this we aim to emphasize the importance of having a precise and up-to-date description of the track geometry in order to guarantee the accuracy of energy-efficient driving strategies. These analyses disregard any correlation between parameter variations. To address this limitation, we utilize different sets of parameters estimated using real data in chapter 3 to calculate the optimal trajectory for each set of parameters. Observing the differences in the optimal trajectories produced from each set of parameters allows to gauge the impact of existing correlations among parameter variations, although the exact extent of these correlations is unknown. This research sheds light on the robustness of optimal trajectories against the mentioned parametric variability. This allows to identify the most critical parameters for train trajectory optimization, that is to say, those that have the largest impact on optimal trajectories. Moreover, this research highlights the need to account for possible parameter variations in TTO and the relevance of train motion model calibration to guarantee the accuracy of the generated optimal trajectories and, ultimately, of energy-efficient driving.

We generate optimal train trajectories by solving a convex quadratically-constrained linear programming problem. This convex TTO algorithm based on the work-energy balance equation that includes extra decision variables to guarantee the convexity of the problem was introduced first in Jibrin et al. (2021b). It was further explored in Jibrin et al. (2021a) and Jibrin et al. (2022), focusing on modelling the hybrid fuel cell system powering of a train performing an optimal trajectory. This method has been applied also on metro trains trajectory optimization (Xiao et al., 2023) and has been combined with Mixed Integer Linear Programming to study the impact of neutral zones on optimal trajectories (Ying et al., 2023). Furthermore, the accuracy of the method has been proven mathematically in Lu et al. (2022) and the same authors compared its performance against other well-known direct train trajectory optimization methods in Feng et al. (2022), finding that not only it is conceptually simpler than similar methods, but also it is significantly faster and allows for more flexibility in the modelling. This algorithm can calculate optimal trajectories in less than a second, making it suitable for real-time applications. We further develop this algorithm, introducing a discretization of the distance variable based on Chebyshev nodes (Stewart, 1996). This reduces the sensitivity of the trajectories calculated with this algorithm to the total number of discretization points when the distance is split in even intervals.

To conclude, the main contributions of this chapter are:

- A sensitivity analysis of the impact of train motion model parameter variations on time and energy-optimal train trajectories.
- Identifying the most critical train motion model parameters for TTO, that is, the parameters that are most relevant for shaping optimal train trajectories.
- Demonstrating the relevance of calibrating the train motion model parameters accurately for maximizing the effectiveness of energy-efficient driving.
- An upgrade to the convex TTO algorithm introduced by Jibrin et al. (2021b) that reduces the sensitivity of the calculated trajectories to the number of discretization points.

The remainder of this chapter is organized as follows. In chapter 6.2 we briefly introduce TTO and the train motion model parameters. In chapter 6.3 we describe the algorithm used in this chapter to generate optimal trajectories. We perform a parameter sensitivity analysis of optimal trajectories in chapter 6.4, where we also show and analyze the results obtained. Last, we draw the main conclusions of this research in chapter 6.5.

6.2 Train Trajectory Optimization

6.2.1 Energy-efficient Train Trajectory Optimization and the train motion model parameters

The Train Trajectory Optimization problem seeks the trajectory with minimum energy consumption among all the possible trajectories of a train between an origin and an end point of the track, s_o and s_e , that arrive at the destination at a prescribed time T . This can be modelled as an optimal control problem as follows (Scheepmaker et al., 2017):

$$\min_{f(s), b(s)} E(s_e) \quad (6.1)$$

subject to

$$m\rho \frac{dv}{ds} = \frac{f(s) - b(s) - R(v(s)) - W(s)}{v(s)}, \quad (6.2)$$

$$\frac{dt}{ds} = \frac{1}{v(s)}, \quad (6.3)$$

$$\frac{dE}{ds} = f(s), \quad (6.4)$$

$$0 \leq f(s) \leq f_{\max}, \quad (6.5)$$

$$0 \leq b(s) \leq f_{\min}, \quad (6.6)$$

$$f(s) \leq p_{\max}/v(s), \quad (6.7)$$

$$\epsilon \leq v(s) \leq v_{\max}(s), \quad (6.8)$$

$$v(s_o) = v_o, v(s_e) = v_e, t(s_o) = 0, t(s_e) = T, E(s_o) = 0. \quad (6.9)$$

Here, $f(s)$ and $b(s)$ (both in N) are the control variables standing for the tractive and brake effort, respectively. $s \in [s_o, s_e]$ is the location of the train (in m), v the speed (in m/s), t the time (in s) and E the energy consumption (in J). Eq. (6.2) describes the dynamics of the speed of the train. There, m is the train mass (in kg) and $\rho > 1$ is the rotating mass factor, a dimensionless constant that accounts for the kinetic energy lost in accelerating the rotating parts of the rolling stock. The running resistance $R(v(s))$ is modelled as a quadratic function of the speed called the Davis equation (Davis, 1926)

$$R(v(s)) = R_0 + R_1 v(s) + R_2 v^2(s), \quad (6.10)$$

where R_0 , R_1 and R_2 are the running resistance parameters (in N, N/(m/s) and N/(m/s)², respectively). The resistance to motion due to the gradients and curves is represented by the term $W(s)$ (Hansen & Pachl, 2014):

$$W(s) = mg \frac{dh(s)}{ds} + mg \frac{k}{r(s)}, \quad (6.11)$$

with $g = 9.81 \text{ m/s}^2$ the gravity acceleration, $\frac{dh(s)}{ds}$ is the derivative of the height profile of the track at the location s , namely the angle of the slope at s . The effect of the curves is introduced as virtual slopes, with $r(s)$ is the radius of the track at s (in m) and k is a

proportional constant (in m). The rate of change of the time and the energy consumption are introduced in Eqs. (6.3) and (6.4). The control variables are bounded in Eqs. (6.5) and (6.6), with $f_{\max}$ and $f_{\min}$ the maximum tractive and brake effort (in N). In Eq. (6.7), $p_{\max}$ bounds the maximum tractive power (in W). The speed is constrained by the speed limits $v_{\max}(s)$. Since the location is used as the independent variable, the inverse of the speed terms would invalidate the Lipschitz condition of Eqs. (6.2) and (6.3) if the speed could be zero. To avoid this, we set a strictly positive minimum speed ϵ . Last, the boundary conditions of the optimal control problem are defined in Eq. (6.9).

Several train motion model parameters characterize the train trajectory optimization problem in Eqs. (6.1)-(6.9): the running resistance parameters R_0 , R_1 , R_2 , the maximum tractive and brake effort $f_{\max}$ and $f_{\min}$, the maximum tractive power $p_{\max}$, the rotating mass factor ρ and the train mass m .

The train motion model parameters may show variations due to the influence of the operating conditions on the train dynamics. Variations of the train mass are mainly produced by its load and composition, so it is approximately constant between each station. An exception to this are diesel-powered locomotives, where the fuel mass decreases along the way although the effect on the total mass is small. The maximum tractive and brake efforts are affected by the adhesion between wheel and rail. Variations in the catenary power and the engine wear can lower the maximum tractive power of the locomotive. The train composition, the direction and strength of the wind, the weather and the adhesion are some sources of variation of the running resistance parameters. Particularly, R_2 accounts for the aerodynamic drag of the train, which increases when running through tunnels due to the generated extra air pressure (Hansen et al., 2017). A deeper discussion on the existing sources of parameter variation can be found in chapter 2.

6.2.2 Minimum time Train Trajectory Optimization

Minimum time trajectories are of special interest for timetablers, since running time supplements are typically calculated as a percentage of the minimum running time. This is important because running time supplements determine the amount of delay that can be recovered without having to reschedule any service. These supplements are added to the target running time to accommodate small disturbances and delays. TTO makes use of these supplements to save energy whenever a train runs on time.

The TTO problem can be modified for calculating the minimum running time driving strategy. Once again, the minimum time trajectory optimization problem can be formulated as an optimal control problem. Evidently, the running time is the variable to be minimized, so it substitutes the energy consumption in the cost function in Eq. (6.1) as follows:

$$\min_{f(s), b(s)} t(s_e). \quad (6.12)$$

Moreover, the energy consumption dynamics in Eq. (6.4) and the boundary conditions on the final running time and the energy consumption can be removed in Eq. (6.9). Therefore, the boundary conditions of the minimum time problem are

$$v(s_o) = v_o, v(s_e) = v_e, t(s_o) = 0. \quad (6.13)$$

Then, the minimum time TTO problem is defined by Eq. (6.12) subject to Eqs. (6.2), (6.3), (6.5)-(6.8), (6.13).

To distinguish the minimum time and the energy-efficient optimal trajectories, we use the following nomenclature: energy-optimal trajectories refer to the solutions of the energy-efficient TTO problem, and time-optimal trajectories refer to the solutions of the minimum time TTO problem. In case of ambiguity, we are referring to energy-optimal trajectories.

6.3 Methodology

6.3.1 Solving the energy-efficient Train Trajectory Optimization problem

In this chapter we reformulate the TTO problem as a convex quadratically-constrained linear program, which is solved to calculate the energy-optimal trajectories. We assume that the geometric characteristics of the track, namely the gradients and curves radii, are piecewise constant. For simplicity, we will not consider the curves in this algorithm, although they could be incorporated easily into it by modelling curves as virtual slopes, as shown in Eq. (6.11).

We discretize the track in an even number of intervals, N , and define the collocation points s_k , $k = 0, \dots, N$, with $s_0 = s_o$ and $s_N = s_e$. These collocation points include all those locations where either the speed limits or the track gradients change, so the track characteristics are constant in each interval. After selecting those points, if more are needed to reach the target N intervals we select Chebyshev nodes between s_o and s_e (Stewart, 1996). Denoting M the remaining number of collocation points needed to reach N intervals, we calculate M Chebyshev nodes x_j , $j = 1, \dots, M$ as follows:

$$x_j = \frac{s_o + s_e}{2} + \frac{s_e - s_o}{2} \cos\left(\frac{(2j-1)\pi}{2M}\right), \quad j = 1, \dots, M. \quad (6.14)$$

Once the Chebyshev nodes have been calculated, the collocation points are obtained by sorting all the mentioned points and nodes. If a Chebyshev node is located within 1 m from one of the collocation points of the original discretization and the latter is not a departure or arrival location, we remove the Chebyshev node. The Chebyshev nodes allocate more points near the beginning and the end of the trajectory. This helps characterizing more precisely the initial traction and the final brake regimes and allow to integrate the time and energy more accurately than if the collocation points were evenly distributed. We observed that the trajectories obtained are sensitive to the number of collocation points when they are evenly distributed. Chebyshev nodes solve this issue. Uneven collocation points distribution are also characteristic of TTO solvers based on pseudospectral methods (Wang & Goverde, 2016; Goverde et al., 2021).

Eqs. (6.2)-(6.4) have to be integrated numerically. At each collocation point $s_k = 0, \dots, N$ we define the speed $v_k = v(s_k)$, and for $k = 0, \dots, N-1$ the control variable $u_k = f(s_k) - b(s_k)$ and the energy consumption variable E_k . The cost function in Eq. (6.1) can

be rewritten as the integral of Eq. (6.4), since $E_0 = 0$. However, the tractive effort $f(s)$ is not a decision variable anymore. To solve this, we introduce the following two linear inequalities at each collocation point,

$$E_k \geq u_k(s_{k+1} - s_k), \quad k = 0, \dots, N-1, \quad (6.15)$$

$$E_k \geq 0, \quad k = 0, \dots, N-1. \quad (6.16)$$

After integrating numerically Eq. (6.4), the resulting cost function is

$$J = \min \sum_{k=0}^{N-1} E_k. \quad (6.17)$$

Notice that if E_k , $k = 0, \dots, N-1$ are decision variables in an optimization problem where their value is minimized, Eq. (6.17) converges to the value of the solution of Eq. (6.4).

The time dynamics in Eq. (6.3) can be integrated similarly,

$$T = \sum_{k=0}^{N-1} \frac{s_{k+1} - s_k}{v_k}. \quad (6.18)$$

The work-energy balance of the train can be obtained by integrating the equation of motion in Eq. (6.2). The terms associated to the kinetic and the potential energy can be calculated analytically, while the work done by the control variable and the losses due to the running resistance have to be calculated numerically. Denoting $h_k = h(s_k)$, $k = 0, \dots, N$, we find

$$\begin{aligned} \frac{1}{2} m \rho (v_{k+1}^2 - v_k^2) = & (s_{k+1} - s_k) \left(u_k - R(v_k) - m \frac{k}{r(s_k)} \right) \\ & - mg(h_{k+1} - h_k), \quad k = 0, \dots, N-1. \end{aligned} \quad (6.19)$$

The constraints on the control variable can be introduced as inequalities

$$-f_{\min} \leq u_k \leq f_{\max}, \quad k = 0, \dots, N-1, \quad (6.20)$$

$$u_k \leq \frac{p_{\max}}{v_k}, \quad k = 0, \dots, N-1. \quad (6.21)$$

When it comes to the speed limits, we can denote $v_{\max,k} = v_{\max}(s_k)$, obtaining

$$\epsilon \leq v_k \leq v_{k,\max}, \quad k = 0, \dots, N. \quad (6.22)$$

Last, the boundary conditions on the speed fix v_0 and v_N ,

$$v_0 = v_o, \quad v_N = v_e. \quad (6.23)$$

The optimal control problem in Eqs. (6.1)-(6.9) has been transformed into a nonlinear programming problem in Eqs. (6.15)-(6.23). Still, this problem is not convex due to the presence of nonlinear terms like the inverse of the speed in the time constraint in Eq. (6.18) and in the maximum power constraint in Eq. (6.21). Moreover, a convex equality constraint requires being affine in the decision variables, which is not the case

in Eq. (6.19) due to the squared speed terms. These nonconvexities can be relaxed by introducing two sets of decision variables, a_k , $k = 0, \dots, N-1$, and b_k , $k = 0, \dots, N$, in the mentioned constraints and by adding two convex quadratic inequalities:

$$a_k v_k \geq 1, \quad k = 0, \dots, N-1, \quad (6.24)$$

$$v_k^2 \leq b_k \leq v_{\max,k}^2, \quad k = 0, \dots, N. \quad (6.25)$$

The decision variables a_k substitute the inverse of the speed in Eqs. (6.18) and (6.21), and b_k substitute the square of the speed in Eq. (6.19) as follows:

$$T = \sum_{k=0}^{N-1} a_k (s_{k+1} - s_k), \quad (6.26)$$

$$u_k \leq p_{\max} \alpha_k, \quad k = 0, \dots, N-1, \quad (6.27)$$

$$\begin{aligned} \frac{1}{2} m \rho (b_{k+1} - b_k) = & (s_{k+1} - s_k) \left(u_k - R_0 - R_1 v_k - R_2 b_k - m \frac{k}{r(s_k)} \right) \\ & - mg(h_{k+1} - h_k), \quad k = 0, \dots, N-1. \end{aligned} \quad (6.28)$$

6.3.2 Solving the minimum time Train Trajectory Optimization problem

Minimum time optimal trajectories can be calculated by solving a modified version of the program formulated in the previous subchapter. Since the arrival time at the destination is the objective to be minimized, we can remove the decision variables relative to the energy consumption E_k , the inequality constraints in Eqs. (6.15) and (6.16), and the time constraint in Eq. (6.26). A minimum time cost function can then be written as

$$\min \sum_{k=0}^{N-1} a_k (s_{k+1} - s_k). \quad (6.29)$$

6.4 Case studies, results and discussion

6.4.1 Description of the case studies

In this chapter, we analyze the volatility of energy and minimum time-optimal trajectories under parameter variations by means of a sensitivity analysis. We consider a Dutch VIRM-IV Intercity train in our case studies, and its nominal train motion model parameters are outlined in chapter 6.4.2. We calculate optimal trajectories that arise from varying each parameter within a certain range while the others are fixed. This allows to analyze the impact of different values of that parameter on the cruise and brake speeds of the optimal trajectory, and on its energy consumption. We perform the mentioned sensitivity analysis of optimal trajectories for each parameter in two flat and straight track scenarios without speed limits: a short track (5 km) and a long track (18.542 km). An energy-optimal train trajectory in a flat track without speed limits can have two different shapes depending on the length of the track and the running time supplement available. If the length of the

track between two stops is short or the running time supplement is small, the optimal trajectory is a concatenation of an initial maximum traction regime, an intermediate coast regime and last, maximum braking. This type of optimal trajectories are typically observed in metro lines, light trains and in regional trains, which have usually several stops over short distances. When the distance between stops is larger, a cruise regime can be observed between the maximum traction and the coast regimes. This long-distance stop pattern is characteristic of long-distance and high-speed trains. We repeat the sensitivity analysis on the volatility of minimum time trajectories on the same two flat track scenarios, exploring also the impact on the arrival time. We add a constant speed limit in the long scenario to avoid reaching speeds that are not realistic. After that, we analyze the impact of gradients on energy-optimal trajectories in the long track scenario. To do this, we place a virtual 500 m gradient in the middle of the coast regime in the long track scenario and perform a sensitivity analysis on energy-optimal trajectories with respect to the value of the gradient.

Furthermore, we use the train motion model parameters estimated in chapter 3 to analyze the energy-optimal trajectories corresponding to each set of estimates in the long flat track scenario. The parameters were estimated using data measured on passenger trains during normal operation in 2021. The estimates represent the existing operating conditions at the time the data was measured. Comparing the energy-optimal trajectories of each set of estimates allows us to determine how these trajectories are affected by varying operating conditions. Moreover, considering the uncertainties and stochasticity of railway operation from a statistical point of view, the parameters can be considered as random variables. In this regard, the estimates are samples of those random variables that include any existing correlation among the values of the parameters. Determining these correlations may be difficult or even impossible due to the virtually infinite number of combinations of parameter variations that should be considered. However, we can still assess the influence of these correlations on the shape of optimal trajectories. In the sensitivity analysis all the parameters but one were fixed and the possible correlations were not considered. This handicap is partially addressed by analyzing the impact of sets of parameter estimates, despite the fact that the existing correlations remain unknown. In turn, the observed variations in the arrival times of the minimum time trajectories evidence that the effectiveness of running time supplements for mitigating delays and accommodating small disturbances may be lower than expected under unfavourable operating conditions.

6.4.2 Rolling stock characteristics

In this chapter we use train motion model parameters corresponding to a VIRM-IV Inter-city train operated by the Dutch railway undertaking NS. Table 6.1 gathers the parameters provided by the rolling stock manufacturer. We study this type of rolling stock since it is widely used in the Netherlands for operating mainline services. Moreover, the train motion model parameters estimated in chapter 3 were obtained by analyzing trajectories of this type of rolling stock.

We study the impact of the mass by varying the train load, starting from the mass of an empty train loaded with the minimum required operational load (242000 kg) until it is fully loaded (287280 kg). When varying any other parameter, the train is considered to be loaded at 35% of its maximum passenger capacity (257848 kg). We consider variations in

the rotating mass factor between 4% to 12%, since this is typically the range of values of this parameter (Hansen et al., 2017). For the remaining parameters we consider variations of 25% around their nominal value and calculate 25 optimal trajectories for different values of each parameter. In chapter 3, parameter variations were estimated in a case study where train load variations were neglected. We observed parameter variations wider than $\pm 25\%$ around the mean in the running resistance parameters and the maximum tractive effort applied, while $p_{\max}$ and $f_{\min}$ showed considerably narrower variability. In some cases, we also observed deviations between the mean of the estimates and the value provided by the manufacturer. Besides the potential coupling between these variations and the train load, we found that the characteristics of manual driving have a strong influence on the parameters, as train drivers usually do not apply traction or brake at the maximum capacity of the rolling stock. Still, the parameter variability observed in chapter 3 is drawn from one case study and therefore it cannot be generalized. In this chapter we aim to provide an overview of the impact of parametric variability on optimal trajectories, which justifies the choice of a $\pm 25\%$ variation range to guarantee the generalizability of the results of the sensitivity analysis.

Parameter	Value
Empty train mass m	242000 kg
Maximum passenger load	45280 kg
Nominal passenger load	35%
Rotating mass factor ρ	6 %
Maximum tractive effort $f_{\max}$	142600 N
Maximum brake effort $f_{\min}$	$0.5m\rho$ N
Maximum tractive power $p_{\max}$	1438000 W
Running resistance parameter R_0	1834.8 N
Running resistance parameter R_1	42.12 N/(m/s)
Running resistance parameter R_2	7.776 N/(m/s) ²

Table 6.1: Train motion model parameters of an NS VIRM-IV Intercity train (Scheepmaker et al., 2020a).

6.4.3 Case study 1: Parameter variations on a long flat track scenario

We consider a 18.542 km long track without gradients and curves. In this scenario optimal trajectories have a cruise regime between the acceleration and the coast regimes. The train motion model parameters in Table 6.1 are fixed and we vary them one by one to perform the sensitivity analysis of optimal trajectories. We present and discuss first the analysis of energy-optimal trajectories and next the analysis of time-optimal trajectories.

Energy-optimal trajectories on a long flat track scenario

Figure 6.1 shows three energy-optimal trajectories corresponding to values of the quadratic running resistance parameter R_2 equal to 75%, 100% and 125% of its nominal value in blue, red and yellow, respectively. The running time in these three trajectories is the

same, 750 s. This parameter is of particular relevance since it is influenced by the wind and the geometry of any tunnels that the train passes through. Several remarkable differences can be observed among the trajectories, which highlight the relevance of determining this parameter accurately for enhancing the benefits of energy-efficient driving. The optimal cruise speed differs by approximately 2.5 m/s (9 km/h) between the lowest and highest value of R_2 . Higher cruise speeds lead to shorter cruise regimes, as the fixed running time has to be compensated. Indeed, large differences can be observed in the length of the cruise regime: while an R_2 equal to 75% of its nominal value leads to an optimal cruise regime that is 1.7 km long, it can be around 4.1 times larger in the case that R_2 is 25% higher than the nominal value. The deceleration of the train when coasting is lower for smaller values of R_2 and the cruise speed is higher, which leads to a much earlier coasting point. In contrast, there is little impact in the initial acceleration and the final brake regimes, but they are slightly longer for lower values of R_2 . This is due to the fact that in an optimal trajectory the train is expected to reach a higher speed so it needs more space to accelerate until that speed. Capacity-wise, the maximum difference in passing times among these three trajectories is 7 s, only 1% of the total running time. If capacity constraints are more demanding, additional timing points could be set at critical points of the infrastructure and an optimal trajectory should be calculated accordingly. Still, this maximum difference takes place around the kilometric point 7. This is probably sufficiently far from the arrival and departure stations, which are typically considered capacity bottlenecks. During the first and last kilometer, the time difference among these trajectories is not larger than 1 s.

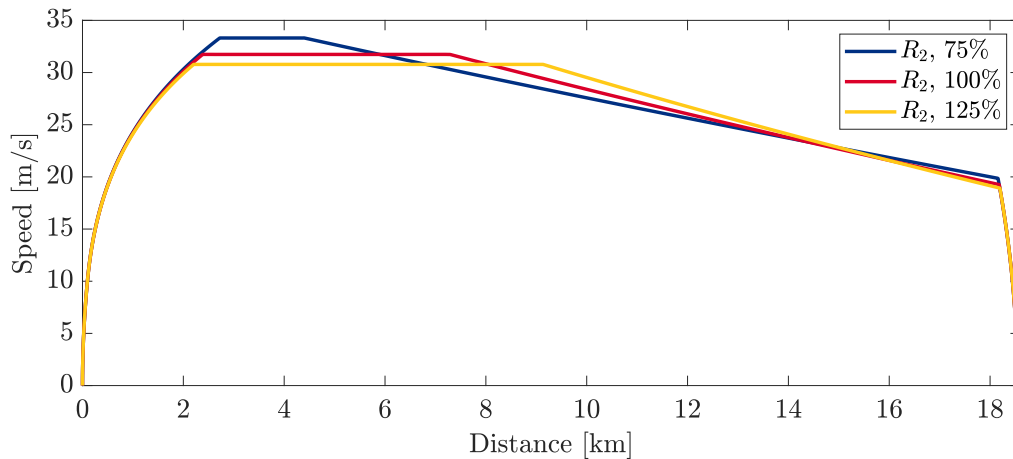


Figure 6.1: Energy-optimal trajectories corresponding to three different values of the running resistance parameter R_2 .

Discretizing the track using Chebyshev nodes allows to reduce the sensitivity of the calculated trajectories with respect to the number of collocation points used. In this chapter we aim to calculate the switching points between driving strategies with a precision of the order of a meter, so we allocate 20000 collocation points along the trajectory in order to obtain smooth results. This is 50 times more collocation points than what is usually needed to obtain accurate trajectories. In practice, this method can calculate optimal trajectories over tracks with variable track gradients and speed limits in less than 0.2 s,

but the large amount of collocation points used increases the computation time to 15 s per trajectory. The quadrature of Eq. (6.2) shown in Eq. (6.19) is based on the Riemann summation rule that utilizes in each interval the value of the integrand function at the left part of the interval whenever an analytic solution of the quadrature is not available. This is the case for the integration of the terms of the applied effort, the running and the curve resistances. The accuracy of this integration is proportional to the square of the integration interval (Süli & Mayers, 2003). The distance between consecutive collocation points has to be kept small. However, we observed that the quality of the optimized trajectories depends more on the integration of the time variable than on the size of the intervals. The time integration is more sensitive to errors in those locations where the train speed is low due to the inverse of the speed term in Eq. (6.3). Therefore, more points have to be allocated next to those locations where the train will probably have low speeds. Chebyshev nodes solve this issue naturally, as they allocate more points at the beginning and at the end of a train trajectory. Furthermore, we also allocate collocation points at every location where either the track gradients or the speed limits change. This is an advantage with respect to TTOs that allocate points in the time domain since they require interpolating routines to approximate the effect of changes in the speed limits or gradients, which add an interpolation error to the calculations (Scheepmaker et al., 2020a).

Figure 6.2 shows, from top to bottom, the influence of each parameter on the energy consumption, the starting location of the cruise, coast and brake regimes, and the optimal cruise and brake speeds.

We define the optimal cruise speed as the mean speed held during the cruise regime and the optimal brake speed as the one observed when the train starts braking in the final brake regime. On the left we show the influence of the mass (represented as a blue full line) and the rotating mass factor (represented as a yellow dashed line), in the middle the influence of the running resistance parameters (R_0 as a yellow full line, R_1 as a dashed blue line and R_2 as a dotted red line) and on the right the influence of the maximum traction and brake parameters ($f_{\max}$ as a yellow full line, $p_{\max}$ as a blue dashed line and $f_{\min}$ as a red dotted line). This symbol and color code will be kept throughout the whole chapter.

We observe that the running resistance parameters and the mass have larger impact on the energy consumption than the other parameters. After all, they determine the losses in the kinetic energy. Among the running resistance parameters, R_2 is by far the one that influences the most the energy consumption, and R_1 influences it the least. The difference in energy consumption between a fully loaded train and an empty one is 15 MJ, which is 1.5 times more than the impact of the extreme values of the rotating mass factor. The traction and brake characteristics show a lower impact. Regarding the initial location of the cruise regime, the maximum tractive power $p_{\max}$ produces the most prominent variations followed by the mass. The maximum tractive power determines how fast the optimal speed can be reached, so for the lowest maximum power considered a train starts cruising 1766 m later than for the largest value. In contrast, the maximum tractive effort regime holds for small speeds at the beginning of the trajectory, hence its minor impact. When it comes to the train load, fully loaded trains are supposed to cruise 800 m later than empty trains.

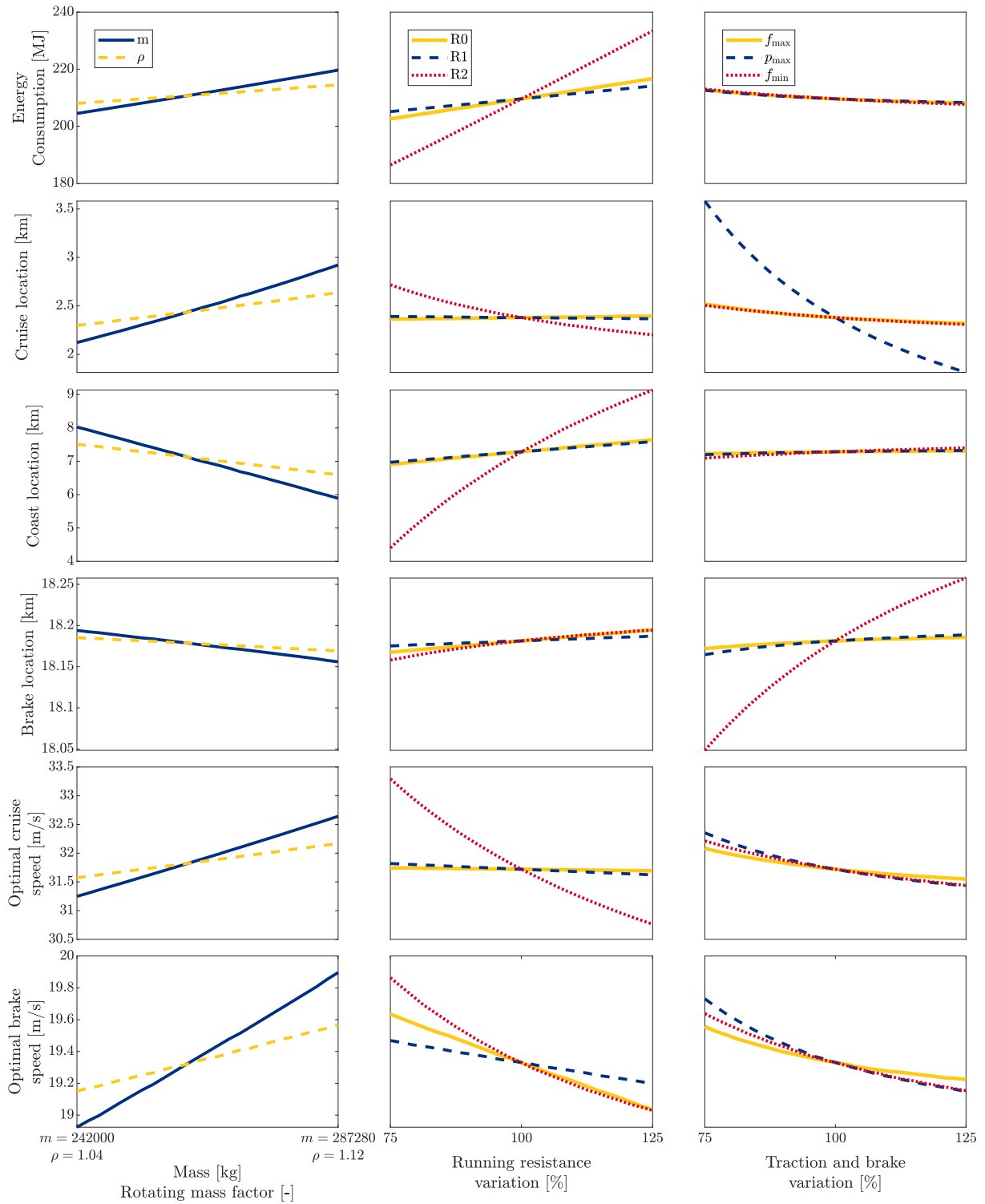


Figure 6.2: Impact on the energy consumption, the starting location of the cruise, coast and brake regimes and optimal cruise and brake speeds of energy-optimal trajectories in the long track scenario.

The quadratic running resistance parameter is again the most critical parameter when it comes to determining the initial location of the coast regime. When R_2 is varied 25% around its nominal value, the coast regime starting location varies over a 4.7 km long interval. With respect to the passenger load, the optimal coast regime starts 2100 m earlier for a fully loaded train than for an empty one. In turn, the initial location of the brake regime shows small differences for most parameters except for the maximum brake effort. Braking 25% harder allows to brake 77 m later, while if its performance is reduced by 25% then a train should start braking 133 m earlier. Still, the brake regime is significantly less affected than the coast or the cruise regime.

On a flat track, the optimal cruise and brake speeds are uniquely determined and depend on the running time supplement available (Albrecht et al., 2016b). The optimal cruise speed is more sensitive to the value of R_2 , and the mass is the second most critical parameter. We observe a 2.5 m/s (9 km/h) difference between the maximum and minimum cruise speeds when varying R_2 , and 1.4 m/s (5.0 km/h) when varying the mass. We also observe that the variations of the optimal cruise speed are approximately the opposite we found for the initial location of the coast regime in Figure 6.2. This is due to the arrival time constraint: the higher the optimal cruise speed is, the sooner a train should start coasting to arrive on time. However, the maximum tractive power had a preponderant role on reaching the cruise speed, as we show in Figure 6.2. We also find that this does not translate into remarkable changes in the cruise speed. While the optimal cruise and brake speeds seem to show a linear dependence on the mass-related parameters, R_0 and R_1 , the dependence on the other parameters looks nonlinear. Moreover, we observe that the mass has the largest impact on the brake speed, followed by R_2 , R_0 and the maximum tractive power.

Time-optimal trajectories on a long flat track scenario

In this scenario we also perform a sensitivity analysis on time-optimal trajectories and study the impact on the energy consumption, on the minimum running time, and on the initial location of the cruise at the speed limit and brake regimes. Figure 6.3 shows the results of the sensitivity analysis on these 4 characteristics from top to bottom. Notice that, unlike energy-optimal trajectories, time-optimal ones do not have a coast regime.

We observe that the impact of the parameters on the energy consumption is similar to the energy-optimal case: R_2 is again the most critical parameter, followed by the mass. The energy consumption is about 2.1 times larger in time-optimal trajectories, so the impact of all parameters is therefore greater in absolute terms. Regarding the running times, the impact of the running resistance parameters on the fastest arrival time is negligible. An empty train only arrives 11 s earlier than a fully loaded one. The major impact on the running time is due to the traction and brake characteristics. This type of time-optimal trajectories consists of accelerating to the maximum speed, cruising at that speed and braking to a standstill. Therefore, the running time is determined by how fast a train can reach the speed limit or how fast it can brake to a standstill, so the traction and brake characteristics take here a predominant role. The maximum tractive power $p_{\max}$ seems to have the largest impact on the minimum running time, closely followed by the maximum brake effort. This highlights that a reduced performance in the traction or brake may affect the expected minimum running time and the running time supplements

allocated in the timetable if they are too tight. Several circumstances may lead to the mentioned reduced performance. For instance, the catenary power may be lower than usual when several trains depart from a station at the same time, adhesion can be lower due to bad rail maintenance, adverse weather conditions or other factors, and the engine power and brake performance may be lower due to mechanical wear or ageing.

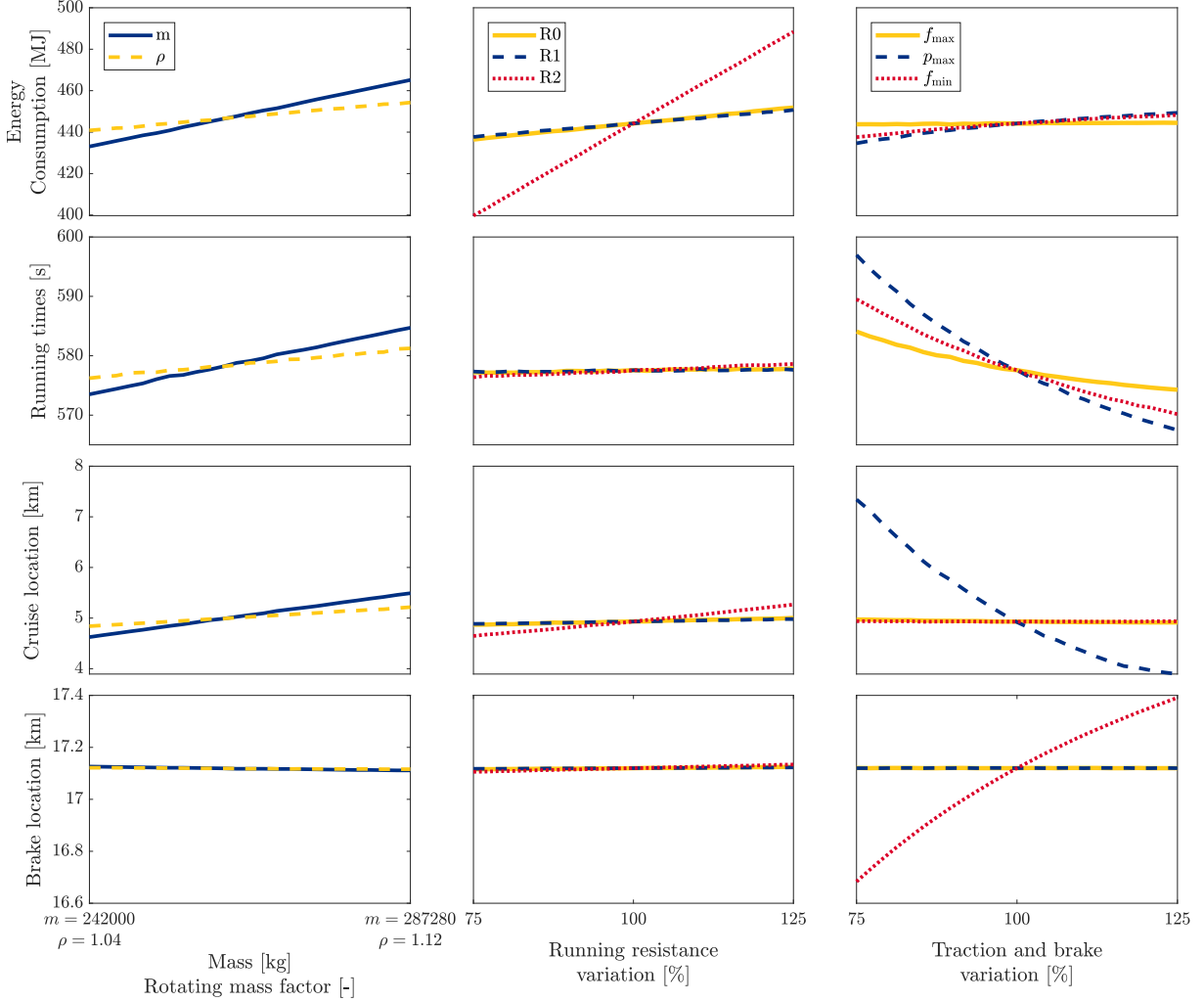


Figure 6.3: Impact on the energy consumption, the running time and the starting location of the cruise and brake regimes of time-optimal trajectories in the long track scenario.

The maximum tractive power mainly determines the location where the speed limit is reached, while the maximum brake effort determines the brake location. In the case of the starting location of the cruise regime, we observe similar results as in Figure 6.2 for most parameters. The influence of the maximum tractive power is larger now due to the longer traction regime, leading to a difference of 3.4 km in the start of the cruise regime. In turn, the brake location is again rather insensitive to the value of most parameters except for the limit on the brake effort, which impact is also magnified with respect to the energy-optimal case. This time the optimal starting location of the brake regime shows a 700 m variation. It is worth mentioning that both locations will be affected more under reduced traction and brake performance conditions.

In summary, R_2 is the most critical parameter on energy-optimal trajectories with a cruise regime when it comes to determining the energy consumption and the coasting regime. In turn, the running time of time-optimal trajectories is more sensitive to the traction and brake characteristics, which also determine the remaining features of both time and energy-optimal trajectories. The mass shows a more relevant role in energy-optimal trajectories than in time-optimal ones. Reflecting on railway planning and operation stages, the results obtained highlight the importance of an accurate calibration of the train motion model and the need to calculate train trajectories that are robust against variations of the motion model parameters.

Energy-optimal trajectories on a long flat track scenario with a gradient

Figure 6.4 shows the speed profiles corresponding to the cases where the track is flat, and when it includes a -3%, and a +3% gradient between kilometric points 15 and 15.5, that is to say, during the coast regime of the long track scenario. These are extreme gradient scenarios that are sometimes found in hilly areas and metro lines, and help to motivate and visualize the relevance of this discussion. Negative gradients are downhill, and positive are uphill. The running time is again set to 750 s. The red line corresponds to the flat track scenario we have been covering in this chapter, the blue line to the -3% steep downhill gradient, and the yellow line to the +3% steep uphill gradient. We observe that the energy-optimal trajectories again consist of an initial maximum traction regime, followed by cruising, coasting and maximum brake regimes. The major difference observed is the length of the cruise and coast regimes. The steep uphill gradient influences the train dynamics by decelerating the train in the track section where the gradient is located. The running time is fixed in all cases, so the train is expected to coast later to compensate for the observed speed loss. The effect of the steep downhill gradient is analogous, but with the opposite sign of the accelerations and the regime lengths.

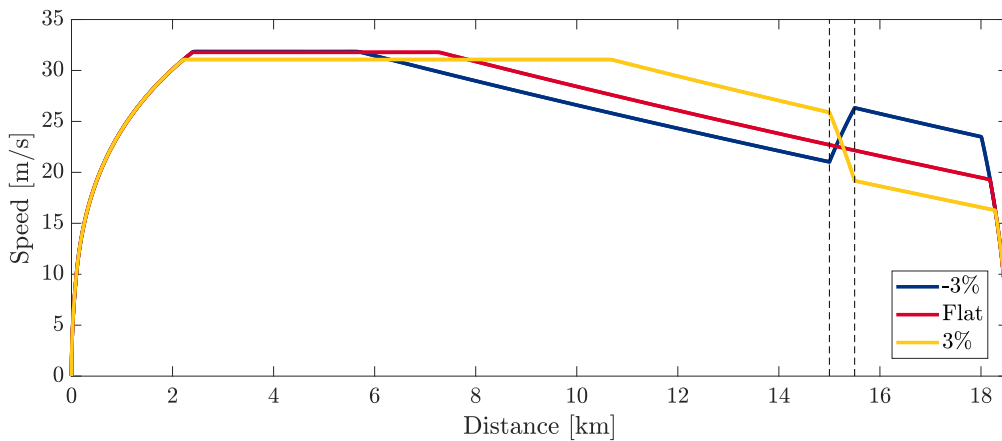


Figure 6.4: Energy-optimal trajectories on tracks with an intermediate -3%, 0% and +3% gradient.

Figure 6.5 shows the sensitivity analysis on the impact of a virtual gradient over 500 m placed at kilometric point 15 on energy-optimal trajectories. The value of the gradient is varied from -3% to 3%, and the running time is 750 s. We observe that the steepest uphill

gradient leads to an optimal trajectory that requires 23% more energy than the steepest downhill gradient. The starting point of the cruise regime and the corresponding cruise speed are more sensitive to uphill than downhill gradients. The optimal cruise speed only varies in 0.82 m/s (3.0 km/h), while the optimal brake speed shows considerably wider variations (7.2 m/s, or 25.9 km/h). Still, this does not impact significantly the location where the train starts braking. Instead, trains are expected to coast later to compensate for the time lost due to the deceleration produced by the steep uphill gradients.

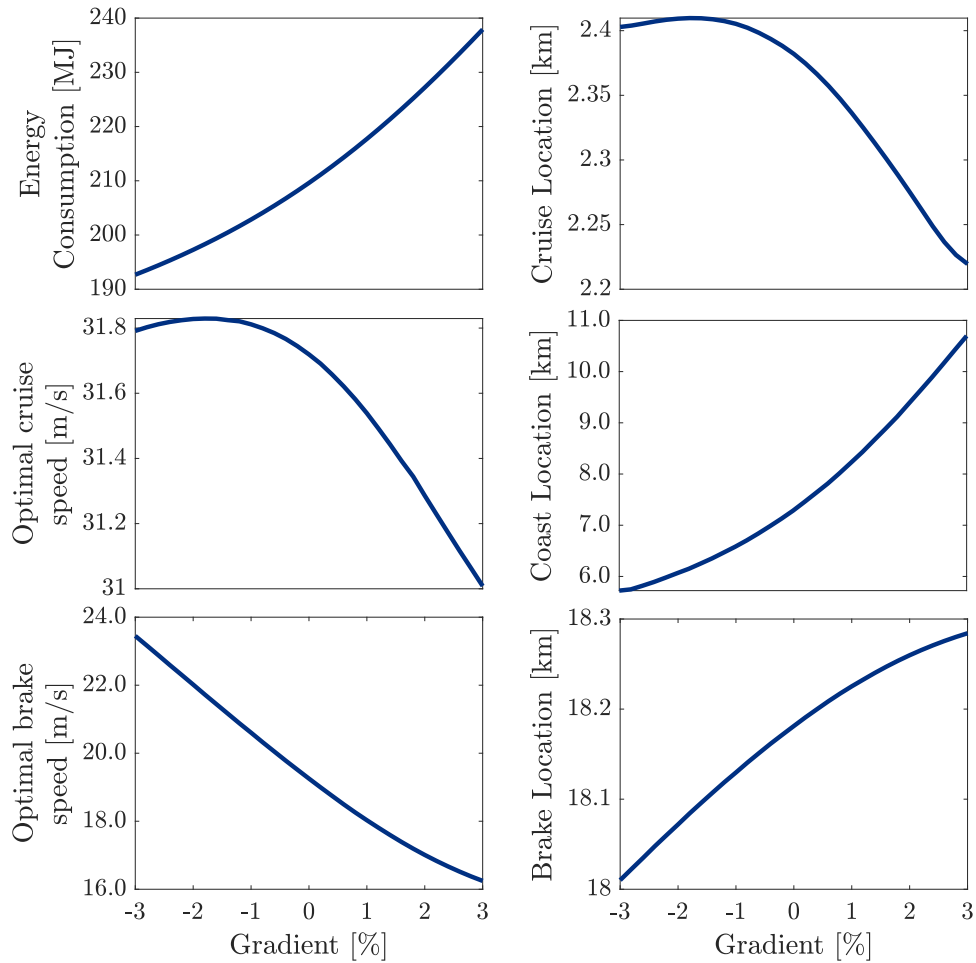


Figure 6.5: Impact of different gradients during the coasting regime on a energy-optimal trajectories in the long track scenario.

6.4.4 Case study 2: Parameter variations on a short flat track scenario

Energy-optimal trajectories on a short flat track scenario

If the distance between two consecutive stops is short and if the running time supplements are small enough, energy-optimal trajectories may not include a cruise regime. We proceed now to study the sensitivity of such energy-optimal trajectories without cruise regime in a 5 km long flat track. Figure 6.6 shows the sensitivity of the energy consumption, the

starting locations of the coast and brake regimes, and the optimal speeds at those locations under parameter variations.

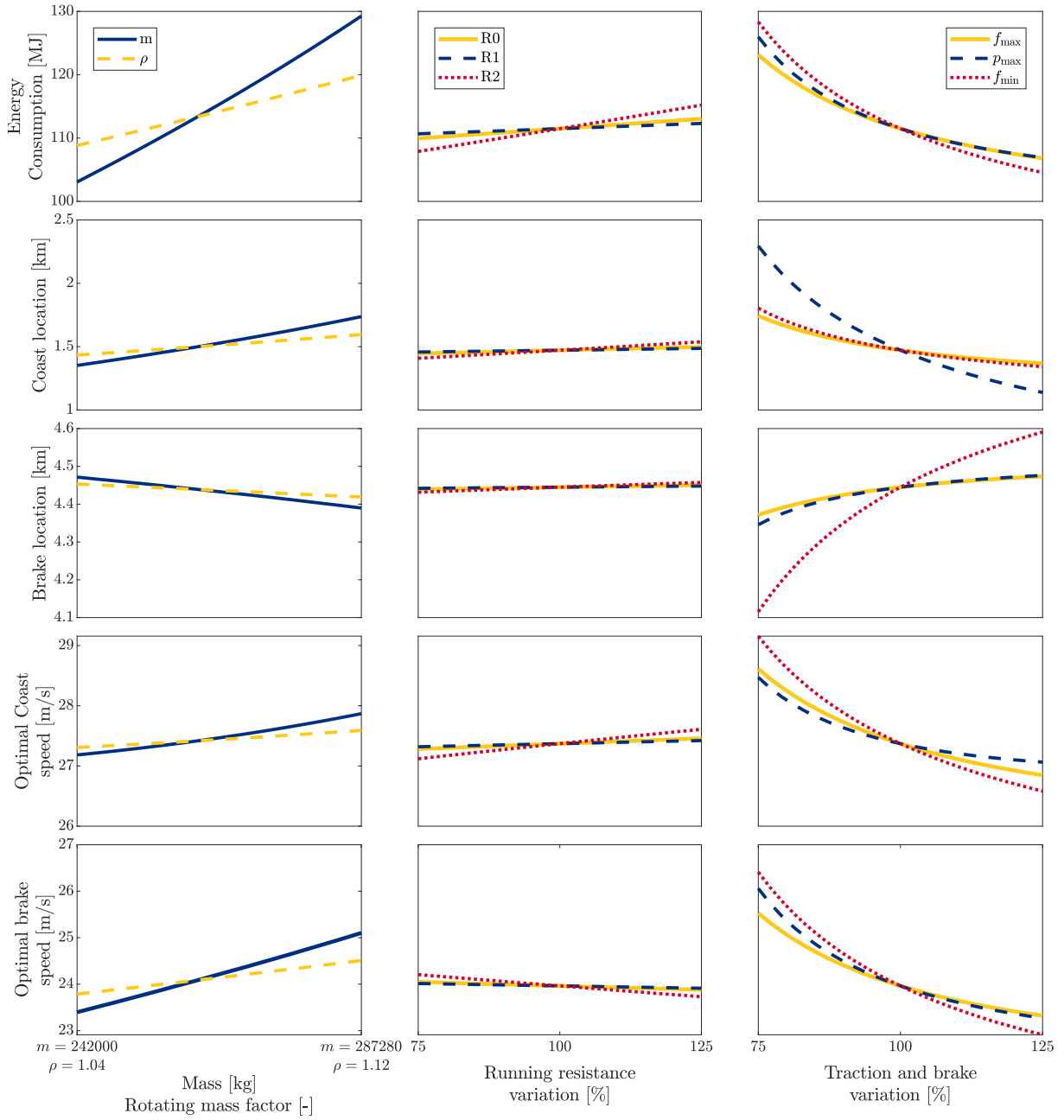


Figure 6.6: Impact on the energy consumption, the starting location of the coast and brake regimes, and optimal cruise and brake speeds of energy-optimal trajectories in the short track scenario.

When compared to the long track scenario, the quadratic running resistance parameter is no longer the most critical parameter. The difference lies in the fact that the maximum speed reached, the coast speed, is 4.4 m/s (16 km/h) lower than the cruise speed of the long track scenario. Therefore, less kinetic energy is lost because the impact of the running resistance is lower. We observe that the energy consumption is affected more by the train load, followed by the maximum brake effort, the maximum tractive power and

the tractive effort. Most of the energy is used to accelerate the train, so the role of the inertial terms is more preponderant.

In the short scenario the coast regime start is determined mainly by the maximum tractive power, although in the long scenario it was determined by R_2 and by the mass. This can be observed by comparing Figures 6.2 and 6.6. Now the traction and brake characteristics show a major role in determining the coast regime. However, the parameter impact on the initial location of the brake regime is similar to what we observed in the long scenario. The maximum brake effort's impact is larger than other parameters'. This is due to the fact that the maximum traction and brake regimes take place over a larger percentage of the total trajectory. The constraints corresponding to the maximum traction and brake characteristics are active during a larger portion of the trajectory, which increases the relevance of these parameters on the optimal trajectories.

Regarding the optimal coast and brake speeds, Figure 6.6 shows the same trend observed throughout the analysis of short track energy-optimal trajectories: the traction and brake characteristics have a major role in determining the optimal coast and brake speeds in detriment of R_2 , being the maximum brake effort the parameter that has the largest impact.

Time-optimal trajectories on a short flat track scenario

Minimum time train trajectories in short track scenarios without a speed limit consist of a maximum traction followed by a maximum braking regime. Again, the length of the track is set to 5 km. Figure 6.7 shows from top to bottom the sensitivity of the energy consumption, running time, the optimal brake speed and the location that separates the traction and the brake regimes of such time-optimal trajectories. Again we observe that the maximum tractive power and brake effort are the parameters that mainly determine the energy consumption, the running time and the shape of the trajectory. The maximum speed reached in this analysis was 38.4 m/s (138 km/h), which is below the 140 km/h speed limit that is generally the maximum speed allowed by the most widely spread Automatic Train Protection systems in the Netherlands (ATB-EG and ATB-NG). We notice again that R_2 is not dominant in this case. This evidences that R_2 's major impact takes place in regimes where the kinetic energy losses play a larger role than the traction and brake performance, that is, during cruise and coast regimes.

We conclude then that in short time- and energy-optimal trajectories without cruise regime, the traction and brake characteristics are the most critical parameters when it comes to determining the features of these trajectories. This means that implementing energy-efficient driving strategies tailored to each train in congested lines requires a consistent driving behaviour, since differences in the amount of traction and brake have the largest influence on the characteristics of optimal trajectories. In this regard, well-implemented ATO technologies constitute an adequate candidate to solve this issue. Furthermore, these insights can be helpful when instructing train drivers. Applying traction and brake closer to the maximum capabilities of the train could help reduce the amount of energy consumed, especially in lines with short distances between stops. We also observe that there is potential for energy-saving in long distance services if the parameter R_2 is carefully estimated and, to a lesser extent, if the mass is estimated or measured, for instance, by means of weigh-in-motion technologies (Pintão et al., 2022).

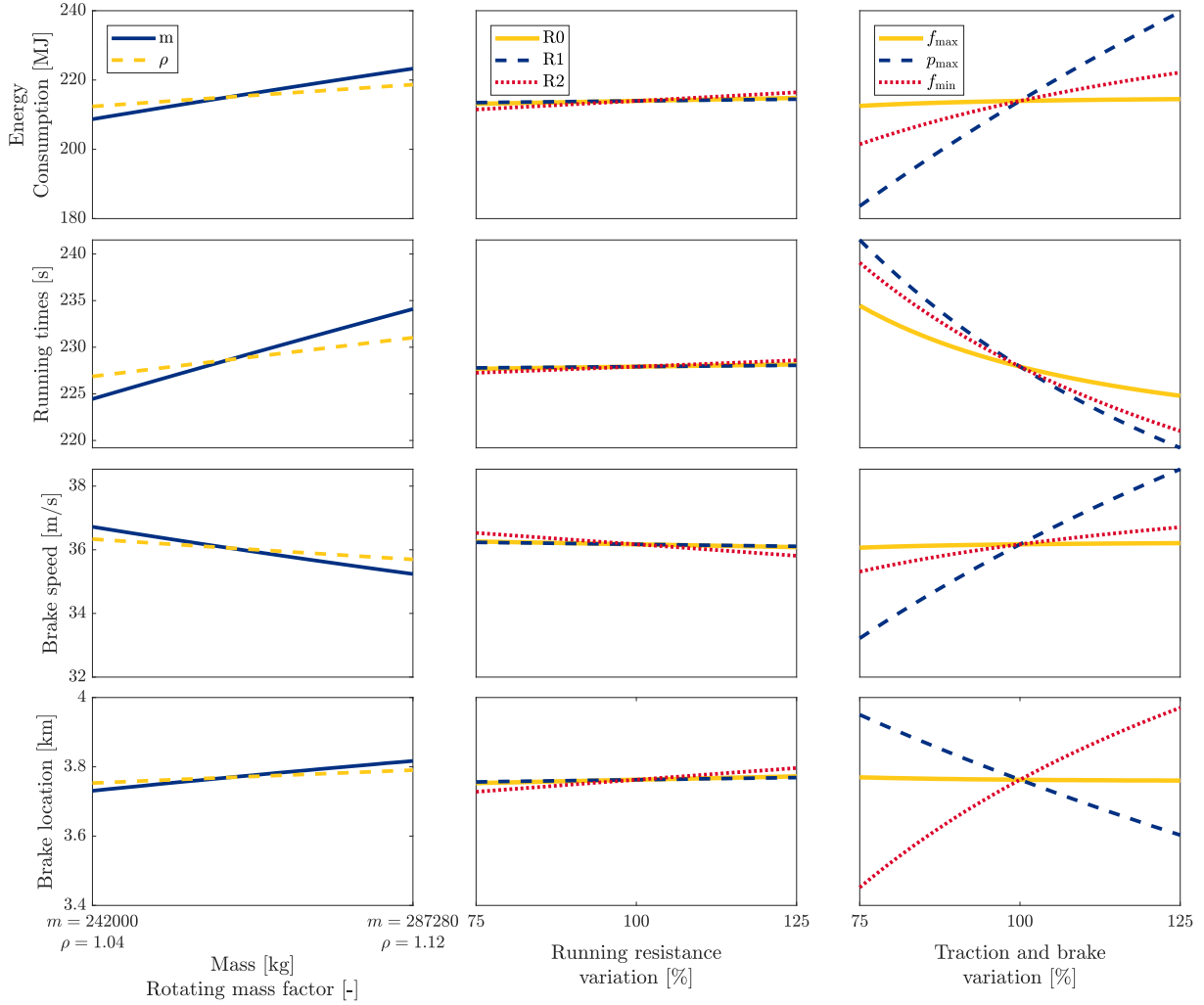


Figure 6.7: Impact on the energy consumption, the running time and the optimal brake location and speed of time-optimal trajectories in the short track scenario.

For short lines, the quadratic running resistance parameter has a lower impact on optimal trajectories than for long lines. This is due to the fact that a train spends less time travelling at high speeds and less kinetic energy is lost along the way due to the running resistance. The train mass plays a larger role when it comes to determining the optimal energy consumption for short lines than for long lines. Traction and brake is applied more often, so more energy is used to overcome the train inertia.

6.4.5 Assessing the impact of observed parameter variations

The sensitivity analysis presented so far did not consider potential correlations among parameters. For instance, a conservative train driver may not only apply a maximum applied tractive effort lower than the maximum capability of the train, but may also brake gently. This means that the $f_{\max}$ and $f_{\min}$ may be correlated, but this not considered in the sensitivity analysis presented so far. To partially address this gap, we show how optimal train trajectories vary under observed parameter variations. In chapter 3 we analyzed 191 trajectories and estimated the train motion model parameters in each of

them. These sets of estimates include some correlations, however, the exact correlations between parameters are unknown. We calculate an optimal trajectory in the flat long scenario for each set of parameter estimates and compare the results obtained with the optimal trajectory calculated using the manufacturer parameters. In that chapter the train load was not available, so we estimated mass-specific parameters, that is to say, the train motion model parameters divided by a factor $m\rho$. Again, we assume a 35% nominal train load in all cases.

Figure 6.8 shows speed-distance diagrams of the energy-optimal trajectories calculated for each of the 191 sets of train motion model parameters that were estimated in chapter 3. We represent in blue the optimal trajectories corresponding to each set of parameter estimates and in red the optimal trajectory obtained by using the parameters provided by the rolling stock manufacturer in Table 6.1. All the speed profiles have been calculated based on the same running time, 750 s, and no delays were considered in the departure times.

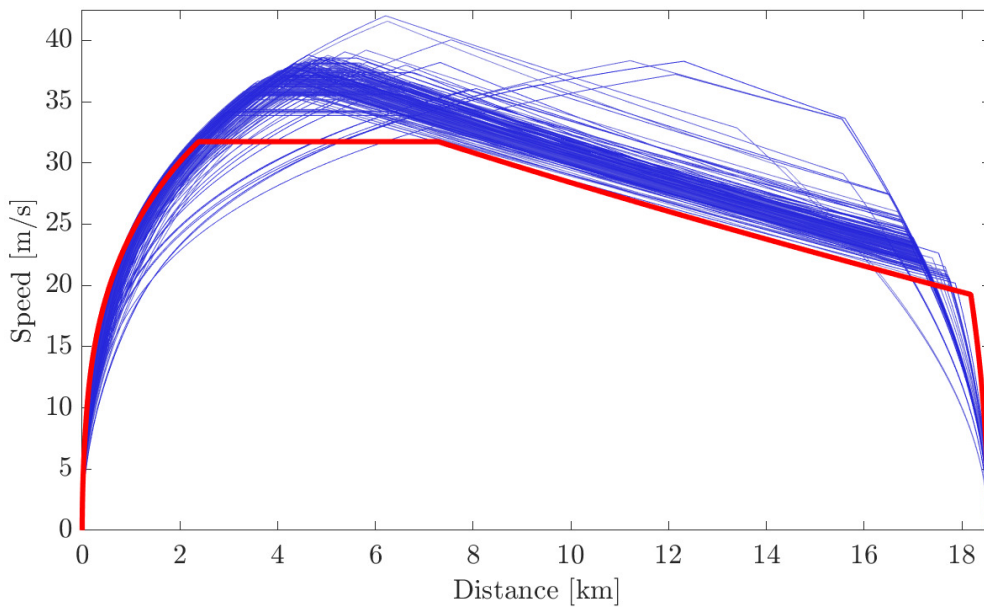


Figure 6.8: Variability of optimal trajectories under observed parameter variations.

We observe that the optimal trajectories obtained from the parameter estimates require reaching higher speeds than the one related to the manufacturer values and than what we observed in Figure 6.2. Moreover, several of these trajectories do not contain a cruise regime. Compared to the results presented in Figure 6.2, the initial traction and the final brake regimes are often considerably longer in the trajectories calculated using estimated parameters. This is caused by several factors. First, the brake performance is always lower than the maximum capacity expected by the manufacturer. Train drivers drive gently, avoiding to use the maximum brake rate. Second, in most cases the traction characteristics of the train are also lower than the manufacturer values. We notice three low traction performance trajectories where the initial traction regime lasted until km 11 and further. This leads to time losses in the traction and brake regimes with respect to the red trajectory, so the blue trajectories will need to reach higher speeds to comply with the expected arrival time. Given that these parameters were estimated from trains in operation, the driving behaviour observed might have been influenced by the actual delay

of the trains at the time the trajectories that were analyzed in chapter 3 were recorded. However, the potential effects of delays on the driving style and the physics of the train are not considered in this chapter, but they are likely to have an impact on the shape of optimal trajectories. When it comes to the running resistance parameters, 88% of the estimates of R_2 are lower than the manufacturer value. Therefore, reaching higher speeds is less energy-consuming than what is expected from the manufacturer values of the parameters. This leads to higher cruise speeds, some of them way too high to be reached given the time and distance constraints. In those cases, a cruise regime is not observed. Speed limits were not included in the calculations of these trajectories to avoid a potential confusion between cruising at the optimal cruise speed and at the speed limit. Five trajectories would hit the speed limit at 140 km/h (38.9 m/s).

Figure 6.9 shows the collection of time-distance diagrams of these trajectories in blue, while the time-distance path corresponding to the manufacturer's parameters is depicted in red. We observe a wide spread in the passing times throughout the whole track. For instance, around kilometric point 8, the fastest passing time is 270 s and the slowest, 364 s. This maximum passing time difference is much larger than the individual maximum differences observed for each individual parameter, which were 7, 8, 9.5, 10 and 16 s for the mass, R_2 , $f_{\max}$, $f_{\min}$ and $p_{\max}$, respectively. Joint variations in the driving style, the mass and R_2 might explain the observed wider difference in passing times. All the optimal trajectories calculated from the parameter estimates run earlier than the one computed using the parameters provided by the manufacturer during the final kilometers of the trajectory. This highlights that the capacity of a line can be lower if the driving styles among trains are not homogeneous. The volatility of the expected passing times highlights that train motion model parameter variations and driving variability must be taken into account in the definition of Train Path Envelopes that provide time constraints to the train trajectory optimization at discrete timing points (Wang et al., 2023). Furthermore, the optimal trajectories should also consider the running time supplement available given the current delay of the train, potentially forcing some of the trains to run faster, which would probably reduce the volatility of the passing times at most locations.

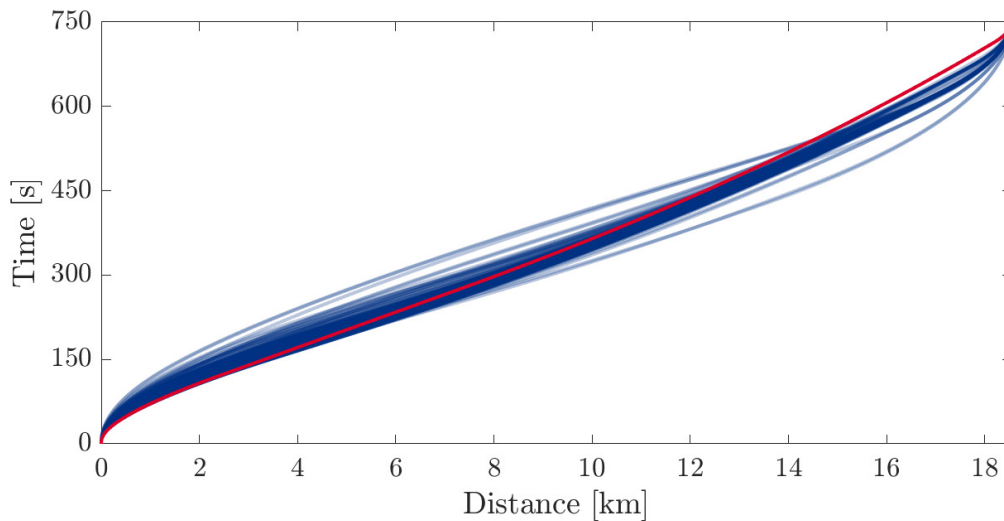


Figure 6.9: Time-distance diagram of all the optimal trajectories calculated using observed parameters.

6.5 Conclusion

Train Trajectory Optimization (TTO) is one of the most popular energy-efficient driving techniques used by railway operators to arrive on time while saving energy. The accuracy of the trajectories produced by TTOs depends on a precise calibration of the parameters of the train motion model that describes the dynamics of a train. Variations of these parameters may impact optimal train trajectories by influencing their shape.

The influence of each train motion model parameter on the characteristics of optimal train trajectories was still to be assessed in the scientific literature. In this chapter we have performed a sensitivity analysis of the volatility of optimal train trajectories with respect to parameter variations. We have calculated energy and time-optimal trajectories for different values of the train motion model parameters, varying each of them one by one while the others are kept fixed. For this purpose, we used an upgraded version of the convex TTO algorithm introduced by Jibrin et al. (2021b), where Chebyshev collocation points are allocated to reduce the sensitivity of the computed trajectories with respect to the number of collocation points. We quantified the impact of each parameter on different features of optimal trajectories, namely the energy consumption, running time, optimal cruise and brake speeds, and the locations where the cruise, coast and brake regimes begin. This allowed us to identify the most critical parameters for TTO and energy-efficient driving, highlighting the need to calibrate them accurately. We considered two scenarios: a 18.542 km long, flat one, that allows to observe a cruise regime in energy-optimal trajectories, and a shorter one, 5 km long, flat scenario without a cruise regime. We also discussed how a single gradient can influence energy-optimal trajectories. All these analyses do not consider any potential correlations among the train motion model parameters. We have analyzed the impact of potential correlations by studying the differences in energy-optimal trajectories by analyzing trajectories calculated using different sets of observed parameter variations and compared them with respect to the results obtained using the parameters provided by the rolling stock manufacturer.

We have identified the most critical parameters for train trajectory optimization, that is to say, those that have the largest impact on optimal trajectories. We observed that the quadratic running resistance parameter R_2 is the most critical parameter that determines the shape of energy-optimal train trajectories with cruise regime. The train mass is the second most critical parameter for this type of trajectories and we observed that the length of the traction and brake regimes are determined by the maximum tractive power and the maximum brake effort, respectively. In time-optimal trajectories over the same distance, the traction and brake characteristics have more impact than the mass and R_2 , although the energy consumption is most sensitive to the latter parameter. Energy-optimal trajectories without a cruise regime may be observed when a train runs over shorter distances. In such a short scenario, the maximum tractive power and brake effort show the largest impact on both energy and time-optimal trajectories. The quadratic running resistance parameter R_2 is less relevant in short trajectories. The shorter distances covered reduce the amount of kinetic energy lost due to the running resistance. The maximum traction and brake characteristics are used more frequently, which explains their predominant relevance in short services. In this regard, ATO can reduce the variability of the amount of traction and brake when driving the train, maximizing the potential for saving energy. Furthermore, this information can be of interest for instructing train

drivers. Still, the train load has a strong impact on the energy consumption. A gradient placed in the middle of the coast regime has a strong impact on the shape of the trajectory. We noticed that the starting brake speed is more sensitive to gradients than to parameter variations. Moreover, the shape of energy-optimal trajectories is more influenced by uphill gradients than by downhill gradients. Regarding the differences between the optimal trajectories obtained from parameters estimated and provided by the manufacturer, we observe that the observed reduced performance in the traction and brake regimes requires reaching higher speeds. The results presented in this chapter will motivate the research presented in chapter 8, where we will address the generation of train trajectories that are robust against parametric variations.

The relevance of the maximum tractive power and brake effort on optimal train trajectories implies that rolling stock and engine maintenance is critical for the performance of energy-efficient driving techniques. Moreover, the maximum applied tractive and brake effort is considerably lower than the maximum capability of the train (see chapter 3). We have demonstrated that ATO and a more aggressive manual driving style lead to more energy savings in services where the stops are shortly-distanced, for instance, in regional and suburban railways. Still, the trade-off between driving more aggressively and passenger comfort should be taken into account when instructing drivers. The quadratic running resistance parameter is directly related to the shape of the head of the train, the wind and the shape of those tunnels where a train runs through. We observed that this parameter is the most critical in long energy-optimal trajectories. Therefore, these factors have to be taken into special consideration for high speed and mainline trains that service lines with distant stops. The train mass and load are also relevant for energy-optimal trajectories between distant stops. Thus, introducing an accurate measurement or estimate of the mass and load will lead to larger energy savings for heavier trains like freight trains.

Chapter 7

Coasting advice based on the analytical solutions of the train motion model

Apart from minor changes, this chapter has been published as Cunillera, A., Jonker, H.H., Scheepmaker, G.M., Bogers, W.H.T.J., Goverde R.M.P. (2023) Coasting advice based on the analytical solutions of the train motion model. *Journal of Rail Transport Planning & Management*, 28, 100412.

7.1 Introduction

Nowadays, trains are equipped with several computers and central processing units that continuously monitor the rolling stock, analyze and store the data measured by sensors implemented on-board, and communicate with external systems such as trackside equipment, the traffic management system and other platforms used by the railway undertaking and the infrastructure manager. All this consumes a large part of the computational power of trains, so efficient software and algorithms are an important asset whenever on-board computations are required.

On-board energy-efficient driving algorithms such as Driver Advisory Systems (DAS) and Automatic Train Operation (ATO) are no exception. Some of these energy-efficient algorithms use Train Trajectory Optimization (TTO) to compute in real time the reference trajectories in order to improve the timeliness and energy usage. Scientific research on TTO aims to minimize total traction energy consumption of a train run given the running times of the timetable. We refer to Scheepmaker et al. (2017) and Yang et al. (2015) for a comprehensive overview on the topic of TTO. A DAS translates the computed trajectories into driving advice that can be accepted and followed by drivers. At *Nederlandse Spoorwegen* (NS), the Dutch railway undertaking, energy-efficient train driving is only applied if a train is not delayed. In turn, ATO algorithms calculate the right amount of tractive and brake effort required to track the reference trajectory. Furthermore, online model calibration is required to maintain over time the accurate performance of the men-

tioned energy-efficient driving algorithms (Cunillera et al., 2022, 2023a), exerting further pressure on the processing units.

In some cases, the restrictive computation power can be a major limitation when updating energy-efficient driving algorithms to enhance their functionalities and increasing their complexity. This was the case of *TimTim*, the in-house real-time route information display app on a tablet for drivers developed by NS. The energy-efficient driving strategy at NS is based on coasting. This is most convenient for the train drivers, and also the energy-efficient driving strategy that has been applied several times in history at NS (i.e., even in the time of the steam engines, train drivers at NS were stimulated to reduce the amount of coals used in order to save energy). Scheepmaker et al. (2020b) showed that for the Dutch case a maximal coasting strategy (following cruising at the speed limit) leads to higher energy savings compared to a timetable based on a reduced cruising speed driving strategy (without coasting), mainly due to the relatively short distance between two stops. For a short distance train with a stop distance of 5 km, mechanical braking and 5% running time supplements, the maximum coasting driving strategy saves about 2.7% more energy than the reduced maximum speed (cruising), while for a long distance train with a stop distance of 50 km, mechanical braking and 5% running time supplements, the maximum coasting strategy saves about 8.2% more energy than the reduced maximum speed strategy (Scheepmaker et al., 2020b). More information about the history of energy-efficient driving at NS can be found in Luijt et al. (2017). The current energy-efficient driving strategy implemented in the DAS of NS, i.e., *Roltijd* on the TimTim app is based on the UZI method (Universal energy-efficient driving idea or in Dutch *Universeel Zuinig rijden Idee*) developed by a train driver at NS (Scheepmaker & Goverde, 2015). The UZI method gives information using static tables to the train driver when to start coasting, depending on the available running time and speed limit between two stops. *Roltijd* computes the coasting curves dynamically in real-time. This is used to indicate to the driver the most appropriate time to start coasting in order to arrive on time at the next stopping point. This contributes to saving energy, while helping drivers to arrive on time and preventing unnecessary delays. Every second the system determines the train speed and location based on GPS measurements and calculates the expected arrival time, provided that the train starts coasting immediately. The difference between the expected arrival time and the target time is displayed to the driver. The coasting advice is therefore provided as a countdown to start coasting. An example of the different screens shown by *Roltijd* during a train ride between two stops can be found in Figure 7.1.

In *Roltijd* the journey between two stops is divided in seven phases, as shown in Figure 7.1. From left to right, we show first the countdown to the scheduled departure time and the report of the actual departure time versus the scheduled one, both in grey. Then, in white, if the train is accelerating, the coasting curve is not calculated. If the train is cruising, the expected arrival time is calculated, provided that the train starts coasting now. If the train would arrive late, the information is shown in white. If the train would arrive on time or early, the information is shown in blue. The displayed information is shown in grey when the train is braking, since the driver must focus on the outside, and a countdown to the scheduled arrival time is shown. Last, the difference between the actual and the scheduled arrival times is displayed once the train has stopped.

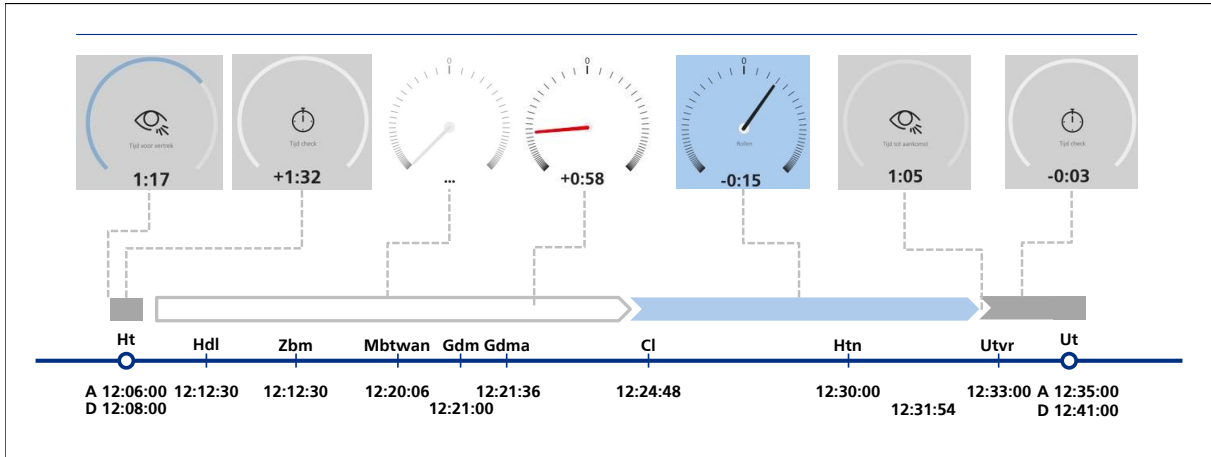


Figure 7.1: *Roltijd* coasting advice during a long distance train run between the stops Ht ('s-Hertogenbosch) and Ut (Utrecht Central). The blue screen (number 5 from right) indicates that the train would arrive 15 s early if it started coasting immediately.

Two main challenges had to be overcome before tackling the plans of the company to expand the functionalities of the DAS. First, the computational burden of *Roltijd* was limiting the implementation of more complex driving advice procedures. The coasting curves were calculated by integrating numerically the train motion model, which is the equation that describes the dynamics of a train running on a track. A small time step was used to keep the calculations accurate enough at the cost of the mentioned high computation time. However, some inaccuracies due to the numerical discretization procedure had been noticed. The forward Euler method was used as an integration method, which has a global error proportional to the integration step (Butcher, 2016). Second, *Roltijd* considered all tracks flat. Despite the fact that the Netherlands is one of the flattest countries in the world, this was leading to non-negligible deviations between the expected and the actual arrival times. For instance, drivers reported a deviation of 17 s at Schiphol Airport station, one of the most critical bottlenecks of the Dutch railway network, because Schiphol is located in a downhill section in a tunnel, and trains reached the brake curve earlier than expected since the track was considered flat in the calculation of the arrival time. Hence, there was a need to find a more efficient way of calculating the speed of a coasting train at a certain location and the time required to reach it. The height profile of the track should also be introduced as input to the algorithm in the new coasting curve calculation method in order to increase the accuracy of the coasting advice.

These issues have been addressed and solved in a joint effort between Team Newton of NS, and the Department of Transport and Planning of Delft University of Technology. In this chapter we present the main results of this collaboration. We have developed a coasting curve calculation method based on the analytical solutions of the train motion model. The new algorithm uses as input the current speed and location of the train, the speed limits of the track, the distance to the next stopping point, the target arrival time, the brake curve to be followed based on the Dutch Automatic Train Protection system ATB, and the location of signs and signals. Regarding the gradients, a description of the height profile of the track consisting of piecewise constant gradients is also used as input.

This assumption allows to find closed explicit expressions of the speed as a function of the time and vice versa, and of the location as a function of the speed and the time. The expressions of the speed and the time as a function of the location have been obtained as implicit formulas, that can be solved by means of a Golden-section Search algorithm (Kiefer, 1953). When the track is considered flat, the intersection between the coast and brake curves can be calculated in one go, reducing the computation time by a factor of 15 with respect to the numerical integration procedure. When gradients are considered, the speed and time have to be computed at the locations where the gradients change. Moreover, the analytical calculation of the coasting curves removes the global integration error of the numerical algorithm proportional to the integration step used.

Some analytical solutions of the train motion model have already been described in the existing literature. Franke et al. (2000) solved the train motion model analytically using the kinetic energy as the state variable instead of the speed. The running resistance is usually modelled as a quadratic function of the speed called the Davis equation (Davis, 1926). The model was simplified by removing the linear term of this speed-dependent running resistance. When the kinetic energy is used instead of the speed in the train motion model, the mentioned linear term is transformed into a term that depends on the square root of the kinetic energy. Thus, this nonlinearity was neglected in that approach. Ye & Liu (2017) solved the train motion model using the speed as the independent variable and piecewise constant gradients and speed limits. However, they modelled the maximum tractive effort and brake as piecewise linear functions of the speed. The maximum effort is generally considered to depend linearly on the speed at low speeds, since adhesion limits the maximum effort that can be applied before the wheels slip. At higher speeds, however, the maximum tractive effort is usually limited by the maximum power of the engine, so the maximum effort depends hyperbolically on the train speed (Hansen & Pachl, 2014). Schank (2011) and Jaekel & Albrecht (2014) modelled the running resistance and the tractive effort as piecewise quadratic functions of the speed. This leads to a quadratic acceleration in the resulting differential equation, which is solved analytically in both references. Becker & Schreckenber (2018) described analytical solutions of the train motion model, studying maximum traction, cruising, coasting and braking separately to compute the running times. They considered constant deceleration when braking, and disregarded the running resistance when applying maximum tractive effort. They modelled the maximum tractive effort as a piecewise quadratic function of the speed for low speeds, and as the power divided by the speed for high speeds. Moreover, they provide a closed expression for the mechanical energy consumed. Hansen & Pachl (2014), derived similar analytical solutions of the train motion model considering also a maximum tractive effort that depends quadratically on the speed for low speeds. They also considered the hyperbolic dependence of the maximum tractive effort on the speed at high speeds, including the running resistances, however, the solutions of the resulting integrals are not shown. These solutions are expanded in Körner & Dahlhaus (European Patent 20 050 025 843, Jul. 25, 2007), describing the acceleration as a polynomial, which is factorized after calculating its roots. This factorization allows to calculate the analytical solutions of the train motion model, however, the analytical formulas of the location as a function of the speed when maximum tractive power is applied, that is to say, when the maximum effort depends hyperbolically on the speed, have not been published yet in the existing literature.

In this chapter we propose a real-time coasting advice generator based on analytical solutions of the train motion model's differential equations. We extend the formulas mentioned in the previous paragraph and calculate the analytical solutions of the train motion model when the maximum tractive and brake effort is limited by the maximum power of the engine, including the mentioned formulas that up to this date were missing in the literature. We use the polynomial factorization introduced in Körner & Dahlhaus (European Patent 20 050 025 843, Jul. 25, 2007) to calculate such formulas. Although the analytic solutions that arise when maximum engine power is applied are not used in the coasting advice algorithm, we include them in this chapter for completeness. These formulas could be used in the future for extending the driving advice calculation including acceleration phases at high speeds that require using the maximum power of the engine. Furthermore, we present a novel analysis of the analytical solutions of the train motion model differential equations based on the theory of dynamical systems (Strogatz, 2018) and the Fundamental Theorem of Algebra (Fine & Rosenberger, 1997). These solutions show bifurcations depending on the value of the gradient and the effort or power applied. The proposed coasting advice algorithm and the formulas implemented are also valid in the case of a constant applied tractive and brake effort, which will allow us to extend these analytical calculations in the near future to provide more complex driving advice.

We have tested the proposed coasting curve calculator in a case study. We have used MATLAB's ode45 function to validate it. This function solves ordinary differential equations using an adaptive fifth-order Runge-Kutta method. We have compared it with the previously-used numerical calculator and the coasting curves performed by a train trajectory optimizer based on a pseudospectral method (Scheepmaker et al., 2020b; Goverde et al., 2021; Wang & Goverde, 2016), finding that the three of them produce similar results, but the proposed method is computationally more efficient than the other two. Although the formulas corresponding to a maximum tractive or brake power have not been implemented in the proposed coasting curve calculator, we also validate them with respect to ode45. The proposed coasting curve calculation method has already been implemented in *Roltijd* and in the current version of *TimTim*, so it is already contributing to transport timely 1.3 million passengers daily in a more sustainable way.

To sum up, the main contributions of this chapter are:

- A computationally-efficient coasting advice algorithm that indicates drivers when to start coasting to arrive on time while saving energy.
- A complete overview of the analytical solutions of the train motion model's differential equations, with closed formulas describing the location as a function of the speed under constant gradient, tractive effort that depends hyperbolically on the speed, and quadratic running resistance.
- A novel analysis of the qualitative behaviour of these solutions, interpreting the train motion model as a vector field and using the theory of dynamical systems to observe the bifurcations described by these solutions. Moreover, we prove the existence of fixed points that constitute terminal speeds of a train under constant gradient and applied tractive effort or power.

The structure of the remainder of this chapter is outlined as follows: In chapter 7.2 we further describe *TimTim*, *Roltijd*, the previous way of calculating the coasting advice, and the train trajectory optimizer used to validate the new method. In chapter 7.3 we derive the analytical solutions of the train motion model's differential equations, analyze their algebraic properties and describe the new coasting curve calculation algorithm. Chapter 7.4 gathers the results of the validation of the proposed algorithm and the analytical formulas with respect to the other two methods. Last, in chapter 7.5 we summarize the main conclusions of this research and mention our future steps.

7.2 Driving advice at Nederlandse Spoorwegen

7.2.1 Onboard driving advice applications: *TimTim* and *Roltijd*

TimTim is a Driver Advisory System application developed by the NS IT department. Drivers use the application on their personal tablet or on a built-in screen in the cabin. *TimTim* informs the driver about the timetable to follow, route settings, temporary speed restrictions and information about surrounding trains. This information helps the driver to drive the train in a punctual and energy efficient way. The application has a wireless connection with the back offices of NS and ProRail (the infrastructure manager) for receiving real time updates of this information.

In addition, *TimTim* contains *Roltijd*, a basic speed profile calculation module that continuously evaluates the current position of the train (based on the tablet or train GPS) with respect to the target arrival at the next station and indicates to the driver when to start coasting in order to arrive on time at the next stop. The expected arrival time is recalculated every second and the corresponding coasting advice is updated in the application, helping drivers to save energy.

Two different stop approaches are considered:

1. If, on the one hand, the train must stop and there are speed reductions due to speed signs or signals, the coasting curve ends at a specific location and then the train brakes following an ATB braking curve with constant deceleration. For instance, this is used to approach a switch or a curve that forces the train to reduce its speed.
2. If, on the other hand, there is no explicit speed reduction, the train coasts until it meets a comfort braking curve, which again is based on a constant deceleration.

The main difference between these two scenarios is that the end of the coasting curve in the latter case is not fixed, but speed-dependent. In both situations, the train is expected to drive at 40 km/h at the start of the platform and stops at the stopping point by following a comfort brake curve. The arrival time is calculated as the sum of coasting time to the braking location, the ATB or comfort braking time until reaching 40 km/h, the time to drive along the platform at that speed and the time to stop from 40 km/h to a standstill following a comfort braking curve. The locations of signs, signals and stopping points are known with an accuracy of a cm.

7.2.2 Train Trajectory Optimization based on a pseudospectral method

We used the solver PROMO to compute the energy optimal train trajectory. The aim of the Train Trajectory Optimization problem is to minimize total traction energy consumption of a train given the amount of running time in the timetable (Scheepmaker et al., 2017), while considering rolling stock and track characteristics such as the speed-dependent running resistance, the maximum tractive and brake effort and power that can be applied, the speed limits and the height profile of the track. PROMO is a solver developed in MATLAB using the GPOPS (General Purpose OPTimal control Software) toolbox version 4.1 (Rao et al., 2010). A Radau pseudospectral method is implemented in GPOPS to solve the optimal control problem (Rao et al., 2011). More details of PROMO can be found in Scheepmaker et al. (2020a). To discretize the optimal control problem, the Radau pseudospectral method applies orthogonal collocation at the Legendre-Gauss-Radau (LGR) points. Afterwards, the optimal control problem is transcribed into to a nonlinear programming (NLP) problem, which can be solved using standard NLP solvers (Betts, 2010). GPOPS has the possibility to evaluate the costates and Hamiltonian to validate whether the results are in line with optimal control theory. Wang & Goverde (2016) and Goverde et al. (2021) give more details regarding the discretization of the optimal control problem. We applied PROMO in different case studies to compute the energy-optimal driving strategy as well as other driving strategies (Scheepmaker et al., 2020b; Scheepmaker & Goverde, 2020) and include the energy-efficient train trajectories into timetable design (Scheepmaker et al., 2020a; Scheepmaker & Goverde, 2021). In this chapter, we use PROMO to validate the coasting advice algorithms.

7.2.3 Numerical coasting curve calculation based on the forward Euler's integration method

Roltijd's coasting curve calculation was performed by integrating numerically the equations of motion of the train (Hansen & Pachl, 2014),

$$m\rho \frac{dv}{dt} = F(t) - R(v) - W(s), \quad (7.1)$$

$$\frac{ds}{dt} = v, \quad (7.2)$$

where s is the train location (in m), v the train speed (in m/s), t the time (in s), m the train mass (in kg), ρ the rotating mass factor that accounts for the inertia of the rotating parts of the train (it is dimensionless), $F(t)$ is the tractive effort and brake applied by the engine (in N), $R(v)$ is the running resistance of the train (in N) and $W(s)$ is the resistance to motion due to the track geometry (in N). In order to simplify Eq. (7.1), we can divide both sides of the equation by $m\rho$, obtaining a mass-specific version of the equation,

$$\frac{dv}{dt} = f(t) - r(v) - w(s), \quad (7.3)$$

where $f(t) = F(t)/(m\rho)$, $r(v) = R(v)/(m\rho)$ and $w(s) = W(s)/(m\rho)$.

The maximum tractive effort that can be applied is usually modelled as constant or linearly-dependent on the speed at low speeds, and limited by the maximum power of the engine $p_{\max}$ at higher speeds, where $p_{\max}$ is in W/kg.

$$f(t) \leq \begin{cases} f_{\max}, & \text{for } 0 \leq v < p_{\max}/f_{\max}, \\ p_{\max}/v, & \text{for } v \geq p_{\max}/f_{\max}. \end{cases} \quad (7.4)$$

Regarding the bounds on the brake effort, we assume for simplicity that the braking capacity of the train is large enough so that the train can follow predefined brake curves at any time and under any operating condition. The analytical solutions of the train motion model presented in chapter 7.3 would also be valid if the maximum brake is modelled similar to the maximum traction.

The running resistance is usually modelled as a quadratic function of the speed, and it is usually called the Davis equation (Davis, 1926),

$$r(v) = r_0 + r_1 v + r_2 v^2, \quad (7.5)$$

where $r_0, r_2 > 0$ and $r_1 \geq 0$, in N/kg, N/(kg(m/s)²) and N/(kg m/s), respectively.

The term related to the track geometry is usually modelled as

$$w(s) = \frac{g}{\rho} \sin(\theta(s)), \quad (7.6)$$

where $g = 9.81 \text{ m/s}^2$ is the gravity acceleration and $\theta(s)$ is the slope in rad of the track at location s . Therefore, $w(s)$ is positive when the train is running on an uphill section, negative on a downhill section and zero on a flat track.

When the train is coasting, f is zero. Then, if a train starts coasting at time t_0 and considering the state of the train at that time, $s_0 = s(t_0)$ and $v_0 = v(t_0)$, the location and speed of the train can be calculated numerically by means of the forward Euler integration method. The acceleration between two consecutive calculations is assumed to be constant, so from Eqs. (7.2) and (7.3) we deduce

$$s_{k+1} = s_k + v_k \Delta t - \frac{1}{2} (r(v_k) + w(s_k)) \Delta t^2, \quad (7.7)$$

$$v_{k+1} = v_k - (r(v_k) + w(s_k)) \Delta t, \quad (7.8)$$

where Δt is the integration time step, $s_k = s(t_k)$, $v_k = v(t_k)$ and $t_k = t_0 + k\Delta t$. In *Roltijd*, Δt is equal to 1 s. Figure 7.2 illustrates the computation of a coast curve using the forward Euler method. Each vertical line represents a time step where the speed is calculated from the speed and location at the previous time step. The dark blue curve in the upper figure represents the train speed during coasting, while the yellow lines represent the speed limit. In the lower figure we show the resistance to motion due to the gradients in red. The full line at zero resistance shows that the old algorithm considers the track flat. As we can observe in this example, the gradients shown as red dotted lines are neglected. This implementation utilizes only four parameters, r_0 , r_1 , r_2 and m , which are known for each rolling stock set and are considered to be constant along each journey. *Roltijd* is based on the assumption that in the last phase of reaching the stopping point location the train is always forced to reduce speed from maximum track speed (or cruising speed)

to 40 km/h through a series of speed reduction signals or signs (e.g. 80, 60, 40). Exact locations of these speed reduction points are known in the data and speed reduction is performed strictly following the ATB braking criteria. This brake curve is precomputed by considering constant deceleration rates. At every location, update Eqs. (7.7) and (7.8) are utilized recursively until the coasting curve intersects the brake curve or it reaches the location where the coasting curve must end. This is used to calculate the expected arrival time at the next stopping point if the train started coasting now, that is to say, at t_0 . The difference between the expected and the target arrival time is displayed to the driver as a countdown to start coasting. *Roltijd* updates this information every second by repeating this procedure.

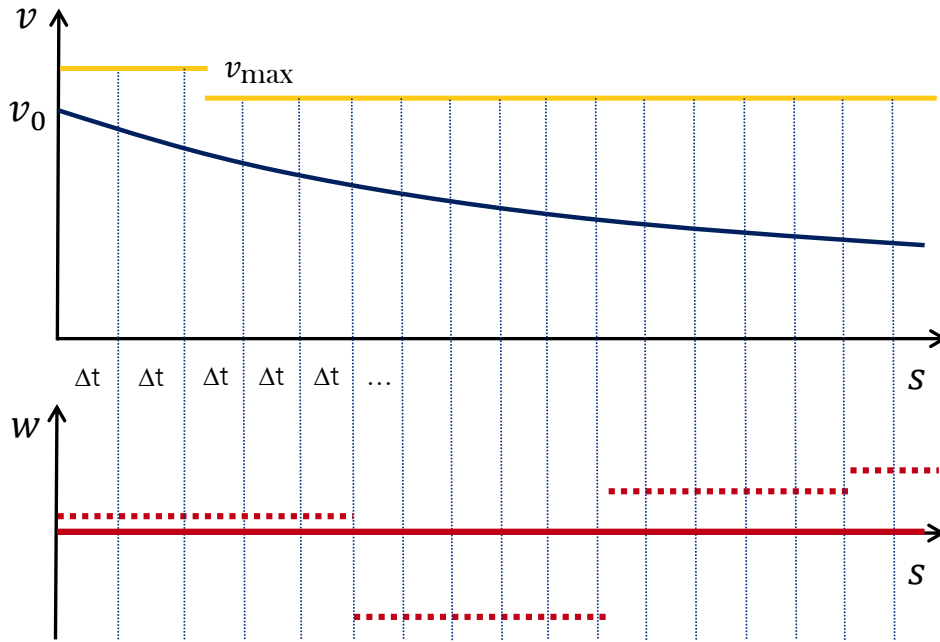


Figure 7.2: Schematic representation of a coast curve calculated by means of the forward Euler method with a time step equal to Δt s.

Despite the simplicity of this algorithm, the error of the forward Euler method per integration step is proportional to the square of the integration time step, while the accumulated error over all the iterations is proportional to the integration time step (Butcher, 2016). Therefore, the error of this algorithm at the brake curve may be in the order of a few seconds. This numerical coasting advice calculation method has some shortcomings. It requires too many computations per second, which compromises the limited computational resources of the tablet. NS aims to extend the coasting advice into driving advice along the entire trip, including multiple coasting sections over intermediate timing points and traction, brake and cruise advice. This will require more efficient calculations, as well as supporting varying track gradients. Moreover, NS moved from a minute based timetable to a 6 second interval based timetable, which requires more accuracy from drivers. Inaccuracies on the coasting advice that lead to deviations of more than 6 seconds are therefore not desirable.

7.3 Generating coasting advice using the analytical solutions of the train motion model

7.3.1 Analysis of the analytical solutions of the train motion model

Constant tractive and brake effort

Eqs. (7.2) and (7.3) can be considered as a dynamical system depending on two parameters: the applied effort and the resistance due to the track geometry. Considering these two parameters to be constant, we proceed to analyze the dependence of solutions of the train motion model on these two parameters using the theory of dynamical systems (Strogatz, 2018). This analysis is essential for calculating the analytical solutions and for the coasting advice algorithm, as we will show later in this chapter.

We can consider the acceleration as a quadratic polynomial with the speed as its independent variable in each track interval where the the applied effort f and the track resistance term w are constant,

$$\frac{dv}{dt} = f - w - r_0 - r_1 v - r_2 v^2. \quad (7.9)$$

In fact, Eq. (7.9) can be interpreted as a one-dimensional vector field when the speed is considered as its independent variable, and we can find easily its fixed points, which are the speeds at which the acceleration is zero. According to the Fundamental Theorem of Algebra (Fine & Rosenberger, 1997), a quadratic polynomial always has two roots and three possible scenarios may occur, depending on the values of f and w :

- **Case a:** The roots of the polynomial are two complex conjugates, $\nu_1, \nu_2 \in \mathbb{C}$, with $\nu_2 = \bar{\nu}_1$.
- **Case b:** The polynomial has a non-positive real double root at the maximum of the parabola described by the polynomial, $\nu_1 = \nu_2 = -\frac{r_1}{2r_2} \in \mathbb{R}$. We define α based on this double root, $\alpha := \frac{r_1}{2r_2} \geq 0$, since $r_1 \geq 0$ and $r_2 > 0$.
- **Case c:** The polynomial has two real, simple solutions, ν_1 and ν_2 , with $\nu_2 > \nu_1$. Moreover, ν_1 will always be negative.

Each of these three cases lead to a different formula when calculating the analytical solution of Eq. (7.9). Therefore, the roots ν_1 and ν_2 of the acceleration polynomial have to be calculated first in order to identify which is the formula to be used.

We have to observe the sign of the acceleration as a function of the train speed in order to understand the physical meaning of each of these three cases. In cases a and b, the acceleration is negative for any value of the speed, except at speed $-\alpha$ in case b, where the acceleration would be zero. However, this speed is negative, so it is not physically meaningful. In case c, the acceleration polynomial has two fixed points: ν_1 , an unstable fixed point, and ν_2 , a stable one. The acceleration is negative for any speed below ν_1 and for any speed larger than ν_2 , and positive between ν_1 and ν_2 . Figure 7.3 shows the phase portrait of case c with positive ν_2 , where the acceleration polynomial is represented in blue in an acceleration-speed diagram.

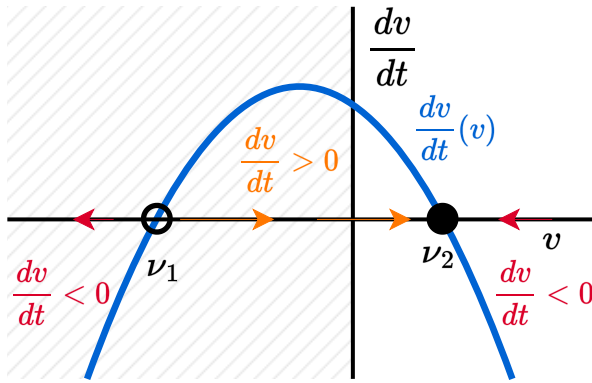
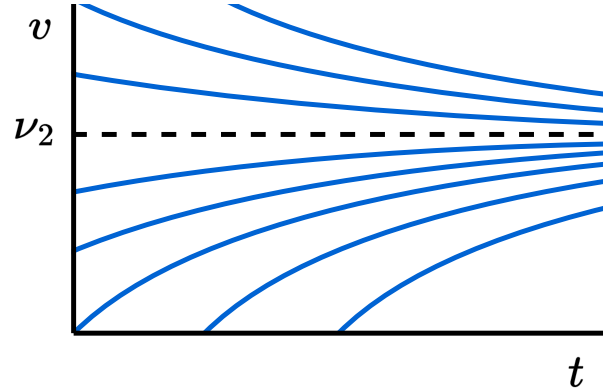


Figure 7.3: Phase portrait of case c.

Figure 7.4: Vector flow around a terminal speed ν_2 .Figure 7.5: Dynamics of the acceleration polynomial in case c, where the stable fixed point ν_2 constitutes a terminal speed.

In Figure 7.3 the negative speeds part of the phase portrait is shaded since it is not physically meaningful. The unstable and stable fixed points, ν_1 and ν_2 are represented by empty and full dots, respectively. ν_2 is the terminal speed for the effort f and the gradient resistance w . This fact is particularly relevant if ν_2 is positive, for instance, when coasting on a steep downhill section or when applying traction on a non-steep section. If a train is running at a speed higher than ν_2 , it will decelerate towards ν_2 , and, if it is running below that speed, it will accelerate towards it. Nevertheless, this terminal speed can never be reached at a finite time if f and w are kept constant, as ν_2 is approached asymptotically. This is due to the fact that ν_2 is a stable fixed point of the vector field in Eq. (7.9). In the coasting advice algorithm we used this fact to avoid calculating the location or passing time at speeds that cannot be reached by coasting under the considered gradient. Figure 7.4 shows the vector flow of Eq. (7.9) around a positive terminal speed ν_2 . Thus, case c can only be found in particular scenarios, i.e. when applying constant traction on non-steep uphill track sections or when a train is coasting on steep downhill sections. The deceleration over speed ν_2 takes place at high speeds, where the applied effort is not enough to compensate the running and track resistances. In turn, cases a and b may take place when the train is braking or coasting at non-steep downhill gradients, or, again, when the applied effort is insufficient to keep a constant speed.

Figure 7.6 shows the bifurcation diagram of Eq. (7.9), where the vertical axis represents the train speed and the horizontal axis the difference between the applied effort and the gradient resistance, $f - w$. The vertical lines represent constant values of $f - w$, and the arrows over them show the sign of the acceleration at those speeds. Since the running resistance parameters are considered constant and fixed, only variations in the term $f - w$ may lead to qualitative changes in the solutions of the dynamical system in Eq. (7.9). The blue line shows the fixed points of the vector field as a function of $f - w$. At any fixed value of $f - w$, the solid dot represents the stable fixed point ν_2 and the open one the unstable fixed point ν_1 . We observe a bifurcation: for negative and the smaller positive values of $f - w$, the dynamical system has no real roots, leading to case a. Then, for a particular value of $f - w$, the dynamical system has only a semistable fixed point, corresponding to

case b, with $\nu_1 = \nu_2$. Last, for higher values of $f - w$, case c arises and the dynamical system has two real simple roots.

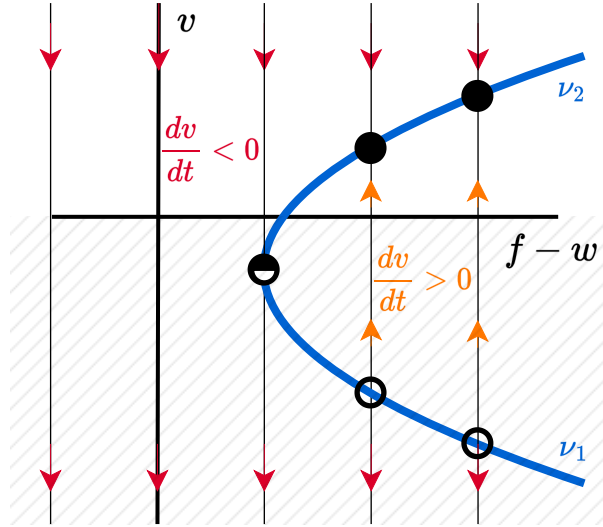


Figure 7.6: Bifurcation diagram of Eq. (7.9).

Constant tractive and brake power

When the train is accelerating at maximum power, $f = p_{\max}/v$, with $p_{\max} > 0$, Eq. (7.3) turns into

$$\frac{dv}{dt} = \frac{p_{\max}}{v} - w - r_0 - r_1 v - r_2 v^2 = \frac{1}{v}(p_{\max} - (w + r_0)v - r_1 v^2 - r_2 v^3). \quad (7.10)$$

Taking $1/v$ as a common factor in the right-hand-side term of Eq. (7.10), we can write it as $1/v$ multiplied by a cubic polynomial. The analysis of Eq. (7.10) is similar to the one presented for the constant effort, so we will cover it briefly. It shows that this vector field always has a real positive root ν_3 , and the other two can be either complex conjugates, a real negative double root, or two real negative simple roots. Again, these three scenarios lead to analytical solutions with different formulas. Regarding the physics of the fixed points, ν_3 is the only one that may be of interest, since it is a stable fixed point that may constitute again a terminal speed if it is large enough.

The case in which a train is braking at the maximum braking power $f = -p_{\max}/v$, with $p_{\max} > 0$ is analogous to the maximum tractive power, but changing the signs of all the roots. A negative real root is always found, and two positive simple roots may be found if the track is sufficiently downhill. The largest of the roots would be a terminal speed where the downhill gradient balances the running resistance and the brake power, and the other positive root would be an unstable fixed point, where any perturbation would deviate the train speed from that fixed point. Nevertheless, the Fundamental Theory of Algebra can be used to prove that braking at maximum power allows for a special case in which a negative triple root is found at $\nu_1 = -2\alpha/3$. This special case can only arise if r_1 is strictly positive in sections where $r_0 + w = 4r_2\alpha^2/3 > 0$ and $p_{\max} = 8r_2\alpha^3/27$.

7.3.2 Analytical solution of the train motion model's differential equations

Constant tractive and brake effort: analytical solutions

Considering a track section in which the applied tractive effort and brake, and the gradient are constant, we may rewrite the right-hand-side term of Eq. (7.3) as a quadratic function of the speed,

$$\frac{dv}{dt} = (f - w - r_0) - r_1 v - r_2 v^2. \quad (7.11)$$

This equation can be integrated easily since its right-hand-side term does not depend on the time. Indeed, since $r_2 > 0$,

$$\int_{v_0}^v \frac{dv}{v^2 + \frac{r_1}{r_2}v - \frac{f-w-r_0}{r_2}} = -r_2 \int_{t_0}^t dt. \quad (7.12)$$

Completing the square of the denominator of the integrand of the speed term, we find

$$\int_{v_0}^v \frac{dv}{\left(v + \frac{r_1}{2r_2}\right)^2 - \left(\frac{r_1}{2r_2}\right)^2 - \frac{f-w-r_0}{r_2}} = -r_2 \int_{t_0}^t dt. \quad (7.13)$$

Taking again $\alpha = \frac{r_1}{2r_2}$ and defining $\beta^2 = \alpha^2 + \frac{f-w-r_0}{r_2}$, we observe that $\alpha \geq 0$ since $r_2 > 0$ and $r_1 \geq 0$. However, β^2 shows a more complex behaviour, since it can be zero, positive or negative. In this latter case, we use $\gamma^2 = -\beta^2$, so γ is real. This leads to three different formulas (Gradshteyn & Ryzhik, 2014). If $\beta > 0$,

$$r_2 \beta(t - t_0) = \frac{1}{2} \log \left(\frac{\beta + v + \alpha}{\beta - v + \alpha} \frac{\beta - v_0 + \alpha}{\beta + v_0 + \alpha} \right), \quad (7.14)$$

where $\log$ is the natural logarithm. If $\beta = 0$, then

$$r_2(t - t_0) = \frac{1}{v + \alpha} - \frac{1}{v_0 + \alpha}. \quad (7.15)$$

If β is complex, then

$$-r_2 \gamma(t - t_0) = \arctan \left(\frac{v + \alpha}{\gamma} \right) - \arctan \left(\frac{v_0 + \alpha}{\gamma} \right). \quad (7.16)$$

This allows to calculate explicitly $v(t)$ and $t(v)$.

From Eqs. (7.2) and (7.3), we can obtain a differential equation that describes the evolution of the train speed with respect to its location, which is particularly relevant in railways since several elements of the track are located at specific locations, not times. Stopping locations, signals, points, gradients and speed limits are some examples.

$$\frac{dv}{ds} = \frac{1}{v} (f(s) - r(v) - w(s)). \quad (7.17)$$

This equation can also be integrated to find formulas that relate the speed and the location. $s(v)$ is found explicitly, however, this is not the case with $v(s)$, which has to be

calculated iteratively by means of a search algorithm such as the Golden-section Search (Kiefer, 1953). The remaining two formulas, $s(t)$ and $t(s)$, can be calculated as $s(v(t))$ and $t(v(s))$, respectively.

If $\beta > 0$, then

$$-2r_2\beta(s - s_0) = \beta \log \frac{(v + \alpha)^2 - \beta^2}{(v_0 + \alpha)^2 - \beta^2} + \alpha \log \left(\frac{\beta + v + \alpha}{\beta - v + \alpha} \frac{\beta - v_0 + \alpha}{\beta + v_0 + \alpha} \right). \quad (7.18)$$

If $\beta = 0$, then

$$-r_2(s - s_0) = \log \frac{v + \alpha}{v_0 + \alpha} + \alpha \left(\frac{1}{v + \alpha} - \frac{1}{v_0 + \alpha} \right). \quad (7.19)$$

If β is complex, then

$$-r_2\gamma(s - s_0) = \frac{\gamma}{2} \log \frac{(v + \alpha)^2 + \gamma^2}{(v_0 + \alpha)^2 + \gamma^2} - \alpha \left(\arctan \left(\frac{v + \alpha}{\gamma} \right) - \arctan \left(\frac{v_0 + \alpha}{\gamma} \right) \right). \quad (7.20)$$

The analysis of the bifurcation of Eq. (7.9) shows that when the train is accelerating β can only be positive, while when it is decelerating β can be real and positive, zero or complex. Thus, the appropriate formulas have been selected based on the value of β when the train decelerates. A special case arises when the forces on the train balance and the acceleration is zero. In this case, the time, location and speed can be calculated by considering the speed to be constant.

Constant tractive and brake power: analytical solutions

For completeness, we show the analytical solutions of Eqs. (7.3) and (7.17) under a constant gradient and $f = p_{\max}/v$. We can rewrite Eqs. (7.3) and (7.17) as

$$\frac{dv}{dt} = \frac{-r_2}{v} (v - \nu_1)(v - \nu_2)(v - \nu_3), \quad (7.21)$$

$$\frac{dv}{ds} = \frac{-r_2}{v^2} (v - \nu_1)(v - \nu_2)(v - \nu_3), \quad (7.22)$$

where ν_1 , ν_2 and ν_3 are the roots of the cubic polynomial that arises after extracting an extra $-r_2/v$ factor from the right-hand-side of Eqs. (7.3) and (7.17). Again, according to the Fundamental Theorem of Algebra, one of these roots (for instance, ν_1) is always real, and the other two can be complex conjugates, a real double root or two real single roots. These three cases lead to the following formulas (Gradshteyn & Ryzhik, 2014), which are explicit only when the speed is the independent variable:

If the polynomial has three real simple roots, ν_1 , ν_2 and ν_3 , then

$$\begin{aligned} -r_2(t - t_0) = & \frac{1}{(\nu_1 - \nu_2)(\nu_1 - \nu_3)(\nu_2 - \nu_3)} \left(\nu_1(\nu_2 - \nu_3) \log \frac{v - \nu_1}{v_0 - \nu_1} + \right. \\ & \left. + \nu_2(\nu_3 - \nu_1) \log \frac{v - \nu_2}{v_0 - \nu_2} + \nu_3(\nu_1 - \nu_2) \log \frac{v - \nu_3}{v_0 - \nu_3} \right), \end{aligned} \quad (7.23)$$

$$\begin{aligned} -r_2(s - s_0) = & \frac{1}{(\nu_1 - \nu_2)(\nu_1 - \nu_3)(\nu_2 - \nu_3)} \left(\nu_1^2(\nu_2 - \nu_3) \log \frac{v - \nu_1}{v_0 - \nu_1} + \right. \\ & \left. + \nu_2^2(\nu_3 - \nu_1) \log \frac{v - \nu_2}{v_0 - \nu_2} + \nu_3^2(\nu_1 - \nu_2) \log \frac{v - \nu_3}{v_0 - \nu_3} \right). \end{aligned} \quad (7.24)$$

If the polynomial has a simple real root ν_1 and a double real one, ν_2 , then

$$-r_2(t - t_0) = \frac{\nu_1}{(\nu_1 - \nu_2)^2} \log \left(\frac{v - \nu_1}{v_0 - \nu_1} \frac{v_0 - \nu_2}{v - \nu_2} \right) + \frac{\nu_2}{\nu_1 - \nu_2} \left(\frac{1}{v - \nu_2} - \frac{1}{v_0 - \nu_2} \right), \quad (7.25)$$

$$\begin{aligned} -r_2(s - s_0) = & \frac{\nu_1^2}{(\nu_1 - \nu_2)^2} \log \frac{v - \nu_1}{v_0 - \nu_1} + \frac{\nu_2^2 - 2\nu_1\nu_2}{(\nu_1 - \nu_2)^2} \log \frac{v - \nu_2}{v_0 - \nu_2} + \\ & + \frac{\nu_2^2}{\nu_1 - \nu_2} \left(\frac{1}{v - \nu_2} - \frac{1}{v_0 - \nu_2} \right). \end{aligned} \quad (7.26)$$

If the polynomial has a real root ν_1 and two complex conjugate roots, $\nu_2 = a + bi$, $\bar{\nu}_2 = a - bi$, then

$$\begin{aligned} -r_2(t - t_0) = & \frac{1}{(\nu_1 - \nu_2)(\nu_1 - \bar{\nu}_2)} \left[\nu_1 \log \frac{v - \nu_1}{v_0 - \nu_1} - \frac{\nu_1}{2} \log \frac{v^2 - 2av + a^2 + b^2}{v_0^2 - 2av_0 + a^2 + b^2} + \right. \\ & \left. + \frac{a^2 + b^2 - \nu_1 a}{b} \left(\arctan \frac{v - a}{b} - \arctan \frac{v_0 - a}{b} \right) \right], \end{aligned} \quad (7.27)$$

$$\begin{aligned} -r_2(s - s_0) = & \frac{1}{(\nu_1 - \nu_2)(\nu_1 - \bar{\nu}_2)} \left[\nu_1^2 \log \frac{v - \nu_1}{v_0 - \nu_1} + \right. \\ & + \frac{a^2 + b^2 - 2a\nu_1}{2} \log \frac{v^2 - 2av + a^2 + b^2}{v_0^2 - 2av_0 + a^2 + b^2} + \\ & \left. + \frac{\nu_1 b^2 + a(a^2 + b^2 - a\nu_1)}{b} \left(\arctan \frac{v - a}{b} - \arctan \frac{v_0 - a}{b} \right) \right]. \end{aligned} \quad (7.28)$$

Last, when applying constant brake power, if the polynomial has a negative triple root, $\nu_1 = -2\alpha/3$, then

$$-r_2(t - t_0) = -\frac{1}{2}\nu_1 \left(\frac{1}{(v - \nu_1)^2} - \frac{1}{(v_0 - \nu_1)^2} \right) - \left(\frac{1}{v - \nu_1} - \frac{1}{v_0 - \nu_1} \right), \quad (7.29)$$

$$\begin{aligned} -r_2(s - s_0) = & -\frac{1}{2}\nu_1^2 \left(\frac{1}{(v - \nu_1)^2} - \frac{1}{(v_0 - \nu_1)^2} \right) - 2\nu_1 \left(\frac{1}{v - \nu_1} - \frac{1}{v_0 - \nu_1} \right) + \\ & + \log \frac{v - \nu_1}{v_0 - \nu_1}. \end{aligned} \quad (7.30)$$

A remark on alternative ways of modelling the tractive and brake effort

In case that the maximum tractive effort is modelled as a linear or quadratic function of the speed, the acceleration polynomial could be carefully rewritten as a quadratic function of the speed and the formulas proposed in chapter 7.3 could still be used (Ye & Liu, 2017; Becker & Schreckenberg, 2018). If the quadratic term is cancelled with r_2 , solving the resulting integrals becomes trivial.

7.3.3 The new coasting advice algorithm

Algorithm 7.1 summarizes the calculation of the coast curve. This algorithm aims to determine the arrival time at the next stop if the train starts coasting at the current location. It runs every second and takes as input the current speed v_0 , location s_0 and time t_0 , the target arrival time, the track gradients and speed limits, the train mass and its running resistance parameters. Since the train is coasting, we consider $f = 0$ in all the calculations. We split the distance to the next stop or target location in sections where the gradient and speed limit are constant and calculate the parameter β or γ in each section, as explained in chapter 7.3.2. Then, the speed and time at the end of each section is calculated by means of Eqs. (7.14)-(7.16), (7.18)-(7.20) until a fixed target location or a comfort brake curve is reached. As we mentioned in chapter 7.2.1, the coast curve must end at certain target locations when there are speed reductions ahead due to speed signs and signals. The train must decelerate after reaching the target location by following an ATB brake curve. In turn, comfort brake curves are used when there are no explicit speed reductions. Both types of brake curves are precomputed and stored beforehand. These brake curves utilize constant deceleration rates, so the speed of a brake curve can be calculated explicitly as a function of the location. It can be introduced in Eqs. (7.18)-(7.20) to obtain the intersection point between the coast and the brake curves. The algorithm also accounts for the possibility of reaching a speed limit while coasting under steep downhill sections. In this scenario, the driver is supposed to brake the train to maintain a speed equal to the mentioned speed limit and avoid overspeeding. After that, the algorithm checks whether a comfort brake curve is reached. The expected arrival time is calculated by summing the time spent coasting, braking to the restricted speed 40 km/h, braking from 40 km/h to a standstill and cruising at 40 km/h between these two latter decelerations. The driver is updated with the difference between the calculated expected arrival time and the target one.

Figure 7.7 illustrates the new algorithm for computing coast curves. The train speed during coasting, the speed limit and the resistance to motion due to track gradients w are shown in dark blue, yellow and red, respectively. The vertical lines show the locations where the new algorithm calculates the state of the train, namely when either the gradients or the speed limits change. Compared to the old algorithm shown in Figure 7.2, the state of the train is only calculated at the locations where the gradients or the speed limit change. Moreover, if the track is not flat, the computations of the old algorithm may differ significantly with respect to the new algorithm's and with reality.

Algorithm 7.1 Coasting advice algorithm based on analytical formulas

```

1: Initialize parameters and measure train state:  $v = v_0$ ,  $s = s_0$ ,  $t = t_0$ 
2: for each section do
3:   Update section gradient, speed limit  $v_{\max}$ , length,  $\beta$  or  $\gamma$ 
4:    $v \leftarrow$  Calculate speed at the end of section using Eqs. (7.18)-(7.20)
5:   if Speed limit exceeded then
6:     Calculate location at which speed limit is reached using Eqs. (7.18)-(7.20)
7:      $v \leftarrow v^{\max}$ 
8:      $v^{\max}$  to be held until end of section
9:   end if
10:  if Comfort brake curve reached then
11:    Calculate intersection between coast and comfort brake curves using
    Eqs. (7.18)-(7.20)
12:     $t \leftarrow t +$  Calculate time until brake curve using Eqs. (7.14)-(7.16) and/or constant speed
13:    go to 18
14:  end if
15:   $t \leftarrow t +$  Calculate time until end of section using Eqs. (7.14)-(7.16) and/or constant speed
16:   $s \leftarrow s +$  section length
17: end for
18:  $t \leftarrow t +$  Calculate remaining time to stopping to a standstill
19: Show difference between  $t$  and target arrival time to the driver

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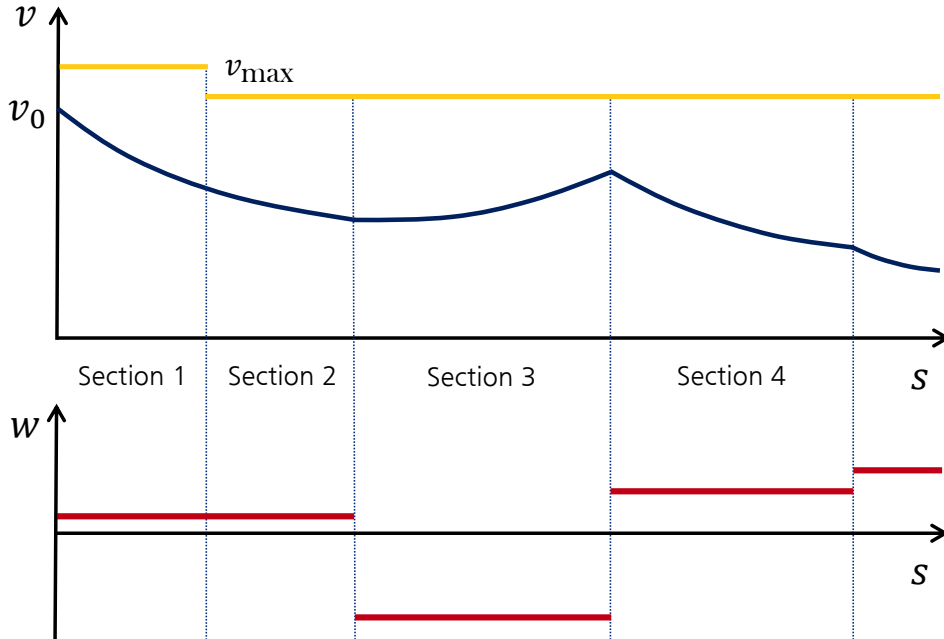


Figure 7.7: Schematic representation of the new coasting advice algorithm, where the time and speed of the train are calculated at the points where either the speed limit or the track gradient change.

7.4 Validation of the new method and the constant tractive and brake power formulas

7.4.1 Validation of the new method

In this chapter we show the results of the validation of the proposed coasting advice algorithm. We have simulated three scenarios with different track geometries and compared the performance and accuracy of the proposed method with the one implemented previously in *Roltijd*, which is based on the forward Euler method, and with the coasting curves generated by PROMO. Table 7.1 gathers the track geometry configuration of these scenarios. In each of them, the track is 5 km long. The track is flat before 1.1 km and after 2.4 km in all scenarios. Regarding the track between 1.1 km and 2.4 km, in the first scenario the track is slightly downhill, with a -0.681 permille slope. This corresponds to the $\beta = 0$ case. The second and third scenarios have a 9.33 permille downhill and uphill slope, between 1.1 km and 2.4 km, respectively. Under flat track and in the considered uphill slope, β^2 is negative, so Eqs. (7.16) and (7.20) have to be used to calculate the speeds and locations on those track intervals. In turn, coasting at the downhill gradient leads to a positive β^2 , so Eqs. (7.14) and (7.18) are the formulas required in this case. Coasting at the -0.681 permille slope requires using Eqs. (7.15) and (7.19). We considered an NS Sprinter train rolling stock type SLT-6 in this case study. Its characteristics are outlined in Table 7.2.

Case	Slope at km 0	Gradient at km 1.1	Gradient at km 2.4
$\beta = 0$ section	0	-0.681	0
Downhill section	0	-9.33	0
Uphill section	0	+9.33	0

Table 7.1: Track geometry of the scenarios used for validating the algorithm. All slopes are in permille, start at the mentioned km and end at the next one, while the last flat track ends at the line end in all scenarios.

Parameter [units]	Value
Train mass [t]	198
Rotating mass supplement [-]	1.08
Max. speed limit [km/h]	140
Train resistance [N] (v: [m/s])	$1375 + 37.48v + 6.75v^2$

Table 7.2: Basic parameters of a NS Sprinter train rolling stock type SLT-6 (Scheepmaker et al., 2020b).

We have implemented both coasting advice algorithms in the same platform, using the same language code. Figures 7.8 and 7.9 show the coasting curves calculated using these three methods. For this comparison, we have included the gradient resistance in the forward Euler method. Figure 7.8 represents the speed curves as a function of the location, and Figure 7.9 the passing times. We have represented in yellow the $\beta = 0$ scenario, in

turquoise the downhill one, and in purple the uphill. We show in full-coloured dashed lines the coasting curves calculated using the forward Euler method and in pale coloured solid lines the coasting curves extracted from the pseudospectral train trajectory optimizer. The blue markers represent the speeds and times calculated using the analytical formulas. The circle marker shows the points calculated in the $\beta = 0$ scenario, the x markers correspond to the downhill scenario, and the + markers to the uphill one. For each track section with constant gradient and constant applied effort, including the case when the train is coasting, the proposed algorithm only needs to calculate the speed and time at the end of the track section. In contrast, the other methods require several intermediate calculations in each track section. We observe that the forward Euler method and the proposed analytical algorithm show a similar accuracy, while there are some small qualitative differences with respect to the coasting curves generated by the train trajectory optimizer, probably due to the sparser and unevenly distributed locations of the points where the trajectory is calculated by the pseudospectral method.

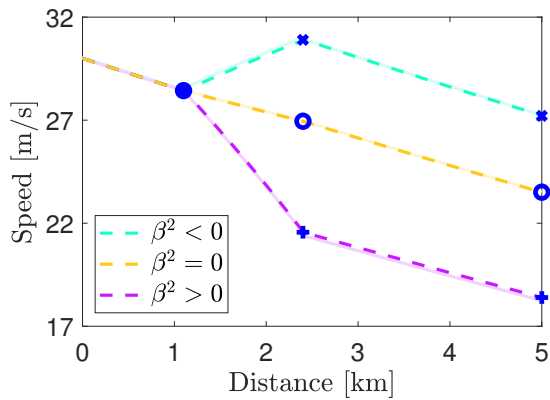


Figure 7.8: Speeds

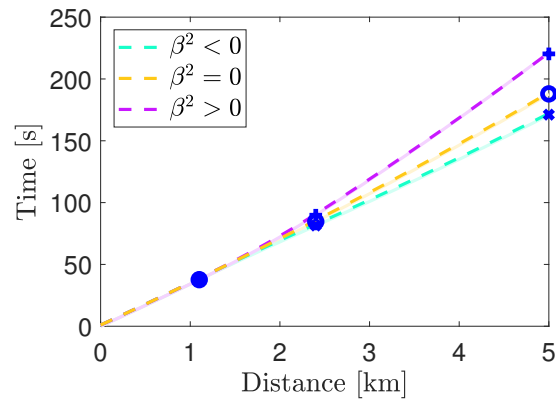


Figure 7.9: Passing times

Figure 7.10: Comparison of the coasting curves generated by the three methods.

Last, Figure 7.11 shows a comparison of the accuracy of these methods with respect to the analytical solutions. In the upper plots we show the error in speed, and in the lower ones, the cumulative time error. Again, we represent in yellow the scenario where $\beta = 0$, in turquoise the downhill, and in purple the uphill. The dashed line corresponds to the error of the forward Euler method with respect to the analytical formulas, and the continuous line, PROMO's error. Furthermore, we compared the analytical formulas with respect to MATLAB's ode45 function, which solves the equations of motion in Eqs. (7.1) and (7.2) using an adaptive Runge-Kutta numerical method (Shampine & Reichelt, 1997). This is represented as a continuous blue line in all the plots. The error of ode45 is several orders of magnitude lower than the other two numerical methods and never exceeds 10^{-4} m/s. This validates the analytical formulas in Eqs. (7.14)-(7.16), (7.18)-(7.20).

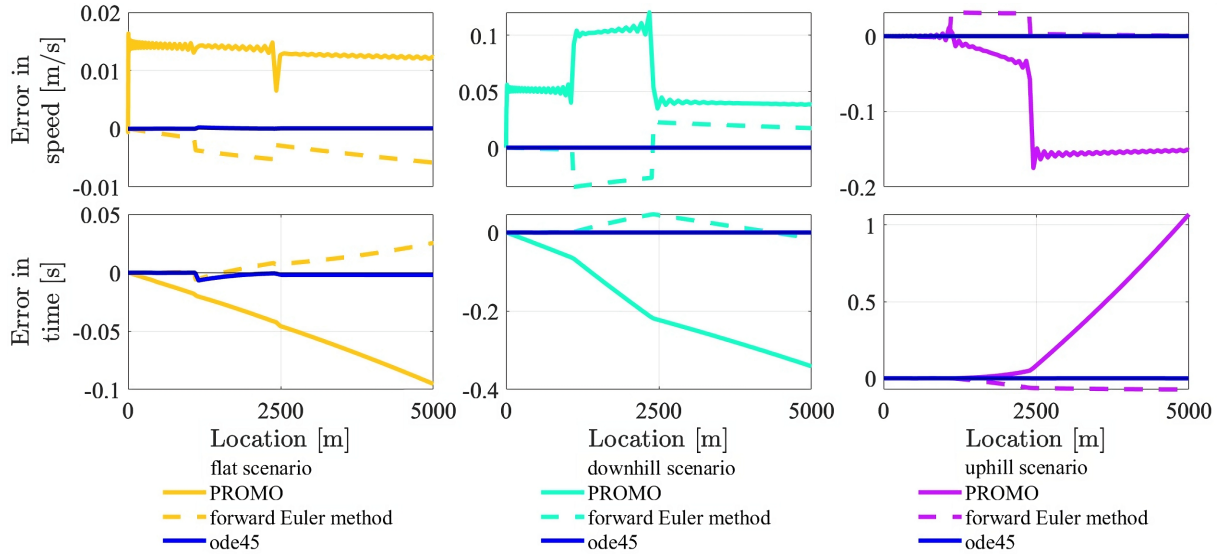


Figure 7.11: Comparison of the error in speed (up) and time (down) at each location with respect to the analytical method for flat (left), downhill (middle) and uphill (right) slopes between 1100 and 2400 m. In each plot the dark blue line represents the error of MATLAB’s ode45 function with respect to the analytical formulas, while the coloured dashed lines represent the error of the forward Euler method, and the coloured continuous line represent PROMO’s error.

When it comes to the other two methods, we observe jumps at the locations where the gradients change. This is due to the fact that both PROMO and the forward Euler method do not calculate the state of the train exactly at these locations. Another source of error is the approximation error in both methods, which makes the error vary linearly in each interval with constant gradient. Particularly, PROMO shows oscillations in the speed since it is approximated as a finite sum of Lagrange polynomials. We also observe speeds up to 0.02 m/s higher than the initial speed while coasting in the first 40 m of the first flat track in the downhill scenario, and up to 0.007 m/s higher during the first 11 m in the scenario where $\beta = 0$. This is another numerical issue inherent in the pseudospectral discretization that constitutes PROMO’s core and biases its error from the beginning of the coast curve. Still, the errors of these three methods are below 1 s and below 0.2 m/s in most cases after 5 km, which demonstrates that the accuracy of the three considered methods is adequate in practice for providing coasting advice.

Regarding the computation times, we used .NET framework’s Stopwatch to determine which method is faster. We implemented both algorithms in the same programming language and run them on the same hardware. We compared the computation times of both algorithms in several scenarios, calculating coasting curves with diverse lengths and initial speeds. We run each scenario 30 times, observing that the computation times became stable after the third run in each scenario. Thus, we disregarded the first three warm-up runs and computed the average computation time. The proposed method was found to be up to 40 times faster than the forward Euler method. When aiming for the accuracy levels required in a practical implementation, the new method is 15 times faster. This relieves the computational burden on the application, allowing us to implement

more complex driving advice routines in the near future. Moreover, the reduction of the computational power needed contributes to extending the battery life of the tablet in which this application is embedded. The computation time of the proposed method depends on the number of changes in gradients and speed limits, so filtering carefully this input data is important to keep the computation times low. Despite this, an accurate description of the track geometry is essential to guarantee the accuracy of the coasting advice. In turn, the forward Euler method's computation time is independent of the number of changes in gradients and speed limits. When it comes to implementing the proposed analytical method, it has been implemented in 250 lines of code, four times more than the forward Euler method. Still, the code is simple and the proposed method can be implemented easily.

7.4.2 Validation of the constant tractive and brake power formulas

We continue by exploring the formulas that are not included in the coasting advice algorithm, i.e., those that involve a constant tractive or brake power term in Eqs. (7.23)-(7.30). We simulate 5 km of coast curves corresponding to the four possible constant power scenarios introduced in chapter 7.3.1: (a) three real simple roots, Eqs. (7.23) and (7.24), (b) a real double root, Eqs. (7.25) and (7.26), (c) two complex roots, Eqs. (7.27) and (7.28), (d) a triple root, Eqs. (7.29) and (7.30). Although these four scenarios may take place for a countless amount of combinations of power and gradient, we only provide one example to test the formulas. Ultimately, validating the formulas is our main scope rather than testing them in a wide variety of scenarios. Besides the basic parameters in Table 7.2, the gradient and the applied power parameters used in the constant power simulations are displayed in Table 7.3. Figure 7.12 shows the error in speed (left) and time (right) at each location of MATLAB's ode45 function with respect to the analytical formulas. We observe that in these four cases the error is kept within 0.002 s or 0.002 m/s in the case of the speed, which validates the analytical formulas in Eqs. (7.23)-(7.30). Still, reaching this accuracy requires solving the mentioned formulas iteratively using a Golden-section Search algorithm. Despite having only a linear convergence rate, it requires only one evaluation of the analytical formulas per iteration and it does not require evaluating their derivative. This step introduces an error in the calculated speed and passing time that is inversely proportional to the number of iterations of the search algorithm and, consequently, to the computation time. The low errors shown in Figure 7.12 allow to reduce the number of iterations while keeping an error that is acceptable in practical implementations. We also noticed that ode45 calculates 30 speed points in the first 10 seconds in all scenarios, which would make it unfit for our practical application due to the limited computation power available unless a lower bound on the integration step is implemented.

Scenario	Power [W]	Gradient [%]	Initial speed [km/h]
(a) Three real simple roots	-800 000	-20	81
(b) A real double root	394 683.2	-10	60
(c) Two complex roots	1 755 000	20	60
(d) A triple root	-42.8	-6.7	80

Table 7.3: Constant tractive and brake power applied and track gradient resistance corresponding to the four different constant power formulas.

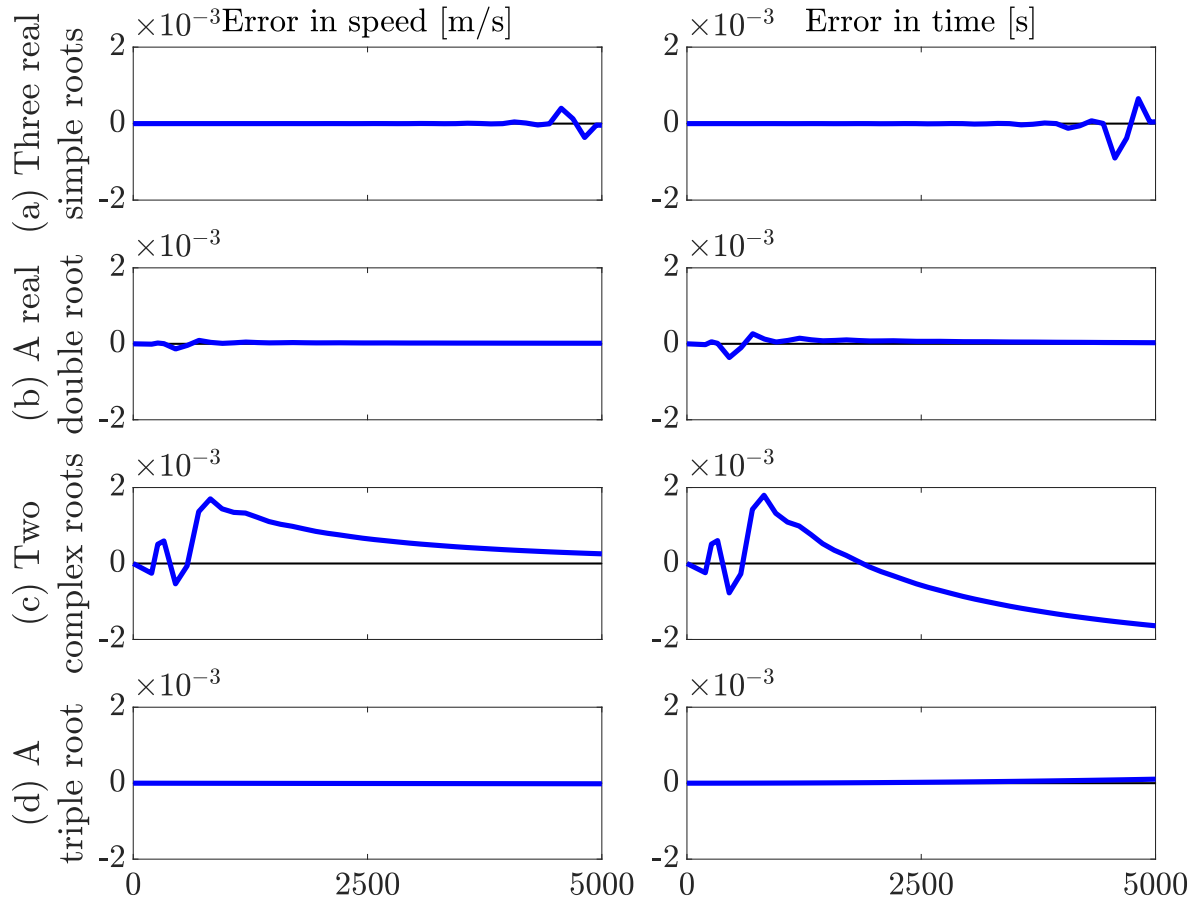


Figure 7.12: Comparison of the error in speed (left) and time (right) at each location of MATLAB's ode45 function with respect to the analytical formulas in the following scenarios: (a) three real simple roots, (b) a real double root, (c) two complex roots and (d) a triple root.

7.5 Conclusion

In this chapter we have presented an accurate and computationally efficient algorithm for calculating coasting curves. We utilize this algorithm every second to calculate the expected arrival time if the train started to coast immediately, and this information is shown to the driver as a countdown to start braking. We utilize a description of the track geometry based on piecewise constant gradients. The analytical solutions of the train motion model are used to calculate the train speed and time at the locations where the gradients change. We show closed formulas for the mentioned analytical solutions, extending the formulas already published in the literature. We present the analytical solutions of the train motion model when the applied tractive and brake effort is limited by the maximum power of the engine, which were still not available in the literature. We have also presented an analysis of the possible solutions based on the theory of dynamical systems and the Fundamental Theorem of Algebra, which is essential in the algorithm to select the formula to be used for the current train speed and gradient. When considered as a dynamical system, the solutions of the train motion model present bifurcations, which are qualitative changes in the dynamics of the solutions. These changes are translated into changes in the closed formulas, and different formulas have to be used in different scenarios. For instance, only one formula can describe the dynamics of a train accelerating under constant effort and gradient, however, three formulas arise when considering a decelerating train under those conditions. The algorithm identifies which of the formulas has to be used at any time, based on whether the train is accelerating or decelerating, and the applied effort and gradient resistance in the current track section. Furthermore, in the maximum power case, we factorized the acceleration term, extracting from it a cubic polynomial which analytical solutions depend on the geometry of its roots.

We have validated the coasting advice algorithm with respect to MATLAB's `ode45` in a simulated case study, and compared its performance and accuracy with the previous method and a train trajectory optimizer based on a pseudospectral method. The proposed method is 15 times faster than the previous algorithm, allowing us in the near future to implement more complex advice and several extra features in the application. Moreover, it shows an improved accuracy over the other two methods. Although the formulas corresponding to a constant tractive or brake power are not implemented in the proposed algorithm, we have also validated their accuracy.

This new coasting advice generator has already being implemented in *Roltijd*, the DAS at NS in the Netherlands, and is contributing to the sustainable mobility of around 1.3 million passengers every day.

The applicability of the analytical solutions of the train motion model goes way beyond a coasting advice algorithm. Their simplicity and accuracy can be exploited in many different areas of the railway industry that require reproducing or predicting the motion of a train. We believe these formulas have the potential to make an impact in the coming years in the field of timetable design and evaluation, simulation, DAS and ATO.

Chapter 8

Robust Train Trajectory Optimization

8.1 Introduction

The stochastic nature of train operation strongly influences the dynamics of trains and the trajectories they perform along their journeys. There are many sources of this stochasticity: rain, wind strength and direction, environmental temperature, available adhesion, and fluctuations in the power of the catenary. The reaction times of human drivers also show considerable volatility (Brandenburger & Jipp, 2017). The train load depends on the number of passengers and the amount of freight on-board, which differs from day to day (Yin et al., 2016b). Even two rolling stock units of the same type may show differences in their dynamics (Norris, 2008), for instance, due to different mechanical wear, faults and engine performance. These stochastic factors influence the dynamics of trains and may produce variations in running and dwell times. If these impacts are large enough, they may cause delays, train path conflicts and disturbances. To mitigate the undesirable effects of stochasticity, timetable planners use a model of the train dynamics to calculate the reference trajectories that the train will follow. They allocate running time supplements and buffers in headway times between train paths and in dwell times, aiming to reduce the potential impact of deviations from the plan during railway operation (Rao et al., 2022). However, disturbances and train path conflicts still occur, since incorporating in the timetable any *a priori* knowledge of the future stochastic factors that will influence the dynamics of a train is a complex task. Data-driven methods to calculate stochastic running times have already been considered to design more robust timetables (Medeossi et al., 2011). In turn, energy-efficient train driving algorithms such as Driver Advisory Systems (DAS) and Automatic Train Operation (ATO) are also considerably affected by stochasticity. For instance, strong headwinds may increase the running resistance of a train, decelerating it more than expected when coasting. This deviation from the planned trajectory may induce an ATO or driver's intervention to reduce the deviation, or even a new reference trajectory may have to be calculated if the deviation is substantially large. These energy-efficient train driving algorithms have to be precise in order to drive a train timely to its destination. The key point is how to calculate train trajectories that are robust against the stochasticity inherent in train operation.

Three main types of stochasticities are usually distinguished: random noise, unmodelled dynamics and parameter variation (Boltyanski & Poznyak, 2011). Characterizing noise statistics is difficult due to the need to eliminate the contribution to the measurement noise (Collantes et al., 2002). In turn, the impact of unmodelled dynamics is also difficult to gauge due to the lack of ground-truth data. For instance, railway networks are full of curves, but still the existing literature is lacking a comprehensive model of the influence of curves on the dynamics of a train (Wu et al., 2022b). Finally, model parameter variation can be determined by means of a wide variety of model calibration algorithms, and the observed variations can sometimes be explained physically (Cunillera et al., 2023a). For instance, the running resistance of a train may increase when running through a tunnel due to the extra aerodynamic drag (Hansen et al., 2017).

ATO, DAS and energy-efficient timetabling rely on Train Trajectory Optimization (TTO) to generate energy-efficient train trajectories that are used as a reference to be tracked in order to arrive on time at a train’s destination with minimal energy consumption. TTO utilizes a train motion model to reproduce and predict the dynamics of a train. The parameters that characterize this model have to be calibrated in order to capture the real behaviour of the train. Several parameter estimation techniques can be used to calibrate the train motion model and fine-tune energy-efficient driving algorithms (Aradi et al., 2015; Cunillera et al., 2022; Rochard & Schmid, 2000). This is a vital step when implementing energy-efficient train driving algorithms, since many aspects of those algorithms require an accurate calibration. A review on train motion model calibration and a deeper discussion on its relevance for energy-efficient train driving algorithms can be found in chapter 2. When it comes to timetabling models and off-board TTO, real time calibration is not possible since the reference trajectories are calculated well before the train starts operation. Such models have to anticipate possible future operating conditions in order to produce plans and reference trajectories that are robust against potential future operating conditions. In this regard, Train Path Envelopes (TPE) containing passing times and time and speed windows can be placed at strategic timing points of the track like junctions, signals and stops (Albrecht et al., 2013; Wang & Goverde, 2016; Wang et al., 2025) to partially compensate for the expected spread in the passing times at those locations. A TPE describes the bandwidths that each train can use at those locations for driving energy-efficiently without hindering neighbouring trains or being hindered by them (Quaglietta et al., 2016).

TTO has historically been mainly addressed deterministically, ignoring any stochasticity in railway operation (Scheepmaker et al., 2017). A few papers on TTO under parameter or model uncertainty can be found in the literature: Wang et al. (2020) used reinforcement learning to calculate train trajectories under uncertainties in the maximum tractive effort and in the running resistance. Yin et al. (2014) used also reinforcement learning to devise an energy-efficient train operation system that does not rely on a train motion model with unknown running resistance. Yin et al. (2016b) combined the previous work with stochastic programming for addressing timetabling rescheduling under passenger load uncertainty. Zhang et al. (2017) combined drivers expert knowledge and operation data to devise a data-driven train operation algorithm. The robustness of the algorithm was tested with respect to traction and brake response time and transmission delay. Yang et al. (2016) proposed a two-phase stochastic program combined with a

simulation-based genetic algorithm for timetable and speed profile optimization, where the train mass is considered a stochastic variable that can take three values with different probabilities corresponding to peak, off-peak and night hours, respectively. Passenger mass uncertainty was also considered in Xing et al. (2022), where a multi-objective genetic algorithm is used to calculate energy-efficient train trajectories. Albrecht et al. (2013) and Goverde et al. (2016) optimized train trajectories with respect to stochastic dwell times using a multi-stage, multi-criteria optimization problem that was solved by dynamic programming. Goodwin et al. (2020) explored multiple train scheduling with trajectories that are robust against stochasticity in the dwell times and in the location of the switching points between driving regimes also by means of a genetic algorithm. Wang et al. (2017) studied robust trajectories with respect to fluctuations in the energy consumption, which is subject to an additive random variable. The TTO problem is modelled as a stochastic constrained shortest path problem that is solved by means of dynamic programming after a Lagrangian relaxation. Cheng et al. (2021) considered the running resistance parameters as random variables, which are introduced in the model as worst-case scenario accelerations and decelerations. Here, the energy consumption is also considered stochastic. A percentile reliability model is used to generate robust trajectories that keep the energy consumption within a confidence interval below the value provided by the objective function. The resulting problem is solved after reformulating it as a linear programming problem. Li et al. (2013b) considered a stochastic quadratic running resistance parameter that is uniformly distributed between two values. An iterative algorithm is utilized to solve the stochastic train trajectory optimization problem. The Pontryagin Maximum Principle is used to show the existence of the usual maximum traction, cruising and maximum braking driving regimes. In turn, coasting is replaced by a quasicosting regime that includes partial traction and braking to compensate the deviations from the mean value of the running resistance parameter. This work is extended in Li et al. (2013a), where the existence of optimal trajectories is analyzed more in depth. The three running resistance parameters are considered stochastic. Among these pioneering approaches, the last two references are the only ones that use mathematical control theory to analyze the TTO problem under uncertainty. The characterization of the uncertainties in most of the mentioned references relies on considering parameters as random variables which distributions are not realistic. This may compromise the robustness of the computed trajectories.

From the analysis of the existing literature on TTO under uncertainty, an open question arises. How can the Robust Train Trajectory Optimization (RTTO) problem be formulated so that it incorporates the uncertainties inherent in train operation in a realistic and practical manner? In turn, the existing literature lacks a definition of a Robust Train Path Envelope (RTPE) against parameter variation that guarantees the drivability of robust-optimal train trajectories.

In this chapter we formulate mathematically the RTTO problem under parametric uncertainty. We consider that different operating conditions impact the train dynamics producing train motion model parameter variation. In our previous research we proposed and applied a method to calibrate the train motion model using operational data from multiple train trajectories between two stations (Cunillera et al., 2023d). From each trajectory, we used location, speed and tractive and brake effort measurements to calculate

a parameter configuration consisting of a set of train motion model parameter estimates that fit best the measurements. Each parameter configuration represents a different scenario corresponding to the operating conditions observed during the associated trajectory. We use these parameter configurations as input of the RTTO problem, which allows to select any scenarios of interest. The RTTO problem is modelled for the first time in the scientific literature as a minimax optimal control problem that seeks to find the robust-optimal control policy that is feasible for all the selected parameter configurations. The model guarantees that the solution of the RTTO problem will be drivable under any of the selected operating conditions. This means that the train will arrive on time at its destination and will not violate any operational constraints if the realized operating condition is among the considered scenarios. When applying the robust-optimal control policy to each parameter configuration, a feasible energy-efficient trajectory is obtained. The collection of all these trajectories constitutes a robust-optimal trajectory bundle. Individually, each trajectory in the bundle is likely to be suboptimal when compared to the solution of the nominal TTO problem associated to its corresponding parameter configuration. However, the optimal control policy obtained from a nominal TTO problem is not necessarily feasible for all other parameter configurations. For instance, the optimal solution of the TTO problem under tailwinds may be not feasible when under headwinds. The running resistance is lower under tailwinds, so the train requires less power to arrive timely at its destination. The same control policy would stop the train before its destination due to the extra aerodynamic drag under headwinds, so it would not be feasible. The robust-optimal control policy obtained by solving the RTTO problem would guarantee its feasibility in any of these two cases.

We apply the Robust Maximum Principle (Boltyanski & Poznyak, 2011) to analyze the main properties of the solutions of the problem. Furthermore, we transform the optimal control problem into a convex optimization problem that is solved to calculate the robust-optimal trajectories. Regarding TPEs, we propose a definition of a Robust Train Path Envelope (RTPE) based on the solution of the RTTO problem. The robust-optimal trajectory bundle is used to establish upper and lower bounds on the time and speed windows. The headway buffer times are distributed evenly among consecutive trains from their respective latest and earliest passing times. The performance of the proposed algorithm is showcased in a case study.

The scientific contributions of this research are fivefold:

1. A mathematical definition of the Robust Train Trajectory Optimization problem.
2. The quantification of different operating conditions as observed variations of train motion model parameters.
3. An analysis of the Robust Train Trajectory Optimization problem based on the Robust Maximum Principle.
4. A numerical algorithm for solving the Robust Train Trajectory Optimization problem based on convex optimization.
5. A formal definition of Robust Train Path Envelopes based on the computed robust-optimal trajectories and general guidelines to set them.

The research in this chapter is organized as follows: In chapter 8.2 we briefly describe the nominal TTO problem. In chapter 8.3 we introduce the RTTO problem and we analyze it mathematically in chapter 8.4. In chapter 8.5 we propose a formal definition of RTPE. In chapter 8.6 we describe the developed algorithm for solving the Robust Train Trajectory Optimization problem. Next, in chapter 8.7 we showcase and analyze the performance of our algorithm in a case study. Last, we outline the main conclusions of this research and our future research in chapter 8.8.

8.2 Nominal Train Trajectory Optimization problem

8.2.1 Nominal energy-efficient Train Trajectory Optimization

The Train Trajectory Optimization (TTO) problem shown in Eqs. (8.1)-(8.9) aims to drive a train timely from s_o to s_e and minimize the consumed energy at the end of the journey in Eq. (8.1) by finding an optimal control policy, that is to say, the tractive and brake effort to be applied at each point of the track (Albrecht et al., 2016b). This optimal control problem is constrained by the dynamics of the train speed in Eq. (8.2), of the time in Eq. (8.3) and of its energy consumption in Eq. (8.4), by some path constraints representing bounds on the tractive and brake effort in Eqs. (8.5)-(8.7) and on the speed in Eq. (8.8), and by boundary conditions in Eq. (8.9).

$$\min_{u_f, u_b} J = E(s_e) \quad (8.1)$$

subject to

$$\frac{dv}{ds} = \frac{u_f(s) - u_b(s) - r(v) - w(s)}{v(s)}, \quad (8.2)$$

$$\frac{dt}{ds} = \frac{1}{v(s)}, \quad (8.3)$$

$$\frac{dE}{ds} = u_f(s), \quad (8.4)$$

$$u_f(s) \leq p_{\max}/v(s), \quad (8.5)$$

$$0 \leq u_f(s) \leq u_{f,\max}, \quad (8.6)$$

$$0 \leq u_b(s) \leq u_{b,\max}, \quad (8.7)$$

$$\epsilon \leq v(s) \leq v_{\max}(s), \quad (8.8)$$

$$v(s_o) = v_o, v(s_e) = v_e, t(s_o) = 0, t(s_e) = t_e, E(s_o) = 0, \quad (8.9)$$

where J [J/kg] is the total traction energy consumed, s [m] is the distance covered by the train, $v(s)$ [m/s] is the speed of the train, $t(s)$ [s] is the time and $E(s)$ [J/kg] is the cumulative energy consumed. Two nonnegative control variables are considered, one corresponding to the traction force, $u_f(s)$, and the other corresponding to the brake force, $u_b(s)$, both in [N/kg]. We do not consider traction losses and regenerative brake in this chapter for readability, although both could be introduced easily in Eq. (8.4) and in the RTTO algorithm in chapter 8.6 as $\frac{dE}{ds} = u_f(s)/\eta_t - u_b(s)\eta_b$, with η_t and η_b the traction and regenerative brake efficiencies, respectively.

The maximum traction force that can be applied is limited by the adhesion of the wheels to the track. Wheels tend to slide if the traction force applied is too high, therefore at low speeds the maximum force that can be applied effectively is usually considered a constant $F_{\max}$ [N]. At higher speeds the maximum traction force that can be applied is limited by the maximum power of the engine $P_{\max}$ [W]. Notice that the traction force, when limited by the maximum engine power, depends hyperbolically on the speed. When considering braking, in this model we assume that the train is able to brake at any speed with a maximum brake rate $F_{\min}$ [N].

In Eq. (8.2), the term $r(v)$ [N/kg] corresponds to the mass-specific running resistance, which is modelled as a quadratic function of the speed called the Davis equation (Davis, 1926), $r(v) = r_0 + r_1 v + r_2 v^2$, where r_0 and r_1 are nonnegative and r_2 is strictly positive, in [N/kg], [N/(kg m/s)] and [N/(kg(m/s)²)], respectively. $w(s) = g \frac{dh(s)}{ds} / \rho$ stands for the mass-specific resistances due to the effect of gradients and curves on the dynamics of the train [N/kg], with g the gravity acceleration and $h(s)$ the height profile of the track. We assume that the gradients $\frac{dh(s)}{ds}$ are smooth, but we neglect the effect of the curves since that data is not available for the instances in our computational results. The gradients' smoothness condition will be relaxed in the implementation proposed in chapter 8.6 to match the piecewise-constant gradients data available in the real case studies that will be showcased in chapter 8.7. This location-based description of the track promotes using the train position as the independent variable of the problem instead of the time, which is used more frequently in physics and engineering as the independent variable. Eq. (8.4) stands for the energy consumption at the wheel, without considering traction losses and regenerative brake. In Eqs. (8.5)-(8.7) we introduce the mass-specific bounds for the control variables, where $u_{f,\max} = F_{\max}/\rho m$, $p_{\max} = P_{\max}/\rho m$ and $u_{b,\max} = F_{\min}/\rho m$, being m [kg] the total mass of the train and $\rho > 1$ a dimensionless rotating-mass factor. The speed is bounded by a piecewise constant speed limit $v_{\max}(s)$ in Eq. (8.8) and we assume that the minimum speed of the train ϵ is strictly positive to ensure the Lipschitz condition of Eqs. (8.2) and (8.3). If the train departs from zero speed or stops to a standstill, the running time and the initial or final location of the train can be corrected by applying the formulas shown in chapter 7. The boundary constraints for the train operation are shown in Eq. (8.9), where s_o and s_e are the initial and final location of the train, respectively. v_o and v_e are the initial and final speed at those locations. The running time T is fixed and given by the timetable.

In Eqs. (8.1)-(8.9), several parameters can be distinguished: the three running resistance parameters r_0 , r_1 and r_2 , the adhesion limit $u_{f,\max}$, the maximum tractive power $p_{\max}$, the maximum brake effort $u_{b,\max}$, the mass m and the rotating-mass factor ρ . The mass-specific description of the TTO problem allows to remove the explicit dependence on the train mass. Any variation in the train mass would be observed as variations in the first six parameters. The rotating-mass factor appears explicitly in Eq. (8.2) only in the term $w(s)$, but in this chapter we will not consider any potential variations of this parameter. This assumption could be relieved, however, we do not have data on variations of the rotating-mass factor. This allows to reduce the number of parameters that may vary by two. In chapter 3, we observed that the maximum tractive power $p_{\max}$ varies slightly, so we will not consider variations of this parameter either. Then, for a given set

of parameters $(r_0, r_1, r_2, u_{f,\max}, u_{b,\max})$, the problem in Eqs. (8.1)-(8.9) constitutes the nominal TTO problem associated to that set of parameters.

8.2.2 Nominal minimum time Train Trajectory Optimization

The minimum time and the energy-optimal Train Trajectory Optimization problems are quite similar from a mathematical point of view. In the minimum time problem, the energy consumption variable, its dynamics in Eq. (8.4), and the terminal time constraint on $t(s_e)$ in Eq. (8.9) are not present. The value of the terminal time is the cost function in the minimum time problem:

$$\min_{u_f, u_b} t(s_e). \quad (8.10)$$

The minimum time TTO problem is Eq. (8.10) subject to Eqs. (8.2), (8.3), (8.5)-(8.8) and the modified boundary conditions in Eq. (8.9).

8.3 Robust Train Trajectory Optimization problem

8.3.1 Energy-efficient Robust Train Trajectory Optimization modelled as a minimax optimal control problem

In this chapter we model the RTTO problem under parametric uncertainty as a minimax optimal control problem. The model parametric uncertainty is introduced as a finite set of observed parameter configurations $\alpha \in \mathcal{A} = \{\alpha_1, \dots, \alpha_q\}$, with $(r_0^\alpha, r_1^\alpha, r_2^\alpha, u_{f,\max}^\alpha, u_{b,\max}^\alpha)$ the parameters corresponding to the α -th parameter configuration and $q \geq 2$ is the number of parameter configurations introduced in the RTTO problem. $\mathcal{A}$ is compact since it is finite and all its elements are bounded in the Euclidean space $\mathbb{R}^5$. Notice the abuse of notation, as α stands both for an element in the set of parameter configurations $\mathcal{A}$ and for a superscript index that indicates a variable or a parameter belonging to a train motion model parameterized with that specific configuration. If $\mathcal{A}$ contained only one parameter configuration, the RTTO problem would be reduced to the nominal scenario introduced in chapter 8.2.

For each parameter configuration $\alpha \in \mathcal{A}$ let $v^\alpha(s)$ be the train speed [m/s], $t^\alpha(s)$ its running time [s] and $E^\alpha(s)$ its energy consumption [J/kg] along the locations $s \in [s_o, s_e]$, in [m]. Let $u_f(s)$ and $u_b(s)$ be the control variables that represent the applied tractive and brake effort, respectively, in [N/kg]. The minimax train trajectory optimization problem is formulated as Eq. (8.11) subject to Eqs. (8.12)-(8.19).

$$\min_{u_f, u_b} J = \min_{u_f, u_b} \max_{\alpha \in \mathcal{A}} E^\alpha(s_e), \quad (8.11)$$

subject to

$$\frac{dv^\alpha}{ds} = \frac{u_f(s) - u_b(s) - r^\alpha(v^\alpha) - w(s)}{v^\alpha(s)}, \quad \alpha \in \mathcal{A}, \quad (8.12)$$

$$\frac{dt^\alpha}{ds} = \frac{1}{v^\alpha(s)}, \quad \alpha \in \mathcal{A}, \quad (8.13)$$

$$\frac{dE^\alpha}{ds} = u_f(s), \quad \alpha \in \mathcal{A}, \quad (8.14)$$

$$u_f(s) \leq p_{\max}/v^\alpha(s), \quad \alpha \in \mathcal{A}, \quad (8.15)$$

$$0 \leq u_f(s) \leq u_{f,\max}^\alpha, \quad \alpha \in \mathcal{A}, \quad (8.16)$$

$$0 \leq u_b(s) \leq u_{b,\max}^\alpha, \quad \alpha \in \mathcal{A}, \quad (8.17)$$

$$\epsilon \leq v^\alpha(s) \leq v_{\max}(s), \quad \alpha \in \mathcal{A}, \quad (8.18)$$

$$v^\alpha(s_o) = v_o, \quad v^\alpha(s_e) \leq v_e, \quad t^\alpha(s_o) = 0, \quad t^\alpha(s_e) \leq t_e, \quad E^\alpha(s_o) = 0, \quad \alpha \in \mathcal{A}, \quad (8.19)$$

where $r^\alpha(v^\alpha) = r_0^\alpha + r_1^\alpha v^\alpha + r_2^\alpha (v^\alpha)^2$ is the running resistance of the train for the parameter configuration α .

Eqs. (8.16) and (8.17) are $2q$ constraints that can be reduced to 2 by observing that only those with minimum maximum tractive and brake effort in $\alpha \in \mathcal{A}$ can be active. Defining $\underline{u}_{f,\max} = \min_{\alpha \in \mathcal{A}} u_{f,\max}^\alpha$ and $\underline{u}_{b,\max} = \min_{\alpha \in \mathcal{A}} u_{b,\max}^\alpha$, we obtain Eqs. (8.20) and (8.21):

$$0 \leq u_f(s) \leq \underline{u}_{f,\max}, \quad (8.20)$$

$$0 \leq u_b(s) \leq \underline{u}_{b,\max}. \quad (8.21)$$

Particularly, the control policy $(u_f(s), u_b(s))$ has to realize the terminal condition in Eq. (8.19), that is to say, for each parameter configuration $\alpha \in \mathcal{A}$ the trajectory $(v^\alpha(s), t^\alpha(s))$ satisfies the boundary conditions. This has a remarkable consequence: the robust-optimal control policy will lead to different intermediate passing times and speeds for each of the parameter configurations, requiring the terminal set of allowed arrival times and speeds at s_e to be sufficiently large to allow for feasible solutions of the RTTO problem. The differences in the dynamics produced by each parameter configuration will lead to a different robust-optimal trajectory for each parameter configuration, although the robust-optimal control policy is unique for all these trajectories. For example, speed deviations arise immediately after departure due to differences in the running resistance parameters, as can be seen after a forward Euler step from the departure speed v_o until location s_1 ,

$$v^\alpha(s_1) = v_o + (s_1 - s_o) \frac{u_f(s_o) - r^\alpha(v_o) - w(s_o)}{v_o}, \quad \alpha \in \mathcal{A}. \quad (8.22)$$

Considering two parameter configurations $\alpha_1, \alpha_2 \in \mathcal{A}$, the speed dispersion between them after a forward Euler step is

$$v^{\alpha_2}(s_1) - v^{\alpha_1}(s_1) = \frac{(s_1 - s_o)}{v_o} (r^{\alpha_1}(v_o) - r^{\alpha_2}(v_o)). \quad (8.23)$$

The deviations in running time can be observed after two forward Euler steps, being equal to $(s_2 - s_1)(1/v^{\alpha_2}(s_1) - 1/v^{\alpha_1}(s_1))$ at s_2 . Consequently, Eq. (8.19) shall allow for an arrival time and speed window large enough to guarantee the feasibility of the RTTO problem, that is to say, the existence of a robust-optimal control policy that can realize such terminal constraints. If the distance to the arrival location is very far from the departure, the trajectories could increasingly deviate along the way. In such cases, the arrival window may not accommodate all the speed and time differences described by each trajectory and the RTTO problem may become infeasible. Selecting carefully the set of operating conditions of interest, calculating the robust-optimal minimum time control policy and defining an adequate arrival window is therefore essential when the distance to

the train's destination is large. Still, in real life a train stops when it reaches zero speed, so in practice the arrival window shall be located before the actual stopping point. A discussion on how to allocate the arrival window is provided in chapter 8.7.

The terminal condition in Eq. (8.19) defines a maximum arrival speed v_e and a latest arrival time t_e . However, in real life a train stops when it reaches zero speed.

We define the robust-optimal trajectory bundle as the family of q trajectories that are produced by the robust-optimal control policy, each of them corresponding to the evaluation of the robust-optimal control policy on the train motion model with each of the parameter configurations introduced in the RTTO problem. This differs from the optimal solution of the nominal TTO problem, as at each point of the track $s \in (s_o, s_e]$ the solution of the RTTO problem will not have only one, but up to q passing speeds and times. This fact constitutes the core of the definition of Robust Train Path Envelopes that will be introduced in chapter 8.5. Still, the robust-optimal control policy has similar properties as the nominal-optimal one, as we will show in chapter 8.4.

The proposed formulation of the RTTO problem relies on a set of observed parameter configurations, each of them representing realized operating conditions. These parameters are estimated from data measured on-board trains while they run on the track under consideration. In this chapter we utilize the parameters estimated in chapter 3, where M-estimation and a sliding median filter were used to estimate the train motion model parameters. A parameter configuration was estimated for each of the realized trajectories considered. The set of estimated parameters will capture the variations in the dynamics of the train produced by the operating conditions at that time. Some examples of the mentioned operating conditions are the weather, the rolling stock load, composition and wear and the track maintenance condition. This allows to associate a physical interpretation to each parameter configuration. Then, those parameter configurations that represent any operating conditions of interest can be introduced as input to the RTTO problem to calculate how to drive the train timely while saving energy despite the existing uncertainty in the actual operating conditions. For example, if it starts raining, the reference to be followed by the train may have to be recalculated by introducing in the RTTO problem several parameter configurations obtained from wet rails or rain condition data instead of dry parameter configurations. This contributes to adapting energy-efficient train operation and planning to the real operational environment, guaranteeing that trains will arrive at their destination within the scheduled arrival time window.

8.3.2 Minimum time Robust Train Trajectory Optimization modelled as a minimax optimal control problem

Parametric model uncertainty can also be introduced in the minimum time TTO problem shown in chapter 8.2.2. The cost function of the nominal problem Eq. (8.10) can be rewritten as a minimax cost as follows:

$$\min_{u_f, u_b} \tau = \min_{u_f, u_b} \max_{\alpha \in \mathcal{A}} t^\alpha(s_e). \quad (8.24)$$

The minimum time RTTO problem is Eq. (8.24) subject to Eqs. (8.12), (8.13), (8.15)-(8.18) and the boundary conditions

$$v^\alpha(s_o) = v_o, \quad v^\alpha(s_e) \leq v_e, \quad t^\alpha(s_o) = 0, \quad \alpha \in \mathcal{A}. \quad (8.25)$$

The cost function in Eq. (8.24) aims to find the fastest driving strategy among all the strategies that are feasible for all the parameter configurations considered. In other words, if we consider as many trains as parameter configurations, this driving strategy would be the fastest to drive all trains to their common destination. The last train to arrive would be the one determining the value of the cost function, that is, the robust-optimal minimum time. Thus, the robust-optimal minimum time can be sensitive to parameter configurations corresponding to outliers or extreme operating conditions.

The importance of the minimum time RTTO problem relies on the fact that a feasible maximum running time shall be used as input in the energy-optimal RTTO problem, and minimum time RTTO is needed to calculate this value. It is also useful for modelling worst-case scenario minimum running times to be used in timetabling for defensive purposes. When the timetable is too tight, underestimating minimum running times can easily lead to delays and train path conflicts, so the robust-optimal minimum running time is a conservative solution when planning tight schedules. In operation, if the train can run faster than the robust-optimal minimum running time, an opportunity arises to save energy while still ensuring its on-time arrival. In this sense, underestimated minimum running times can be viewed as supplement times optimally allocated under parametric model uncertainty assumptions.

8.4 Robust Maximum Principle analysis of the Robust Train Trajectory Optimization problem

8.4.1 Analysis of the energy-efficient Robust Train Trajectory Optimization problem

In this chapter we analyze the RTTO problem by means of the Robust Maximum Principle introduced in Boltyanski & Poznyak (2011) with the aim to provide insights on the structure of robust-optimal control policies. Similar analyses of the nominal TTO problem based on the Maximum Principle can be found in Albrecht et al. (2016b) and Goverde et al. (2021). From now on, we only show the dependence on the independent variable s where relevant to simplify the notation.

We define nonnegative Lagrange multipliers μ_f , μ_{f_0} , μ_b , μ_{b_0} , and μ_p^α , μ_v^α , $\alpha \in \mathcal{A}$, and the following Complementary Slackness Conditions (CSC) relative to the path constraints:

$$\mu_f(\underline{u}_{f,\max} - u_f) = 0, \quad (8.26)$$

$$\mu_{f_0} u_f = 0, \quad (8.27)$$

$$\mu_b(\underline{u}_{b,\max} - u_b) = 0, \quad (8.28)$$

$$\mu_{b_0} u_b = 0, \quad (8.29)$$

$$\mu_p^\alpha(p_{\max} - u_f v^\alpha) = 0, \quad \alpha \in \mathcal{A}, \quad (8.30)$$

$$\mu_v^\alpha(v_{\max} - v^\alpha) = 0, \quad \alpha \in \mathcal{A}. \quad (8.31)$$

A Lagrange multiplier is zero if its associated constraint is inactive, and strictly positive when it is active.

We compute the augmented Hamiltonian associated to the energy-optimal RTTO control problem:

$$\begin{aligned}
 H = \sum_{\alpha \in \mathcal{A}} & \left(\frac{\lambda_v^\alpha}{v^\alpha} (u_f - u_b - r^\alpha(v^\alpha) - w) + \frac{\lambda_t^\alpha}{v^\alpha} + \lambda_E^\alpha u_f + \right. \\
 & \left. + \mu_p^\alpha (p_{\max} - u_f v^\alpha) + \mu_v^\alpha (v_{\max} - v^\alpha) \right) + \\
 & + \mu_f (u_{f,\max} - u_f) + \mu_{f_0} u_f + \mu_b (u_{b,\max} - u_b) + \mu_{b_0} u_b,
 \end{aligned} \tag{8.32}$$

where λ_v^α , λ_t^α and λ_E^α are the costate variables of v^α , t^α and E^α , respectively.

The Euler-Lagrange equations for these costate variables are

$$\frac{d\lambda_v^\alpha}{ds} = -\frac{\partial H}{\partial v^\alpha} = \frac{\lambda_t^\alpha}{(v^\alpha)^2} + \frac{\lambda_v^\alpha}{v^\alpha} \left(\frac{u_f - u_b - r^\alpha(v^\alpha) - w}{v^\alpha} + (r^\alpha)'(v^\alpha) \right) + \mu_p^\alpha u_f + \mu_v^\alpha, \tag{8.33}$$

$$\frac{d\lambda_t^\alpha}{ds} = -\frac{\partial H}{\partial t^\alpha} = 0, \Rightarrow \lambda_t^\alpha(s) = \lambda_t^\alpha(s_e) \text{ is constant in } s, \tag{8.34}$$

$$\frac{d\lambda_E^\alpha}{ds} = -\frac{\partial H}{\partial E^\alpha} = 0, \Rightarrow \lambda_E^\alpha(s) = \lambda_E^\alpha(s_e) \text{ is constant in } s. \tag{8.35}$$

The Robust Maximum Principle states that for the robust-optimality of a control policy (u_f^*, u_b^*) it is necessary that real values of the endpoints of the costate variables exist, λ_v^α , λ_t^α , λ_E^α , $\forall \alpha \in \mathcal{A}$, and nonnegative real functions exist, $\nu_v(\alpha)$, $\nu_t(\alpha)$, $\nu_E(\alpha)$, such that the following conditions are satisfied (Boltyanski & Poznyak, 2011):

- The augmented Hamiltonian in Eq. (8.32) is maximized for the robust-optimal control policy (u_f^*, u_b^*) ,

$$(u_f^*, u_b^*) = \arg \max H. \tag{8.36}$$

- The Complementary Slackness Conditions for the terminal speed, time and energy of the train give

$$\nu_v(\alpha)(v^\alpha(s_e) - v_e) = 0, \alpha \in \mathcal{A}, \tag{8.37}$$

$$\nu_t(\alpha)(t^\alpha(s_e) - t_e) = 0, \alpha \in \mathcal{A}, \tag{8.38}$$

$$\nu_E(\alpha)(E^\alpha(s_e) - J) = 0, \alpha \in \mathcal{A}. \tag{8.39}$$

- The Transversality Condition for the endpoints of the costate variables

$$\lambda_v^\alpha(s_e) + \nu_v(\alpha) = 0, \alpha \in \mathcal{A}, \tag{8.40}$$

$$\lambda_t^\alpha(s_e) + \nu_t(\alpha) = 0, \alpha \in \mathcal{A}, \tag{8.41}$$

$$\lambda_E^\alpha(s_e) + \nu_E(\alpha) = 0, \alpha \in \mathcal{A}. \tag{8.42}$$

- For at least one parameter configuration $\alpha \in \mathcal{A}$ the Nontriviality Condition shall hold:

$$\exists \alpha \in \mathcal{A} : \sqrt{(\lambda_v^\alpha(s_e))^2 + (\lambda_t^\alpha(s_e))^2 + (\lambda_E^\alpha(s_e))^2} + \nu_v(\alpha) + \nu_t(\alpha) + \nu_E(\alpha) > 0. \tag{8.43}$$

Notice that $\nu_v(\alpha)$, $\nu_t(\alpha)$ and $\nu_E(\alpha)$ are functions with domain $\mathcal{A}$. The parameter configuration $\alpha \in \mathcal{A}$ is their independent variable, not the train location s like most of the other variables in this chapter. To reflect this, in these variables α is shown explicitly as the independent variable and not as a superscript.

From Eq. (8.14), we infer that all the E^α are equal for all values of $\alpha \in \mathcal{A}$ since $E^\alpha(s_o) = 0$, $\forall \alpha \in \mathcal{A}$. Therefore, from Eq. (8.11), $J = E^\alpha(s_e) \forall \alpha \in \mathcal{A}$. This means that $\nu_E(\alpha)$ is strictly positive $\forall \alpha \in \mathcal{A}$ in virtue of Eq. (8.39) and that $\lambda_E^\alpha(s_e)$ is strictly negative $\forall \alpha \in \mathcal{A}$ due to Eq. (8.42). From the strictly positive values of $\nu_E(\alpha)$ and strictly negative of $\lambda_E^\alpha(s_e)$, $\forall \alpha \in \mathcal{A}$ we deduce that the Nontriviality Condition in Eq. (8.43) holds $\forall \alpha \in \mathcal{A}$. We can simplify the notation by denoting $E^\alpha(s) = E(s) \forall s \in [s_o, s_e], \alpha \in \mathcal{A}$. Without loss of generality, we can take $\lambda_E^\alpha = -1/q \forall \alpha \in \mathcal{A}$ by renormalizing the Lagrange multipliers, costate variables and functions $\nu_v(\alpha)$ and $\nu_t(\alpha)$. The augmented Hamiltonian in Eq. (8.32) is simplified accordingly:

$$\begin{aligned} H = & -u_f + \sum_{\alpha \in \mathcal{A}} \left(\frac{\lambda_v^\alpha}{v^\alpha} (u_f - u_b - r^\alpha(v^\alpha) - w) + \frac{\lambda_t^\alpha}{v^\alpha} + \mu_p^\alpha (p_{\max} - u_f v^\alpha) + \right. \\ & \left. + \mu_v^\alpha (v_{\max} - v^\alpha) \right) + \mu_f (\underline{u}_{f,\max} - u_f) + \mu_{f_0} u_f + \\ & + \mu_b (\underline{u}_{b,\max} - u_b) + \mu_{b_0} u_b. \end{aligned} \quad (8.44)$$

If the speed is equal to the speed limit, then a parameter configuration $\tilde{\alpha} \in \mathcal{A}$ exists, for which $v^{\tilde{\alpha}} = v_{\max}$ and $\mu_v^{\tilde{\alpha}} > 0$. This constrained speed driving regime may happen if the resistance to motion due to the track gradients w and the bounds on the maximum traction and brake allow $u_f - u_b = r^{\tilde{\alpha}}(v_{\max}) + w$ to hold. From now on, we will consider the speed unconstrained $\forall \alpha \in \mathcal{A}$ in the analysis of the potential driving regimes in a robust-optimal control policy, so $\mu_v^\alpha = 0 \forall \alpha \in \mathcal{A}$.

From Eq. (8.36), the augmented Hamiltonian in Eq. (8.44) shall be stationary with respect to the robust-optimal control policy (u_f^*, u_b^*) . Writing $\lambda = \sum_{\alpha \in \mathcal{A}} \lambda_v^\alpha / v^\alpha$ and $\mu_{fp} = \mu_f + \sum_{\alpha \in \mathcal{A}} \mu_p^\alpha v^\alpha$, the Stationary Conditions become

$$0 = \frac{\partial H}{\partial u_f} = -1 + \lambda - \mu_{fp} + \mu_{f_0}, \quad (8.45)$$

$$0 = \frac{\partial H}{\partial u_b} = -\lambda - \mu_b + \mu_{b_0}. \quad (8.46)$$

μ_{fp} can also be viewed as a Lagrange multiplier because of its nonnegativity. If $\mu_{fp} = 0$, then $\mu_f = 0$ and $\mu_p^\alpha = 0 \forall \alpha \in \mathcal{A}$. $\mu_{fp} > 0$ implies that either μ_f or one of the μ_p^α is strictly positive while the others are zero. This means that either $u_f = \underline{u}_{f,\max}$ or $u_f = p_{\max}/v^\alpha$, for one of the $\alpha \in \mathcal{A}$. Therefore, $\mu_{fp} > 0$ implies that $u_f = \min\{\underline{u}_{f,\max}, p_{\max}/v^{\alpha_1}, \dots, p_{\max}/v^{\alpha_q}\}$. The value of λ determines the sign of combinations of the Lagrange multipliers μ_{fp} , μ_{f_0} , μ_b and μ_{b_0} via the Stationary Conditions in Eqs. (8.45) and (8.46). Since the Lagrange multipliers are nonnegative, the possible driving regimes in a robust-optimal control policy can be drawn from the CSC relative to the path constraints in Eqs. (8.26)-(8.30):

- If $\lambda > 1$, then $\mu_{fp} - \mu_{f_0} > 0$ and $\mu_{b_0} - \mu_b > 0$. All Lagrange multipliers are nonnegative, therefore $\mu_{fp} > 0$, $\mu_{b_0} > 0$ and $\mu_{f_0} = \mu_b = 0$. From the CSC in Eq. (8.26) and

(8.30) follows that $u_f = \min\{\underline{u}_{f,\max}, p_{\max}/v^{\alpha_1}, \dots, p_{\max}/v^{\alpha_q}\}$ and from Eq. (8.29), $u_b = 0$. This corresponds to a **maximum traction** regime at the maximum feasible rate for all parameter configurations.

- If $0 < \lambda < 1$, the Stationary Conditions show that $\mu_{f_o} - \mu_{fp} > 0$ and $\mu_{b_o} - \mu_b > 0$. The nonnegativity of the Lagrange multipliers implies that $\mu_{f_o} > 0$, $\mu_{b_o} > 0$ and $\mu_{fp} = \mu_b = 0$. The CSC in Eq. (8.27) and Eq. (8.29) lead to $u_f = u_b = 0$, which corresponds to a **coasting** regime.
- If $\lambda < 0$, the Stationary Conditions show that $\mu_{f_o} - \mu_{fp} > 0$ and $\mu_b - \mu_{b_o} > 0$, which holds only if $\mu_{f_o} > 0$, $\mu_b > 0$ and $\mu_{fp} = \mu_{b_o} = 0$. From the CSC in Eq. (8.27) and Eq. (8.28) we observe that $u_f = 0$, $u_b = \underline{u}_{b,\max}$. Negative λ leads to a **maximum braking** regime at the maximum brake rate feasible for all parameter configurations.

The observed maximum traction and brake regimes are conservative regimes, in the sense that the amount of traction and brake determined by the Robust Maximum Principle is the highest possible that is feasible for all parameter configurations, that is, the minimum among all bounds in traction and brake. Like in nominal TTO, coasting is also a driving regime in a robust-optimal trajectory (Albrecht et al., 2016b).

If λ is 0 or 1, singular control regimes arise if λ stays constant over a non-trivial interval of the track. This means that $\frac{d\lambda}{ds} = 0$ inside the whole interval. From Eq. (8.33) and (8.12),

$$\frac{d\lambda}{ds} = \sum_{\alpha \in \mathcal{A}} \frac{d(\lambda_v^\alpha / v^\alpha)}{ds} = \sum_{\alpha \in \mathcal{A}} \left(\frac{\lambda_t^\alpha}{(v^\alpha)^3} + \frac{\lambda_v^\alpha (r^\alpha)'(v^\alpha)}{v^\alpha} + \frac{\mu_p^\alpha u_f + \mu_v^\alpha}{v^\alpha} \right). \quad (8.47)$$

A deeper analysis is required to analyze the singular control regimes. We will need to determine the value of the costate variables at the arrival location by assessing how the trajectories in the robust-optimal bundle arrive at destination. We will mainly proceed by proof of contradiction, assuming that a control policy is robust-optimal, modifying it sequentially while keeping its feasibility, ending up with a feasible control policy with lower energy consumption than the one we originally assumed to be robust-optimal and concluding that the control policy that we started with was not robust-optimal. We turn maximum traction into coasting or braking in infinitesimal or sufficiently small intervals of the track, reducing the total energy consumption and the arrival speed while increasing the arrival time. In turn, maximum braking is transformed into coasting in small intervals, which increases the arrival speed and decreases the arrival time while having no impact on the total energy consumption.

We proceed now to demonstrate that in a robust-optimal control policy, at least one $\alpha \in \mathcal{A}$ shall exist for which λ_t^α is nonzero. We aim to prove by contradiction that if this was not the case, $\nu_t(\alpha) = 0 \forall \alpha \in \mathcal{A}$ in virtue of Eq. (8.41) and from Eq. (8.38) would follow that the maximum arrival time constraint in Eq. (8.19) is inactive $\forall \alpha \in \mathcal{A}$, so the trajectories of all the parameter configurations considered arrive earlier than the maximum arrival time specified in Eq. (8.19). Two possible scenarios exist: one in which the maximum arrival speed constraint is saturated for at least one $\alpha \in \mathcal{A}$, so $v^\alpha(s_e) = v_e$ for at least one $\alpha \in \mathcal{A}$, and another in which this constraint is not saturated $\forall \alpha \in \mathcal{A}$, that is, $v^\alpha(s_e) < v_e \forall \alpha \in \mathcal{A}$. Without loss of generality, we assume that the α

saturating any constraint is unique. In the first scenario, the arrival speed constraint is saturated for one of the parameter configurations. We cut off traction into coasting in a small interval so the arrival time increases for all parameter configurations with reduced energy consumption. This interval is small enough to ensure that the arrival time does not increase beyond saturating the maximum arrival time boundary condition for any of the parameter configurations. If initially the minimum speed in Eq. (8.18) was saturated at the terminal location s_e for one of the parameter configurations, we can redefine the minimum speed by taking half of the minimum speed ϵ in Eq. (8.18). The modification of the robust-optimal control policy described above can be performed in a small interval without violating the redefined minimum speed constraint. The resulting modified control policy is a feasible control policy that is more energy-efficient than the one we assumed to be robust-optimal. Therefore, the first scenario cannot take place. Now that we have shown that the first scenario cannot happen we move on to the second scenario. If the arrival speed constraint is not saturated for any parameter configuration, we can modify the robust-optimal control policy by turning brake into coasting until the arrival speed constraint is saturated for one of the parameter configurations. The modified control policy leads to arriving earlier to s_e with higher arrival speeds for all parameter configurations. The energy consumption remains invariant in this modification. This modified control policy has transformed the second scenario into the first scenario without altering the energy consumption, and we have already shown in the first scenario that we can build a more energy-efficient control policy. This shows that the original robust-optimal control policy of the second scenario was not robust-optimal, so this scenario cannot take place. Therefore, in a robust-optimal trajectory the boundary condition on the maximum arrival time shall be saturated for at least one $\alpha \in \mathcal{A}$. From Eq. (8.38), an $\alpha \in \mathcal{A}$ exists so that $\nu_t(\alpha)$ is strictly positive. From Eq. (8.41), λ_t^α is strictly negative for that parameter configuration. All the other λ_t^α 's will be zero, so the term $\sum_{\alpha \in \mathcal{A}} \lambda_t^\alpha / (v^\alpha)^k$ is strictly negative for any nonnegative power k .

Now that we have shown that the arrival time constraint must be saturated for at least one parameter configuration, we will show that the constraint on the maximum arrival speed must also be saturated in a robust-optimal control policy. If this statement was false, we could modify the robust-optimal control policy to show that it was not truly optimal. We cut brake into coasting in an interval so the arrival speed increases until it saturates its boundary condition. The arrival time will be reduced while the consumed energy does not vary. This is again reduced to the first scenario discussed in the previous paragraph, so the control policy can be modified again to make it more energy-efficient than the one we originally assumed to be robust-optimal. Therefore, in a solution of the RTTO problem both boundary conditions on the arrival time and speed shall be saturated for at least one parameter configuration, which may or may not be the same configuration.

We can analyze now the singular control regimes arising from $\lambda = 0$ or 1 , which have a complex physical interpretation.

- If $\lambda = 1$, from the Stationary Conditions we obtain $\mu_{fp} - \mu_{fo} = 0$ and $\mu_{bo} - \mu_b = 1$. The nonnegativity of the Lagrange multipliers and Eqs. (8.26)-(8.30) imply that $\mu_{fp} = \mu_{fo} = \mu_b = 0$ and $\mu_{bo} = 1$. From the CSC in Eqs. (8.26)-(8.30) follows that $0 < u_f < \min\{\underline{u}_{f,\max}, p_{\max}/v^{\alpha_1}, \dots, p_{\max}/v^{\alpha_q}\}$ and $u_b = 0$. This is a **partial traction** regime.

If $\lambda = 1$ in a nontrivial track interval, it will remain constant inside the interval. As we have just shown, two parameter configurations exist, $\alpha_1, \alpha_2 \in \mathcal{A}$, being not necessarily different, so that $\lambda_t^{\alpha_1} < 0$, $\lambda_t^\alpha = 0 \forall \alpha \neq \alpha_1$, $\lambda_v^{\alpha_2}(s_e) < 0$ and $\lambda_v^\alpha(s_e) = 0 \forall \alpha \neq \alpha_2$.

If the maximum arrival speed and time constraints are saturated by the same parameter configuration, that is, if $\alpha_1 = \alpha_2$, then $\lambda = \lambda_v^{\alpha_1}/v^{\alpha_1} = 1$ because in virtue of Eq. (8.33), $\lambda_v^\alpha = 0 \forall s \leq s_e \forall \alpha \neq \alpha_1$. Eq. (8.47) shows that

$$0 = \frac{d\lambda}{ds} = \frac{\lambda_t^{\alpha_1}}{(v^{\alpha_1})^3} + \frac{\lambda_v^{\alpha_1}}{v^{\alpha_1}} \frac{(r^{\alpha_1})'(v^{\alpha_1})}{v^{\alpha_1}} = \frac{\lambda_t^{\alpha_1}}{(v^{\alpha_1})^3} + \frac{(r^{\alpha_1})'(v^{\alpha_1})}{v^{\alpha_1}}. \quad (8.48)$$

Since $r(v) = r_0 + r_1 v + r_2 v^2$, with r_0 and r_1 nonnegative and r_2 strictly positive, then $v^2 r'(v) = r_1 v^2 + 2r_2 v^3$ is strictly positive and strictly increasing in v . Therefore, like in nominal TTO (Khmelnitsky, 2002), Eq. (8.48) can hold only for a unique positive speed $\tilde{v}^{\alpha_1}$ for the model with parameter configuration α_1 .

Although in nominal TTO this would correspond to a cruise phase at this speed if the gradients w are not steep uphill or downhill for this speed, in RTTO the concept of cruising at a specific constant speed for all parameter configurations does not exist. At any location of the track after the train's departure, the robust-optimal trajectory applied to the train model with different parameter configurations leads to as many passing speeds as configurations considered in the RTTO problem. Applying a certain amount of partial traction that would lead to holding a constant speed for one of the parameter configurations considered does not necessarily mean that the speed will remain constant for all the other parameter configurations. However, since the value of $\lambda_t^{\alpha_1}$ fixes the value of the cruise speed $\tilde{v}^{\alpha_1}$ in Eq. (8.48), on nonsteep track sections, the necessary amount of traction required to keep a constant speed $\tilde{v}^{\alpha_1}$ is also fixed for the train model with the parameter configuration α_1 , $\tilde{u}_f = r^{\alpha_1}(\tilde{v}^{\alpha_1}) + w$. If this amount of traction is applied on Eq. (8.12) for the train motion model with any other parameter configuration $\alpha \neq \alpha_1$ over a sufficiently long distance with constant gradients, we would observe that v^α asymptotically approaches a constant value (Cunillera et al., 2023c), provided that r_0^α is small enough to allow for a unique speed v^α that verifies $0 = r^{\alpha_1}(\tilde{v}^{\alpha_1}) - r^\alpha(v^\alpha)$. We can therefore speak of an asymptotic cruise regime for all parameter configurations. Furthermore, this regime with partial traction corresponding to cruising at $\tilde{v}^{\alpha_1}$ and asymptotic cruising at other speeds for other parameter configurations can only take place if α_1 is the parameter configuration that saturates the arrival time and speed boundary conditions.

If the maximum arrival speed and time constraints are saturated by different parameter configurations, that is, if $\alpha_1 \neq \alpha_2$, then $\lambda_t^{\alpha_1} < 0$, $\lambda_t^\alpha = 0 \forall \alpha \neq \alpha_1$, $\lambda_v^{\alpha_2}(s_e) < 0$ and $\lambda_v^\alpha(s_e) = 0 \forall \alpha \neq \alpha_2$. In fact, $\lambda_v^\alpha(s) = 0 \forall s \leq s_e \forall \alpha \neq \{\alpha_1, \alpha_2\}$. The nonzero costate variables of the speed are related through λ :

$$\lambda = \lambda_v^{\alpha_1}/v^{\alpha_1} + \lambda_v^{\alpha_2}/v^{\alpha_2} = 1. \quad (8.49)$$

If λ stays constant over a nontrivial track interval, from Eq. (8.47), we observe

$$0 = \frac{d\lambda}{ds} = \frac{\lambda_t^{\alpha_1}}{(v^{\alpha_1})^3} + \frac{\lambda_v^{\alpha_1}}{v^{\alpha_1}} \frac{(r^{\alpha_1})'(v^{\alpha_1})}{v^{\alpha_1}} + \frac{\lambda_v^{\alpha_2}}{v^{\alpha_2}} \frac{(r^{\alpha_2})'(v^{\alpha_2})}{v^{\alpha_2}}. \quad (8.50)$$

The physical interpretation of this regime is more complicated. This is a scenario that can only be observed for unconstrained speeds if the model with the slowest parameter configuration under the robust-optimal control is not the one ending with the highest speed at destination. Furthermore, Eqs. (8.49) and (8.50) describe the relation between the speeds of the parameter configurations α_1 and α_2 during the regime $\lambda = 1$ and their corresponding costate variables. In this scenario, the speed is not necessarily constant for any of the considered parameter configurations. As we will see in chapter 8.7, this partial traction regime shows up when reaching higher speeds would jeopardize arriving at destination within the time and speed windows. Applying partial traction in the middle of a robust-optimal trajectory when driving at moderate speeds reduces the dispersion in the passing speeds of the trajectory bundle, leading to lower dispersion in the arrival speeds.

- If $\lambda = 0$, from the Stationary Conditions, $\mu_{f_o} - \mu_{f_p} = 1$ and $\mu_{b_o} - \mu_b = 0$. Since the Lagrange multipliers are nonnegative and Eqs. (8.26)-(8.30) hold, the equalities above hold if and only if $\mu_{f_p} = \mu_b = \mu_{b_o} = 0$ and $\mu_{f_o} = 1$. Introducing this information into the CSC in Eqs. (8.26)-(8.30) allows to observe that $u_f = 0$ and $0 < u_b < \underline{u}_{b,\max}$, so $\lambda = 0$ can only hold in a **partial brake** regime.

If $\lambda = 0$ in a nontrivial track interval, its derivative will also be zero inside this interval. Like in the previous scenario, at most two parameter configurations exist, $\alpha_1, \alpha_2 \in \mathcal{A}$, such that $\lambda_t^{\alpha_1} < 0$, $\lambda_t^\alpha = 0 \ \forall \alpha \neq \alpha_1$, $\lambda_v^{\alpha_2}(s_e) < 0$, $\lambda_v^\alpha(s_e) = 0 \ \forall \alpha \neq \alpha_2$. In fact, $\lambda_t^\alpha = 0$ and $\lambda_v^\alpha = 0 \ \forall s \leq s_e$, $\forall \alpha \neq \{\alpha_1, \alpha_2\}$.

If the maximum arrival speed and time constraints are saturated by the same parameter configuration, that is, if $\alpha_1 = \alpha_2$, then $\lambda = \lambda_v^{\alpha_1}/v^{\alpha_1}$ and Eq. (8.47) shows that

$$0 = \frac{d\lambda}{ds} = \frac{\lambda_t^{\alpha_1}}{(v^{\alpha_1})^3}, \quad (8.51)$$

which cannot hold since $\lambda_t^{\alpha_1}$ is strictly negative. Therefore, $\lambda = 0$ cannot hold in a nontrivial track interval if $\alpha_1 = \alpha_2$. If a speed limit constraint was active, $\lambda = 0$ would take place when partial brake is required to prevent overspeeding with any of the parameter configurations.

If the maximum arrival speed and time constraints are saturated by different parameter configurations, that is, if $\alpha_1 \neq \alpha_2$, then

$$\lambda = \lambda_v^{\alpha_1}/v^{\alpha_1} + \lambda_v^{\alpha_2}/v^{\alpha_2} = 0, \quad (8.52)$$

with $\lambda_v^{\alpha_1}(s_e) = 0$ and $\lambda_v^{\alpha_2}(s_e) < 0$. λ cannot be trivially zero due to its components λ_v^α being zero. $\lambda_v^{\alpha_2}(s)$ remains strictly negative for all $s < s_e$. If it became zero eventually at any location, its derivative in Eq. (8.33) and all its higher order derivatives would also be zero at that location, so $\lambda_v^{\alpha_2}(s) = 0 \ \forall s \leq s_e$, contradicting the fact that $\lambda_v^{\alpha_2}(s_e) < 0$. Then, from Eq. (8.47) we obtain again the same Eq. (8.50) linking the costate variables of the time and speed, and the speeds v^{α_1} and v^{α_2} . The physical interpretation of this scenario is again difficult and it may happen only when the parameter configurations realizing the maximum arrival time and the maximum speed are different while their speeds and costates verify Eqs. (8.50) and (8.52). In

practice, this driving regime may happen under steep downhill gradients to limit the arrival speed dispersion in the trajectory bundle, favouring the existence of a feasible robust-optimal control policy that fulfills maximum arrival speed constraint for all parameter configurations.

8.4.2 Analysis of the minimum time Robust Train Trajectory Optimization problem

The Robust Maximum Principle can also be used to analyze the structure of the robust-optimal control policies that solve the minimum time RTTO problem. We follow a similar approach as in the analysis of the energy-efficient RTTO problem and several of the equations shown in the previous chapter are also used in this one. For simplicity, in this chapter we do not change the notation with respect to the energy-efficient problem. However, if any function or variable is redefined in this chapter, the text or any equation will accordingly refer to the new definition.

The Complementary Slackness Conditions (CSC) relative to the path constraints in Eqs. (8.26)-(8.30) shall also be fulfilled in the minimum time RTTO problem and its related Lagrange multipliers are also nonnegative.

The augmented Hamiltonian of the minimum time RTTO problem is

$$\begin{aligned}
 H = \sum_{\alpha \in \mathcal{A}} \left(\frac{\lambda_v^\alpha}{v^\alpha} (u_f - u_b - r^\alpha(v^\alpha) - w) + \frac{\lambda_t^\alpha}{v^\alpha} + \mu_p^\alpha (p_{\max} - u_f v^\alpha) + \right. \\
 \left. + \mu_v^\alpha (v_{\max} - v^\alpha) \right) + \mu_f (\underline{u}_{f,\max} - u_f) + \mu_{f_0} u_f + \\
 + \mu_b (\underline{u}_{b,\max} - u_b) + \mu_{b_0} u_b.
 \end{aligned} \tag{8.53}$$

The Euler-Lagrange equations in Eqs. (8.33), (8.34) and (8.47) for the costate variables λ_v^α , λ_t^α , and λ , respectively, remain unchanged.

The Robust Maximum Principle asserts that the necessary conditions for a robust-optimal minimum time control policy (u_f^*, u_b^*) are the existence of real values of the end-points of the costate variables λ_v^α and λ_t^α , $\forall \alpha \in \mathcal{A}$, the existence of nonnegative functions $\nu_v(\alpha)$ and $\nu_t(\alpha)$ with domain in $\mathcal{A}$, and the verification of the following conditions:

- The robust-optimal control policy (u_f^*, u_b^*) is the one that maximizes the augmented Hamiltonian in Eq. (8.53)

$$(u_f^*, u_b^*) = \arg \max H. \tag{8.54}$$

- The Complementary Slackness Conditions for the terminal speed and time,

$$\nu_v(\alpha)(v^\alpha(s_e) - v_e) = 0, \forall \alpha \in \mathcal{A}, \tag{8.55}$$

$$\nu_t(\alpha)(t^\alpha(s_e) - \tau) = 0, \forall \alpha \in \mathcal{A}, \tag{8.56}$$

where τ was defined in Eq. (8.24).

- The Transversality Condition at the endpoint s_e :

$$\lambda_v^\alpha(s_e) + \nu_v(\alpha) = 0, \alpha \in \mathcal{A}, \tag{8.57}$$

$$\lambda_t^\alpha(s_e) + \nu_t(\alpha) = 0, \alpha \in \mathcal{A}. \tag{8.58}$$

- At least one $\alpha \in \mathcal{A}$ the Nontriviality Condition shall hold:

$$\exists \alpha \in \mathcal{A}: \sqrt{(\lambda_t^\alpha(s_e))^2 + (\lambda_v^\alpha(s_e))^2} + \nu_t(\alpha) + \nu_v(\alpha) > 0. \quad (8.59)$$

Maximizing the Hamiltonian in Eq. (8.53) provides the following Stationary Conditions:

$$0 = \frac{\partial H}{\partial u_f} = \lambda - \mu_{fp} + \mu_{f_0}, \quad (8.60)$$

$$0 = \frac{\partial H}{\partial u_b} = -\lambda - \mu_b + \mu_{b_0}, \quad (8.61)$$

with $\lambda = \sum_{\alpha \in \mathcal{A}} \lambda_v^\alpha / v^\alpha$ and $\mu_{fp} = \mu_f + \sum_{\alpha \in \mathcal{A}} \mu_p^\alpha v^\alpha$.

If the speed hits a speed limit for one of the parameter configurations, $\tilde{\alpha}$, then $v^{\tilde{\alpha}} = v_{\max}$ and $\mu_v^{\tilde{\alpha}} > 0$. Holding this speed limit for the parameter configuration $\tilde{\alpha}$ will be possible under a gradients resistance w if the traction and brake forces verify $u_f - u_b = r^{\tilde{\alpha}}(v_{\max}) + w$ while being within their maximum bounds. The analysis of the possible driving regimes in a minimum time robust-optimal control policy will continue for unconstrained speeds, so $\mu_v^\alpha = 0 \ \forall \alpha \in \mathcal{A}$.

Again, the possible driving regimes in a minimum time robust-optimal control policy are determined by the value of λ along the train journey.

- If $\lambda > 0$, from the Stationary Conditions follows that $\mu_{fp} - \mu_{f_0} > 0$ and $\mu_{b_0} - \mu_b > 0$. All these Lagrange multipliers are nonnegative, so these inequalities can hold if and only if $\mu_{fp} > 0$, $\mu_{b_0} > 0$ and $\mu_{f_0} = \mu_b = 0$. The CSC in Eqs. (8.26), (8.29) and (8.30) constrain the values of the control policy. In this case, $u_f = \min\{\underline{u}_{f,\max}, p_{\max}/v^{\alpha_1}, \dots, p_{\max}/v^{\alpha_q}\}$ and $u_b = 0$, which corresponds to a **maximum traction** regime at the maximum traction rate feasible for all parameter configurations.
- If $\lambda < 0$, the Stationary Conditions show that $\mu_{f_0} - \mu_{fp} > 0$ and $\mu_b - \mu_{b_0} > 0$. The nonnegativity of the Lagrange multipliers forces $\mu_{f_0} > 0$, $\mu_b > 0$ and $\mu_{fp} = \mu_{b_0} = 0$. From the CSC in Eqs. (8.27) and (8.28) we observe that $u_f = 0$ and $u_b = \underline{u}_{b,\max}$, which corresponds to a **maximum brake** regime at the maximum feasible rate among all parameter configurations.

The minimum time robust-optimal control policy is a concatenation of maximum traction and brake regimes at the most conservative maximum traction and brake rates among all the considered parameter configurations. Still, a singular control regime is possible for $\lambda = 0$.

- If $\lambda = 0$, from the Stationary Conditions, $\mu_{fp} - \mu_{f_0} = 0$ and $\mu_b - \mu_{b_0} = 0$. These equalities hold if and only if all Lagrange multipliers are zero due to Eqs. (8.26)-(8.30), so $0 < u_f < \min\{\underline{u}_{f,\max}, p_{\max}/v^{\alpha_1}, \dots, p_{\max}/v^{\alpha_q}\}$ and $0 < u_b < \underline{u}_{b,\max}$. If $\lambda = 0$, over a nontrivial interval of the track, it is constant in that interval. Its rate of change in Eq. (8.47) is zero inside the interval.

Since $\mathcal{A}$ is a finite set, the set of values $t^\alpha(s_e)$, $\alpha \in \mathcal{A}$, is a finite subset of $\mathbb{R}$ and therefore is compact and its maximum value τ is achieved by one of the elements

of the subset. This means that for a minimum time robust-optimal control policy, at least one of the parameter configurations leads to the maximum running time among the running times of all the configurations. Therefore, an α exists for which the value of $\nu_t(\alpha)$ is strictly positive in virtue of Eq. (8.56), fulfilling the Nontriviality Condition. From Eq. (8.58), a parameter configuration exists so its corresponding λ_t^α is strictly negative, while for all the other parameter configurations it is zero. Without loss of generality, we can assume that $\lambda_t^\alpha = -1$, in line with the nominal case, and renormalize the λ_v^α 's accordingly without changing the notation.

We assume now that the minimum time robust-optimal control policy does not saturate the arrival speed constraint for any α . This is not possible, because transforming the robust-optimal control policy by turning braking into traction in a small interval increases the arrival speed for all parameter configurations and the running time is decreased. Therefore, a minimum time robust-optimal control policy saturates also the arrival speed constraint for at least one $\alpha \in \mathcal{A}$. From Eqs. (8.55) and (8.57), an α exists for which $\lambda_v^\alpha(s_e)$ is strictly negative, and for all the other parameter configurations it is zero.

Therefore, like in the energy optimization case, up to two parameter configurations α_1 and $\alpha_2 \in \mathcal{A}$ exist, for which $\lambda_t^{\alpha_1} \neq 0$, $\lambda_t^\alpha = 0 \forall \alpha \neq \alpha_1$ and $\lambda_v^{\alpha_2}(s_e) \neq 0$, $\lambda_v^\alpha(s_e) = 0 \forall \alpha \neq \alpha_2$. In fact, $\lambda_v^\alpha \neq 0 \forall s \leq s_e$ only for α_1 and α_2 because from Eq. (8.33), $\frac{d\lambda_v^\alpha}{ds} = 0 \forall s \leq s_e$, $\forall \alpha \neq \{\alpha_1, \alpha_2\}$ and therefore $\lambda_v^\alpha = 0, \forall s \in [s_o, s_e] \forall \alpha \neq \{\alpha_1, \alpha_2\}$. α_1 and α_2 may or not refer to the same parameter configuration and we will study each case separately.

If the maximum arrival speed and time constraints are saturated by the same parameter configuration, that is, if $\alpha_1 = \alpha_2$, the analysis is similar to nominal minimum time TTO. In this case $\lambda = \lambda_v^{\alpha_1}/v^{\alpha_1} = 0$. If $\lambda = 0$ holds over a nontrivial track interval, it leads to $\frac{d\lambda}{ds} = 0$ inside the interval. Then, from Eq. (8.47), we observe

$$0 = \frac{d\lambda}{ds} = \frac{\lambda_t^{\alpha_1}}{(v^{\alpha_1})^3} + \frac{\lambda_v^{\alpha_1}}{v^{\alpha_1}} \frac{(r^{\alpha_1})'(v^{\alpha_1})}{v^{\alpha_1}} = \frac{\lambda_t^{\alpha_1}}{(v^{\alpha_1})^3}, \quad (8.62)$$

contradicting that $\lambda_t^{\alpha_1}$ is nonzero. Therefore, λ cannot be zero in a nontrivial interval of the track and this value does not represent any meaningful driving regime for unconstrained speeds. If the speed limit is hit for one of the parameter configurations, $\lambda = 0$ would be possible as an overspeed prevention driving regime.

If the maximum arrival speed and time constraints are saturated by different parameter configurations, that is, if $\alpha_1 \neq \alpha_2$, then $\lambda = \lambda_v^{\alpha_1}/v^{\alpha_1} + \lambda_v^{\alpha_2}/v^{\alpha_2} = 0$, with $\lambda_v^{\alpha_1}(s_e) = 0$ and $\lambda_v^{\alpha_2}(s_e) < 0$. In virtue of Eq. (8.33), $\lambda_v^{\alpha_2}$ cannot be zero at any location, so $\lambda_v^{\alpha_1}$ will also be nonzero when $\lambda = 0$. Then,

$$0 = \frac{d\lambda}{ds} = \frac{\lambda_t^{\alpha_1}}{(v^{\alpha_1})^3} + \frac{\lambda_v^{\alpha_1}}{v^{\alpha_1}} \frac{(r^{\alpha_1})'(v^{\alpha_1})}{v^{\alpha_1}} + \frac{\lambda_v^{\alpha_2}}{v^{\alpha_2}} \frac{(r^{\alpha_2})'(v^{\alpha_2})}{v^{\alpha_2}}. \quad (8.63)$$

This scenario can only hold for minimum time robust-optimal trajectories in which the parameter configuration with maximum arrival time is not the one arriving with the highest speed possible according to the arrival speed boundary condition. It can

only happen at specific speeds v^{α_1} and v^{α_2} , and values of the costate variable $\lambda_v^{\alpha_1}$ determined by Eq. (8.63). In practice, in chapter 8.7 we will show that partial traction is observed in minimum time robust-optimal trajectories when driving at moderate speeds favours arriving at destination within the arrival speed window.

8.5 Robust Train Path Envelopes

We proceed with modelling Robust Train Path Envelopes (RTPE). These are timing points, and time and speed windows allocated at strategic locations of the track that provide a real-time traffic plan aiming to guarantee the drivability of train trajectories under train motion model parametric uncertainty without delays with respect to the scheduled passing times or affecting adjacent trains. Let T_j represent train j , $j \in \mathcal{J}$, with $\mathcal{J}$ the set of trains running in the network. Its Train Path Envelope (TPE) is a collection of W_j timing points and windows,

$$\{(s_w^j, [t_{\min,w}^j, t_{\max,w}^j], [v_{\min,w}^j, v_{\max,w}^j])\}_{w=1}^{W_j}, \quad (8.64)$$

where the timing points are ordered locations along the route, $s_w^j \leq s_{w+1}^j$, $t_{\min,w}^j \leq t_{\max,w}^j$ and $v_{\min,w}^j \leq v_{\max,w}^j$, $\forall w = 1, \dots, W_j$, $\forall j \in \mathcal{J}$. TPEs allow for energy-efficient driving by allocating the buffer times between successive trains to avoid conflicting train paths. They are typically based on the expected earliest and latest passing times and speeds at specific critical locations of the track (Albrecht et al., 2013; Wang & Goverde, 2016). For convenience, when a time window reduces to a time target, that is, where $t_{\min,w}^j = t_{\max,w}^j = t_w^j$, we just provide the time target t_w^j , and similar for the speed windows that reduce to a target speed.

In RTTO, passing times and speeds are concepts that are not unequivocally defined after the departure of the train, as we mentioned in chapter 8.3. Instead, we can calculate from the robust-optimal trajectory bundle as many passing times and speeds as the number of parameter configurations introduced in the RTTO problem. The definition of a TPE has to be adapted accordingly. Broadly speaking, TPEs define timing points with associated time windows by allocating time margins from the passing times of a train and its adjacent trains at strategic locations. In turn, RTPEs define time windows by allocating time margins from the maximum and minimum passing times of the trajectory bundle of all trains at such strategic locations. RTPE's definition adds robustness against train motion model parametric uncertainty.

In this chapter we do not aim to provide an extensive definition of all the possible scenarios where RTPEs can be allocated, but rather provide general guidelines on how to define them based on the robust-optimal trajectory bundle's passing times and speeds and to highlight the main differences between TPE and RTPE. Figures 8.1-8.4 gather time-distance graphs showing examples of timing points and windows based on our proposed definition of RTPE. For the sake of simplicity and to make it easier for the reader to compare RTPE and nominal TPE, we provide the same typical examples at stations and junctions as Wang & Goverde (2016): stopping at stations (Fig. 8.1), passing through stations (Fig. 8.2), overtaking another train (Fig. 8.3) and passing through junctions (Fig. 8.4).

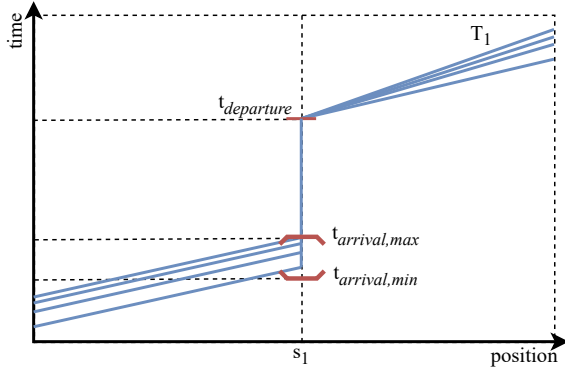


Figure 8.1: RTPE for stops.

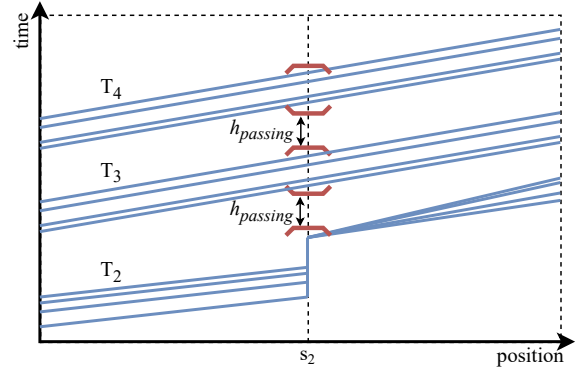


Figure 8.2: RTPE for passing through stations.

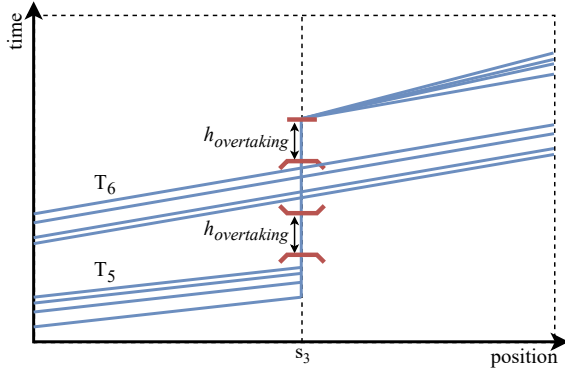


Figure 8.3: RTPE for overtaking.

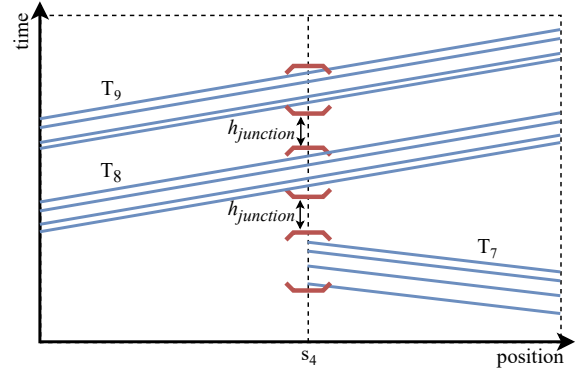


Figure 8.4: RTPE for passing through junctions.

Figure 8.1 shows an RTPE timing point describing the stop at a station. When departing from a stop, a timing point $(s_1, t_{\text{departure}}, 0)$ can be set at the stop location s_1 at the scheduled departure time $t_{\text{departure}}$. While early departures are not allowed, RTPEs can account for late departures by turning the timing point into a window of width equal to the late departure margin δ , $(s_1, [t_{\text{departure}}, t_{\text{departure}} + \delta], 0)$. When arriving at a stopping location, a timing point with a fixed arrival time no longer guarantees the feasibility of the RTTO problem since each of the trajectories of the bundle arrives at the stopping location at a different time. Therefore, an arrival time window has to be set at the stopping location $(s_1, [t_{\text{arrival,min}}, t_{\text{arrival,max}}], 0)$, with $t_{\text{arrival,min}} < t_{\text{arrival,max}}$. The time window defined by $t_{\text{arrival,min}}$ and $t_{\text{arrival,max}}$ shall be sufficiently wide to accommodate all the arrivals of the robust-optimal trajectory bundle. Its width may depend on the parameter configuration set $\mathcal{A}$. An acceptable late arrival threshold can be defined around the scheduled arrival time to calculate the upper bound of the arrival time window $t_{\text{arrival,max}}$ and it shall be equal to the latest arrival time introduced in the RTTO problem. If desired, a late departure buffer could be allocated as well, turning the departure timing point into a time window.

Figures 8.2-8.4 exemplify RTPE timing windows at intermediate passing locations. In these scenarios, supplements in the running times are distributed among consecutive trains based on the earliest and latest passing times of the robust-optimal trajectory bundle for each train, taking into account headways, priorities and other operational criteria. In this chapter we assume for simplicity that all trains have the same priority. We also assume

that all operational constraints on the maximum and minimum speed are included in the constraints on the speeds in Eq. (8.18) in the RTTO problem, so speed windows are defined directly from the robust-optimal trajectory bundle. Speed windows in the RTPE may be optional when the train is not supposed to stop. In case allocating speed windows is not desired, a wide window can be set from zero speed to the maximum speed at that location of the track. Temporary speed limits or other operational bounds on the speed can be added by lowering the maximum speed of the window. Lower bounds on the speed window can be set at places with operational constraints at low speeds. Setting a minimum speed may be desired in some scenarios, like to reduce capacity consumption at bottlenecks like junctions, to avoid excessive mechanical wear when running at low speeds in curves, and to prevent low adhesion issues at low speeds and at standstill on steep gradients and tunnels. As a general rule, in each timing point we consider the maximum and minimum passing times and speeds of the robust-optimal trajectory bundle. Buffer times are distributed among consecutive trains according to their respective priority.

Figure 8.2 shows RTPE timing points and windows of 3 trains: T_2 , which stops and departs at that station, and T_3 and T_4 , which pass through the station without stopping. The passing windows of trains T_3 and T_4 are depicted in red. For example, the robust passing time window of train T_3 is defined from the scheduled departure time of train T_2 , D_{T_2} , the earliest and latest passing times of the robust-optimal trajectory bundle of T_3 , $P_{T_3}^-$ and $P_{T_3}^+$, and the earliest passing time of the trajectory bundle of its following train T_4 , $P_{T_4}^-$. The minimum headway between consecutive trains at the station located at s_2 is $h_{passing}$. For simplicity, we use the same notation for the minimum headway between a train departing before a train passes through the station, and for the minimum headway between consecutive trains passing through the station. These minimum headways are usually different and may potentially include for example a margin to account for late departures in the departure-passing scenario. The passing time window of train T_3 is calculated as $[(D_{T_2} + P_{T_3}^- + h_{passing})/2, (P_{T_3}^+ + P_{T_4}^- - h_{passing})/2]$. In this case, the buffer between each train is distributed evenly among them. The departure time window of T_2 accounting for potential late departures would be $[D_{T_2}, (D_{T_2} + P_{T_3}^- - h_{passing})/2]$ in this case.

Figure 8.3 shows timing points and windows of trains T_5 and T_6 . The latter overtakes the earlier, which stops at location s_3 to be overtaken. An overtaking RTPE time window is allocated at s_3 for train T_6 , aiming for improved capacity. Passing too early through that location is not desired because T_5 has to stop there and a rail switch has to move before the T_6 can pass through that location. In turn, a late passing would delay the departure of T_5 . Denoting $h_{overtaking}$ the minimum headway time between the stop of T_5 and the earliest possible passing time of T_6 . To keep the notation simple, we assume that the minimum headway between consecutive passing and departures is also $h_{overtaking}$. This minimum headway time may allocate some buffer for late arrival of T_5 . However, early departure is not allowed for T_5 . If D_{T_5} is the scheduled departure time of T_5 , $A_{T_5}^+$ its latest arrival time according to its robust-optimal trajectory bundle and $P_{T_6}^-$ the earliest passing time among all the trajectories in the bundle of T_6 , the overtaking time window of T_6 is $[(A_{T_5}^+ + P_{T_6}^- + h_{overtaking})/2, D_{T_5} - h_{overtaking}]$.

Figure 8.4 shows time windows of trains T_7 , T_8 and T_9 , the earlier of them deviating to another track at the junction s_4 after coming from the opposite direction of the other two

trains. We denote by $h_{junction}$ the minimum headway time between consecutive trains at that junction, neglecting likely differences in this headway depending on the direction of the junctions and use the same notation as in previous examples for the earliest and latest passing times of the robust-optimal trajectory bundles of trains T_7 , T_8 and T_9 . The RTPE time window of T_8 at the junction located at s_4 is $[(P_{T_7}^+ + P_{T_8}^- + h_{junction})/2, (P_{T_8}^+ + P_{T_9}^- - h_{junction})/2]$.

RTPEs are by definition more robust and accurate than TPEs based on nominal trajectories as defined in Wang & Goverde (2016). They increase the amount of trajectories that are drivable within the target points and windows, as well as the robustness of railway operation. If the realized operating condition is included into the set of observed ones, the RTPE will guarantee by definition the drivability of the robust-optimal control policy, that is to say, it will lead to a feasible realized trajectory. This realized trajectory will correspond to one of the trajectories in the robust-optimal bundle, so the passing times and speeds will always be feasible. However, nominal TPEs do not offer this robustness against parametric uncertainty in the train motion model. TPE arrival timing points are an archetypal scenario: if the running resistance is underestimated when creating the TPE and the running time to the destination is tight with respect to the minimum running time, arriving on time may not be possible, violating the arrival time window at the timing point. In case the operating conditions vary from the ones considered, RTPE windows still add some robustness to the operation. Robust windows will guarantee the drivability of energy-efficient trajectories unless the operating conditions differ significantly from the ones considered in the RTTO problem. For example, a train can depart on time or within its departure window, but driving under very adverse environmental conditions may slow the train to the extent that eventually it is no longer possible to drive within the allocated passing times at an RTPE timing point, or to arrive within the RTPE arrival window. The robust-optimal trajectory bundle can accommodate some deviations from the operating conditions considered. Moreover, the extra buffer time in each window helps mitigating the impact of larger deviations on the drivability of the RTPEs.

Capacity-wise, RTPEs utilize the same minimum headways as their nominal counterparts. Nevertheless, in very dense networks, significant deviations from the nominal operating condition considered may compromise the drivability of nominal TPEs leading to knock-on delays and train path conflicts, while RTPEs may accommodate such deviations in some cases. Including extreme scenarios in the RTTO and RTPE computation could also be possible to add even more robustness. However, that would not be a good practice in most cases due to the trade-off between robustness and operational efficiency. In the event of such extreme scenarios, train dispatching should intervene instead to mitigate any impact in an updated timetable. Then, nominal or RTTO could support train dispatching, but they are not tools that can substitute the latter.

8.6 Solving the Robust Train Trajectory Optimization problem

8.6.1 Energy-efficient Robust Train Trajectory Optimization

In this chapter we describe an algorithm based on convex optimization for solving the RTTO problem. Several of the existing algorithms for solving the nominal TTO problem (Scheepmaker et al., 2017) could be adapted for RTTO. Indirect optimization methods turn the TTO problem into a boundary value problem that is solved instead of the original one. Direct optimization methods discretize the continuous variables of the TTO problem, rewrite algebraically the relations between the discretized variables and solve the resulting nonlinear programming problem. Genetic algorithms and other metaheuristics are also frequently used to solve the mentioned nominal problem. When compared to the nominal TTO problem, RTTO is more complex due to its higher dimensionality. Boundary value problems become substantially more difficult to solve when their dimensionality increases. In turn, higher dimensionality or complexity does not usually hinder metaheuristics and direct optimization methods, so both methods are candidates for solving the RTTO problem. Since metaheuristics are considerably slower, in this chapter we opt for an RTTO implementation by means of direct optimization.

We transform this problem into a convex quadratically constrained quadratic program with linear objective function. The location variable s_k , $k = 0, \dots, N$, is discretized in N intervals. Particularly, we set a discretization point in those locations where either the track gradient or the speed limit change, so that the track characteristics are constant in each interval. We assume that N is much larger than the number of points where the track gradients or the speed limits change. The remaining discretization points are selected as Chebyshev nodes (Stewart, 1996). We observed that using a uniform interval discretization leads to results that are sensitive to the number of intervals N . However, Chebyshev-based discretization eliminates the observed sensitivity with respect to the number of discretization points. We denote by Δs_k the length of the k -th interval, and let $s_0 = s_o$, $s_N = s_f$, $s_{k+1} = s_k + \Delta s_k$, for $k = 0, \dots, N-1$. The speed corresponding to the parameter configuration α at each discretized location is $v_k^\alpha = v^\alpha(s_k)$, for $k = 0, \dots, N$, $\alpha \in \mathcal{A}$. Regarding the tractive and brake effort, we use the variable $u_k = u_f(s_k) - u_b(s_k)$, for $k = 0, \dots, N-1$. We assume that u_k is constant along the whole interval $[s_k, s_{k+1})$. The energy consumption at each interval is denoted by E_k , $k = 0, \dots, N-1$.

The cost function in Eq. (8.11) can be rewritten as

$$\min \sum_{k=0}^{N-1} E_k. \quad (8.65)$$

Eq. (8.12) can be integrated and rewritten as the work-energy balance in each interval.

$$\frac{1}{2}(v_{k+1}^\alpha)^2 + \left(\Delta s_k r_2^\alpha - \frac{1}{2}\right)(v_k^\alpha)^2 + \Delta s_k r_1^\alpha v_k^\alpha + \Delta s_k r_0^\alpha - \Delta s_k u_k + \frac{g}{\rho}(h_{k+1} - h_k) = 0, \quad (8.66)$$

$$k = 0, \dots, N-1, \quad \forall \alpha \in \mathcal{A},$$

with h_k the height of the track at s_k , $k = 0, \dots, N$. In Eq. (8.66), some terms are integrated exactly and remaining ones are integrated by means of Euler's forward method.

Eq. (8.13) can be integrated as a Riemannian sum and it can be transformed into an inequality constraint by adding the boundary condition on the arrival time window:

$$t^l \leq \sum_{k=0}^{N-1} \frac{\Delta s_k}{v_k^\alpha} \leq t^h, \quad \forall \alpha \in \mathcal{A}, \quad (8.67)$$

where t^l and t^h are lower and upper bounds to the running time of the train, that stand respectively for the lowest acceptable early arrival time, and for the largest acceptable delay. These bounds constitute a typical arrival window of a RTPE. The accuracy of the numerical integration shown in Eq. (8.67) requires a fine discretization mesh at those locations where the speed is expected to be very slow. Chebyshev nodes fulfill this requirement by allocating more discretization nodes next to stopping locations.

Eq. (8.14) can be conveniently transformed by introducing as decision variables for the energy consumption at each interval E_k , $k = 0, \dots, N-1$, with $E_0 = 0$. These energy variables will be minimized in virtue of Eq. (8.65) and two sets of inequalities in Eqs. (8.68) and (8.69). The inequalities guarantee that the energy consumption will fulfill Eq. (8.11) in an optimal solution of the problem. The energy regenerated while braking could be easily introduced in Eq. (8.69), although we do not consider regenerative braking in this chapter.

$$E_k \geq \Delta s_k u_k, \quad k = 0, \dots, N-1, \quad (8.68)$$

$$E_k \geq 0, \quad k = 0, \dots, N-1. \quad (8.69)$$

The bound on the tractive power in Eq. (8.15) is speed-dependent,

$$u_k \leq p_{\max}/v_k^\alpha, \quad k = 0, \dots, N-1, \quad \forall \alpha \in \mathcal{A}, \quad (8.70)$$

Moreover, Eqs. (8.20) and (8.21) also bound the control variable,

$$-\underline{u}_{b,\max} \leq u_k \leq \underline{u}_{f,\max}, \quad k = 0, \dots, N-1. \quad (8.71)$$

The speed limit $v_{\max}(s)$ can also be discretized at each point of the track, $v_{\max,k} = v_{\max}(s_k)$, $k = 0, \dots, N$. Then, Eq. (8.18) can be rewritten as

$$\epsilon \leq v_k^\alpha \leq v_{\max,k}, \quad k = 0, \dots, N, \quad \forall \alpha \in \mathcal{A}. \quad (8.72)$$

The boundary conditions on the arrival speed window in Eq. (8.19) can be written as a function of the speed at s_0 and s_N , respectively,

$$v_0^\alpha = v_o, \quad v_e^l \leq v_N^\alpha \leq v_e^h, \quad \forall \alpha \in \mathcal{A}, \quad (8.73)$$

where $v_e^l < v_e^h$ are lower and upper bounds to the final speed of the train. This speed window shall be large enough to ensure the existence of a robust-optimal control policy that solves the RTTO problem.

Solving the nonlinear programming problem Eq. (8.65) subject to Eqs. (8.66)-(8.73) is equivalent to solving the RTTO problem introduced in chapter 8.3. Still, the square of the speed in the equality constraint in Eq. (8.66) and the inverse of the speed in the inequalities in Eqs. (8.67) and (8.70) prevent the problem from being convex. In optimization problems convexity is a desirable property since any local optimal solution

is guaranteed to be the global optimum, so efficient local optimization methods can be used to solve convex problems. If the problem is not convex, global optimality is not guaranteed for a local solution. Either a suboptimal solution may be found or longer computation times are required for exploring more broadly the solution space. In a convex optimization problem the inequalities must be linear or quadratic, and, in the latter case, any bilinear form should be written as positive semidefinite on the lower side of the inequality, and any equality constraint should be affine in the decision variables. Two sets of variables can be defined to make the problem convex, a_k^α , $k = 0, \dots, N-1$, and b_k^α , $k = 0, \dots, N$, $\alpha \in \mathcal{A}$. The first ones relate to the inverse of the speed and the second ones to the speed squared. Constraints shown in Eqs. (8.66), (8.67) and (8.70) are reformulated as linear or affine in the decision variables by means of a_k^α and b_k^α as follows:

$$\frac{1}{2}b_{k+1}^\alpha + \left(\Delta s_k r_2^\alpha - \frac{1}{2}\right)b_k^\alpha + \Delta s_k r_1^\alpha v_k^\alpha + \Delta s_k r_0^\alpha - \Delta s_k u_k + \frac{g}{\rho}(h_{k+1} - h_k) = 0, \quad (8.74)$$

$$k = 0, \dots, N-1, \forall \alpha \in \mathcal{A},$$

$$t^l \leq \sum_{k=0}^{N-1} \Delta s_k a_k^\alpha \leq t^h, \quad \forall \alpha \in \mathcal{A}, \quad (8.75)$$

$$u_k \leq p_{\max} a_k^\alpha, \quad k = 0, \dots, N-1, \forall \alpha \in \mathcal{A}, \quad (8.76)$$

Last, we have to relate a_k^α and b_k^α to the speed adequately, keeping the convexity of the problem. This can be done easily by means of the quadratic constraints shown in Eqs. (8.77)-(8.79):

$$a_k^\alpha v_k^\alpha \geq 1, \quad k = 0, \dots, N-1, \forall \alpha \in \mathcal{A}, \quad (8.77)$$

$$b_k^\alpha \geq (v_k^\alpha)^2, \quad k = 0, \dots, N, \forall \alpha \in \mathcal{A}, \quad (8.78)$$

$$b_k^\alpha \leq v_{\max,k}^2, \quad k = 0, \dots, N, \forall \alpha \in \mathcal{A}, \quad (8.79)$$

The decision variables of the resulting convex problem are v_k^α , b_k^α , $k = 0, \dots, N$, $\alpha \in \mathcal{A}$, a_k^α , for $k = 0, \dots, N-1$, $\alpha \in \mathcal{A}$, and u_k and E_k for $k = 0, \dots, N-1$. If the solution of the problem is optimal, the nonlinear constraints in Eqs. (8.77) and (8.78) will be saturated.

The energy-efficient RTTO problem reformulated as a direct convex optimization method is therefore Eq. (8.65) subject to Eqs. (8.68), (8.69) and (8.71)-(8.79). In chapter 8.7 we will show the performance of the proposed convex program for solving the RTTO problem.

8.6.2 Adding intermediate timing points and windows as constraints

Intermediate target points and windows can be included in the quadratically constrained quadratic program as linear inequality constraints. For each target point or window located at $s = s_w$ we place a collocation point at that location. Let $s_{k_w} = s_w$ be the mentioned collocation point. Denoting by t_w^l and t_w^h the lower and upper bounds on the time window at $s = s_w$ and by v_w^l and v_w^h the corresponding bounds on the train speed, time and speed windows are described as inequality constraints

$$t_w^l \leq \sum_{k=0}^{k_w-1} \Delta s_k a_k^\alpha \leq t_w^h, \quad \forall \alpha \in \mathcal{A}, \quad (8.80)$$

$$v_w^l \leq v_w^\alpha \leq v_{k_w}^h, \forall \alpha \in \mathcal{A}. \quad (8.81)$$

Notice that if target points are considered, for instance, when setting the departure time at an intermediate station, the lower and upper time bounds will be equal.

8.6.3 Minimum time Robust Train Trajectory Optimization

The minimum time RTTO problem can also be reformulated as a convex optimization problem. Since the energy consumption is not considered in the minimum time problem, the energy variables E_k and its related constraints in Eqs. (8.68) and (8.69) are removed. The arrival window in Eq. (8.75) is removed as well. The cost function in Eq. (8.65) is replaced by the minimization of a new decision variable, τ , representing the latest arrival time at s_e among all the considered parameter configurations,

$$\min \tau. \quad (8.82)$$

In virtue of Eq. (8.24), τ is the maximum over the arrival times of all parameter configurations, $\tau = \max_{\alpha \in \mathcal{A}} t^\alpha(s_e)$, and it can be modelled in a convex manner as follows:

$$\sum_{k=0}^{N-1} \Delta s_k a_k^\alpha \leq \tau, \forall \alpha \in \mathcal{A}. \quad (8.83)$$

An equality will hold in Eq. (8.83) for the parameter configuration with the latest arrival time. A similar trick can be used for solving the energy-efficient RTTO problem in case there are also parameter variations in the traction and regenerative brake efficiencies.

Therefore, the convex optimization formulation of the minimum time RTTO problem is Eq. (8.82) subject to Eqs. (8.71)-(8.74), (8.76)-(8.79), (8.83) and optionally also intermediate time and speed windows in Eqs. (8.80) and (8.81). The decision variables of this optimization problem are τ , v_k^α , b_k^α , $k = 0, \dots, N$, $\alpha \in \mathcal{A}$, a_k^α , for $k = 0, \dots, N-1$, $\alpha \in \mathcal{A}$, and u_k for $k = 0, \dots, N-1$. Again, the nonlinear constraints in Eqs. (8.77) and (8.78) shall be saturated in an optimal solution of the minimum time problem.

8.7 Results and discussion

In this chapter we analyze the performance of the proposed Robust Train Trajectory Optimization (RTTO). First, we motivate the usage of RTTO over nominal TTO in chapter 8.7.1. The robust time- and energy-optimal trajectories calculated are compared and discussed with respect to nominal-optimal trajectories. Both robust and nominal-optimal trajectories are calculated by means of the RTTO algorithm presented in chapter 8.6. When computing nominal trajectories, only 1 parameter configuration is introduced in the RTTO problem. An example showcasing a new singular control regime that does not appear in nominal Train Trajectory Optimization (TTO) is presented in chapter 8.7.2. Gradients and speed limits are considered in two real case studies in chapter 8.7.3, where we showcase potential practical applications of the proposed method under different sources of parametric uncertainty. We test RTTO on simulated flat track scenarios without speed limits, exploring its performance and limitations in chapter 8.7.4. Last, Robust Train Path Envelopes (RTPEs) are computed from the calculated robust-optimal

trajectories in a cyclic timetable and are compared with the nominal TPEs in chapter 8.7.5.

When the robust-optimal control policy is applied to the train motion model for each parameter configuration introduced in the RTTO problem, a different trajectory is described in the time-speed-distance space. These trajectories share a common initial speed, but differences in the train motion model parameters create deviations in their speeds along the train journey. Therefore, in the context of RTTO, the robust-optimal control policy leads to as many possible trajectories as parameter configurations used in the RTTO problem. The collection of the trajectories that arise from each parameter configuration when tracking a robust-optimal control policy constitutes a robust-optimal trajectory bundle. The time and speed deviations along the train's journey in the trajectory bundle require defining a sufficiently large arrival time and speed window in Eq. (8.19) in order to guarantee the existence of feasible robust-optimal trajectories. The terminal set, that is, the arrival time and speed windows introduced in the RTTO problem, has to be large enough to allow each individual speed profile in the trajectory bundle to end within these windows at the arrival location. For example, an acceptable or configurable delay margin can be added to the scheduled arrival time to define the upper bound of the arrival time window. The arrival speed window is more complex since it shall be sufficiently large to accommodate possible speed differences among trajectories in the bundle, allowing to end a trajectory with nonzero speed even at stopping locations. The longer the distance between the initial and final location in the RTTO problem is, the larger the arrival speed window shall be to accommodate the speed deviations arising from model parameter differences. This implies that the existence of a robust-optimal trajectory might not be guaranteed for RTTO problems with very long distances to be covered and with large parameter differences, both from a theoretical and practical point of view. Considering that stopping locations take place at platforms that usually have reduced maximum speed limits, the endpoint of a robust-optimal trajectory can be placed at the beginning of the mentioned platform's reduced speed limit, and assume that the train will automatically reaccelerate up to the speed limit or the brake curve before decelerating to stop at the required spot in the platform. Both the reacceleration and the deceleration to stop are expected to take place at the most conservative acceleration and deceleration rates among the parameter configurations considered, in line with the possible driving regimes in a robust-optimal control policy. The running time spent in this stopping procedure can be calculated from the lowest bound of the arrival speed window, and the upper bound of the arrival time window must be adjusted according to this correction. For simplicity, this correction is not shown graphically in chapters 8.7.1 to 8.7.4, but it is included in all calculations.

8.7.1 Motivating the usage of RTTO over nominal TTO

Robust-optimal trajectories are more robust against parametric uncertainty than nominal-optimal trajectories, assuming that the parametric uncertainty is accurately estimated and that the realized parameter configuration has been covered in the RTTO problem. Under parametric model uncertainty, nominal-optimal trajectories may not be drivable in the sense that tracking the speed of the optimal trajectory accurately may not be possible. For example, strong tailwinds decrease the running resistance. If the nominal-

optimal trajectory is calculated for a parameter configuration corresponding to zero wind speed operating conditions, a train tracking this trajectory under strong tailwinds will reach high speeds earlier than expected and will decelerate less when coasting. This translates into deviations in the expected speed and passing times, and into meeting speed limits and brake curves earlier than expected, requiring early driver or ATO brake intervention. Similarly, differences in the applicable traction and brake rates may also jeopardize the tracking of nominal-optimal trajectories. While robust-optimal trajectories utilize conservative traction and brake rates that are achievable for all the parameter configurations considered, nominal-optimal trajectories use the maximum traction and brake rates allowed by the parameter configuration used. In the event of low adhesion, reduced engine performance or different rolling stock types used, high traction and brake rates may not be applicable, leading to differences between the expected and the realized passing speeds and times. To sum up, parametric uncertainty may lower the effectiveness of energy-optimal trajectories in terms of energy savings and expected timeliness.

Figures 8.5 and 8.6 exemplify this phenomena in an early planning scenario with uncertainty in the rolling stock to be used in operation. Figure 8.5 shows two nominal-optimal trajectories calculated in a 10 km long flat track with varying speed limits: one represented in blue, corresponding to an Intercity VIRM VI train, and the other in yellow, corresponding to a Sprinter SLT VI train. Both trains are commonly operated by NS in the Netherlands. They are used in long-distance services between major cities, although Sprinter trains stop at more intermediate stations. Occasionally, a Sprinter train performs the service scheduled for an Intercity train and vice versa, if the rolling stock availability requires so. Their parameter configurations are shown in Table 8.1. In this example we ignore the different train lengths. The terminal speed is set to 50 km/h for a comparison with RTTO. An intermediate 60 km/h speed limit is placed between kms 4 and 5, mimicking an intermediate station without scheduled stop. In other parts of the track, the maximum speed is set to 120 km/h. The speed limits are shown in Figure 8.5 as thin black dashed lines. We calculated first the robust-optimal minimum running time in this scenario, which is 462.8 s. The arrival time is set to 10% over this time for both trains when calculating their nominal-optimal trajectories. The nominal-optimal trajectory of each train is tracked by the other train by introducing the optimal control policy in the other train's motion model by means of Eq. (8.74). The trajectory performed by the Intercity train when tracking the Sprinter's optimal trajectory is depicted as a dotted magenta line, and the trajectory performed by the Sprinter train is depicted as a dotted light blue line. In the lower part of Figure 8.5, the nominal-optimal control policies calculated for the VIRM VI Intercity and the SLT VI Sprinter trains are represented as full blue and dotted yellow lines, respectively.

This naive tracking strategy shows that the VIRM VI Intercity train cannot successfully track the SLT VI Sprinter's optimal trajectory, although both nominal-optimal trajectories are very similar in structure and in the location of the switching points between driving regimes. When the Intercity tracks the trajectory calculated for the Sprinter, the intermediate reduced speed limit is violated due to the lower braking capability of the Intercity. The Intercity also arrives at its destination with a speed higher than the specified arrival speed when tracking the other trajectory. In turn, the Sprinter brakes harder than what is expected for the Intercity. This is not a problem in practice, because during

Train	r_0 [N/kg]	r_1 [N/(kg m/s)]	r_2 [N/(kg(m/s) ²)]	$u_{f,\max}$ [N/kg]	$u_{b,\max}$ [N/kg]	$p_{\max}$ [W/kg]
VIRM VI	6.93e-03	1.11e-04	2.00e-05	0.547	0.364	5.52
SLT VI	6.95e-03	1.89e-04	3.41e-05	0.859	0.500	5.52

Table 8.1: Parameter configurations corresponding to the rolling stock characteristics of 2 trains in the fleet of NS: an Intercity VIRM VI (above) and a Sprinter SLT VI (below) (Scheepmaker et al., 2020a).

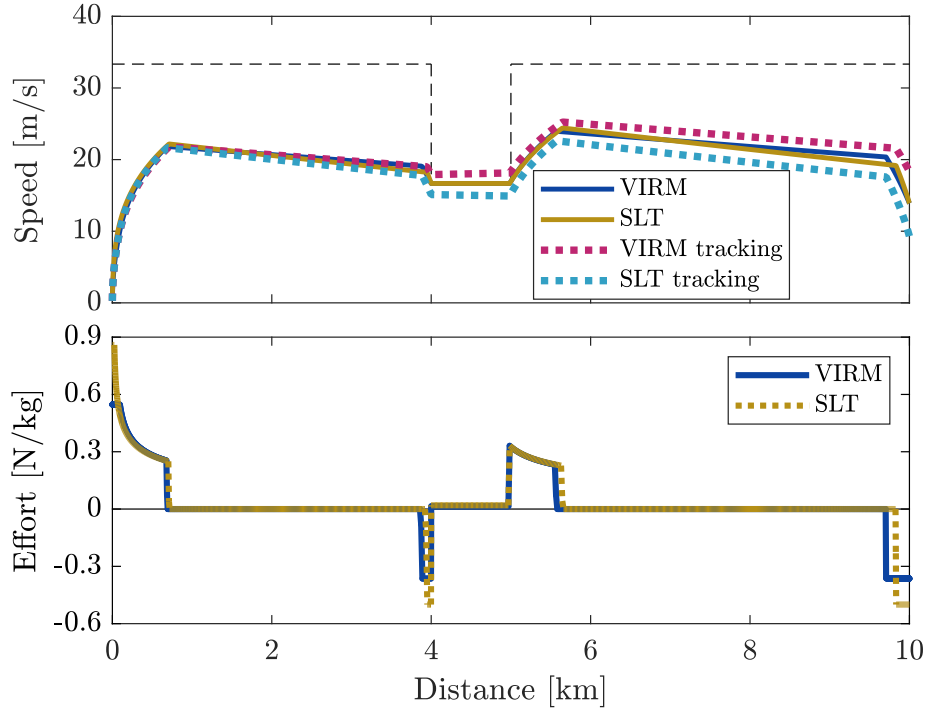


Figure 8.5: Nominal-optimal control policies may not be tracked successfully by a different train.

real operation the train could brake softer by following, for instance, a brake curve to the speed reduction. Small deviations are observed when the Sprinter coasts due to its higher aerodynamic resistance r_2 . The unsuccessful tracking of the Sprinter’s optimal trajectory performed by the Intercity train evidences that parametric model uncertainty in nominal TTO can lower the accuracy and drivability of the calculated optimal trajectories.

RTTO can be used to mitigate the impact of parametric model uncertainty in optimal trajectories. A robust-optimal trajectory is calculated for both parameter configurations under the same conditions used for the nominal-optimal trajectories. In the calculated robust-optimal trajectory bundle shown in Figure 8.6, the trajectories corresponding to the Intercity and the Sprinter trains are represented in blue and yellow, respectively. We track with both trains the robust-optimal control policy shown in blue in the lower part of Figure 8.6. The trajectory described by the Intercity train during this tracking is shown as a dotted magenta line and the one described by the Sprinter train is shown as a dotted light blue line. As can be observed in Figure 8.6, tracking the robust-optimal

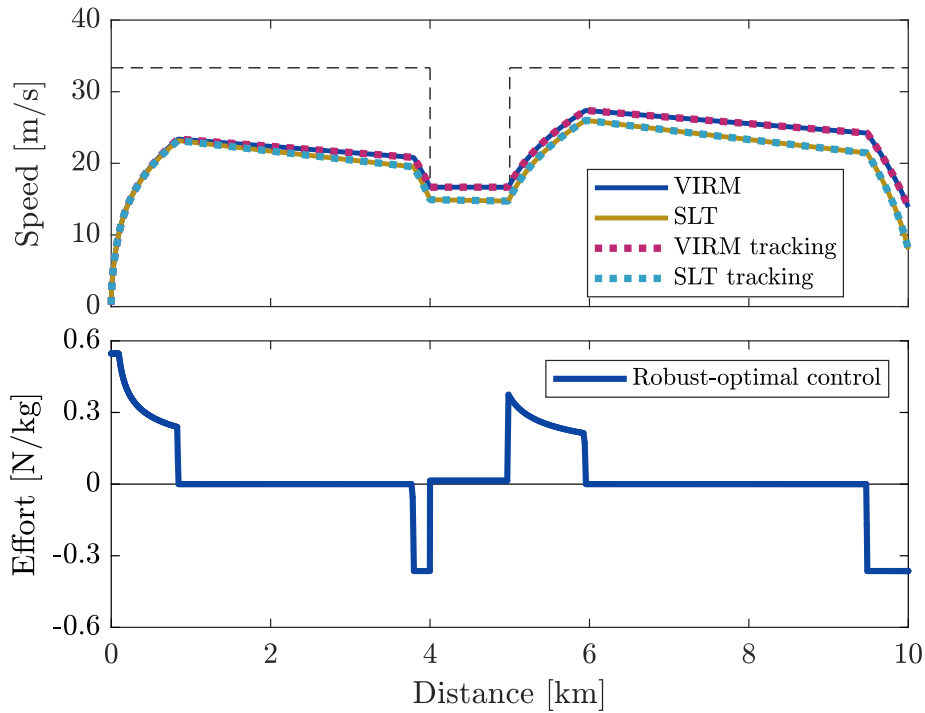


Figure 8.6: Robust-optimal control policies can be tracked by all the trains that were introduced in the RTTO problem.

control policy allows to arrive at destination without significant deviations from each of the trajectories in the robust-optimal bundle, which was not the case when tracking the nominal trajectories. In the end, each trajectory in the robust-optimal bundle is the evaluation of the robust-optimal control policy on the train motion model with each parameter configuration introduced in RTTO. This evaluation is exactly the naive tracking used in these two examples. RTTO guarantees that the trajectories in the robust-optimal bundle will be the result of tracking the robust-optimal control policy. The constraints in the RTTO problem enforce arriving within the arrival time and speed windows, not violating any speed limit or bound on the available traction and brake efforts of each train. Thus, RTTO is able to eliminate the impact of parametric uncertainty observed in the nominal scenario.

This example highlights that the accuracy of the trajectories calculated with nominal TTO using a train motion model that is not adequately calibrated to the rolling stock used or the operating conditions may be low. These deviations from the nominal-optimal trajectories translate into driver and ATO brake interventions, potential delays and energy consumptions over the predicted minima. RTTO can be used to calculate trajectories that are robust against parametric model uncertainty. This is of particular relevance in planning stages, for instance, when the rolling stock to be deployed in a line is still uncertain or when trajectories are calculated under a large operational uncertainty. In real operation, fast nominal TTO could be used iteratively as a Model Predictive Control strategy to adapt on-the-go to deviations of the train performance with respect to the computed trajectories, provided that there is enough traction and brake capability avail-

able to mitigate those deviations. Also, Automatic Train Protection systems supervise speed limits and brake curves violations, eventually being able to trigger the emergency brake if necessary. However, in this chapter we do not consider any live feedback on the train performance while the train is in motion.

8.7.2 Robust-optimal control policies produce new singular driving regimes

In chapter 8.4 we showed that a minimum time robust-optimal trajectory is a concatenation of different driving regimes: maximum traction, maximum brake and a singular driving regime corresponding to reduced traction or brake. In turn, energy-efficient robust-optimal trajectories consist of maximum traction, coasting, maximum brake, partial traction and partial brake regimes. The latter two are also singular driving regimes. As we discussed during the Robust Maximum Principle analysis, the singular driving regimes that may arise in robust-optimal control policies constitute the main difference with respect to the structure of nominal-optimal trajectories. In nominal TTO, the state variables are the speed and the running time. Each state variable has an associated costate variable. The target arrival time and speed constraints are verified by these two state variables. However, in RTTO, each parameter configuration leads to a different pair of speed and running time state variables, and to a pair of costates. We have shown that at most one costate variable of the time is nonzero, and up to two costate variables of the speed are nonzero independently of the number of parameter configurations introduced in the RTTO problem. In physical terms, one of the trajectories in the robust-optimal trajectory bundle determines the robust-optimal minimum running time or, in the case when the energy consumption is minimized, it determines the latest arrival time. In fact, the robust-optimal minimum running time is the latest arrival time among the trajectories in the minimum time robust-optimal trajectory bundle. Then, either the trajectory with latest arrival time or a different trajectory will be arriving at destination with a speed equal to the maximum arrival speed allowed by the boundary conditions of the RTTO problem. If the trajectory defining the latest arrival time is not the one arriving at the maximum arrival speed, very specific values of the speed variables of both trajectories and their associated costates exist, for which partial traction or brake singular driving regimes that do not exist in nominal optimal trajectories may appear.

Figure 8.7 shows one of these novel singular driving regimes in a minimum time robust-optimal control policy. The parameter configurations used are presented in Table 8.1. The 10 km long track is flat. The speed limit is 38.9 m/s (140 km/h) over the whole track except between kms 4 and 5, where the maximum speed is reduced to 16.7 m/s (60 km/h). The speed limits are represented as thin dashed black lines in the figure. The arrival speed window is set from 0.28 to 13.9 m/s (1 to 50 km/h). In the robust-optimal trajectory bundle, the trajectory described by the Intercity train is represented in blue, while the one described by the Sprinter is depicted as a yellow dashed line.

The minimum time robust-optimal control policy is a concatenation of maximum traction, constant partial traction during unconstrained speeds, maximum brake, constant partial traction while the Intercity train's speed is constrained by the speed limit, maximum traction, constant partial traction during unconstrained speeds and maximum

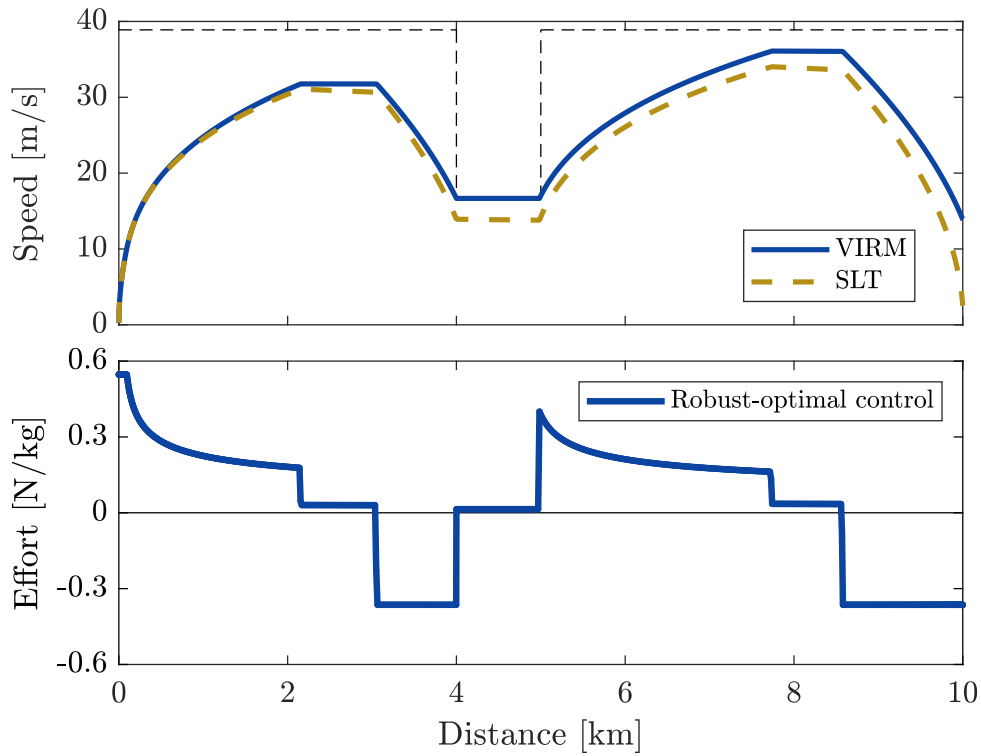


Figure 8.7: Minimum time robust-optimal control policy with a singular driving regime.

brake until arrival. The singular partial traction regimes lead to two different unconstrained constant speeds for the Intercity train, one at each side of the reduced speed limit. In nominal energy-efficient TTO, unconstrained constant speed regimes always take place at a unique cruise speed in each journey without stops (Scheepmaker et al., 2020a). This is different in the robust-optimal trajectory bundle, as we observe that the partial traction regimes at each side of the speed limit take place at different unconstrained speeds. The singular partial traction regimes appear in RTTO because driving at lower speeds reduces the speed dispersion in the trajectory bundle, which favours arriving within the specified speed window for all the trajectories. In fact, we observed that if the maximum arrival speed is reduced to 8.3 m/s (30 km/h), this phenomenon becomes more pronounced. In that scenario, the minimum time robust-optimal control policy leads to a trajectory bundle that never exceeds 16.7 m/s (60 km/h) anywhere in the trajectory. The robust-optimal trajectory bundle consists of maximum traction and brake regimes, linked by partial traction. At the end of the maximum traction regime, approximately around km 0.320, the speed deviation is 0.06 m/s (0.2 km/h). The brake regime starts approximately at km 9.733, where the speed deviation grows up to approximately 2.28 m/s (8.2 km/h). In turn, the speed deviation at the arrival location is equal to 7.87 m/s (28.7 km/h), considering that the minimum arrival speed was 1 m/s (0.28 km/h). To conclude, the structure of the optimal control policies in RTTO and nominal TTO show major differences due to the more strict feasibility requirement in the arrival speeds in RTTO.

8.7.3 Practical applications of Robust Train Trajectory Optimization in real case scenarios

Robust Train Trajectory Optimization under rolling stock uncertainty

Robust-optimal control policies can also be calculated in real scenarios with gradients. In early planning stages, preliminary energy-efficient timetables can be computed from robust-optimal trajectories using as input parameter configurations corresponding to the different candidate rolling stock types that could be potentially deployed in that line. We exemplify now how RTTO can calculate energy-efficient trajectory bundles under rolling stock uncertainty for planning purposes. We consider an 8 km long mainly ascending line with varying gradients and speed limits, and 2 typical rolling stock units used in the Dutch railways: an Intercity VIRM VI and a Sprinter SLT VI train. For simplicity, we assume that all trains have the same length. This assumption could be relieved by modifying the speed limits according to the longest rolling stock unit, so trains can reaccelerate once the rear end of the longest train is already past the speed restriction. The train motion model parameters are shown in Table 8.1 (Scheepmaker et al., 2020a). A reduced 11.1 m/s (40 km/h) speed limit starts at the beginning of a platform at km 7.9 and stays constant until the stopping location. We set the arrival speed window at the beginning of this reduced speed limit, from 0.28 to 11.1 m/s (1 to 40 km/h). The maximum arrival time is 5% over the calculated robust-optimal minimum time, 391.1 s. We observed that the minimum time robust-optimal trajectory bundle stays a few m/s below the highest speed limits to keep its speed dispersion low enough to guarantee the feasibility of the robust-optimal control policy. The energy-efficient robust-optimal trajectory bundle calculated under uncertainty in the rolling stock to be deployed with varying gradients and speed limits is shown in Figure 8.8. The trajectories corresponding to the Intercity and Sprinter trains are represented as a full blue line and as a dashed yellow line, respectively. Speed limits are shown as thin dashed black lines. In the lower plot, the robust-optimal control policy is shown in blue and the height profile in light grey.

We observe that the robust-optimal control policy accelerates until the robust-optimal trajectory bundle hits an 11.1 m/s (40 km/h) speed limit, reaccelerating right after this speed restriction. A few meters before km 3, a coasting regime starts before until the final brake. The trajectory of the Intercity VIRM VI shows higher speeds than the Sprinter's over most part of the journey. The maximum passing time and speed dispersion between the trajectories in the robust-optimal bundle take place at the arrival window, being 30.7 s and 6.9 m/s (24.8 km/h) respectively. The observed dispersion in the arrival time and speed is the price to pay for calculating by means of RTTO a control policy that is robust against an ulterior rolling stock choice. As we observed in the previous example, the calculated robust-optimal control policy will be drivable, no matter which of these two trains is deployed in the future. Therefore, RTTO can eliminate the impact of rolling stock deployment uncertainty in early planning stages.

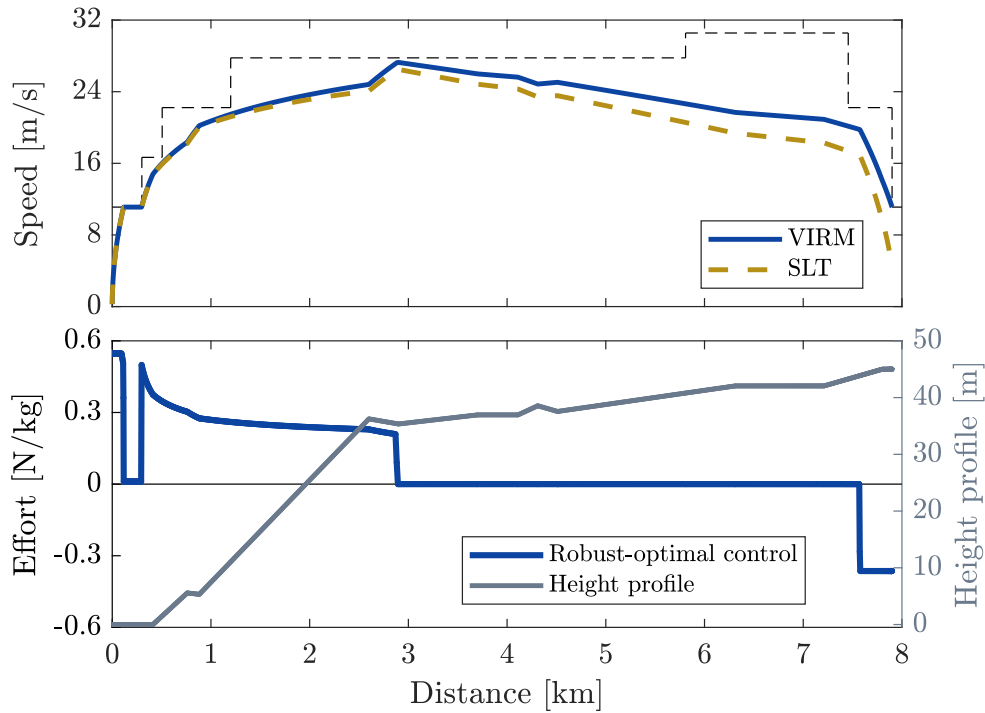


Figure 8.8: Robust-optimal trajectory bundle in a scenario with uncertainty in the rolling stock to be deployed.

Robust Train Trajectory Optimization under uncertain operating conditions

RTTO can be used to relieve the effect of operational uncertainty in optimal train trajectories. We consider now a VIRM IV Intercity train, for which 191 parameter configurations have been estimated for different operating conditions in chapter 3. Among these estimates, we select two as shown in Table 8.2. They correspond to the parameter configurations whose running resistance parameters are the furthest away in Euclidean distance among all available parameter configurations. Physically speaking, we have selected the two parameter configurations that represent the most different operating conditions among all the estimates. For instance, one may represent dry and calm operating conditions (lower running resistance), and the other wet and windy conditions (higher running resistance). Figure 8.9 shows in blue the robust-optimal trajectory bundle calculated in this scenario. The track gradients and speed limits used are those in the Arnhem Centraal-Nijmegen corridor, located in the Eastern part of the Netherlands, where the data to estimate the 191 parameter configurations in chapter 3 was collected. This corridor shows varying gradients and speed limits, which are represented in grey in the lower plot, and as thin black dashed lines in the upper plot, respectively. The robust-optimal control policy is depicted in blue in the lower plot, while the trajectories in the robust-optimal bundle are depicted as a dashed yellow line and a full blue line in the upper one. The arrival speed window is again from 0.28 to 11.1 m/s (1 to 40 km/h). We located it at the beginning of the platform, where a reduced speed limit starts. This is 17.656 km far from the departure location. The calculated robust-optimal minimum running time is

671.4 s. When calculating the energy-efficient robust-optimal control policy shown in the figure, the maximum arrival time is this minimum time plus a 20% margin.

r_0 [N/kg]	r_1 [N/(kg m/s)]	r_2 [N/(kg(m/s) ²)]	$u_{f,\max}$ [N/kg]	$u_{b,\max}$ [N/kg]	$p_{\max}$ [W/kg]
11.7e-03	2.00e-04	2.11e-05	0.270	0.162	5.49
2.21e-03	9.26e-04	1.49e-05	0.274	0.166	5.49

Table 8.2: Parameter configurations characterizing different operating conditions drawn from chapter 3.

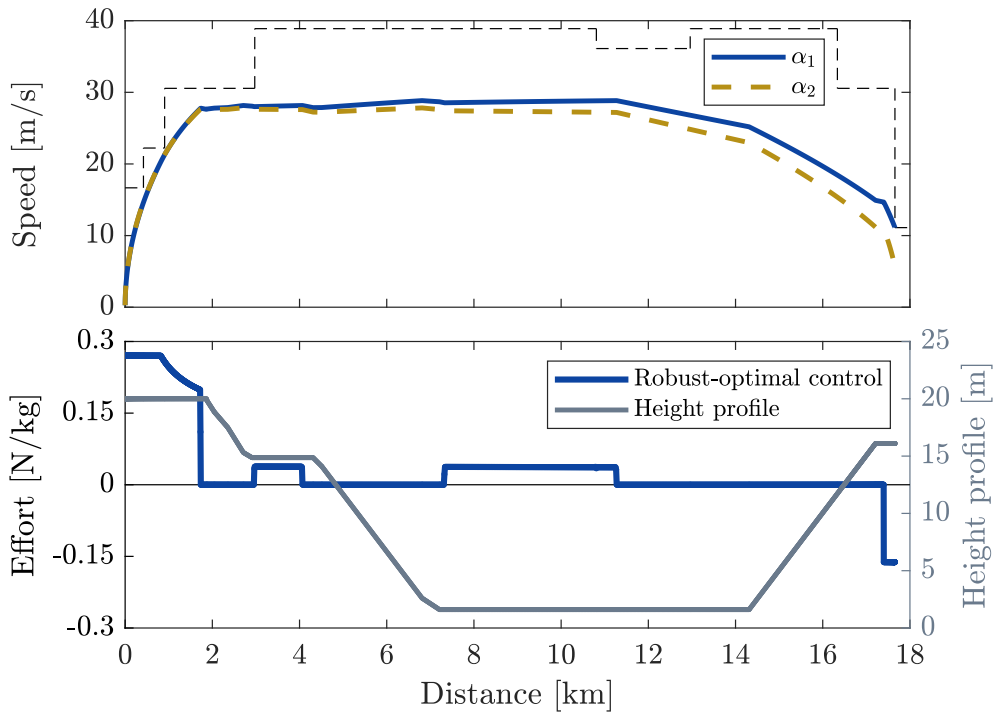


Figure 8.9: Robust-optimal trajectory bundle in a scenario with uncertainty in the operating conditions.

The observed dispersion in the arrival times and speeds is 62.3 s and 5.56 m/s (20 km/h), respectively, and corresponds to the maximum deviations observed along the whole journey. These differences are larger than in the uncertain rolling stock scenario. However, in this example we are introducing in the RTTO problem two diametrically-opposed parameter configurations that were observed in chapter 3. Intuitively-speaking, the dispersions should be smaller when more similar parameter configurations are introduced in the RTTO problem. We will analyze this further later in this chapter. The track used in this example is more than two times longer than in the previous example. Despite the observed passing time and speed deviations, the calculated robust-optimal control policy is guaranteed to be drivable if any of the considered antagonistic operating conditions take place when the train tracks this control policy. Thus, RTTO can be used to mitigate the impact of uncertain operating conditions like the weather on the performance of the control policy to be tracked by a train.

8.7.4 Performance analysis of Robust Train Trajectory Optimization on flat tracks

Minimum time Robust Train Trajectory Optimization

Now that we have motivated and showcased some applications of RTTO, we proceed to analyze its performance in simpler scenarios. Figure 8.10 exemplifies a minimum time robust-optimal trajectory bundle over a 10 km long flat track with no speed limits and 2 parameter configurations shown in Table 8.3. One trajectory is represented as a blue full line, and the other as a yellow dashed line. The robust-optimal control policy is represented below in blue. 1000 collocation points have been used in the calculations. The 2 parameter configurations utilized have been computed by k -means clustering the running resistance estimates in chapter 3 based on the squared Euclidean distance (Arthur & Vassilvitskii, 2007). The corresponding maximum traction and brake are calculated by taking the mean values in each cluster. The maximum traction power used is equal to the mean value observed in chapter 3 for both parameter configurations. The arrival speed window is set from 0.28 to 4.17 m/s (1 to 15 km/h).

r_0 [N/kg]	r_1 [N/(kg m/s)]	r_2 [N/(kg(m/s) ²)]	$u_{f,\max}$ [N/kg]	$u_{b,\max}$ [N/kg]	$p_{\max}$ [W/kg]
4.85e-03	4.49e-04	1.95e-05	0.341	0.168	5.49
8.41e-03	1.84e-04	2.51e-05	0.347	0.167	5.49

Table 8.3: Parameter configurations obtained by a k -means clustering the running resistance of the 191 estimates in chapter 3 based on the squared Euclidean distance (Arthur & Vassilvitskii, 2007), with 2 clusters.

We observe a minimum time robust-optimal control policy consisting of a maximum traction regime followed by maximum brake. The robust-optimal arrival time is 411.7 s, which is 1.1 s slower than the arrival time of the fastest trajectory in the bundle. In the upper plot the robust-optimal trajectory bundle reaches speeds slightly over 40 m/s. No speed differences can be observed with the naked eye in this figure because the maximum speed difference is 0.25 m/s (1 km/h), which takes place right at the arrival location. The fastest trajectory is the one arriving with a speed equal to the upper bound of the arrival speed window. The nominal minimum time trajectories ending at the upper bound of the arrival time window for each individual parameter configuration are respectively 1.1 and 1.9 s faster than the robust-optimal trajectory. The small difference between nominal and robust-optimal minimum running times hints that the latter could be used to obtain more robustness against model parametric uncertainty in timetabling and in energy-efficient train driving.

Figure 8.11 shows how the quadratic constraints in Eqs. (8.77) and (8.78) are verified for each parameter configuration, one depicted as full blue lines, the other as yellow dashed lines, in the solution of the minimum time RTTO problem. In both plots the constraints are close to zero, so we can conclude that the solution found is optimal.

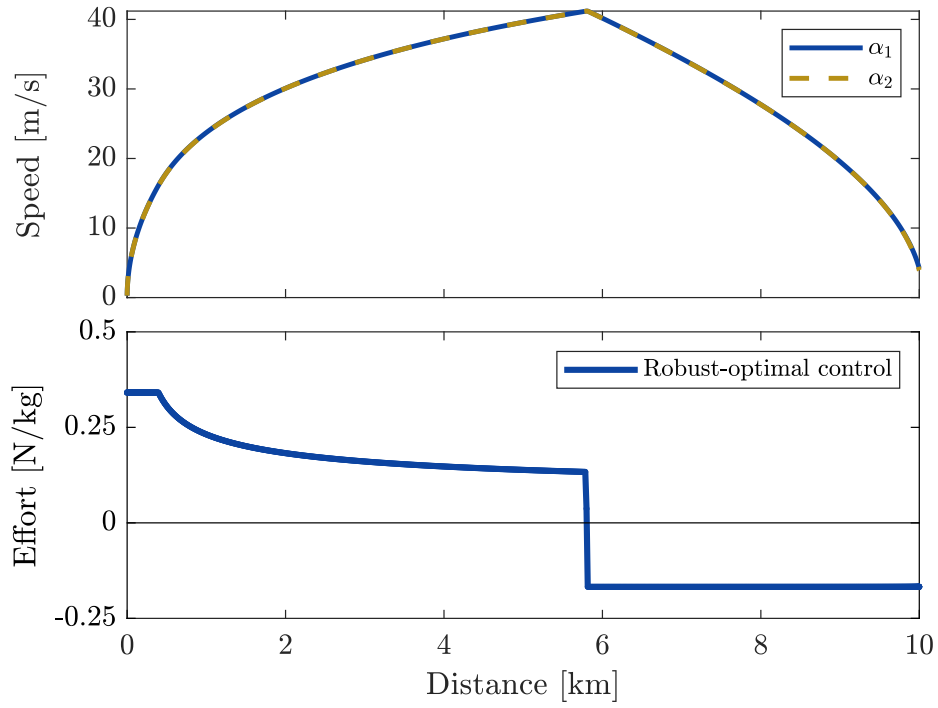


Figure 8.10: Example of a minimum time robust-optimal control policy in a flat track.

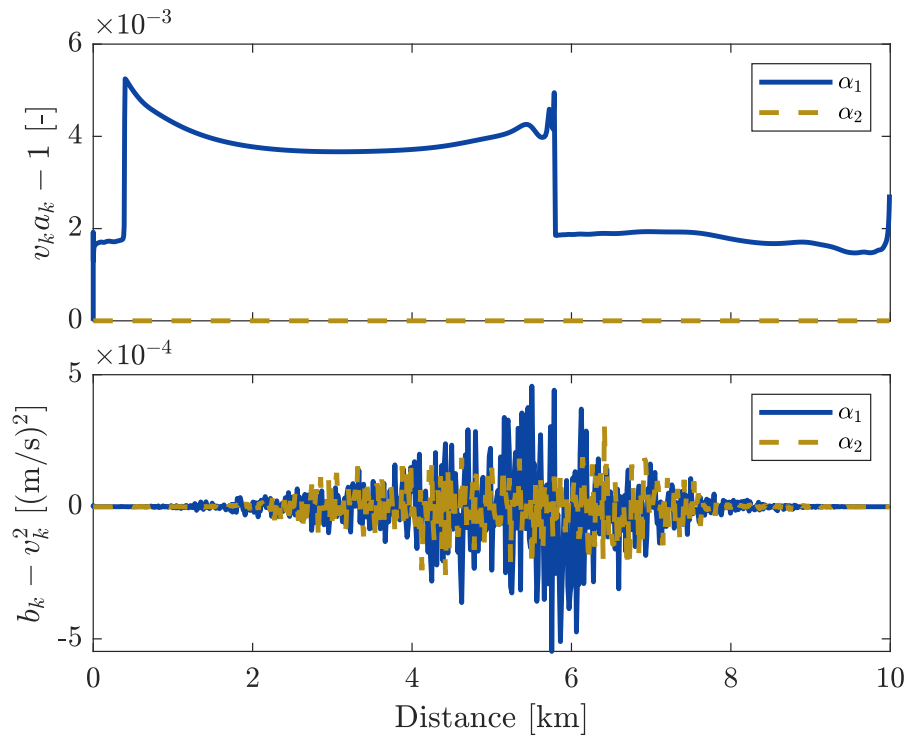


Figure 8.11: Optimality check of the quadratic constraints in Eqs. (8.77) and (8.78) in a minimum time robust-optimal control policy.

Energy-efficient Robust Train Trajectory Optimization

Figure 8.12 shows an example of a robust-optimal trajectory bundle calculated for 2 parameter configurations over a 10 km long flat track without speed limits and 1000 collocation points. The parameter configurations used in Figure 8.12 are shown again in Table 8.3. The trajectories in the robust-optimal trajectory bundle are represented as a blue full line and as a yellow dashed line, and the robust-optimal trajectory, which is common for both, is represented in blue. The size of the arrival speed window set from 0.28 to 4.17 m/s (1 to 15 km/h) because smaller windows were found to lead to infeasible RTTO problems. A 20% running time supplement over the robust-optimal minimum time is selected as the maximum arrival time.

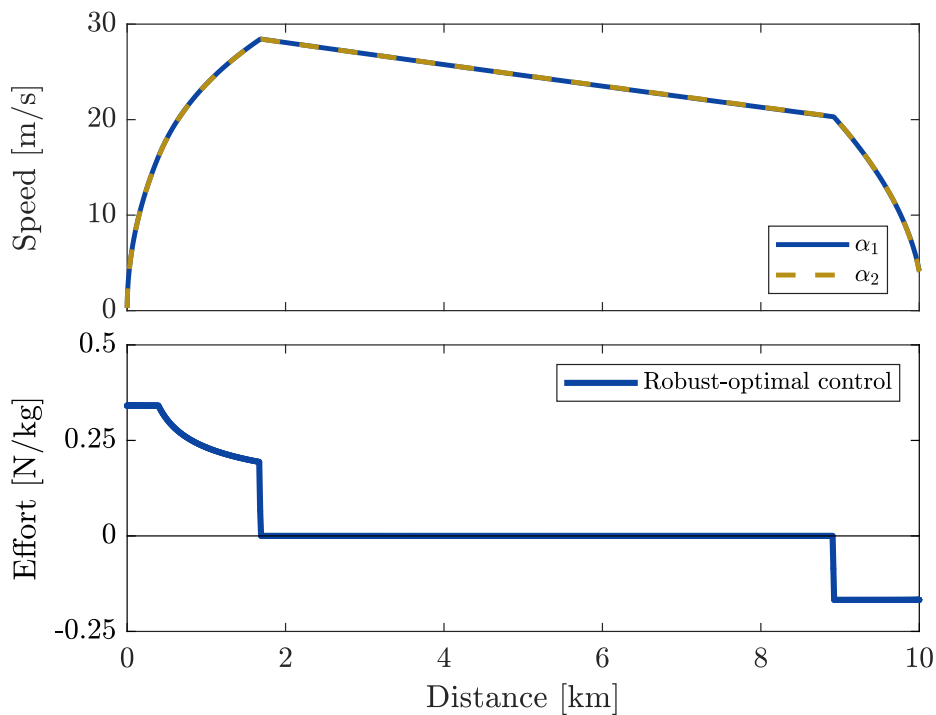


Figure 8.12: Example of an energy-efficient robust-optimal control policy in a flat track.

In the calculated robust-optimal control policy we identify three driving regimes: maximum traction, coasting and maximum brake, with both traction and brake at the minimum feasible rate for all parameter configurations. We observed that larger maximum arrival times lead to robust-optimal control policies consisting of a consecutive concatenation of maximum traction, partial traction leading to constant speed for one of the trajectories of the bundle, coasting and maximum brake. Like in the minimum time robust-optimal example, the largest speed difference in the trajectory bundle takes place at the arrival location, but no differences are observed with the naked eye. One of the trajectories in the bundle reaches its destination with the maximum allowed arrival speed, 4.17 m/s, while the other is 0.1 m/s slower. Small differences are found in the passing times, where the largest difference is found again at the arrival location. The fastest trajectory is 0.2 s faster than the specified maximum arrival time. When compared to

the nominal-optimal trajectories corresponding to each of the parameter configurations used in the RTTO problem, the robust-optimal control policy is 0.12% and 0.51% less efficient than the respective nominal-optimal trajectories that arrive at the maximum arrival time and at the maximum arrival speed. The marginal increase in energy consumption described by the robust-optimal control policy is caused by the feasibility requirement for all parameter configurations and by the fact that one of the trajectories in the robust-optimal bundle arrives half minute earlier than its nominal counterpart. Robust-optimal control policies use traction and brake more conservatively than in nominal scenarios, forcing the robust-optimal trajectory bundle to reach higher speeds and increasing the kinetic energy losses due to the aerodynamic running resistance. Still, the robust-optimal control policy is mathematically the most energy-efficient among all control policies feasible for both parameter configurations. The small difference observed in the energy consumption shows that the robust-optimal control policy calculated for the two centroids of the 2 clusters of the running resistance estimates in chapter 3 is very close to their respective nominal-optimal control policies.

In Figure 8.13 the quadratic constraints in Eqs. (8.77) and (8.78) are again shown as full blue and yellow dashed lines for each parameter configuration. We observe in this figure that these constraints are verified in the solution of the energy-efficient RTTO problem.

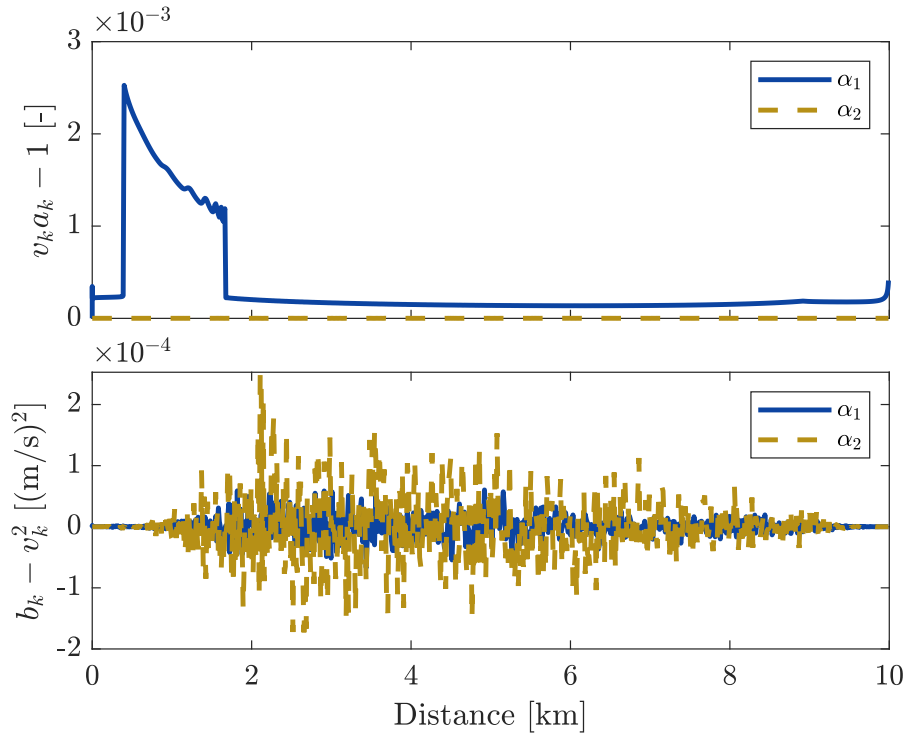


Figure 8.13: Optimality check of the quadratic constraints in Eqs. (8.77) and (8.78) in an energy-efficient robust-optimal control policy.

Different track lengths

The feasibility of the RTTO problem requires the existence of a control policy that is able to drive a train into the arrival time and speed window for all the parameter configurations considered in the RTTO problem. For different lengths of a flat track with a 140 km/h speed limit over the whole track, we have calculated first the robust-optimal minimum running time and a robust-optimal energy-efficient control policy using a 10% running time supplement over the robust minimum time the maximum arrival time. For fair comparison purposes, we have set an arrival speed window from 0.28 to 5.56 m/s (1 to 20 km/h) for all track lengths. Each robust-optimal control policy has been calculated using 1000 collocation points, both for the minimum time and the energy-efficient RTTO problems. Using a higher number of collocation points (up to 10 times more) did not significantly increase the accuracy of the solutions obtained, so we used the same number of collocation points in each scenario. The parameter configurations used in the calculations are again the two centroids gathered in Table 8.3. Table 8.4 shows for each track length considered the calculated robust-optimal minimum running time, the dispersion in the arrival time and speed observed in the energy-efficient robust-optimal trajectory bundles, the expected robust-optimal energy consumption and the difference in percentage with the energy used in nominal trajectories that arrive at destination at the maximum arrival time with a speed equal to the upper bound of the speed window.

Track length [km]	Robust-optimal minimum running time [s]	Dispersion in arrival times in energy-efficient robust trajectories [s]	Dispersion in arrival speeds in energy-efficient robust trajectories [m/s]	Robust-optimal energy consumption [J/kg]	Difference w.r.t. nominal energy consumption [%]
0.5	69.2	0.7	0.17	35.0	2.19-3.25
1	105.8	1.1	0.26	63.6	3.10-3.73
1.5	134.5	1.4	0.31	92.7	3.36-3.72
2	159.0	1.5	0.33	121.7	3.33-3.46
3	200.8	1.5	0.32	179.3	2.70-2.86
4	236.9	1.3	0.28	236.1	1.96-2.28
5	269.4	1.0	0.22	292.7	1.32-1.74
7.5	341.3	0.2	0.06	433.8	0.24-0.77
10	406.4	0.4	0.04	573.7	0.19-0.72
15	535.6	0.8	0.11	806.6	0.26-0.66
20	664.8	0.6	0.08	1018.9	0.13-0.37
30	923.0	0.1	0.03	1456.2	0.04-0.16
40	1181.1	0.5	0.06	1901.7	0.05-0.42
50	1439.1	0.9	0.09	2349.3	0.05-0.17

Table 8.4: Robust-optimal minimum running times in different track length scenarios and performance of robust-optimal control policies calculated with a 10% running time supplement over the robust-optimal minimum running times.

The results obtained show that the dispersion in the arrival times and speeds of the calculated energy-efficient robust-optimal trajectories is bounded and small. The dispersion is kept below 2 s and 1.5 km/h for each track length, which is sufficiently low for tracking the calculated robust-optimal control policy in a speed regulator or in a Driver Advisory System. In practice, the small dispersions observed indicate that RTTO can be used in both urban transit railway systems with short distances between stations, and in regional and intercity railways with larger distances between stops. A larger amount of collocation points can be used in planning stages when targetting high resolution in the robust-optimal trajectory bundle, or whenever real-time calculations are not required. For example, a robust-optimal trajectory for 2 parameter configurations is calculated in under a second when using 1000 collocation points with a laptop powered by a Intel i7 processing unit at 1.9 GHz. We observed that infeasible or wrong results due to numerical instability may arise if the arrival speed window is too small. While for the shorter track lengths the width of the arrival window can be kept below 2 m/s, larger windows are required for longer tracks, although their whole width is not fully used by the robust-optimal trajectory bundle. The arrival window can be kept equal to 20 km/h in most cases for flat tracks. The brake curve of the train can be calculated before solving the RTTO problem and the arrival window can be placed at the desired maximum arrival speed in the brake curve. When it comes to the energy consumption, larger differences up to 4% are observed with respect to the nominal-optimal trajectories in the shorter tracks. In the longer tracks, the difference in energy consumption is up to 2 orders of magnitude smaller than in shorter tracks. This is due to the fact that, when aiming to arrive at a distant location at a given time with minimal energy consumption, the arrival time constraint has a predominant role when shaping the optimal trajectory, lowering the impact of parametric uncertainty. This flat track scenario with a constant high speed limit allows to analyze the performance of the proposed method under simple, general conditions. However, as we will discuss later, in more complex scenarios with gradients and lower and varying speed limits, we observed arrival time dispersions that scale with the length of the track.

Different number of parameter configurations

As we mentioned earlier, differences in the parameter configurations lead to speed deviations when tracking a control policy. If the number of parameters introduced in the RTTO problem is large, or if the parameter configurations show big differences, the existence of a feasible control policy that drives the train into the arrival time and speed window for all the parameter configurations may be jeopardized. In Figure 8.14 we assess how the number of parameter configurations q introduced in the RTTO problem impacts robust-optimal control policies. We have randomly sampled 100 sets of q parameter configurations among the 191 estimates obtained in chapter 3. We have introduced each set in the RTTO problem and calculated their robust-optimal minimum running time and their energy-efficient robust-optimal control policy over a 10 km long flat track. For each set, the maximum arrival time is 20% over the robust-optimal minimum running time and the arrival window is set from 0.28 to 11.1 m/s (1 to 40 km/h). A 38.9 m/s (140 km/h) constant speed limit is set over the whole track. We obtained a statistic representation of the dispersion in the arrival times and speeds for each number of parameter configura-

rations q . Figure 8.14 shows a box-and-whiskers diagram of the observed dispersions in the arrival times in the left hand side, and in the arrival speeds in the right hand side. The observed medians are represented as yellow lines, the interquartile ranges in blue, the whiskers in black and the identified outliers as yellow dots.

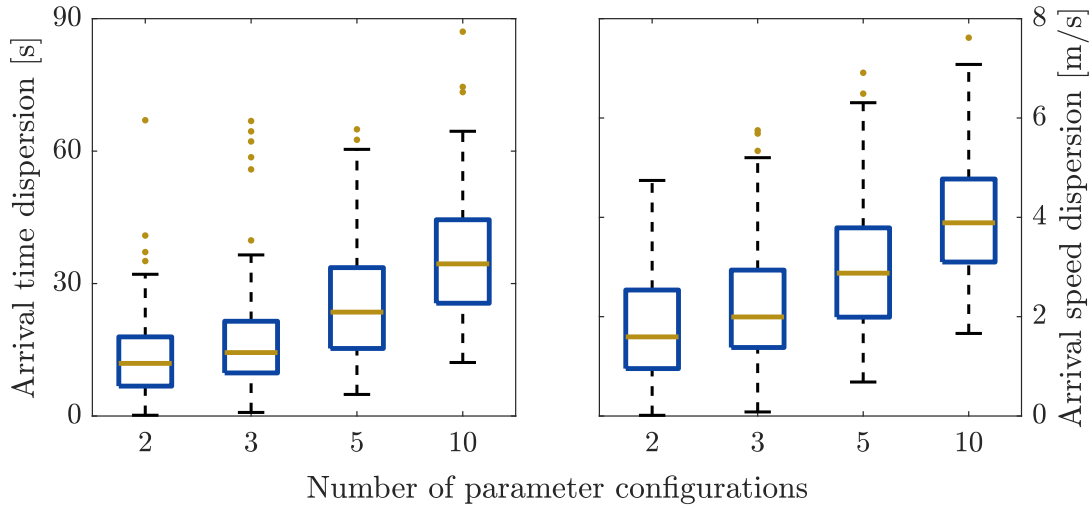


Figure 8.14: Observed dispersion in robust-optimal arrival times and speeds for different number of parameter configurations.

The results obtained show that the dispersions in the arrival time and speed among the trajectories in the robust-optimal trajectory bundle grow the more parameter configurations are introduced in the RTTO problem. Regarding the dispersions in the arrival times, the interquartile ranges observed are approximately equal for $q = 2, 3$, and 50% larger for $q = 5, 10$. This means that the distributions of the arrival time dispersions become wider for the larger q . Despite 75% of the arrival time dispersions fall below 18.0, 22.5, 33.6 and 44.5 s for $q = 2, 3, 5, 10$, respectively, outliers over 60 s are observed for all q , including an 87 s outlier for $q = 10$. Considering that the length of the track used in this analysis is 10 km, in practice less than 10 parameter configurations should be introduced in the RTTO problem. When it comes to the observed distributions of the dispersions in the arrival speeds, the interquartile ranges remain approximately constant for all q . The distributions mainly drift towards larger speed dispersions as q increases. When compared to the time dispersion distributions, the speeds are more symmetrically distributed. 75% of the speed dispersions are below 2.5, 2.9, 3.8 and 4.8 m/s (9.1, 10.6, 13.6 and 17.2 km/h) for $q = 2, 3, 5$ and 10. Less outliers are identified in the speed dispersions. Overall, this analysis highlights that the number of parameter configurations introduced in the RTTO problem shall be kept preferably low, below 10, to keep the arrival time and speed dispersions low enough for practical implementations. Also, the dispersions observed at arrival depend on how distant the parameter configurations are in the parameter space. Since the main purpose of RTTO is to mitigate the impact of parametric uncertainty in the optimal trajectories, it is essential to select carefully the parameter configurations to be introduced in the RTTO problem.

In chapter 6.4.5 we observed deviations up to 94 s in the passing times among the nominal-optimal trajectories calculated for the 191 parameter configurations estimated in

chapter 3. All these trajectories were calculated on a 18.5 km flat track with a target running time equal to 750 s. The maximum time deviation takes place around km 8, while in RTTO, the maximum deviations are observed at the final location. This remark shows that deviations are also observed among nominal-optimal trajectories calculated with different sets of parameter estimates and that their order of magnitude is similar to those deviations in the robust-optimal trajectory bundle. While nominal-optimal trajectories deviate more around the middle of the track section, robust-optimal trajectories show more dispersion at the arrival location. We will discuss next how to select smartly a small number of parameter configurations of interest.

Optimality check for the convex hull of the parameter configurations introduced in the Robust Train Trajectory Optimization problem

Next, we analyze computationally whether the robust-optimal trajectory calculated for any fixed set of parameter configurations in the RTTO problem is also robust-optimal for any other parameter configuration inside the convex hull of the original set. From the 191 parameter configurations obtained in chapter 3, we randomly sample 100 sets of 3 parameter configurations. The convex hull of each set of 3 parameter configurations is a triangle in the parameter space. We interpolate 12 parameter configurations as shown graphically in Figure 8.15. The convex hull triangle is divided in 16 smaller triangles of equal size and we take all their vertices except for the vertices of the convex hull. In this diagram, the 3 round dots represent the 3 sets of parameter configurations randomly sampled among the mentioned 191 parameter configurations, the full black line shows their convex hull, and the crosses show the parameter configurations interpolated inside the convex hull.

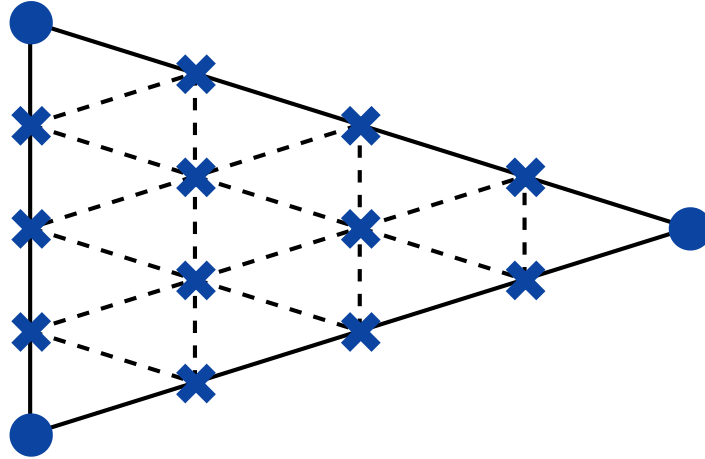


Figure 8.15: Convex hull diagram in the parameter space.

For each randomly sampled set of 3 parameter configurations we calculate their robust-optimal minimum running time on a 10 km long flat track. A 38.9 m/s (140 km/h) speed limit is set over the whole track and the arrival speed window is set from 0.28 to 11.1 m/s (1 to 40 km/h). We calculate the energy-efficient robust-optimal control policy of these 3 parent parameter configurations with a maximum arrival time 10% over their calculated minimum running time. Then, each of the parameter configurations

interpolated inside the convex hull is individually added to the RTTO problem along with its 3 parent parameter configurations. Their energy-efficient robust-optimal control policy is computed with a maximum arrival time again 10% over the robust minimum running time of the parent parameter configurations. Both robust-optimal control policies are compared by means of Dynamic Time Warping (DTW) based on the Euclidean distance (Paparrizos et al., 2024) and by calculating the sum of the absolute differences between both control policies at each collocation point. The first method is an alignment-based metric that measures shape differences between the robust-optimal control policies. DTW is able to match the different driving regimes even if their switching points are not located exactly at the same collocation points. If the compared control policies show different driving regimes, DTW will produce a high value. Instead, the second method is more sensitive to variations in the locations of the switching points between driving regimes.

The DTW comparison shows that the distances between the robust-optimal control policies calculated with a parameter configuration interpolated inside the convex hull and their parent control policy stay bounded below $1.27\text{e-}08$ N/kg. This small number indicates that all the calculated control policies show the same driving regimes as their parent robust-optimal control policy. The maximum observed sum of the absolute distances between control policies at each collocation point is $1.05\text{e-}03$ N/kg. This means that the switching points between driving regimes are located at the same collocation points as in their parent control policies. The observed absence of outliers indicates that convex hulls can be used to reduce the number of parameter configurations to be introduced in the RTTO problem for calculating robust-optimal control policies. This improves the performance and computation time of the RTTO algorithm presented in this chapter. From a practical perspective, the different parameter configurations can be segmented according to the operating conditions they represent, like the weather, and use their convex hull to calculate robust-optimal trajectories.

8.7.5 Robust Train Path Envelopes: a case study

Last, we showcase Robust Train Path Envelopes (RTPEs) in a cyclic timetable and compare them with nominal TPEs. We consider a fictitious corridor which contains 5 stations: S_A , S_B , S_C , S_D and S_E . The stopping points at each of these stations are located 5.877, 10.309, 16.054 and 18.500 km far from the departure location at station S_A , respectively. Varying speed limits and gradients are considered in this example. The timetable shows a cyclic 3-train pattern from S_A to S_E repeating every 20 minutes, alternating one train stopping at all intermediate stations, T_1 , and two trains driving non-stop between both stations, T_2 and T_3 . The aim of this example is to showcase RTPEs for different situations. For instance, non-stopping train T_2 overtakes T_1 at station S_D . In this timetable, an Intercity VIRM IV will perform the non-stopping services. A Sprinter SLT VI is usually expected to be used in the multiple-stop service T_1 , although occasionally an Intercity VIRM VI will take its place for rolling stock circulation purposes. Table 8.5 shows the scheduled arrival, departure and dwell times in the cyclic timetable. Figure 8.16 shows a time-distance diagram with the scheduled train paths.

We calculate RTPEs in this cyclic timetable. We introduce the following mix of parametric uncertainty sources. The non-stopping services T_2 and T_3 are performed by Intercity VIRM IV trains with uncertain operating conditions. The 3 parameter configu-

Station	T_1 : SLT/VIRM VI		T_2 : VIRM IV		T_3 : VIRM IV		Scheduled dwell time [s]
	Arrival time	Departure time	Arrival time	Departure time	Arrival time	Departure time	
	[mm:ss]	[mm:ss]	[mm:ss]	[mm:ss]	[mm:ss]	[mm:ss]	
S_A	—:—	00:00	—:—	08:04	—:—	15:20	—
S_B	04:48	05:48	—:—	—:—	—:—	—:—	60
S_C	09:25	10:25	—:—	—:—	—:—	—:—	60
S_D	15:03	21:03	—:—	—:—	—:—	—:—	360
S_E	24:38	—:—	22:23	—:—	29:39	—:—	—

Table 8.5: Scheduled arrival and departure times in the cyclic timetable.

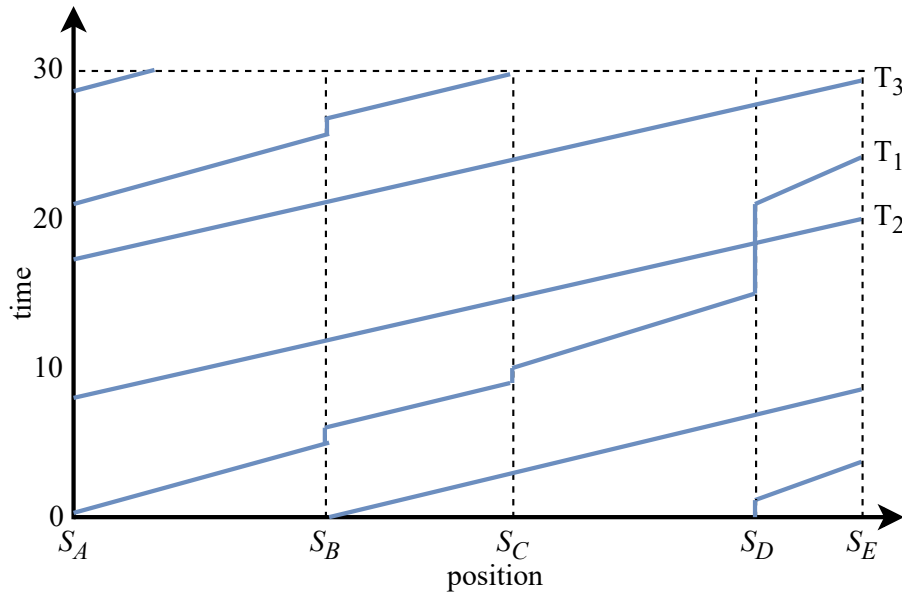


Figure 8.16: Time-distance diagram of the cyclic 3-train pattern from S_A to S_E .

rations used for T_2 and T_3 are drawn from the estimates in chapter 3 and are presented in Table 8.6. Given the uncertainty in the rolling stock to be used in the overtaken train service T_1 , we introduce in the RTTO problem the train motion model parameters presented in Table 8.1, which correspond to an Intercity VIRM VI and a Sprinter SLT VI. Energy-efficient robust-optimal control policies are calculated for all trains according to the scheduled arrival, departure and dwell times shown in Table 8.5. The maximum arrival times are equal to the upcoming departure times minus their associated minimum dwell times. The maximum arrival speed is set 11.1 m/s (40 km/h) in all cases. The minimum arrival speeds are set for T_1 to 2.78 m/s (10 km/h) at stations S_B , S_C and S_D , and to 9.72 m/s (35 km/h), at S_E . For T_2 and T_3 , it is set to 4.17 m/s (15 km/h) at the arrival at station S_E . The arrival windows are located in each case before the actual stopping location, where all trains can brake to stop at a standstill at the scheduled point by applying the highest brake rate feasible for all trains from the maximum arrival speed. The arrival times introduced in the RTTO problem are corrected using kinematics by subtracting the time required to reaccelerate from the lowest allowed arrival speed and brake to a standstill using the highest feasible rates for all trains. Figure 8.17 shows the

robust-optimal control policy calculated for T_1 and Figure 8.18 the robust-optimal control policy of trains T_2 and T_3 .

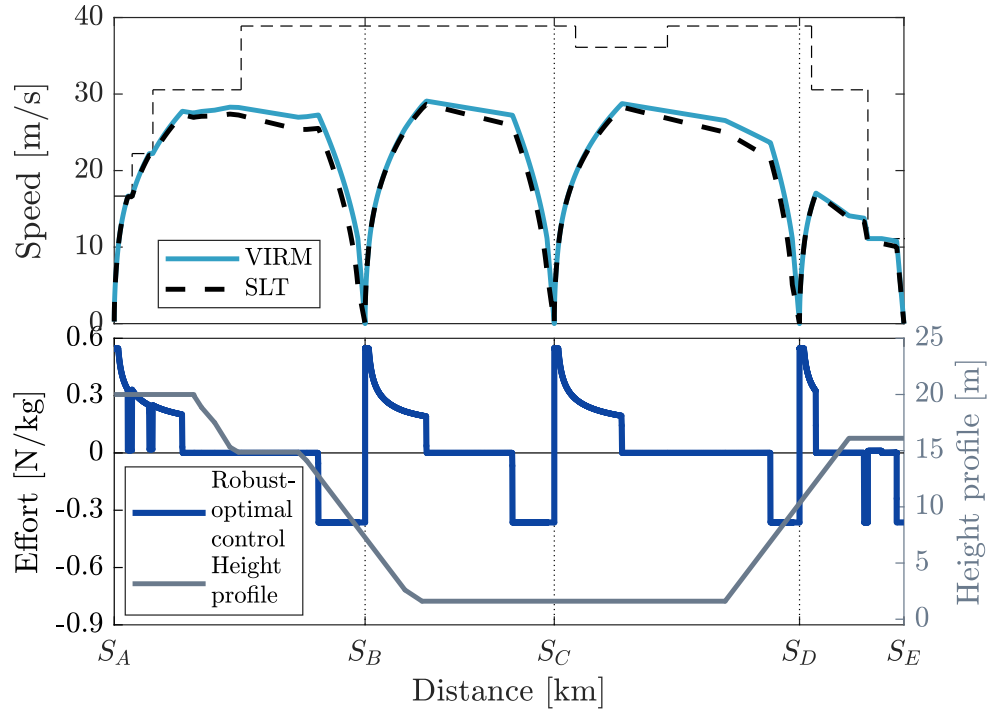


Figure 8.17: Robust-optimal trajectory bundle and control policy calculated for T_1 .

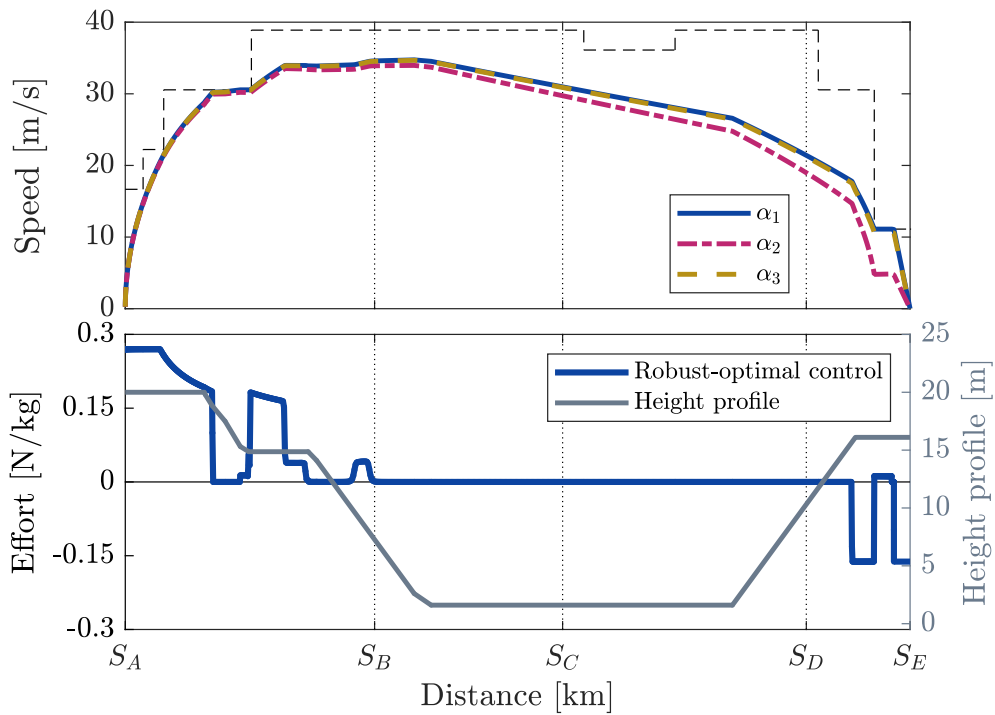


Figure 8.18: Robust-optimal trajectory bundle and control policy calculated for T_2 and T_3 .

r_0 [N/kg]	r_1 [N/(kg m/s)]	r_2 [N/(kg(m/s) ²)]	$u_{f,\max}$ [N/kg]	$u_{b,\max}$ [N/kg]	$p_{\max}$ [W/kg]
5.66e-03	3.19e-04	2.01e-05	0.270	0.166	5.49
8.94e-03	3.36e-04	2.10e-05	0.364	0.166	5.49
6.43e-03	3.18e-04	1.97e-05	0.341	0.162	5.49

Table 8.6: Parameter configurations characterizing different operating conditions of an Intercity VIRM VI train obtained in chapter 3.

In turn, we also calculate the nominal TPEs in this scenario to compare them later with RTPEs. The uncertainty in the rolling stock to be used for T_1 is removed by using the Sprinter's parameters shown in Table 8.1 in the nominal TTO problem. When it comes to T_2 and T_3 , the average of the estimates presented in Table 8.6 is used to characterize their dynamics. All the nominal-optimal trajectories are calculated according to the scheduled times in Table 8.5. The arrival speeds used are equal to the maximum arrival speeds used in the RTTO case. The arrival times and locations are corrected by assuming that the train will keep braking from the arrival speed with the maximum brake rate of the train until coming to a standstill. The overtaking window introduced for T_2 in the RTTO problem is used again in the nominal case. Figures 8.19 and 8.20 show the nominal-optimal trajectories calculated and their corresponding optimal control policies for T_1 , and T_2 and T_3 , respectively.

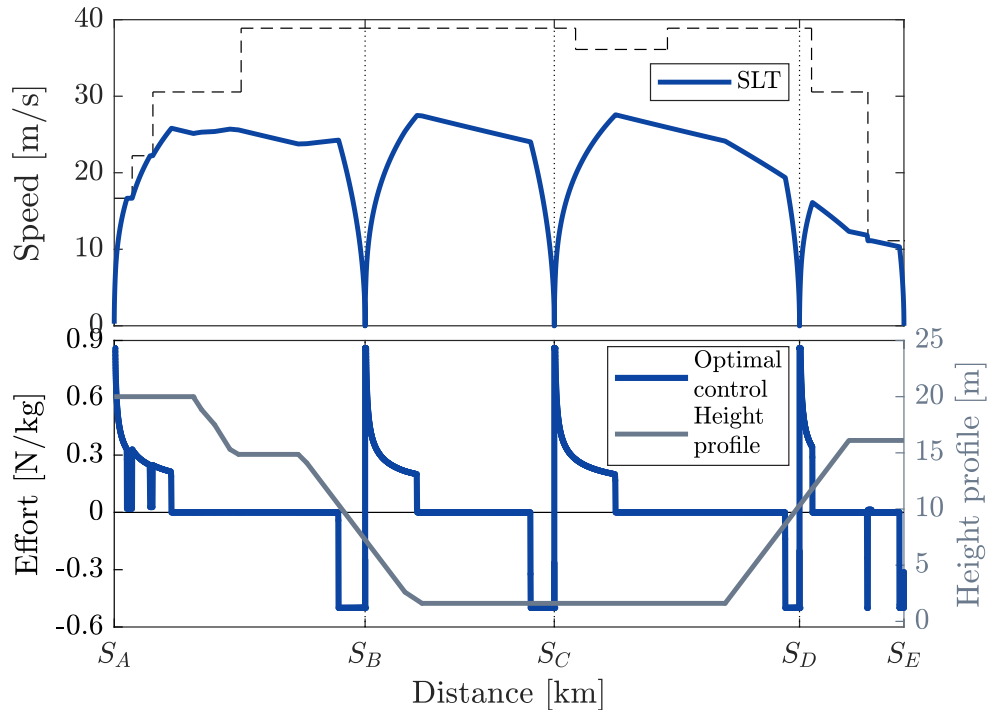


Figure 8.19: Nominal-optimal trajectory and control policy calculated for T_1 .

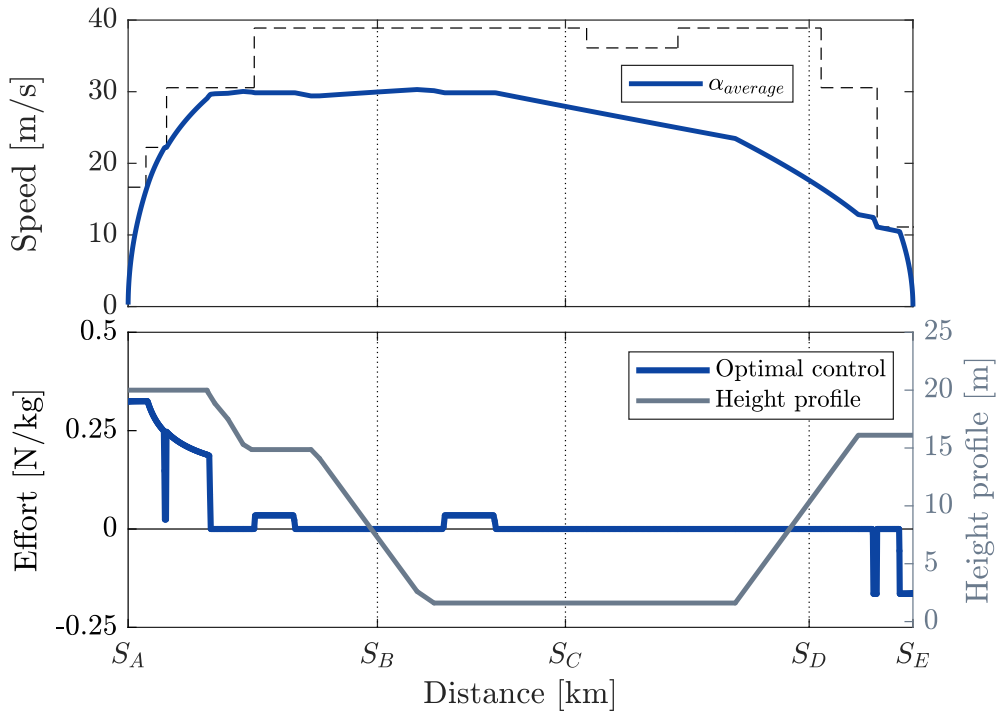


Figure 8.20: Nominal-optimal trajectory and control policy calculated for T_2 and T_3 .

The calculated robust-optimal trajectory bundles and nominal-optimal trajectories allow to compute their associated RTPEs and nominal TPEs, respectively. Time-distance diagrams obtained with RTTO (left) and nominal TTO (right) are presented in Figures 8.21 and 8.22, where the trajectories corresponding to T_1 are represented in blue, the ones corresponding to T_2 are shown as full yellow lines, and those of T_3 , as dashed yellow lines. Train services of other timetable cycles are shown in black. RTPEs are computed based on the earliest and latest passing times of both trains at each station. In nominal trajectories, the passing time of a train with nonzero speed is unequivocally defined at any location. Passing times are used as input to compute RTPEs and nominal TPEs along with the scheduled arrival and departure times, headway times and speed constraints. We consider a 120 s minimum overtaking headway $h_{overtaking}$ between trains T_1 and T_2 at station S_D , a 120 s minimum passing headway $h_{passing}$ between T_1 's departure from S_D and T_3 's earliest passing time, and a 180 s minimum headway $h_{arrival}$ between the consecutive arrivals at S_E of T_1 and T_3 . Early departures are not allowed, although we allow for a 30 s late departure from stations S_A , S_B and S_C for all trains in both RTPEs and nominal TPEs. To compute the RTPEs of train T_k at station X , we denote D_{X,T_k} its departure time from that station, A_{X,T_k}^- and A_{X,T_k}^+ its earliest and latest arrival times, and P_{X,T_k}^- and P_{X,T_k}^+ its earliest and latest passing times through the station. At station S_D , the upper bound of T_1 's RTPE depends on the latest arrival time of T_1 and the earliest passing time of T_2 , $(A_{S_D,T_1}^+ + P_{S_D,T_2}^- - h_{overtaking})/2$. Its departure window at S_D is computed considering that no early departures are allowed and the earliest passing time of T_3 , $[D_{S_D,T_1}, (D_{S_D,T_1} + P_{S_D,T_3}^- - h_{passing})/2]$. The overtaking window T_2 at S_D is calculated taking into account that T_1 will not depart early, $[(A_{S_D,T_1}^+ + P_{S_D,T_2}^- +$

$h_{overtaking})/2, D_{S_D, T_1} - h_{overtaking}]$. T_3 's lower bound on its passing window is computed as $(D_{S_D, T_1} + P_{S_D, T_3}^- + h_{passing})/2$. At station S_E , the arrival window of T_1 is constrained by the latest arrival time of T_2 and the earliest arrival time of T_3 , $[(A_{S_E, T_2}^+ + A_{S_E, T_1}^- + h_{arrival})/2, (A_{S_E, T_1}^+ + A_{S_E, T_3}^- - h_{arrival})/2]$. Consequently, the upper bound of T_2 's arrival window at S_E is $(A_{S_E, T_2}^+ + A_{S_E, T_1}^- - h_{arrival})/2$ and the lower bound of T_3 's is $(A_{S_E, T_1}^+ + A_{S_E, T_3}^- + h_{arrival})/2$. When computing nominal TPEs, these formulas are also used with the difference that the earliest and latest passing or arrival times of a train at any station are the same. In nominal TPEs, we allow to arrive up to 30 s late at stations S_B and S_C , assuming a minimum dwell time of 30 s. Likewise, a 30 s early arrival is allowed at S_B , S_C and S_D . In RTPEs, we define also a 60 s time window at such arrivals, centered at the midpoint between the expected earliest and latest arrivals of the robust-optimal trajectory bundle. Alternatively, these arrival windows could be computed from the passing times of adjacent trains. For simplicity, the lower bound of T_2 's arrival window at S_E and the upper bounds of T_3 's passing window at S_D and arrival window S_E are not calculated. Table 8.7 gathers the computed RTPEs and nominal TPEs.

Location	Type	T_1 : RTPEs		T_1 : nominal TPEs	
		Time window	Speed window	Time window	Speed window
S_A	Departure	[00:00, 00:30]	0	[00:00, 00:30]	0
S_B	Arrival	[04:08, 05:08]	0	[04:18, 05:18]	0
S_B	Departure	[05:48, 06:18]	0	[05:48, 06:18]	0
S_C	Arrival	[08:50, 09:50]	0	[08:55, 09:55]	0
S_C	Departure	[10:25, 10:55]	0	[10:25, 10:55]	0
S_D	Arrival	[14:27, 15:27]	0	[14:52, 15:52]	0
S_D	Departure	[21:03, 21:35]	0	[21:03, 22:00]	0
S_E	Arrival	[24:28, 25:16]	0	[24:31, 26:09]	0

Location	Type	T_2 : RTPEs		T_2 : nominal TPEs	
		Time window	Speed window	Time window	Speed window
S_A	Departure	[08:04, 08:34]	0	[08:04, 08:34]	0
S_D	Overtaking	[17:27, 19:03]	[0, 140]	[17:52, 19:03]	[0, 140]
S_E	Arrival	[—:—, 22:28]	0	[—:—, 22:31]	0

Location	Type	T_3 : RTPEs		T_3 : nominal TPEs	
		Time window	Speed window	Time window	Speed window
S_A	Departure	[15:20, 15:50]	0	[15:20, 15:50]	0
S_D	Passing	[24:35, —:—]	[0, 140]	[25:00, —:—]	[0, 140]
S_E	Arrival	[27:16, —:—]	0	[28:09, —:—]	0

Table 8.7: Calculated RTPEs and nominal TPEs for the timetable in Table 8.5.

The main noticeable difference between Figures 8.21 and 8.22 is the dispersion shown by the trajectories in the robust-optimal trajectory bundle. It becomes larger close to the arrival locations because the robust-optimal trajectory bundle spends more time there with different low speeds. Still, this dispersion does not hinder the operation of adjacent trains.

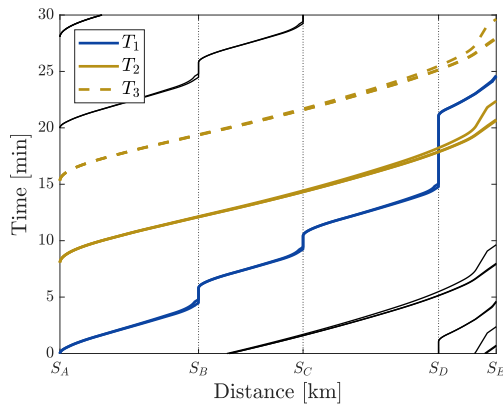


Figure 8.21: Cyclic timetable with a 20 minutes cycle pattern calculated using RTTO.

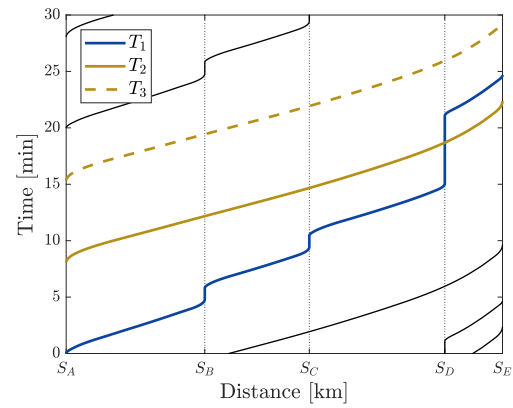


Figure 8.22: Cyclic timetable with a 20 minutes cycle pattern calculated using nominal TTO.

We observe more noteworthy differences between the reported RTPEs and nominal TPEs in Table 8.7. In the first stations, S_A , S_B and S_C , RTPEs and nominal TPEs are either equal or show a slight shift because in RTPEs arrival windows are computed from the midpoint of the arrival times of the robust-optimal trajectory bundle. However, the most remarkable difference takes place at stations S_D and S_E , where the overtaking, passing and arrival RTPEs and TPEs have been computed. RTPEs allocate shorter time windows for T_1 than nominal TPEs at those locations. Likewise, RTPE's time windows of trains T_2 and T_3 are larger than their nominal counterparts. The lower and upper bounds of the time window of a train's RTPE are calculated respectively from the earliest and latest passing times of the robust-optimal trajectory bundle of that train. Therefore, the width of the time window is proportional to the running time dispersion of the bundle at that location. This dispersion is mainly cumulative as the train drives further from its departure location, so it is larger close to the arrival point. As can be observed in Figure 8.21, the running time dispersion scales with the track length in real scenarios with complex gradients and varying reduced speed limits. In such real scenarios, RTPE time windows are roughly proportional to the distance covered from departure. The observed distance-dependent size of the RTPE time windows is of particular relevance in railway operation. Concept-wise, TPEs are a real-time traffic plan, defining bounds on the passing times at strategic locations, so trains can drive within these bounds without delays or affecting any adjacent train. In shorter track sections, there is less margin to deviate from the expected trajectory than in longer track sections. RTPE smartly allocates tighter windows for trains running over shorter track sections, and wider windows for trains covering longer distances. This can help railway operators to reduce the likelihood of primary and knock-on delays. RTPEs provide more robustness than nominal TPEs against factors that produce volatility in the running times of trains, including the dispersion produced by parametric uncertainty. The theoretical capacity usage remains unaltered since the same amount of trains can be operated for both nominal TPE and RTPE, and the same minimum headways between consecutive trains are used in both cases.

8.8 Conclusion

In this chapter we have presented a new mathematical formulation of the RTTO problem under model parametric uncertainty as a minimax optimal control problem that aims to find the most energy-efficient trajectory that is feasible for all the parameter configurations considered. The robust-optimal control policy specifies the amount of traction and brake to be applied by the train at each track location, no matter which is the operating condition realized when the train starts its operation. However, speed and running time deviations arise after departure when applying the same control policy on train motion models with different parameter configurations. This implies that the train will arrive at its destination with different speeds and at different times for each parameter configuration. Consequently, the arrival conditions have to be relaxed in the RTTO problem and arrival speed and time windows shall be defined. The arrival time and speed windows can be placed, for instance, at the beginning of the platform or at the expected brake curve to the stopping location. The robust-optimal control policy will lead to as many trajectories in the speed-time-distance space as the number of parameter configurations introduced in the RTTO problem. We have called this ensemble of trajectories originated by the robust-optimal control policy the robust-optimal trajectory bundle. We have also formulated the minimum time RTTO problem that aims to find the control policy that is the fastest to drive the train to its destination among all the control policies feasible for all the considered parameter configurations considered.

We have analyzed the RTTO problem by means of the Robust Maximum Principle (Boltyanski & Poznyak, 2011), showing that a robust-optimal control policy solving the RTTO problem is a concatenation of up to 5 different driving regimes: maximum traction, maximum brake, coasting, and partial traction and brake. The latter two are singular driving regimes that take place to guarantee the feasibility of the robust-optimal solutions. The mentioned maximum traction and brake are conservative driving regimes, as they take place at the maximum rates feasible for all the parameter configurations considered. The switching points between driving regimes are determined by the robust-optimal control policy, so an Automatic Train Operation system can track the calculated tractive and brake effort instead of the speed profile.

The RTTO problem can be solved by means of some of the existing techniques commonly used for solving the nominal TTO problem. We have designed a computational method to solve the RTTO problem based on direct optimization, in which the track is discretized in several intervals where the equations of motion are transformed into algebraic equations. The robust-optimal control and the speed corresponding to each parameter configuration are calculated at each point of the discretized track.

A definition of Robust Train Path Envelopes (RTPEs) has also been proposed in this chapter. The main conceptual difference with respect to nominal Train Path Envelopes (TPEs) is the fact that in the robust-optimal trajectory bundle the passing speed and time at any location after departure is not unequivocally defined. The headways between consecutive trains are allocated from the earliest or latest passing times of the robust-optimal trajectory bundle. This definition showed a major benefit over nominal TPEs besides the additional robustness against parametric uncertainty and deviations from the nominal-optimal trajectory: larger time windows are provided by RTPEs to those trains that are expected to show more running time dispersions along their journey.

We have shown that parametric uncertainty may reduce the accuracy of nominal TTO, producing trajectories that are not drivable if the parameter configuration realized in operation differs from the one that was introduced in the nominal TTO problem. RTTO solves this issue and provides additional robustness against the parameter configurations in the convex hull of the input configurations at the cost of a slight increase in the energy consumption. RTTO applies conservative traction and brake rates in maximum traction and brake regimes. This implies that more time is spent at lower speeds, so in order to arrive within the arrival time window, higher speeds are reached to compensate for the extra time spent at lower speeds. We observed that sufficiently large, yet reasonably-sized arrival speed windows shall be defined to guarantee the existence of robust-optimal trajectories. If the arrival speed window is too small, a solution of the RTTO problem may not exist. New singular driving regimes arise in RTTO when the arrival speed window is small. Driving at reduced speeds favours the existence of feasible robust control policies for all the parameter configurations, as the speed dispersion of the trajectory bundle is smaller at lower speeds. The performance and limitations of minimum time and energy-efficient robust-optimal control policies have also been explored on flat tracks. We have discussed two potential applications of RTTO: mitigating the impact of uncertain operating conditions like the weather and calculating robust trajectory bundles in strategic planning stages with uncertainty in the rolling stock to be deployed. We have calculated RTPEs in a cyclic timetable, finding that when compared with nominal TPEs, more bandwidth is allocated in the time windows of those trains covering longer distances and are likely to show larger running time dispersion due to the parametric uncertainty.

This new RTTO problem definition and the numerical optimization method proposed to solve it aim to impact the railway industry on multiple levels. Parameter estimation can be performed to observe and quantify the impact of different operating conditions on the dynamics of trains. Historical data logged and stored on-board can be used to calculate the parameters that are used as input in the RTTO problem. This approach allows to characterize physically the considered uncertainties and to have a clear intuition of their meaning and influence. Those operational conditions of interest can be precisely selected to be used as input. For instance, if rain is expected, the parameter configurations corresponding to wet driving conditions can be used, leading to smoother accelerations and decelerations. Likewise, unlikely operating conditions can be left out of the calculations in favour of efficiency and feasibility. Small differences were observed in energy consumption between nominal- and robust-optimal control policies. RTTO is slightly less energy-efficient than nominal TTO for regional and intercity railways. However, the observed difference is small and the additional robustness provided by RTTO can be a key factor to break some of the potential initial resistances in the industry against implementing RTTO. The absence of an unequivocally-defined passing speed for each track location in the robust-optimal trajectory bundle does not constitute a problem when aiming to execute the robust-optimal control policy. Instead, trains can track the traction and brake effort specified at each track location by the robust-optimal control policy. The research showcased in this chapter contributes to increasing the robustness and reliability of railway operation and long-term plans, and to optimizing the energy usage for a more sustainable railway planning and operation. RTTO can be implemented in different railway systems: from mass rapid transit trains to Intercity services. It can

be used to mitigate rolling stock uncertainty and support strategic planning by exploring trajectory bundles that are robust against rolling stock choice and composition. This can help timetable planners and decision makers in long-term planning. The proposed method can be more useful in planning than in operational stages, since in operation feedback on the real-time performance of the train is available, and Model Predictive Control based on fast nominal TTO and real-time model calibration can be used to adapt the driving strategies according to the available feedback. Our research also paves the way to establishing a rigorous and reliable framework that will allow the development of Driver Advisory Systems and Automatic Train Operation that are robust against varying operating conditions.

The RTTO problem presented in this chapter opens several research questions. The proposed method could be extended to solve the robust two-train trajectory optimization problem. It could rely on the problem proposed in this chapter while adding appropriate headway constraints. An alternative formulation of the RTTO problem using as control variables the percentage of the available traction and brake of each train could be of interest. This would allow all trains to apply the traction and brake at their maximum available rates. However, larger speed and running time deviations should be expected if the trains differ significantly in their traction and brake capability. An alternative method for solving the RTTO problem based on indirect optimization methods or metaheuristics would also be of interest. Last, the integration of RTTO in Driver Advisory Systems and Automatic Train Operation should also be explored in order to bring robustness closer to the industry.

Chapter 9

Conclusions

9.1 Introduction

This thesis is devoted to solving the Train Trajectory Optimization under uncertain operating conditions.

The main objective of this thesis is to develop models and algorithms to calculate robust train trajectories that optimize the running time and energy consumption while considering uncertainties and stochastic variations in the model input parameters.

Train motion model calibration has been used to quantify the impact of the uncertainties and the stochasticity inherent in railway operation on the train dynamics. Estimates of the train motion model parameters are obtained and introduced in the Train Trajectory Optimization problem to calculate trajectories that are robust against the considered uncertain operating conditions.

This is the final chapter of the thesis. In chapter 9.2, the research questions that were outlined in chapter 1.2 are answered based on the main findings of this research. Several practical recommendations and open research topics are proposed in chapters 9.3 and 9.4, respectively.

9.2 Main findings

This chapter aims to answer the research questions outlined in chapter 1.2.

1. **How can the main uncertainties and variations in the characteristics of trains and railway operations be quantified?**

Data measured during real operation can be used to quantify the impact of the main uncertainties and stochasticity present in railway operation by calibrating the train motion model. A deep overview and discussion of the techniques used for train motion model calibration was showcased in chapter 2. These techniques can be classified in two main categories: offline techniques that analyze large quantities of data at once, and online techniques that update the calibration dynamically, as

soon as a new set of measurements is available. We observed that the existing literature was missing accurate online calibration methods for train motion models and offline techniques that produce estimates that are robust against measurement outliers or noise. Bridging both literature gaps is essential to advancing automation in the railway industry.

The robust offline estimation problem was tackled in chapter 3, where M-estimation was used to calibrate the running resistance parameters and a sliding median filter was utilized to calibrate the remaining train motion model parameters. The results obtained demonstrate that M-estimation outperforms the existing train motion model calibration techniques when it comes to producing robust estimates. Ten different M-estimation error measures were compared, and the most accurate ones were identified. Furthermore, statistics on the obtained parameter estimates were drawn and analyzed, and their individual values were used as input in chapters 6 and 8.

Online calibration was addressed in chapters 4 and 5. In chapter 4, an Unscented Kalman Filter was used to calibrate the train motion model parameters that are required as input for Train Trajectory Optimization. We observed that tractive and brake effort measurements are required to calibrate the train motion model accurately. The quality of the data used for calibrating the train motion model parameters was an issue in chapters 3 and 4. Chapter 5 focused on estimating the running resistance parameters. The parameter related to the aerodynamic drag is known to be more volatile than the other two running resistance parameters which quantify the mechanical resistances. An estimation method based on Recursive Least Squares with multiple forgetting factors was used to capture the different volatilities of the parameters successfully if the measurement sampling time was lower than or equal to 1 s. We tested this method in a numerical case study in which we were able to select the variations of the running resistance parameters and compare the accuracy of the proposed method with respect to the real values of the parameters. This method was also found to be robust against small measurement noise, and the produced estimates were shown to be usable for enhancing Train Trajectory Optimization.

2. What is the influence of these variations on optimized train trajectories?

The most critical parameters for Train Trajectory Optimization and energy-efficient driving were identified in chapter 6. A sensitivity analysis of the volatility of energy and time-optimal trajectories with respect to train motion model parameter variations was performed, observing that the quadratic running resistance parameter has a predominant role in shaping energy-optimal train trajectories with a cruise regime. The amount of energy consumed, the starting location of the coast phase, and the optimal cruise and brake speeds of energy-optimal train trajectories are mostly influenced by the quadratic value of the running resistance parameter. The train mass has an impact smaller than the quadratic running resistance parameter in this type of trajectories. The traction and brake characteristics are the most critical parameters in time-optimal trajectories and trajectories in which a cruise regime is absent, which is typically the case for short stops of regional trains.

3. How can Driver Advisory Systems incorporate dynamic, inaccurate and limited real-time information?

We elaborated on this research question in chapter 7. We showed in the context of Driver Advisory Systems how to calculate the expected arrival time at a train's destination dynamically based on the current train GPS location and speed, and on the gradients of the track. For the first time in the scientific literature, the analytic solutions of the train motion model differential equations have been fully described and analyzed by means of bifurcation theory and the Fundamental Theorem of Algebra. The formulas described in chapter 7 have been integrated in the recursive coasting advice calculator used in operation by NS. Bifurcation theory is used in the calculations to select which formula shall be used based on the train speed and current gradient, and to determine which speeds can be accessed by coasting from the current speed. The formulas are used to calculate the expected arrival time of the train, assuming that the train is coasting or will start coasting immediately. This calculation is performed every second using the current speed of the train and its location, which is measured by the GPS of the driver's tablet or of the cabin. The calculated expected arrival time is compared to the scheduled arrival time and driving advice is provided to the driver as a countdown to start coasting. The frequent recalculation of the coasting advice mitigates the adverse effects of inaccurate input data and accommodates immediately any scheduled arrival time update. Energy savings are realized by maximizing the amount of time spent coasting, so the amount of time in which the train driver applies traction is reduced while on-time arrival is guaranteed. The proposed formulas constitute an upgrade over the forward Euler method used by NS, which did not account for the track gradients. Furthermore, the algorithm was found by NS to be computationally more efficient than their previous calculation method. This is essential to conserve the driver's tablet battery, ensuring it lasts for an entire shift, as well as to allow to implement more complex driving advice in the future. The algorithm developed in chapter 7 was implemented in July 2022 in TimTim, the Driver Advisory System developed at NS. It is currently in use and is helping thousands of NS' drivers to transport 1.3 million passengers every day timely and sustainably by providing accurate driving advice to their drivers in real time.

4. How can the uncertainties present in train operation be integrated in a real-time train trajectory optimizer?

The Train Trajectory Optimization problem can incorporate the uncertainties present in train operation by modelling these uncertainties as variations in the train motion model parameters. In other words, Train Trajectory Optimization can be addressed under parametric uncertainty in the train motion model. These variations can be estimated using real data, like in chapter 3, and the estimates obtained can be included in the Train Trajectory Optimization problem by demanding the feasibility of its optimal solution for all the sets of parameters included in the problem. We developed this theory in chapter 8, where the Robust Train Trajectory Optimization problem under model parametric uncertainty is formulated. The solution of the Robust Train Trajectory Optimization problem is a robust-optimal control

policy, that is, the amount of traction and brake to be applied at each location of the track that would drive the train to its destination if the realized set of train motion model parameters falls within the convex hull of the parameters introduced in the problem. When applying the robust-optimal control policy on the train motion model, each of the sets of parameters introduced in the problem leads to a different speed profile. Thus, robust-optimal control policies lead to a trajectory bundle formed by each of the individual trajectories. The speed and time deviations along the train's path require defining an arrival condition consisting of arrival speed and time windows. The feasibility requirement will force all the trajectories in the bundle to arrive within both windows. In practice, trains track the speed in the optimal trajectories. Since the passing speed is not univocally defined after departure in Robust Train Trajectory Optimization, the robust-optimal control policy shall be tracked instead. We analyzed the Robust Train Trajectory Optimization problem mathematically by means of the Robust Maximum Principle. Five different driving regimes have been identified: traction and brake at the maximum rates feasible for all the sets of parameters introduced in the Robust Train Trajectory Optimization problem, coasting, and partial traction and brake. The latter two take place not only to avoid overspeeding and to reduce the maximum speed in sufficiently long track sections like in nominal Train Trajectory Optimization, but also to keep the speed dispersion along the train's journey low enough to favour the feasibility of the robust-optimal control policy. The Robust Train Trajectory Optimization problem was reformulated as a convex nonlinear optimization problem, which can be solved in less than a second. We showed in a computational example that tracking energy-efficient trajectories calculated by means of nominal Train Trajectory Optimization is not always possible under parametric uncertainty, but Robust Train Trajectory Optimization solves this issue. The proposed framework allows railway operators, and tactical and strategic planners to select the parametric uncertainty to be introduced in the Robust Train Trajectory Optimization problem. We showed this by calculating energy-efficient robust-optimal trajectory bundles under uncertainty in the rolling stock to be deployed and robust control policies that mitigate the impact of uncertain operating conditions on optimal train trajectories.

5. How can Train Path Envelopes be computed, so that they are robust against these variations and drivable train trajectories are guaranteed?

Robust Train Path Envelopes have been defined in chapter 8. The main difference with respect to nominal Train Path Envelopes is that headways between consecutive trains are allocated from the earliest and latest passing times specified by the robust-optimal trajectory bundle. As shown in a simulated case study, this strategy allocates wider time windows to trains travelling longer distances and showing larger running time variations due to the parametric uncertainty. These trains are also the ones that are more likely to show larger driving variations. Consequently, the traffic plans computed by means of Robust Train Path Envelopes provide more robustness against driving variations and parametric uncertainty, lowering the likelihood of primary and knock-on delays.

9.3 Recommendations for practice

The following practical recommendations are drawn from the research presented in this thesis:

- Special care should be dedicated to calibrating the train motion model accurately in order to guarantee the precise performance of railway tools, since the values provided by rolling stock manufacturers are in general not accurate. In this regard, the literature review presented in chapter 2 and the methods proposed in chapters 3-5 are of special help for railway practitioners and scholars. Offline calibration should be used to calibrate models and tools that use static train motion model parameters along the whole trajectory. Online calibration should complement offline calibration whenever spatiotemporal phenomena lead to parameter variations that reduce the performance of a statically-parametrized train motion model and for tools that adapt and update their outputs in real time, like advanced versions of Driver Advisory Systems and Automatic Train Operation.
- Railway undertakings should promote training train drivers to drive efficiently by applying higher traction and brake rates. Coasting strategies are frequently described in energy-efficient driving trainings, but the impact of high traction and brake rates in energy-efficiency is often overlooked. Although applying high traction consumes more energy than low traction, the time gained by accelerating faster allows to drive at lower speeds and to coast for longer, so the time spent at higher speeds with higher losses due to the running resistance is lower. The results presented in chapters 3 and 6 show that an adequate driving style has potential for saving energy and for increasing the capacity of railway networks. Energy-efficient driving techniques may be complemented with other measures that account for the interaction among trains, like traction and brake synchronization and power peak shaving strategies. When it comes to robust manual driving, the efficient driving training mentioned is still relevant despite Robust Train Trajectory Optimization prescribes using worst-case scenario traction and brake rates. The energy-efficient driving style of each driver must be analyzed to capture the traction and brake rates performed by drivers under each operating condition of interest. The observed driving characteristics shall be introduced in the Robust Train Trajectory Optimization problem to ensure the adequacy, accuracy and efficacy of the computed robust driving strategies.
- Infrastructure managers should make an effort to have an up-to-date and accurate description of the track geometry, and make it publicly accessible. It would allow an effective implementation of energy-efficient driving measures and automation, as well as benefit research.
- Robust Train Trajectory Optimization can be a valuable tool for the industry, particularly in strategic and tactical planning stages. However, the industry must go step by step. Nominal Train Trajectory Optimization should be implemented first across the various industry actors before aiming to implement more advanced technologies like Robust Train Trajectory Optimization. Still, the first practical implementations

of Robust Train Trajectory Optimization could take place in early planning stages with uncertainty in the rolling stock to be deployed. The robust energy-optimal trajectory bundles computed for a group of different rolling stock units could support decision-making and strategic planning, allowing to optimize long-term planning and resource allocation also from an energy-efficient perspective.

- Railway operators should use Robust Train Path Envelopes to compute real-time traffic plans. Robust Train Path Envelopes provide robustness against parametric uncertainty and different operating conditions. Time windows are smartly allocated, offering more width to trains that travel longer distances, which are more likely to show more driving deviations that cause primary delays or hinder adjacent trains causing knock-on delays.

9.4 Recommendations for future research

Last, several open research questions are outlined.

- A quantitative assessment of the impact of weather on the dynamics of a train through parameter estimation would be of interest to highlight the need to tailor the timetables and the main driving strategies used in Driver Advisory Systems and Automatic Train Operation to the actual operating conditions. Highly accurate measurements of the train location, speed and tractive and brake effort would be necessary to obtain high quality estimates. Furthermore, the measurement sampling rate should be at least 1 measurement per second. Historical weather data should be used to classify the estimates obtained. These estimates could be classified in terms of the existing weather conditions at the time the measurements were recorded and potential correlations between the estimates and the weather conditions could be identified. Wind strength and direction, and wet or dry rail conditions are some examples of the weather characteristics that could be considered in such study. Further open research questions regarding train motion model calibration can be found in chapter 2.
- Train Trajectory Optimization under uncertainty could also be addressed by means of stochastic optimal control. This method can be considered as an alternative to the Robust Train Trajectory Optimization approach presented in this thesis. The data analysis and noise characterization challenges outlined in chapter 1.1 should be overcome before comparing the performance of this method and Robust Train Trajectory Optimization.
- Analyzing which factors are preventing railway companies to incorporate Train Trajectory Optimization and other energy-efficient driving measures would also be of interest. I hypothesize that more competition between railway undertakings, market liberalization and decentralization may help Train Trajectory Optimization to permeate railways, since external pressure can help overcome the historical inertia of the whole railway industry towards innovation.

Appendix: List of Estimates

r_0 [N/kg]	r_1 [N/(kg m/s)]	r_2 [N/(kg(m/s) ²)]	$u_{f,\max}$ [N/kg]	$u_{b,\max}$ [N/kg]	$p_{\max}$ [W/kg]
6.54e-03	3.95e-04	2.04e-05	0.520	0.243	5.46
6.55e-03	4.96e-04	2.00e-05	0.263	0.162	5.47
2.92e-03	5.47e-04	2.04e-05	0.426	0.162	5.49
9.54e-03	1.18e-04	3.05e-05	0.447	0.081	5.52
2.32e-03	9.51e-04	1.07e-05	0.319	0.162	5.44
3.30e-03	5.93e-04	2.04e-05	0.516	0.162	4.75
8.88e-03	0.95e-04	3.10e-05	0.326	0.162	5.46
2.26e-03	7.19e-04	1.75e-05	0.348	0.166	5.59
4.79e-03	4.97e-04	2.11e-05	0.277	0.162	5.49
6.34e-03	4.86e-04	1.94e-05	0.236	0.081	5.54
10.2e-03	3.11e-04	2.13e-05	0.263	0.243	5.45
8.89e-03	0.82e-04	3.15e-05	0.481	0.118	5.52
7.11e-03	3.66e-04	2.08e-05	0.520	0.155	5.49
8.94e-03	3.36e-04	2.10e-05	0.364	0.166	5.48
4.23e-03	6.70e-04	1.44e-05	0.272	0.162	4.21
7.78e-03	3.02e-04	2.24e-05	0.319	0.169	5.51
5.62e-03	5.23e-04	1.28e-05	0.253	0.162	2.95
4.41e-03	7.03e-04	1.13e-05	0.409	0.166	5.55
6.19e-03	2.97e-04	2.39e-05	0.229	0.166	5.43
6.41e-03	1.97e-04	2.36e-05	0.270	0.162	5.49
7.13e-03	3.59e-04	1.86e-05	0.312	0.166	5.52
8.05e-03	0.96e-04	2.51e-05	0.270	0.166	5.54
5.24e-03	3.34e-04	2.12e-05	0.249	0.162	5.49
3.83e-03	4.58e-04	1.64e-05	0.367	0.243	5.47
7.33e-03	1.76e-04	2.26e-05	0.263	0.166	5.50
10.7e-03	0.89e-04	2.34e-05	0.357	0.162	5.47
6.50e-03	4.15e-04	1.50e-05	0.381	0.162	5.39
4.57e-03	5.46e-04	1.76e-05	0.416	0.162	5.45
7.84e-03	1.57e-04	2.75e-05	0.263	0.162	5.43
7.67e-03	0.42e-04	2.90e-05	0.346	0.243	5.47
5.98e-03	1.65e-04	2.64e-05	0.291	0.162	3.02
8.91e-03	0.91e-04	2.69e-05	0.423	0.243	5.56
5.56e-03	3.92e-04	1.91e-05	0.513	0.162	5.43

8.81e-03	2.17e-04	2.22e-05	0.190	0.162	5.53
9.20e-03	1.68e-04	2.17e-05	0.391	0.250	5.40
6.08e-03	1.48e-04	2.54e-05	0.478	0.169	5.43
10.4e-03	0.37e-04	2.73e-05	0.291	0.166	5.53
5.96e-03	3.52e-04	1.44e-05	0.256	0.162	5.51
9.18e-03	0.28e-04	2.36e-05	0.208	0.166	5.47
7.94e-03	3.37e-04	1.90e-05	0.256	0.166	5.55
8.87e-03	1.25e-04	2.77e-05	0.343	0.162	3.84
8.41e-03	1.19e-04	2.74e-05	0.263	0.199	5.52
6.67e-03	3.96e-04	1.65e-05	0.305	0.166	5.54
9.35e-03	3.02e-04	2.23e-05	0.263	0.166	5.49
7.56e-03	2.59e-04	2.62e-05	0.360	0.166	5.51
8.50e-03	1.87e-04	2.53e-05	0.270	0.162	3.44
1.07e-03	0.42e-04	2.60e-05	0.478	0.162	5.24
4.61e-03	3.08e-04	2.63e-05	0.416	0.162	5.49
7.03e-03	1.79e-04	2.61e-05	0.229	0.162	5.47
6.14e-03	4.62e-04	1.83e-05	0.478	0.162	5.54
6.71e-03	2.16e-04	2.15e-05	0.398	0.247	4.49
7.43e-03	1.68e-04	2.45e-05	0.520	0.162	4.45
6.40e-03	2.27e-04	2.46e-05	0.357	0.162	5.50
3.30e-03	7.14e-04	1.19e-05	0.371	0.162	5.52
7.39e-03	3.81e-04	1.77e-05	0.384	0.166	5.53
6.15e-03	2.88e-04	2.24e-05	0.336	0.166	2.99
4.26e-03	5.09e-04	1.51e-05	0.516	0.162	2.83
3.88e-03	6.15e-04	1.52e-05	0.294	0.162	5.49
6.14e-03	3.00e-04	1.99e-05	0.520	0.247	5.51
5.01e-03	5.00e-04	1.41e-05	0.516	0.162	5.48
8.30e-03	1.23e-04	2.23e-05	0.336	0.166	5.51
7.11e-03	3.51e-04	1.89e-05	0.301	0.166	4.09
6.50e-03	1.98e-04	2.40e-05	0.520	0.162	5.49
6.43e-03	3.18e-04	1.97e-05	0.341	0.162	5.52
7.63e-03	2.15e-04	2.17e-05	0.412	0.169	5.47
2.85e-03	5.84e-04	1.47e-05	0.409	0.169	5.47
7.62e-03	2.18e-04	2.20e-05	0.426	0.184	4.99
11.7e-03	2.00e-04	2.11e-05	0.270	0.162	5.50
8.28e-03	1.22e-04	2.07e-05	0.170	0.166	4.82
7.18e-03	5.39e-04	1.78e-05	0.215	0.118	5.46
6.05e-03	5.05e-04	1.54e-05	0.301	0.162	5.42
7.98e-03	1.84e-04	2.34e-05	0.371	0.081	5.48
3.40e-03	7.45e-04	1.28e-05	0.263	0.162	2.40
9.52e-03	2.60e-04	2.44e-05	0.450	0.162	5.52
7.35e-03	5.67e-04	1.36e-05	0.329	0.162	5.39
9.21e-03	1.78e-04	2.53e-05	0.305	0.162	5.51
6.90e-03	3.05e-04	2.18e-05	0.298	0.162	5.41
9.54e-03	0.15e-04	2.90e-05	0.436	0.162	5.50

8.07e-03	1.01e-04	2.78e-05	0.381	0.162	5.53
8.51e-03	1.27e-04	2.48e-05	0.395	0.162	5.45
4.22e-03	6.04e-04	1.33e-05	0.284	0.162	4.57
8.53e-03	4.44e-04	1.42e-05	0.353	0.162	5.49
6.86e-03	3.57e-04	2.04e-05	0.256	0.162	5.51
5.66e-03	3.19e-04	2.01e-05	0.270	0.166	2.85
7.47e-03	2.61e-04	2.19e-05	0.513	0.162	5.49
8.75e-03	2.12e-04	1.79e-05	0.374	0.162	5.47
8.64e-03	2.90e-04	1.93e-05	0.197	0.162	5.39
7.97e-03	1.81e-04	1.83e-05	0.416	0.162	5.48
8.41e-03	1.00e-04	1.95e-05	0.364	0.162	5.47
2.45e-03	5.45e-04	1.81e-05	0.225	0.166	5.55
8.00e-03	2.40e-04	1.85e-05	0.364	0.162	5.51
9.24e-03	0.05e-04	2.50e-05	0.277	0.162	5.44
2.38e-03	6.96e-04	1.44e-05	0.346	0.213	5.57
8.95e-03	1.95e-04	2.26e-05	0.263	0.162	5.52
9.87e-03	2.00e-04	1.66e-05	0.364	0.162	5.53
2.21e-03	9.26e-04	1.49e-05	0.274	0.166	4.22
3.98e-03	4.99e-04	1.94e-05	0.450	0.166	5.52
4.42e-03	5.57e-04	2.10e-05	0.256	0.166	5.47
7.92e-03	2.35e-04	2.32e-05	0.398	0.162	5.11
3.10e-03	3.63e-04	2.05e-05	0.263	0.162	5.51
5.72e-03	4.80e-04	1.74e-05	0.402	0.162	5.40
6.92e-03	2.17e-04	2.57e-05	0.308	0.162	5.46
6.50e-03	3.99e-04	2.02e-05	0.305	0.162	5.56
4.72e-03	5.95e-04	1.53e-05	0.388	0.162	5.51
6.45e-03	2.92e-04	2.60e-05	0.267	0.166	5.44
4.23e-03	4.70e-04	1.99e-05	0.419	0.162	5.50
2.98e-03	4.79e-04	1.64e-05	0.312	0.162	5.56
4.72e-03	3.80e-04	2.15e-05	0.263	0.166	5.51
10.1e-03	1.36e-04	2.32e-05	0.312	0.166	5.45
6.90e-03	2.62e-04	2.15e-05	0.388	0.162	5.52
10.5e-03	0.27e-04	2.34e-05	0.284	0.166	5.52
7.79e-03	1.26e-04	2.38e-05	0.450	0.162	5.48
5.61e-03	2.67e-04	2.27e-05	0.520	0.162	5.55
5.12e-03	3.08e-04	1.59e-05	0.270	0.162	5.49
4.96e-03	5.15e-04	1.38e-05	0.329	0.162	5.59
10.5e-03	1.01e-04	2.38e-05	0.506	0.162	5.50
4.99e-03	4.55e-04	1.29e-05	0.218	0.162	5.43
5.18e-03	5.72e-04	1.49e-05	0.398	0.162	5.47
4.76e-03	5.27e-04	1.17e-05	0.222	0.166	5.56
9.77e-03	1.76e-04	1.93e-05	0.339	0.162	5.46
4.16e-03	3.98e-04	1.69e-05	0.336	0.162	5.46
6.55e-03	7.15e-04	1.30e-05	0.381	0.162	4.41
9.09e-03	2.87e-04	2.08e-05	0.270	0.162	2.76

10.7e-03	0.96e-04	2.76e-05	0.409	0.166	4.83
9.78e-03	1.10e-04	2.71e-05	0.305	0.162	5.51
6.60e-03	2.17e-04	2.71e-05	0.450	0.162	5.49
6.17e-03	3.54e-04	2.16e-05	0.332	0.162	4.55
7.87e-03	1.61e-04	2.60e-05	0.457	0.162	4.71
7.48e-03	2.54e-04	2.47e-05	0.506	0.162	4.64
5.36e-03	4.53e-04	2.73e-05	0.229	0.166	4.67
7.63e-03	0.44e-04	3.08e-05	0.454	0.166	4.63
7.25e-03	5.23e-04	1.92e-05	0.423	0.166	4.67
9.28e-03	1.04e-04	2.86e-05	0.298	0.162	4.73
4.62e-03	6.02e-04	2.02e-05	0.263	0.162	4.57
6.77e-03	3.63e-04	2.11e-05	0.263	0.166	4.63
3.59e-03	7.77e-04	1.00e-05	0.431	0.162	4.50
6.71e-03	4.00e-04	1.52e-05	0.520	0.166	4.64
5.41e-03	3.19e-04	1.88e-05	0.291	0.169	4.56
7.96e-03	0.46e-04	3.03e-05	0.378	0.162	4.60
4.15e-03	5.23e-04	2.03e-05	0.284	0.162	4.65
5.86e-03	3.41e-04	2.38e-05	0.319	0.162	4.32
6.81e-03	3.21e-04	2.68e-05	0.378	0.166	4.57
7.01e-03	2.25e-04	2.30e-05	0.454	0.162	4.55
5.63e-03	2.85e-04	2.36e-05	0.369	0.166	4.58
3.90e-03	2.43e-04	2.92e-05	0.391	0.161	4.62
6.47e-03	1.95e-04	2.34e-05	0.287	0.166	4.56
7.66e-03	0.23e-04	2.92e-05	0.436	0.162	5.47
7.95e-03	0.40e-04	2.99e-05	0.343	0.162	5.49
5.98e-03	1.44e-04	3.09e-05	0.249	0.166	5.47
4.19e-03	4.21e-04	1.98e-05	0.256	0.162	5.37
6.10e-03	6.30e-04	1.44e-05	0.232	0.232	5.43
5.11e-03	3.43e-04	2.41e-05	0.277	0.166	5.46
9.20e-03	0.73e-04	2.68e-05	0.284	0.162	5.36
3.53e-03	4.86e-04	1.79e-05	0.454	0.287	5.49
10.2e-03	0.63e-04	3.01e-05	0.384	0.162	4.49
2.27e-03	6.03e-04	2.13e-05	0.367	0.162	5.61
7.76e-03	0.72e-04	2.94e-05	0.326	0.162	5.51
3.67e-03	6.96e-04	1.76e-05	0.263	0.166	5.17
8.96e-03	0.14e-04	3.05e-05	0.298	0.162	5.48
6.23e-03	3.53e-04	2.39e-05	0.274	0.166	5.47
4.66e-03	5.39e-04	1.83e-05	0.423	0.162	5.48
3.31e-03	7.28e-04	2.14e-05	0.263	0.162	5.49
7.30e-03	3.61e-04	1.70e-05	0.350	0.162	5.46
5.15e-03	4.07e-04	2.31e-05	0.357	0.166	5.45
8.99e-03	0.07e-04	3.37e-05	0.319	0.162	5.50
5.88e-03	0.51e-04	3.35e-05	0.326	0.166	5.52
9.24e-03	0.94e-04	2.85e-05	0.246	0.162	5.53
4.52e-03	4.11e-04	2.65e-05	0.305	0.166	5.50

7.91e-03	1.83e-04	2.66e-05	0.263	0.166	5.21
8.20e-03	0.57e-04	3.08e-05	0.332	0.247	5.50
4.98e-03	4.14e-04	1.93e-05	0.488	0.162	5.56
4.53e-03	2.64e-04	2.31e-05	0.329	0.162	5.57
11.2e-03	0.06e-04	2.70e-05	0.343	0.166	5.48
8.27e-03	1.40e-04	2.64e-05	0.464	0.166	5.53
8.62e-03	0.84e-04	3.03e-05	0.166	0.166	5.51
8.80e-03	1.05e-04	2.49e-05	0.291	0.166	5.51
3.30e-03	3.10e-04	2.45e-05	0.291	0.162	5.49
7.25e-03	0.44e-04	2.98e-05	0.284	0.166	5.56
8.93e-03	2.05e-04	2.49e-05	0.343	0.166	5.52
2.41e-03	5.92e-04	1.67e-05	0.305	0.155	5.66
7.99e-03	1.66e-04	2.16e-05	0.263	0.162	5.49
6.36e-03	1.55e-04	2.61e-05	0.298	0.169	5.43
7.96e-03	0.83e-04	2.78e-05	0.513	0.166	5.43
5.15e-03	3.37e-04	2.23e-05	0.506	0.162	5.46
8.48e-03	1.27e-04	2.55e-05	0.440	0.247	5.51
3.30e-03	5.61e-04	1.29e-05	0.253	0.166	5.51
6.51e-03	0.96e-04	3.07e-05	0.218	0.166	5.08
8.46e-03	1.14e-04	2.77e-05	0.350	0.247	5.53
7.37e-03	3.05e-04	2.27e-05	0.423	0.118	5.55
7.16e-03	0.73e-04	3.06e-05	0.274	0.162	5.53
10.4e-03	2.02e-04	2.26e-05	0.239	0.166	5.51

Table A.1: 191 parameter estimates obtained in Chapter 3 using the Fair measure (Cunillera et al., 2023d).

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List of Abbreviations

The following abbreviations are used in this thesis:

ATB-EG	<i>Automatische Treinbeïnvloeding - Eerste Generatie,</i> Automatic Train Protection - First Generation
ATB-NG	<i>Automatische Treinbeïnvloeding - Nieuwe Generatie,</i> Automatic Train Protection - New Generation
ATO	Automatic Train Operation
ATP	Automatic Train Protection
CEN	<i>Comité Européen de Normalisation,</i> European Committee for Standardization
CFD	Computational Fluid Dynamics
CSC	Complementary Slackness Conditions
DAS	Driver Advisory Systems
DTW	Dynamic Time Warping (DTW)
EEA	European Environmental Agency
EKF	Extended Kalman Filter
ERTMS	European Rail Traffic Management System
ETCS	European Train Control System
GA	Genetic Algorithm
GHG	Greenhouse Gas
GNSS	Global Navigation Satellite System
GPOPS	General Purpose OPTimal control Software
GPS	Global Positioning System
HVAC	Heating, Ventilation and Air Conditioning
IEA	International Energy Agency
ILI	Iterative Learning Identification
IT	Information Technology
LGR	Legendre-Gauss-Radau
LSR	Least Squares Regression
OLS	Ordinary Least Squares
MILS	Multi-Innovation Least Squares
MLE	Maximum Likelihood Estimation
NLP	Nonlinear Programming
NS	<i>Nederlandse Spoorwegen,</i> the Dutch railways
TPE	Train Path Envelope
TTO	Train Trajectory Optimization/Optimizer
PROMO	PseudospectRal Optimal train control MOdel

PSO	Particle Swarm Optimization
RLS	Recursive Least Squares
RMSE	Root Mean Square Error
RTPE	Robust Train Path Envelope
RTTO	Robust Train Trajectory Optimization/Optimizer
SA	Simulated Annealing
SLT	Sprinter Light Train
SQP	Sequential Quadratic Programming
UIC	<i>Union internationale des chemins de fer</i> , International Union of Railways
UKF	Unscented Kalman Filter
UT	Unscented Transformation
UZI	<i>Dutch Universeel Zuinig rijden Idee</i> , Universal energy-efficient driving idea
V-MILS	Interval-Varying Multi-Innovation Least Squares
VIRM	<i>Verlengd InterRegio Materieel</i> , Lengthened interregional rolling stock

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Summary

In railway operation, punctuality is prioritized over energy savings. However, the current political sustainability goals, the cost of energy and the benefits of corporate environmental responsibility on public image drive railway operators to implement energy-efficient driving strategies. Train Trajectory Optimization can be used to calculate the reference speed to be tracked by a train at each location of the track in order to arrive on time with minimum energy consumption. Driver Advisory Systems and Automatic Train Operation utilize these trajectories to provide driving advice or to drive the train timely and energy-efficiently. In practice, the stochasticity of railway operation lowers the accuracy of the train motion model used in Train Trajectory Optimization to predict and reproduce the dynamics of the train, hindering the performance of Train Trajectory Optimizers. For instance, headwinds increase the aerodynamic drag of trains and their energy consumption, while tailwinds reduce them. In light of this challenge, this thesis is devoted to characterizing the impact of the uncertainties and the stochasticity inherent in railway operation and to developing models and algorithms that calculate train trajectories that are robust against them.

The uncertainties and stochasticity of railway operation like weather, aerodynamic drag and the variability in manual driving have an impact on the train dynamics and in the applied traction and brake rates. Train motion model calibration is used to characterize these variations in the dynamics through estimation of the train motion model parameters. We reviewed the existing train motion model calibration techniques, finding that they can be classified in two main categories. Offline calibration techniques analyze data measured in one or more train trajectories, calculating a set of parameter estimates that fit best for the whole dataset or for each of the trajectories. Online calibration techniques analyze data in real time, as soon as new measurements are available, and are able to capture how parameters vary along the train's trajectory. From the assessment of the state of the art in train motion model calibration we proposed a research agenda and practical recommendations. We explored both offline and online calibration techniques in this thesis. In offline calibration, measurement outliers or wrong data can bias the estimation, reducing the accuracy of the obtained parameters. M-estimation is an offline estimation technique we used to calibrate the running resistance of a train. It produces estimates robust against measurement outliers because its mathematical properties allow it to leave outliers out of the estimation or to give them less weight than to normal measurements. We complemented this technique with a sliding mean filter, which was used to calibrate the bounds of the traction and brake characteristics, and the brake rates applied by the train at different speeds. In total, 191 trajectories of the same train were analyzed, obtaining 191 sets of parameter estimates that would be used later in other

chapters of this thesis. We observed that M-estimation outperformed other train motion calibration techniques in terms of robustness against outliers and calculated probability distributions for the calculated parameter estimates. Regarding online estimation, we used an Unscented Kalman Filter to calibrate the train motion model parameters in real time. We integrated it in a framework that distinguishes whether the train is applying traction to accelerate, holding a constant speed, coasting or braking. We found that this method requires accurate measurements of the applied traction and brake to calibrate the running resistance. During our data analysis for train motion model calibration, we noticed that the lack of ground-truth data on the parameters hinders assessing the accuracy of the estimates. We estimated the running resistance parameters by means of Recursive Least Squares with multiple forgetting factors in several computational case studies that included different parameter variations and noise measurements. We quantified the accuracy of the estimates, observing that the running resistance can be accurately estimated after an adequate initial guess of their values. Furthermore, we found the proposed method robust against additive Gaussian measurement noise and different measurement sampling rates.

After calibrating the train motion model parameters, we assessed the influence of each parameter on the shape of optimal train trajectories. We used a real-time Train Trajectory Optimizer to perform a sensitivity analysis of the volatility of time- and energy-optimal trajectories with respect to individual variations of each of the train motion model parameters. This analysis allowed us to observe that in long trajectories that contain a cruise speed regime, the quadratic running resistance parameter has a predominant role in determining the optimal cruise speed and the locations of the switching points between driving regimes. In turn, in shorter trajectories, traction and brake characteristics have a larger impact on the shape of optimal trajectories. Manually-driven trains show larger variations in the traction and brake rates applied, so manual driving-induced variations have an impact in the energy consumption of optimal trajectories. Optimal trajectories were calculated using each of the 191 parameter estimates computed using M-estimation, observing large variations in the shape of the optimal trajectories due to the dispersion in the maximum traction and brake applied by the drivers.

We explored how to incorporate dynamic, inaccurate and limited real-time information on Driver Advisory Systems. The train motion model is a nonlinear dynamical system that can be solved analytically. We calculated and provided a complete overview of the analytical solutions of the train motion model for the first time in the scientific literature. These analytical solutions have different mathematical formulas, depending on the values of the train motion model parameters, the amount of traction or brake applied and the resistance to motion due to the track gradients. Bifurcation theory shows the existence of a terminal speed, and which speeds can be accessed under constant track gradients and traction or brake force applied. We incorporated these formulas in July 2022 in TimTim, the Driver Advisory System currently used by the drivers of NS. They are helping thousands of drivers to transport 1.3 million passengers daily on time and sustainably. The formulas proposed are used to calculate from the current GPS location data and train speed when the train would stop if it started coasting immediately. This expected arrival time is compared with the timetable and driving advice is provided as a countdown to when to start coasting to arrive on time at the next stop. This driving

advice is recalculated recursively and updated every second to mitigate negative effects due to measurement noise or other sources of inaccurate and limited data. The proposed driving advice calculation method improved the accuracy and computation time of the previous version of TimTim, which did not consider track gradients. Additionally, a lower battery usage of the tablets in which TimTim is installed was observed due to the lower computation times.

We have also formulated the Robust Train Trajectory Optimization problem under parametric uncertainty. The parametric uncertainty is introduced as sets of parameter estimates representing different operating conditions of interest. This problem aims to calculate the most energy-efficient driving strategy that is feasible for all the considered sets of parameter estimates. That is, this driving strategy would successfully drive a train to its destination under each of the considered operating conditions. For instance, the estimates introduced can be those that were calculated by means of M-estimation. However, introducing different sets of parameters in the train motion model and executing the same amount of traction and brake in each parametrized model leads to a different speed and passing time for each set of parameters. This means that the speed and the running time is not univocally defined at each location after departure. In Robust Train Trajectory Optimization, the optimal train trajectory concept is extended to a robust-optimal trajectory bundle that gathers the individual trajectories that arise from each set of parameters. These differences in the speeds and running times require more flexibility in how arriving at destination is defined in the Robust Train Trajectory Optimization problem. The train will no longer arrive at destination with a certain speed and at a specific time, but at different speeds and times for each set of parameters. Thus, an arrival time and speed window shall be defined in the Robust Train Trajectory Optimization problem to guarantee the existence of robust-optimal driving strategies. We analyzed the Robust Train Trajectory Optimization problem by means of the Robust Maximum Principle, deducing that up to 5 driving regimes can be observed in a robust-optimal driving strategy. These are traction and brake at the maximum rates allowed by all parameter configurations, coasting, and partial traction and brake. The maximum traction and brake are more conservative regimes than their nominal counterparts. Partial traction and brake may arise in robust-optimal driving strategies in order to control the speed dispersion by driving at reduced speeds, aiming to guarantee the feasibility of optimal trajectories when the arrival speed window is small. Our simulations showed that Robust Train Trajectory Optimization can be used to mitigate the impact of uncertain operating conditions like the weather, and in strategic planning to calculate early timetables under uncertainty in the rolling stock to be deployed.

Last, we have proposed a definition of Robust Train Path Envelopes against parametric uncertainty based on Robust Train Trajectory Optimization. In railway operation, Train Path Envelopes are based on real-time traffic plans to specify passing times and windows to individual trains that ensure smooth operation without hindering other trains. Thus, they are designed to prevent primary and knock-on delays. The headways between consecutive trains are computed from the passing times specified by the optimal trajectory. However, the passing times are not univocally defined after departure in the robust-optimal trajectory bundle. Robust Train Path Envelopes compute the minimum headways between adjacent trains from the earliest and latest passing times predicted by

the robust-optimal trajectory bundle. This definition ensures robustness against parametric uncertainty. The simulations performed showed that a larger time bandwidth is allocated in the Robust Train Path Envelopes of those trains that cover longer distances and are more likely to show driving speed and time dispersions, providing additional robustness against volatility in driving and running times.

Finally, this thesis paves the way for designing and implementing more accurate, robust and sustainable Driver Advisory Systems and Automatic Train Operation and to reducing the impact of the large uncertainty present in strategic planning stages.

Samenvatting

In spoorwegverkeer krijgt punctualiteit voorrang op energiebesparing. De huidige politieke duurzaamheidsdoelstellingen, de energiekosten en de voordelen van maatschappelijk verantwoord ondernemen voor het publieke imago stimuleren spoorwegondernemingen echter om energiezuinige rijstrategieën te implementeren. Trajectorieoptimalisering voor treinen (Train Trajectory Optimization) kan worden gebruikt om de referentiesnelheid te berekenen die een trein op elke locatie van het spoor moet aanhouden om op tijd aan te komen met een minimaal energieverbruik. Machinistadvies-systemen (Driver Advisory Systems) en automatische treinbesturing (Automatic Train Operation) gebruiken deze trajectoriën om rijadvies te geven of om de trein tijdig en energiezuinig te besturen. In de praktijk vermindert de stochasticiteit van het treinverkeer de nauwkeurigheid van het treinbewegingsmodel dat bij trajectorieoptimalisering wordt gebruikt om de dynamiek van de trein te voorspellen en te reproduceren, wat de prestaties van trajectorieoptimalisering belemmert. Tegenwind verhoogt bijvoorbeeld de aerodynamische weerstand van treinen en hun energieverbruik, terwijl rugwind deze vermindert. Met het oog op deze uitdaging is dit proefschrift gewijd aan het karakteriseren van de impact van de onzekerheden en de stochasticiteit die inherent zijn aan de spoorwegverkeer, en aan het ontwikkelen van modellen en algoritmen die treintrajectoriën berekenen die hiertegen bestand zijn.

De onzekerheden en stochasticiteit van de het spoorwegverkeer, zoals het weer, aerodynamische weerstand en de variabiliteit in handmatig rijden, hebben een impact op de treindynamiek en op de toegepaste tractie- en remkrachten. Kalibratie van het treinbewegingsmodel wordt gebruikt om deze variaties in de dynamiek te karakteriseren door middel van het schatten van de parameters van het model. We hebben de bestaande technieken voor de kalibratie van treinbewegingsmodellen beoordeeld en vastgesteld dat deze in twee hoofdcategorieën kunnen worden onderverdeeld. Offline kalibratietechnieken analyseren gegevens die zijn gemeten tijdens één of meer treintrajectoriën, waarbij een set parameterschattingen wordt berekend die het beste past bij de gehele dataset of bij elk van de trajectoriën. Online kalibratietechnieken analyseren gegevens in realtime, zodra er nieuwe metingen beschikbaar zijn, en zijn in staat om vast te leggen hoe parameters variëren langs de trajectorie van de trein. Op basis van de beoordeling van de huidige stand van de techniek in de kalibratie van treinbewegingsmodellen hebben we een onderzoeksagenda en praktische aanbevelingen voorgesteld. In dit proefschrift hebben we zowel offline als online kalibratietechnieken onderzocht. Bij offline kalibratie kunnen uitschieters in de metingen of onjuiste gegevens de schatting beïnvloeden, wat de nauwkeurigheid van de verkregen parameters vermindert. M-estimation is een offline schattingstechniek die we hebben gebruikt om de rijweerstand van een trein te kalibreren. Het levert schattingen op die robuust zijn tegen uitschieters, omdat de wiskundige eigenschappen ervan

toelaten om uitschieters buiten de schatting te laten of ze minder gewicht te geven dan normale metingen. We hebben deze techniek aangevuld met een ‘sliding mean filter’, dat werd gebruikt om de grenzen van de tractie- en remkenmerken te kalibreren, evenals de remkrachten die door de trein bij verschillende snelheden worden toegepast. In totaal werden 191 trajectoriën van dezelfde trein geanalyseerd, wat 191 configuraties parameterschattingen opleverde die later in andere hoofdstukken van dit proefschrift zouden worden gebruikt. We stelden vast dat M-estimation beter presteerde dan andere kalibratietechnieken voor treinbewegingen in termen van robuustheid tegen uitschieters, en we berekenden kansverdelingen voor de berekende parameterschattingen. Wat betreft online schatting hebben we een ‘Unscented Kalman Filter’ gebruikt om de parameters van het treinbewegingsmodel in realtime te kalibreren. We hebben dit geïntegreerd in een raamwerk dat onderscheid maakt of de trein tractie toepast om te versnellen, een constante snelheid aanhoudt, uitrolt of remt. We ontdekten dat deze methode nauwkeurige metingen van de toegepaste tractie en remkracht vereist om de rijweerstand te kalibreren. Tijdens onze data-analyse voor de kalibratie van het treinbewegingsmodel merkten we dat het gebrek aan ‘ground-truth’ data over de parameters het beoordelen van de nauwkeurigheid van de schattingen belemmert. We hebben de parameters van de rijweerstand geschat door middel van ‘Recursive Least Squares’ met meerdere vergeetfactoren in verschillende casestudy’s, die verschillende parametervariaties en meetruis bevatten. We hebben de nauwkeurigheid van de schattingen gekwantificeerd en geconstateerd dat de rijweerstand nauwkeurig kan worden geschat na een adequate initiële inschatting van hun waarden. Bovendien vonden we de voorgestelde methode robuust tegen additieve Gaussische meetruis en verschillende steekproefgroottes.

Na het kalibreren van de parameters van het treinbewegingsmodel hebben we de invloed van elke parameter op de vorm van optimale treintrajectoriën beoordeeld. We hebben een realtime trajectorieoptimaliseringsalgoritme gebruikt om een gevoeligheidsanalyse uit te voeren naar de volatiliteit van tijd- en energie-optimale trajectoriën met betrekking tot individuele variaties van elk van de parameters van het treinbewegingsmodel. Deze analyse stelde ons in staat om te observeren dat bij lange trajectoriën met een kruissnelheidsregime, de kwadratische rijweerstandsparemeter een overheersende rol speelt bij het bepalen van de optimale kruissnelheid en de locaties van de overgangspunten tussen rijregimes. Bij kortere trajectoriën hebben tractie- en remkenmerken daarentegen een grotere impact op de vorm van optimale trajectoriën. Handmatig bestuurdde treinen vertonen grotere variaties in de toegepaste tractie- en remkrachten, waardoor door handmatig rijden veroorzaakte variaties invloed hebben op het energieverbruik van optimale trajectoriën. Optimale trajectoriën werden berekend met behulp van elk van de 191 parameterschattingen die via M-estimation waren berekend, waarbij grote variaties in de vorm van de optimale trajectoriën werden waargenomen als gevolg van de spreiding in de maximale tractie en remkracht die door de machinisten werd toegepast.

We hebben onderzocht hoe dynamische, onnauwkeurige en beperkte realtime informatie kan worden geïntegreerd in machinistadvies-systemen (Driver Advisory Systems). Het treinbewegingsmodel is een niet-lineair dynamisch systeem dat analytisch kan worden opgelost. We hebben voor het eerst in de wetenschappelijke literatuur een volledig overzicht van de analytische oplossingen van het treinbewegingsmodel berekend en verstrekt. Deze analytische oplossingen hebben verschillende wiskundige formules, afhankelijk van

de waarden van de parameters van het treinbewegingsmodel, de hoeveelheid toegepaste tractie of remkracht en de bewegingsweerstand als gevolg van de helling van het spoor. Bifurcatietheorie toont het bestaan van een eindsnelheid aan, en welke snelheden bereikbaar zijn onder constante hellingen en toegepaste tractie- of remkracht. We hebben deze formules in juli 2022 geïntegreerd in TimTim, het machinistadvies-systeem dat momenteel door de machinisten van de NS wordt gebruikt. Ze helpen duizenden machinisten om dagelijks 1,3 miljoen passagiers op tijd en duurzaam te vervoeren. De voorgestelde formules worden gebruikt om op basis van de huidige GPS-locatiegegevens en treinsnelheid te berekenen wanneer de trein zou stoppen als deze onmiddellijk zou beginnen met uitrollen. Deze verwachte aankomsttijd wordt vergeleken met de dienstregeling, waarna rijadvies wordt gegeven in de vorm van een countdown tot het moment waarop het uitrollen moet beginnen om op tijd bij de volgende halte aan te komen. Dit rijadvies wordt recursief herberekend en elke seconde bijgewerkt om negatieve effecten door meetruis of andere bronnen van onnauwkeurige en beperkte gegevens te beperken. De voorgestelde berekeningsmethode voor rijadvies verbeterde de nauwkeurigheid en de rekentijd van de vorige versie van TimTim, die geen rekening hield met hellingen. Daarnaast werd een lager batterijverbruik waargenomen van de tablets waarop TimTim is geïnstalleerd, dankzij de kortere rekentijden.

We hebben ook het probleem van robuuste treintrajectorieoptimalisering (Robust Train Trajectory Optimization) geformuleerd onder parametrische onzekerheid. De parametrische onzekerheid wordt geïntroduceerd als configuraties van parameterschattingen die verschillende relevante bedrijfsomstandigheden vertegenwoordigen. Dit probleem heeft tot doel de meest energie-efficiënte rijstrategie te berekenen die haalbaar is voor alle parameterschattingen. Dat wil zeggen dat deze rijstrategie een trein succesvol naar zijn bestemming zou leiden onder elk van de meegenomen bedrijfsomstandigheden. Deze geïntroduceerde schattingen kunnen bijvoorbeeld de schattingen zijn die door middel van M-estimation zijn berekend. Het introduceren van verschillende configuraties parameters in het treinbewegingsmodel en het uitvoeren van dezelfde hoeveelheid tractie en remkracht in elk geparametriseerd model leidt echter tot een verschillende snelheid en doorkomsttijd voor elke configuratie parameters. Dit betekent dat de snelheid en de rijtijd na vertrek op elke locatie niet eenduidig zijn gedefinieerd. In Robust Train Trajectory Optimization wordt het concept van het optimale treintraject uitgebreid naar een robuust-optimale trajectoriebundel, die de individuele trajectoriën verzamelt die voortvloeien uit elke configuratie parameters. Deze verschillen in snelheden en rijtijden vereisen meer flexibiliteit in de manier waarop het aankomen op een bestemming wordt gedefinieerd in het robuuste optimalisatieprobleem. De trein komt niet langer aan op de bestemming met een bepaalde snelheid en op een specifiek tijdstip, maar op verschillende snelheden en tijden voor elke configuratie parameters. Daarom moet een venster voor aankomsttijd en -snelheid worden gedefinieerd om het bestaan van robuust-optimale rijstrategieën te garanderen. We hebben het Robust Train Trajectory Optimization-probleem geanalyseerd met behulp van het Robust Maximum Principle, waaruit we hebben afgeleid dat er tot vijf rijregimes kunnen worden waargenomen in een robuust-optimale rijstrategie. Dit zijn tractie en remmen met de maximale kracht die is toegestaan door alle parameterconfiguraties, uitrollen (coasting), en gedeeltelijke tractie en remkracht. De maximale tractie en remkracht zijn conservatievere regimes dan hun nominale tegenhangers. De laatste twee regimes worden

anders gebruikt dan bij nominale treintrajectorieoptimalisering. Gedeeltelijke tractie en remkracht kunnen voorkomen in robuust-optimale rijstrategieën om de snelheidsspreiding te beheersen door met lagere snelheden te rijden, met als doel de haalbaarheid van optimale trajectoriën te garanderen wanneer het venster voor de aankomstsnelheid klein is. Onze simulaties toonden aan dat Robust Train Trajectory Optimization kan worden gebruikt om de impact van onzekere bedrijfsomstandigheden, zoals het weer, te verzachten, en in de planning om vroege dienstregelingen te berekenen onder onzekerheid over het in te zetten rollend materieel.

Ten slotte hebben we een definitie voorgesteld voor robuuste treinpad-bandbreedtes (Robust Train Path Envelopes) tegen parametrische onzekerheid, gebaseerd op robuuste treintrajectorieoptimalisering. In het treinverkeer worden treinpadvensters gebruikt op basis van realtime verkeersplannen om doorkomsttijden en vensters specificeren die een soepele treinenloop garanderen zonder andere treinen te hinderen. Zo voorkomen ze primaire vertragingen en volgvertragingen. De opvolgtijden tussen opeenvolgende treinen worden berekend op basis van de doorkomsttijden die in het optimale trajectorie zijn gespecificeerd. In de robuust-optimale trajectoriebundel zijn de doorkomsttijden na vertrek echter niet eenduidig gedefinieerd. Robuuste treinpadvensters berekenen de minimale opvolgtijden tussen aangrenzende treinen op basis van de vroegste en laatste doorkomsttijden die door de robuust-optimale trajectoriebundel worden voorspeld. Deze definitie garandeert robuustheid tegen parametrische onzekerheid. De uitgevoerde simulaties toonden aan dat er een grotere tijdbandbreedte wordt toegewezen aan treinen die langere afstanden afleggen en waarbij de kans op spreiding in rijnsnelheid en rijtijd groter is, wat extra robuustheid biedt tegen volatiliteit in het rijgedrag en de rijtijden.

Ten slotte baant dit proefschrift de weg voor het ontwerpen en implementeren van nauwkeurigere, robuustere en duurzamere bestuurdersadvies-systemen en automatische treinbesturing, en voor het verminderen van de impact van de grote onzekerheid die aanwezig is in strategische planningsfasen.

Resumen

En la operación ferroviaria, la puntualidad tiene mayor prioridad sobre el ahorro energético. Sin embargo, los actuales objetivos políticos de sostenibilidad, el coste de la energía y los beneficios de la responsabilidad ambiental corporativa en la imagen pública impulsan a las operadoras ferroviarias a implementar estrategias de conducción eficientes. Optimización de trayectorias puede utilizarse para calcular la velocidad de referencia que debe seguir un tren en cada punto de la vía para llegar a tiempo a su destino con un consumo mínimo de energía. Los Sistemas de Ayuda a la Conducción y el ATO utilizan estas trayectorias para proporcionar ayudas a la conducción o para conducir el tren de forma puntual y eficiente. En la práctica, la estocasticidad de la operación ferroviaria reduce la precisión del modelo de movimiento del tren que se utiliza en la optimización para predecir y reproducir su dinámica, lo que dificulta el rendimiento de los optimizadores de trayectorias. Por ejemplo, el viento en contra aumenta la resistencia aerodinámica y el consumo, mientras que el viento a favor los reduce. Ante este reto, el objetivo de esta tesis es caracterizar el impacto de la incertidumbre y la estocasticidad inherentes a la operación ferroviaria y a desarrollar modelos y algoritmos que calculan trayectorias robustas frente a ellas.

Las incertidumbres de la operación, como la meteorología, la resistencia aerodinámica y la variabilidad típica de la conducción manual, influyen en la dinámica del tren y en la cantidad de tracción y freno aplicadas. La calibración del modelo de movimiento del tren se utiliza en esta tesis para caracterizar las mencionadas variaciones mediante estimación de parámetros. Hemos revisado las técnicas existentes de estimación de parámetros, las cuales pueden clasificarse en dos categorías principales. Aquellas técnicas de calibración que analizan en diferido datos medidos en una o más trayectorias, calculando el conjunto de estimaciones que se ajustan mejor a todo el conjunto de datos o a cada una de las trayectorias. Las técnicas de calibración a tiempo real analizan los datos a medida que se reciben nuevas mediciones, y son capaces de captar cómo varían los parámetros a lo largo del trayecto del tren. A partir de la evaluación del estado del arte, propusimos una agenda de investigación y recomendaciones prácticas. En esta tesis exploramos ambas técnicas. En la calibración en diferido, los valores atípicos o los datos erróneos pueden sesgar la estimación. La M-estimación es una técnica de calibración en diferido que utilizamos para estimar la resistencia al avance del tren. Produce estimaciones robustas porque permite excluir los valores atípicos o darles un menor peso. Complementamos esta técnica con un filtro de media móvil para calibrar los límites de las características de tracción y freno, y las tasas de frenado aplicadas por el conductor a diferentes velocidades. En total se analizaron 191 trayectorias del mismo tren, obteniendo 191 estimaciones. La M-estimación superó a otras técnicas similares en robustez. También calculamos distribuciones de probabilidad

para los parámetros obtenidos. En cuanto a la estimación online, utilizamos un Filtro de Kalman Unscented para calibrar los parámetros en tiempo real, integrándolo en un marco que distingue si el tren está traccionando, si mantiene su velocidad, si deriva o está frenando. Hemos observado que este método requiere mediciones precisas de la tracción y el freno aplicados. También notamos que la falta de datos reales de referencia de cada uno de los parámetros dificulta evaluar precisión real de las estimaciones. Hemos utilizado Mínimos Cuadrados Recursivos con múltiples factores de olvido para estimar la resistencia al avance en varios casos de estudio computacionales, observando que la resistencia puede estimarse con gran precisión si se inicializa adecuadamente el valor de las estimaciones. Además, observamos que este método es robusto frente a ruido gaussiano aditivo en las mediciones y frente a diferentes tasas de muestreo.

Tras la calibración del modelo de movimiento del tren, evaluamos la influencia de cada parámetro en la forma de las trayectorias óptimas. Utilizamos un optimizador de trayectorias en tiempo real para realizar un análisis de sensibilidad de la volatilidad de las trayectorias óptimas en tiempo y energía respecto a variaciones individuales de cada uno de los parámetros del modelo. Esto nos permitió observar que, en trayectorias largas con régimen de velocidad de crucero, el parámetro cuadrático de la resistencia al avance es el más determinante en la determinación de la velocidad de crucero y los puntos de cambio entre los distintos regímenes de conducción. En trayectorias más cortas, las características de tracción y freno tienen un mayor impacto. Los trenes conducidos manualmente muestran mayores variaciones en las tasas de tracción y freno aplicadas, lo que repercute en el consumo energético. Se calcularon trayectorias óptimas utilizando cada una de las 191 estimaciones obtenidas mediante M-estimación, observando grandes variaciones en su forma debido a la dispersión en el uso de la tracción y el freno por parte de los maquinistas.

Exploramos cómo incorporar información en tiempo real, dinámica e imprecisa en sistemas de ayuda a la conducción. El modelo de movimiento del tren es un sistema dinámico no lineal que puede resolverse analíticamente. Calculamos y proporcionamos una descripción completa de sus soluciones analíticas por primera vez en la literatura científica. Estas soluciones dependen de los parámetros del modelo, la fuerza aplicada y la resistencia al avance por pendiente de la vía. La teoría de bifurcaciones muestra la existencia de una velocidad terminal y qué velocidades son alcanzables bajo condiciones constantes. En julio de 2022, incorporamos estas fórmulas en TimTim, el sistema a la conducción utilizado actualmente por los maquinistas de NS. Ayudan a miles de maquinistas a transportar diariamente a 1,3 millones de pasajeros de forma puntual y sostenible. Las fórmulas calculan, a partir del GPS y la velocidad, cuando se detendría el tren si empezara a derivar inmediatamente. Este tiempo estimado se compara con el horario y se ofrece un consejo de conducción en forma de cuenta atrás para iniciar la deriva. El tiempo estimado de llegada mediante deriva se recalcula cada segundo para mitigar efectos negativos de ruido o datos limitados. Este método mejoró la precisión y el tiempo de cálculo de la versión anterior, que no consideraba las pendientes de la vía, observándose además un menor consumo de batería en las tabletas gracias a la mayor eficiencia computacional.

También hemos formulado el problema de la optimización robusta de trayectorias bajo incertidumbre paramétrica. Esta incertidumbre se introduce en la optimización como conjuntos de estimaciones de parámetros que representan diferentes condiciones opera-

tivas de interés. El objetivo es calcular la estrategia de conducción más eficiente que sea factible para todos los conjuntos de parámetros considerados; es decir, una estrategia que lleve al tren a su destino sin importar qué condición operativa sucederá cuando se ejecute la estrategia. Por ejemplo, pueden introducirse las estimaciones calculadas mediante M-estimación. Sin embargo, introducir distintos parámetros en el modelo del tren y aplicar la misma tracción y frenado sobre cada uno de ellos produce variaciones en las velocidades y tiempos de paso tras la salida, lo que significa que estos no están definidos unívocamente tras la salida. En la Optimización Robusta, el concepto de trayectoria óptima se extiende a un haz de trayectorias robusto-óptimas que agrupa las trayectorias individuales que surgen de cada conjunto de parámetros. Estas diferencias exigen más flexibilidad al definir lo que es llegar a destino: el tren ya no llega a una hora y velocidad fijas, sino dentro de rango. Por ello, debe definirse una ventana de tiempo y velocidades de llegada en el problema de optimización robusta de trayectorias para garantizar la existencia de estrategias de conducción robusto-óptimas. Analizamos este problema mediante el Principio del Máximo Robusto, deduciendo que pueden observarse hasta cinco regímenes de conducción: tracción y frenado máximos (a la mayor tasa permitida por todas las configuraciones), deriva, y tracción y frenado parciales. Estos dos últimos regímenes toman un papel distinto respecto a la optimización de trayectorias determinista, ya que surgen para controlar la dispersión de la velocidad del haz de trayectorias conduciendo a bajas velocidades, garantizando la viabilidad cuando la ventana de llegada es estrecha. Las simulaciones demostraron que esta teoría reduce el impacto de condiciones de operación inciertas como la meteorología y ayuda en la planificación estratégica de horarios cuando existe incertidumbre sobre el material rodante a emplear.

Por último, hemos propuesto una definición de Envoltentes de Paso Robustas (Robust Train Path Envelopes) frente a la incertidumbre paramétrica. En la operación ferroviaria, estas envoltentes se basan en planes de tráfico en tiempo real para especificar ventanas de paso a cada tren que aseguran una operación fluida y están diseñadas para evitar retrasos primarios y en cadena. Los intervalos mínimos entre trenes se calculan habitualmente según los tiempos de paso de la trayectoria óptima. Sin embargo, estos tiempos de paso no están unívocamente definidos en el haz de trayectorias robusto-óptimas. Las Envoltentes de Paso Robustas asignan los intervalos mínimos basándose en los tiempos de paso más tempranos y tardíos predichos por el haz. Las simulaciones mostraron que el método propuesto asigna un mayor ancho de banda temporal a los trenes que recorren distancias largas y son más propensos a sufrir dispersión de tiempos y velocidades, lo que aporta una robustez adicional frente a la volatilidad en los tiempos de marcha.

En conclusión, esta tesis allana el camino para diseñar e implementar sistemas de asistencia a la conducción y de ATO más precisos, robustos y sostenibles, y para reducir el impacto de la gran incertidumbre presente en las etapas de planificación estratégica.

Summary in Chinese

在铁路运营中，准点率优先于节能。然而，当前可持续发展的政策目标、能源成本以及企业环境责任对公众形象的益处，正促使铁路运营商实施节能驾驶策略。列车路径优化可用于计算列车在轨道上每个点必须保持的参考速度，以确保列车准时到达且能耗最低。驾驶员辅助系统和自动驾驶系统利用这些路径提供驾驶建议，或以高效节能的方式驾驶列车。实际上，铁路运营的随机性会降低路径优化中用于预测和重现列车动力学的运动模型精度，从而影响路径优化器的性能。例如，逆风会增加列车的空气阻力和能耗，而顺风则会降低这些阻力和能耗。面对这一挑战，本论文着重于：

刻画铁路运营中固有的不确定性和随机性的影响，

并开发能够计算出对这些不确定性和随机性列车轨迹的稳健模型和算法。

铁路运营中的不确定性和随机性，例如天气、空气动力阻力和人工驾驶的差异，都会影响列车动力学以及牵引和制动速率。列车运动模型校准用于通过预估模型参数来表征这些动力学变化。我们回顾了现有的列车运动模型校准技术，发现它们可以分为两大类。离线校准技术分析沿一条或多条列车轨迹测量的数据，计算一组最能拟合整个数据集或每条单独轨迹的参数估计值。在线校准技术在获得新的测量数据后立即进行实时分析，能够捕捉参数沿列车路径的变化情况。基于对列车运动模型校准技术现状的评估，我们提出了研究议程和实用建议。在此我们探讨了离线和在线校准技术。在离线校准中，测量异常值或错误数据会使估计值产生偏差，从而降低所获得参数的精度。M估计是一种离线估计技术，我们用它来校准列车的滚动阻力。该方法能够有效应对测量异常值，因为其数学特性允许将异常值从估计中排除或赋予其低于正常测量值的权重。我们采用移动平均滤波器来补充该技术，该滤波器用于校准牵引和制动特性的极限，以及列车在不同速度下施加的制动率。总共分析了同一列车的191条轨迹，得到了191组参数估计值，这些估计值将在本论文的其他章节中使用。我们观察到，就参数估计值的稳健性和计算出的概率分布而言，M估计优于其他列车运动校准技术。关于在线估计，我们使用UKF卡尔曼滤波器实时校准列车运动模型参数。我们将此方法集成到一个框架中，该框架能够区分列车是在加速、保持恒定速度、滑行还是制动状态下施加牵引力。我们发现，该方法需要精确测量施加的牵引力和制动力，才能校准滚动阻力。在列车运动模型校准的数据分析过程中，我们发现由于缺乏参数的真实世界数据，难以评估估计值的准确性。我们利用递归最小二乘法，结合多个遗忘因子，在多个计算案例研究中估计了滚动阻力参数，这些案例研究涵盖了不同的参数变化和噪声测量。我们量化了估计值的准确性，发现只要对滚动阻力参数进行适当的初始估计，就可以对其进行精确估计。此外，我们还发现所提出的方法对噪声具有稳健估测。

在校准列车运动模型参数后，我们评估了每个参数对最优列车轨迹形状的影响。使用实时列车轨迹优化器，对时间最优和能量最优轨迹的波动性进行了敏感性分析，以研究列车运动模型中每个参数的单独变化。分析结果表明，在包含巡航速度段的长轨迹中，二次运行阻力参数在决定最优巡航速度和驾驶模式切换点的位置方面起着主

导作用。而在较短的轨迹中，牵引和制动特性对最优轨迹的形状影响更大。人工驾驶的列车在牵引和制动速率方面表现出更大的变化，因此人工驾驶引起的变化会对最优轨迹的能耗产生影响。利用M估计计算得到的191个参数估计值，计算了最优轨迹。观察到最优轨迹形状存在较大差异，这是由于驾驶员施加的最大牵引力和制动力存在分散性所致。

我们探讨了如何将动态的、不精确的和有限的实时信息纳入驾驶员辅助系统。列车运动模型是一个非线性动力系统，可以通过解析方法求解。我们计算并提供了列车运动模型解析解的完整概述，这在科学文献中尚属首次。这些解析解具有不同的数学公式，取决于列车运动模型参数的值、施加的牵引力或制动力的的大小以及轨道坡度引起的运动阻力。分岔理论表明存在一个终端速度，以及在恒定轨道坡度和施加的牵引力或制动力下可以达到的速度。我们在2022年7月将这些公式整合到了TimTim中，TimTim是NS公司目前为司机提供的驾驶员辅助系统。这些公式帮助数千名司机每天准时且可持续地运送130万名乘客。这些公式用于根据当前的GPS位置数据和列车速度计算列车如果立即滑行，何时会停下来。（即预计到达时间）系统会将这个预计到达时间与时刻表进行比较，并提供驾驶建议，以倒计时的形式提示何时开始滑行，以便准时到达下一站。此驾驶建议会递归地重新计算，并每秒更新一次，以减轻测量噪声或其他不准确或有限的数据来源造成的负面影响。所提出的驾驶建议计算方法提高了TimTim先前版本的准确性和计算速度，先前版本未考虑轨道坡度。此外，由于计算时间缩短，安装了TimTim的平板电脑的电池消耗量也降低了。

我们还构建了参数不确定性下的列车轨迹稳健优化问题。参数不确定性以参数估计集的形式引入，这些参数估计集代表了不同的运行工况。该问题的目标是计算在所有考虑的参数估计集下都可行的最节能驾驶策略。也就是说，该驾驶策略能够在每种考虑的运行工况下成功地将列车驶至目的地。例如，这些估计值可以通过M估计计算得出。然而，在列车运动模型中引入不同的参数集，并在每个参数化模型中执行相同的牵引和制动量，会导致每组参数对应的速度和通过时间不同。这意味着列车出发后，在每个位置的速度和运行时间并非唯一确定。在列车轨迹稳健优化中，最优列车轨迹的概念被扩展到稳健最优轨迹束，该轨迹束汇集了由每组参数产生的单个轨迹。速度和运行时间的差异要求在列车轨迹稳健优化问题中，对到达目的地的定义方式更加灵活。列车不再以特定速度和时间到达目的地，而是根据不同的参数组合，以不同的速度和时间到达。因此，需要定义一个到达时间和速度窗口，以保证存在最优的稳健驾驶策略。我们运用最大值原理分析了列车轨迹稳健优化问题，得出结论：在最优的稳健驾驶策略中，最多可以观察到5种驾驶状态。这些状态包括：以所有参数配置允许的最大速率进行牵引和制动、滑行以及部分牵引和制动。最大牵引和制动状态比其标称对应状态更为保守。后两种状态在标称列车轨迹优化中不会出现。部分牵引和制动可能出现在最优的稳健驾驶策略中，目的是通过降低速度来控制速度分散，从而在到达速度窗口较小时保证最优轨迹的可行性。我们的模拟表明，稳健的列车轨迹优化可以用于减轻天气等不确定运行条件的影响，并在战略规划中用于计算在待部署的铁路车辆存在不确定性的情况下制定的早期时刻表。

之后，我们基于列车轨迹稳健优化，提出了一种应对参数不确定性的稳健列车路径包络线定义。在铁路运营中，列车路径包络线用作实时交通规划，指定列车通过时间和窗口，以确保列车平稳运行，避免对其他列车造成干扰。因此，它们可以防止主要延误和连锁延误。相邻列车之间的间隔时间由最优轨迹指定的通过时间计算得出。然而，在稳健最优轨迹束中，列车出发后的通过时间并非唯一确定。列车稳健路径包络线根据最优稳健轨迹束预测的最早和最晚通过时间计算相邻列车之间的最小间隔时间。该定义确保了对参数不确定性的稳健性。仿真结果表明，对于行驶距离较长且行

驶速度和时间波动较大的列车，交通规划中分配的时间带宽更大，从而进一步增强了对行驶时间和运行时间波动的稳健性。

最后，本论文为设计和实施更准确、更稳健、更可持续的驾驶员咨询系统和自动列车运行系统铺平了道路，并减少了战略规划阶段中存在的巨大不确定性的影响。

About the author

Alejandro (Alex) Cunillera Pérez was born in Palma de Mallorca, Spain, in 1995. In 2017 he obtained his BSc degrees in Mathematics and Physics at Universidad Complutense de Madrid, Spain. In 2019 he obtained his MSc degree in Physics of Complex Systems at the Institute for Cross-Disciplinary Physics and Complex Systems in Palma de Mallorca with a master thesis on reconstructing time series by means of reservoir computing. In 2024 he obtained a postgraduate degree in Finance at the Massachusetts Institute of Technology.

In 2019 Alex worked as a research consultant at the Instituto de Investigación Tecnológica in Madrid, designing energy-efficient speed profiles for metro, high speed and regional trains, and developing train simulators and ERTMS capacity calculators.

Pursuing his dream to become a full-time researcher, in 2020 he moved to the Netherlands to start his PhD research at the Department of Transport and Planning of Delft University of Technology. His PhD research focused on fast algorithms to calculate train trajectories that are robust against the uncertainties and stochastic variations inherent to railway operation. His research interests include model calibration, mathematical modelling, optimization, optimal control and smart mobility.

From 2024 he works at Stadler Signalling in Switzerland, leading the company efforts in energy-efficient driving and developing Automatic Train Operation for CBTC.



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