Online physical model identification for database-driven safe fight envelope prediction of damaged aircraft

Zhang, Y; de Visser, CC; Chu, QP

DOI
10.2514/6.2016-2011

Publication date
2016

Document Version
Accepted author manuscript

Published in
Proceedings of the AIAA atmospheric flight mechanics conference

Citation (APA)
Zhang, Y., de Visser, CC., & Chu, QP. (2016). Online physical model identification for database-driven safe fight envelope prediction of damaged aircraft. In s.n. (Ed.), *Proceedings of the AIAA atmospheric flight mechanics conference* (pp. 1-11). American Institute of Aeronautics and Astronautics Inc. (AIAA). https://doi.org/10.2514/6.2016-2011

Online Physical Model Identification for Database-driven Safe Flight Envelope Prediction of Damaged Aircraft

Y. Zhang[*], C.C. de Visser[†], and Q.P. Chu[‡]

Delft University of Technology, Delft, Zuid-Holland, 2629HS, The Netherlands.

This paper presents a novel approach for online identification of the physical condition of damaged aircraft by monitoring and analysing the change of dynamic characteristics of each damage case. As a crucial building block of the database-driven safe flight envelope prediction system, this approach is intended to link the online identification to the offline generated database by finding out the specific key for onboard information retrieval of safe flight envelopes. Simulation results have successfully demonstrated the feasibility of the proposed concept and architecture.

I. Introduction

According to the surveys of past accidents, the damage to aircraft can lead to fatal aerodynamic instability and even loss of control. Adverse effects like aerodynamic change, structural degradation, engine damage and reduced flight control effectiveness may present during a damage situation and surprise the pilot. Any combinations of these will worsen the situation and eventually overwhelm a pilots ability to control an aircraft back to a norminal flight condition. Whereas the damage in some cases causes immediate, unrecoverable loss of control, others resulted in recoverable situations,[1] given that the pilot could have a clear awareness of the current situation. To aid the pilot and automation system to maintain controlled flight from an otherwise completely non-recoverable situation, a safe flight envelope prediction system is needed. However, barriers and limitations exist in several aspects, making online realisation of safe flight envelope extremely difficult. The calculation efficiency of safe flight envelope based on reachability analysis using numerical level set method is relatively low for real time applications. Besides, the limited range of new measurements after damage hinders the update of flight model in a global way.

To address these challenging problems, a variety of researches have been conducted in the past few years due to the paramount importance of knowledge of the safe flight envelope to airliners. Particularly, the development of safety-critical system attracts a lot of attentions from researchers. Belcastro from NASA Langley Research Center[2] presents a general framework for the validation and verification of complex integrated safe flight envelope protection system for preventing aircraft loss-of-control accidents. In his papers, Belcastro also gives a comprehensive summary of key technical challenges associated with the realisation of the system, including data acquisition and modelling of physics-based abnormal conditions, as well as the replication of loss-of-control environment, etc. Wilborn and Foster[3] try to define the loss-of-control accidents of commercial transport with a quantitative criteria composed of five envelopes relating to airplane flight dynamics, aerodynamics, structural integrity and flight control use. This quantitative approach is designed to reduce the occurrence of abnormal conditions, and is further studied by Chongvisal[4] and applied to the development of in-flight loss-of-control prediction and prevention systems to assist the pilot in flying the aircraft through adverse environmental conditions.[4] Whereas this quantitative approach may not be applicable

*PhD Student, Control and Simulation Section, Faculty of Aerospace Engineering, Delft University of Technology; Kluyverweg 1, 2629HS, Delft, The Netherlands.

†Assistant Professor, Control and Simulation Section, Faculty of Aerospace Engineering, Delft University of Technology; Kluyverweg 1, 2629HS, Delft, The Netherlands, Member.

‡Associate Professor, Control and Simulation Section, Faculty of Aerospace Engineering, Delft University of Technology; Kluyverweg 1, 2629HS, Delft, The Netherlands.

to the post-damage case when the nominal model is no longer valid, and the new safe operation boundaries need to be redefined.

The determination of safe flight envelope under off-nominal conditions in some works in this area is derived from the calculation of backward and forward reachable sets, which is connected to optimal control theory and level set method based on the continuous-time nonlinear models.[5–7] One major bottleneck of this algorithm is the feasibility of onboard computations. Alternatively, some researchers[8]approach to this issue with discrete-time linearized models to execute a progression of several small transitions through trim state space, which is said to enable fast prediction of recoverable sets. Another interesting idea is utilising a database as a part of safe flight envelope protection system, which is firstly mentioned in the paper of Tang et al.[9] However, no detailed discussions or further investigations on the implementation of such a database-driven prediction system is included in the paper. Similar ideas of storing different scales and types of aircraft system failures in a database are proposed by Moncayo et al.,[10] but the case of structural damages are not dealt with. The Database-driven method is key to the development our safe flight envelope system because it can circumvent the problem of online calculation and the lack of globally updated model. The key issue in this system is how to retrieve the right safe flight envelope given limited local measurements onboard. One inspiring idea is to use a representative physical model as the index to the database, which requires the online detection and isolation of faults and damages.

Numerous methods have been developed for fault detection and actuator health monitoring systems such as model-based residual-threshold[11] method and data-based artificial-immune-system approach,[10] which can precisely tell where and how bad the fault is. The development of load detection can also monitor the completeness of actuators in case they are partially damaged or have completely separated. Structural damage, however, is much more complicated to predict compared with other contributions to aircraft loss-of-control, because the destruction of airframe can introduce highly nonlinear components, mass property shifts and non-symmetric aerodynamics to the model, which is difficult to analytically estimate. Though it triggers many motivations and interests in this area, not many solid solutions are provided or applied to flight vehicles, due to their complex dynamic environment. A comprehensive review of the state-of-art methods for structural damage monitoring and assessment can be found in the work of Lopez and Sarigul-Klijn,[12] in which they elaborately described the challenges and opportunities of flight vehicle structural damage monitoring, diagnosis and control with uncertainties. In conjunction with a fault tolerant control system, Lombaerts et al. developed a nonlinear reconfigurable flight control method based on the identification of physical model of damaged aircraft, in which the valuable information contained in parameters is expected to indicate the physical state of the aircraft.[11]

Motivated by the challenges of the issue and inspired by the research work described above, we intend to design a system that integrates database techniques together with the identification of physical damages so that the online and offline activities can be separated from and interact with each other at the same time. The research work presented in this paper is a proof-of-concept study on an indispensable part of the system that links the online identified models to the retrieval of offline database.

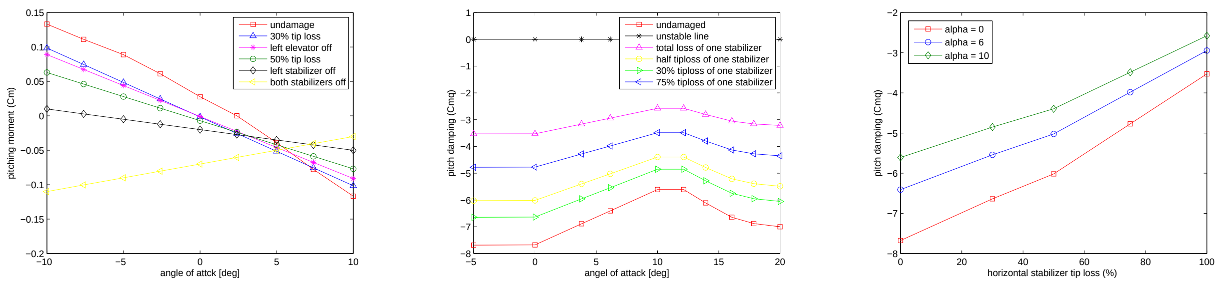
II. Aerodynamic Effects of Structural Damage

In order to correctly identify the type of damages, we should firstly know what to expect in each damage case, i.e., the aerodynamic characteristics of the partially damaged aircraft, which requires the damaged data from destructive tests. With the increasing concern for the safety of civil airliners, a number of experiments have been conducted to satisfy the expanding need for multiple sources of data (empirical, computational, and experimental). Earlier researches done by NASA Langley Research Center are a series of wind tunnel tests on a swept-wing airplane model to determine the effects of simulated wing damage on the aerodynamic characteristics of the model at Mach numbers larger than one. Wing damage was simulated by removal of either a leading-edge or a trailing-edge portion or an entire wing panel.[13] In recent years, the Generic Transport Model (GTM), a 5.5% scale model of a commercial aircraft has been the subject of a series of extensive wind tunnel tests and CFD experiments[14] undertaken by NASA[15] for the exploration of loss-of-control events involving various contributions such as stall/ high-angle-of-attack, airframe damages and icing, etc.[16]

The effect of structural damage is mainly a combination of aerodynamic changes, mass property shifts and control degradations. Fortunately, the control effectiveness can be evaluated in a relatively independent way through the isolation of actuator faults by advanced actuators fault detection systems. Mass properties, though coupled with moments and forces, seem not have a substantial effect on flight characteristics relative to the aerodynamic and control effects, according to an experiments conducted by Shah,[17] in which a large, asymmetric mass change (physical separation of an engine) was modeled. In this paper we mainly focus on the aerodynamic effects of structural damages based on the wind tunnel tests conducted by Shah et al.,[1,17] in which they model the effects of damages in the form of partial or total tip loss applied to the wings, horizontal tails, and vertical tail.

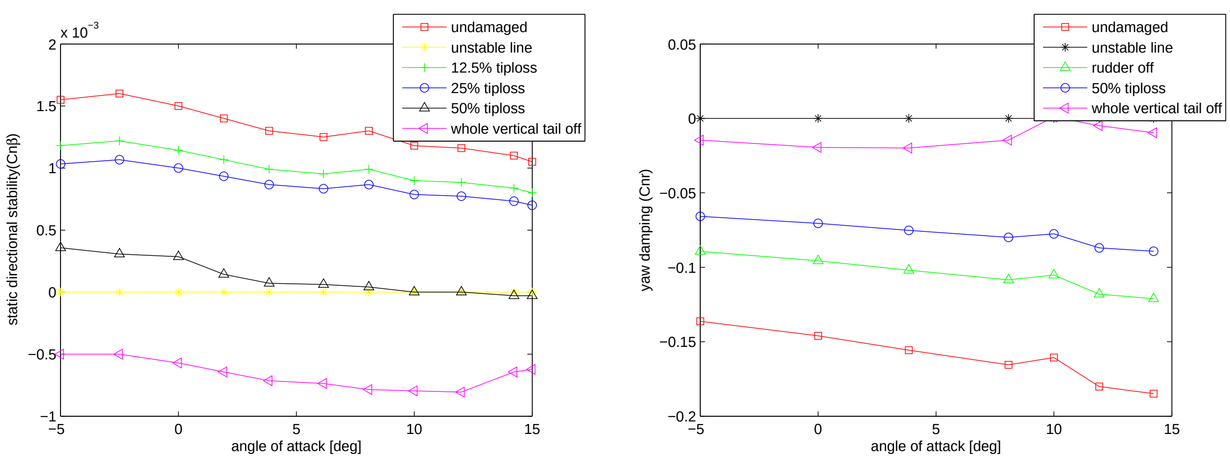
A. Horizontal Stabilizer Damage

The only stabilizing contribution to pitching moment is from horizontal stabilizers, so the damage of which will cause significant change in both static and dynamic longitudinal stability. Figure 1(a) shows static longitudinal stability in terms of pitching moment versus angle of attack for several damage cases.[1] A steady decrease in pitching moment and a trend from negative to positive (unstable) in C_{m_α} is observed with increasing area of damages. An important indication of dynamic stability, i.e., pitching damping C_{m_q} over the angle of attack range is depicted in figure 1(b). It is clearly shown in figure 1(c) that the reduction in pitch damping is roughly proportional to the scale of tip loss. Due to geometric asymmetry after one side damage or unequal damage of both tails, an incremental rolling moment is generated, which is another important feature of this kind of damage.



(a) (b) (c)

Figure 1. aerodynamic effects of horizontal tail damage



(a) (b)

Figure 2. aerodynamic effects of vertical tail loss

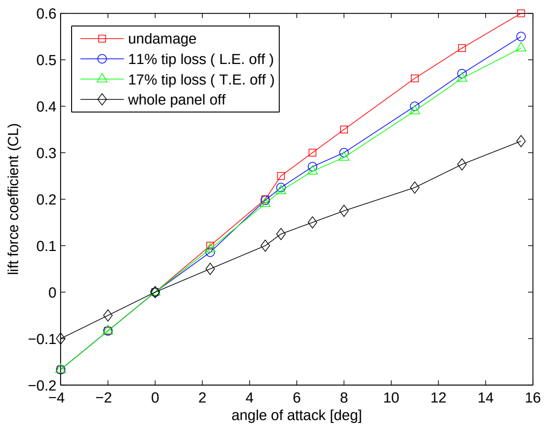
B. Vertical Tail Loss

The effects of vertical tail damage on directional characteristics is similar to those of horizontal tail damage on pitch axis, except that it rarely induces incremental rolling moments since the damage is basically symmetric.

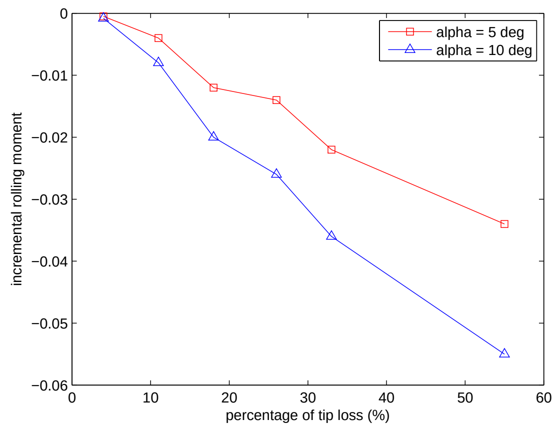
The change in lateral static and dynamic stability is illustrated in figure 2, from which we can see a decrease in yawing moment at the same angle of sideslip with increasing size of tip loss. As to static stability, the value C_{n_β} experiences an progressive decrease proportional to the scale of damage at the same angle of attack, but the change with respect to α remains almost the same. Yaw damping, as is displayed in figure 2(b), steadily decreases and approaches to zero (unstable limit) as larger area of tip loss is modeled. The trend, however, changes little throughout the whole range of α.

C. Wing Damage

One important difference between wing damage and tail damage is the progressive reduction of lift curve slope versus angle of attack, as displayed in figure 3(a). Similar to horizontal stabilizer damage, wing tip loss also results in asymmetrical aerodynamics, mostly incremental rolling moment, but in a more significant way. Beside, according to wind tunnel tests, the incremental rolling moments, which are generated by unequal normal force contributions from left to right wings, are roughly proportional to damaged scale at given angle of attack, which can be seen in figure 3(b).



(a)



(b)

Figure 3. aerodynamic effects of wing tip loss

III. Online Damaged Aircraft Model Identification

This section presents an early-stage development of a detection and diagnosis system for aircraft structural damages based on the two-step nonlinear aircraft model identification. The system will first measure the current forces and moments to tell if there exist a significant change that might be caused by structural damages. If the change is over a certain level, a diagnosis system will be initiated, using estimation of aerodynamics parameters to identify the specific type and the severity of each damage case based on its unique aerodynamic feature, as is investigated in the previous section. Since it is impractical (and impossible) to consider or predict all potential damage conditions, the study only focuses on tip damage to the wings, horizontal stabilizers and vertical tails.

A. Online Nonlinear Damaged Aircraft Model Identification

The identification method used in our study is called two-step method, which has been continuously developed at Delft University of Technology over the last 20 years.[11] What makes this method successful and eligible for online applications is that it decouples the joint of nonlinar state-estimation and model parameter estimation problem into two separate optimization problems. At first step, the nonlinear part is isolated in the aircraft state estimation using Kalman filter, which is always referred to as flight path reconstruction. After this the identification of aerodynamic model in the second step can be simplified as an equation error problem. Compared with other methods, the two-step method seems to be the most appropriate solution in this context,[11]because in structural damage cases, significant changes always occur to the aerodynamic model of the aircraft. If the system can produce accurate estimated states of the current flight condition at the first step, an a-priori onboard aerodynamic model can be updated independently in case of a model mismatch due to damage cases. Each aerodynamic moment model is a combination of a series of nonlinear functions of aircraft states and actuator deflection. Once the structure of the approximation model is determined, the aerodynamic coefficients can be calculated using least square or other parameter estimation methods. Among all the methods, multivariate simplex splines are always used for approximating aerodynamic model due to its high accuracy and flexibility in locally updating nonlinear models with uncertainties.[18–20]

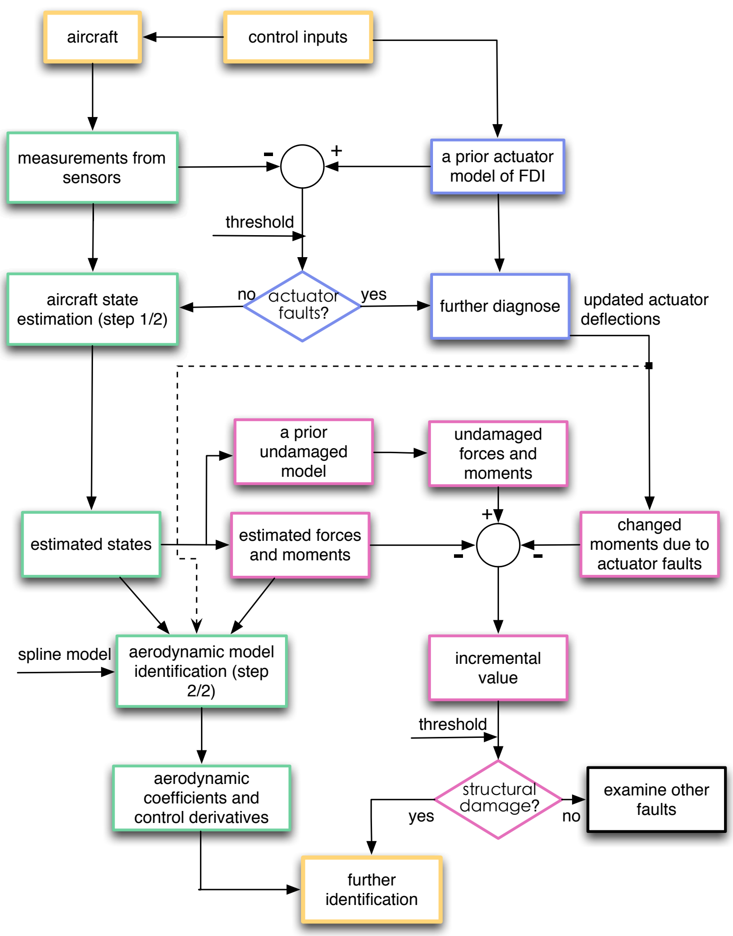


Figure 4. general process of damage detection and isolation

B. Damage Detection and Isolation Process

Various contributive factors can cause changes in forces and moments to different extends, and structural damage, like actuator faults, is one of them. If we can successfully separate these effects from the total contribution, then the structural damage can be accurately detected with appropriate thresholds. A preliminary damage detection scheme is illustrated in figure 4, on the assumption that sensor can function well and the change of mass properties are either trivial or perfectly known. The whole process is mainly composed of three parts, which are fault detection and isolation (FDI) system, aircraft model identification section based on two-step method described above, and structural damage detection that we propose in this paper. When the pilots command is given and control inputs are fed in, the measurements from sensor will be filtered to estimated current states and instrumentation biases through kinematic and observation model of the aircraft, which is the first step of two-step method. At the same time the system also need to examine the health of actuators by comparing the residual between actual deflections measured by sensors and the expected values from a prior actuator model with a predefined or adaptive threshold.

In the second step, the results of state estimation procedure and the inertial measurement unit properties are the available data for the following identification procedure. During this process, the observed moment coefficients and the expected forces and moments at the same flight condition via an a prior undamaged aerodynamic model or look-up table are calculated from the measured and filter states:

$$C_l = \frac{\dot{p}I_{xx} + qr\left(I_{zz} - I_{yy}\right) - \left(pq + \dot{r}\right)I_{xz}}{\frac{1}{2}\rho V^2 S b} \tag{1}$$

$$C_m = \frac{\dot{q}I_{yy} + rp\left(I_{xx} - I_{zz}\right) + \left(p^2 - r^2\right)I_{xz}}{\frac{1}{2}\rho V^2 S \bar{c}} \tag{2}$$

$$C_n = \frac{\dot{r}I_{zz} + pq\left(I_{yy} - I_{xx}\right) + \left(qr - \dot{p}\right)I_{xz}}{\frac{1}{2}\rho V^2 S b} \tag{3}$$

However, before making any abrupt decisions, we have to take into consider the contribution of actuator faults, because changes in either control effectiveness or the angle of deflections will arise variations in the total moments. So if a certain actuator fault cannot respond to the command as expected, the condition will soon be detected by the FDI system, and the part of moment induced by actuators will be evaluated. By subtracting it from the residual between calculated ideal value and measured actual value of moments, we can isolate the influence of structural damage from actuator faults, as shown below:

$$\Delta C_{dmg} = C_{actural} - C_{nominal} - \Delta C_{fault} \tag{4}$$

If the modified residual is deviated from a predefined or adaptive threshold, a structural damage can be declared in the respective steering channel. Together with the identified aerodynamic model using polynomial model or more advanced spline model in step two, further estimation of damage type and severity will be processed.

C. Physical Model Identification

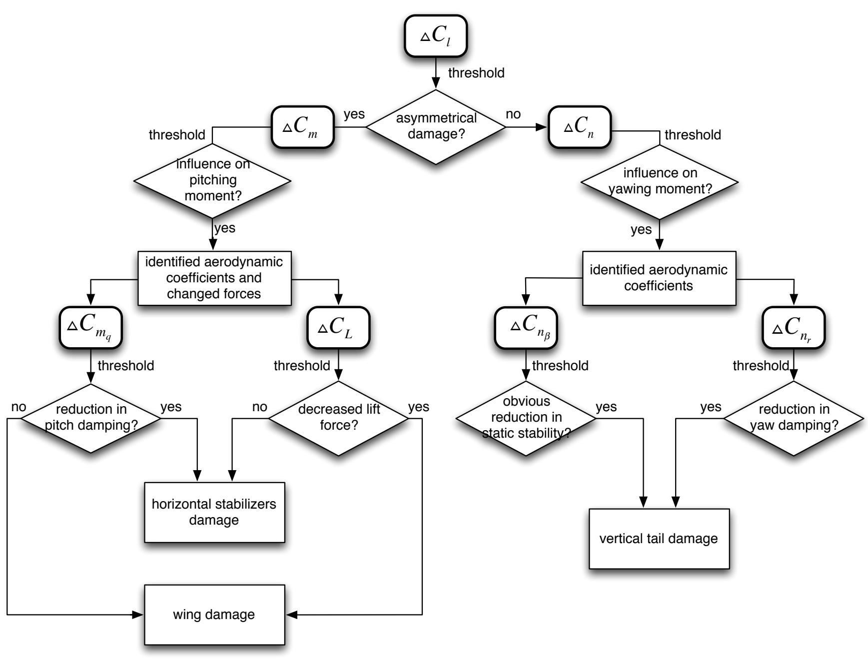
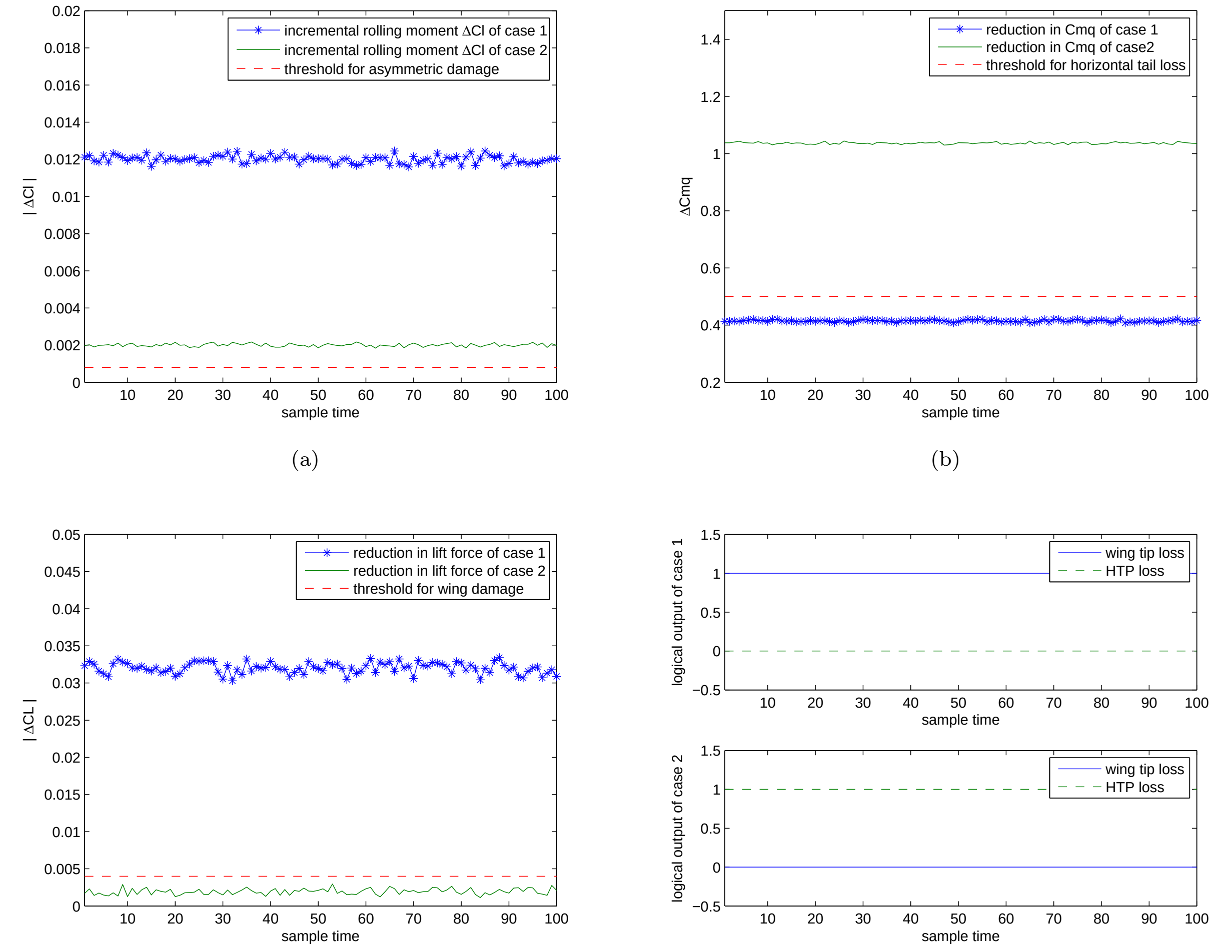


Figure 5. physical model identification

Assuming only one damage case occur at one time, and the changes of aerodynamic forces, moments and coefficients caused by structural damages have been isolated and calculated, then the identification of



(a) (b)

(c) (d)

Figure 6. the identification results of case 1 and case 2

physical model can be realised by monitoring and comparing the aerodynamic components that each damage case has the most dominant and unique effects on.

As is displayed in figure 5, the three main categories we want to classify are horizontal stabilizer damage, wing damage and vertical tail damage. Only the first two of them will result in asymmetrical damages and generate an incremental rolling moment. Hence, if the absolute value of ΔC_l is not obviously detected, we can roughly classify the current situation as vertical tail loss. However, in case of trivial ΔC_l caused by system errors or rare situations where almost the same damage occur to each side of the wings or horizontal stabilizers, we should also look into other important criteria that can reflect lateral stability. If reductions in both static and dynamic lateral stability (which are ΔC_{n_β} and yawing damping ΔC_{n_r} respectively), together with a total decrease in yawing moment are observed over certain thresholds, the current condition can be estimated as vertical tail loss. On the other hand, the detection of incremental rolling moments above a certain threshold will lead to further identifications on horizontal stabilizers and wings damage, and the sign of the incremental value will indicate which side of the wings or stabilizers has a more severe condition than the other. Given the same flight condition, variations will be seen in aerodynamic responses of these two damage cases, which can be used to distinguish one from the other. The horizontal stabilizer is in most cases almost the only source of longitudinal stability of the aircraft, so if large reductions in pitch damping Δm_q and static longitudinal stability C_{m_α} are detected, the dominate reason should be horizontal tail loss. Similarly, both experimental data and flight dynamics theories support that the reduction of lift force is most likely to be contributed to the tip loss of wings rather than horizontal stabilizers.

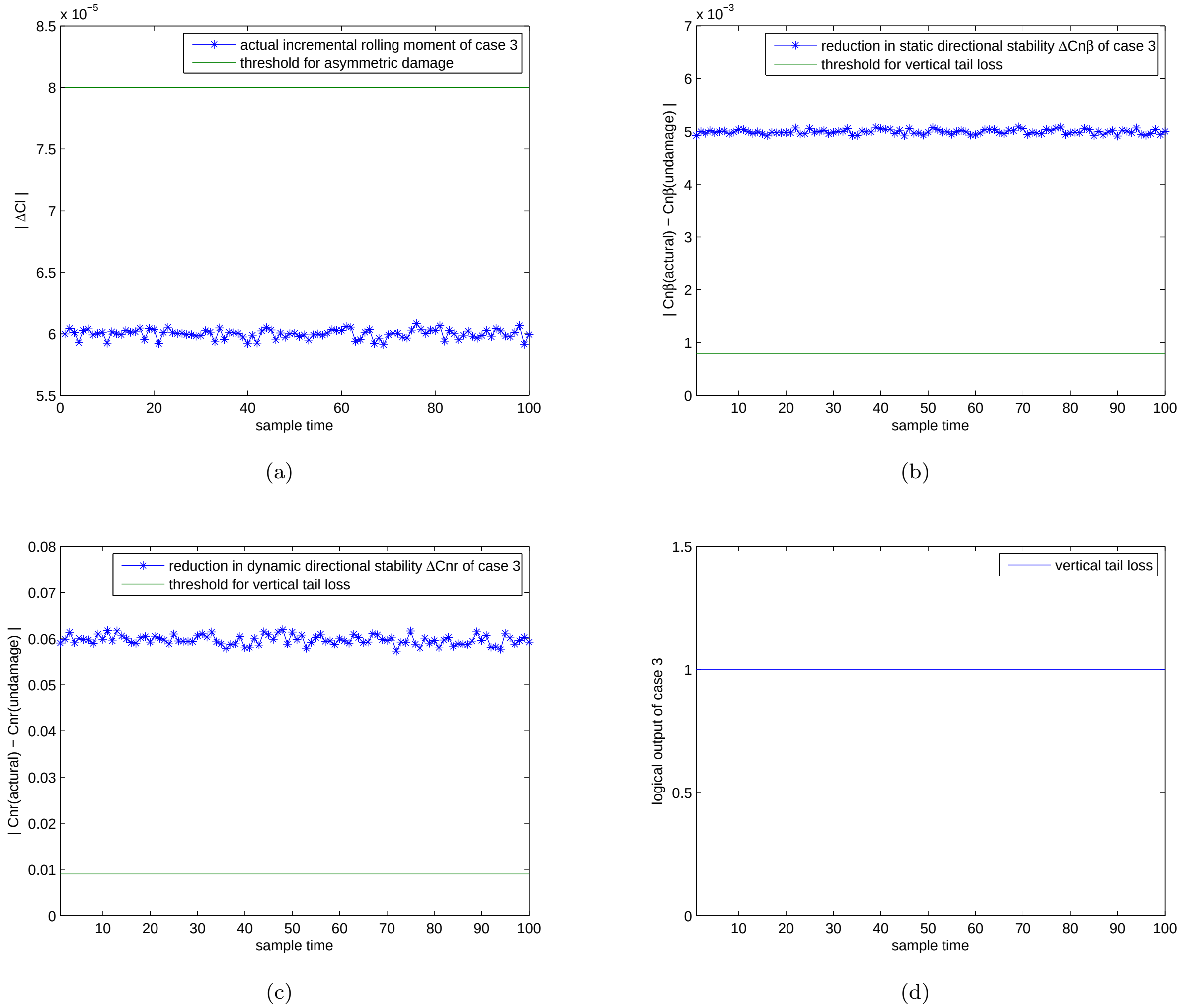


Figure 7. the identification results of case 3

To test the feasibility of the scheme, three damage cases are designed in the simulation test given the same flight condition, which are wing damage, horizontal tail loss and vertical tail loss respectively. The data of damaged aerodyanmic efficients are generated by scaling the magnitude of the nominal aerodynamic from the look-up table, like:

$$C_{act}\left(x\right) = [1 + a]C_{nominal}\left(x\right) \tag{5}$$

The polynomial model is used to approximate the damaged aerodynamic model and the estimation of coefficients is done by least square method. By monitoring the residual between identified values of representative aerodynamic coefficients and undamaged coefficients, and then comparing it with the corresponding threshold, the method can successfully identify the right damage case, as is clearly shown in figure 6 and figure 7. The thresholds are chosen based on several tests with or without damages.

1. Evaluation of Damage Severity

Once the specific location of the damage has been identified, the next step is to estimate how bad the damage is, i.e., the approximate percentage of tip loss. Since absolute values have less significance than its change compared to its initial value, at each sample time, the system calculates the residual between the undamaged baseline value and the estimated actual damaged value of representative aerodynamic coefficients, after which the residual will be divided by the baseline value to evaluate the severity of the actual damage. Take pitch damping for example, the scale of the damage with respect to undamaged values is calculated as:

$$\frac{C_{m_q damage} - C_{m_q undamage}}{C_{m_q undamage}} \times 100\% \tag{6}$$

Then the calculated scale will be used to compare and interpolate with experiment data in the look-up table of damaged aircraft. In this article we use wind tunnel data of GTM[1,17] as the database. As can be seen in figure 8(left), the relation between aerodynamic changes and the percentage of tip loss is almost linear in a local region, so the approximate scale of current structural damage can be roughly estimated by multiplying a proportional factor. As a proof of concept demonstration, the estimation results of an example of 25% tip loss of horizontal stabilizer are shown in figure 8(right). Suppose the damaged aircraft can only reach a limited local region of $\alpha \in [-3.8, 6.1]\, deg$ and no actuator or sensor faults are detected, simulation results have proved that the method described in this section is able to estimate the severity of the damage in a simple but efficient way.

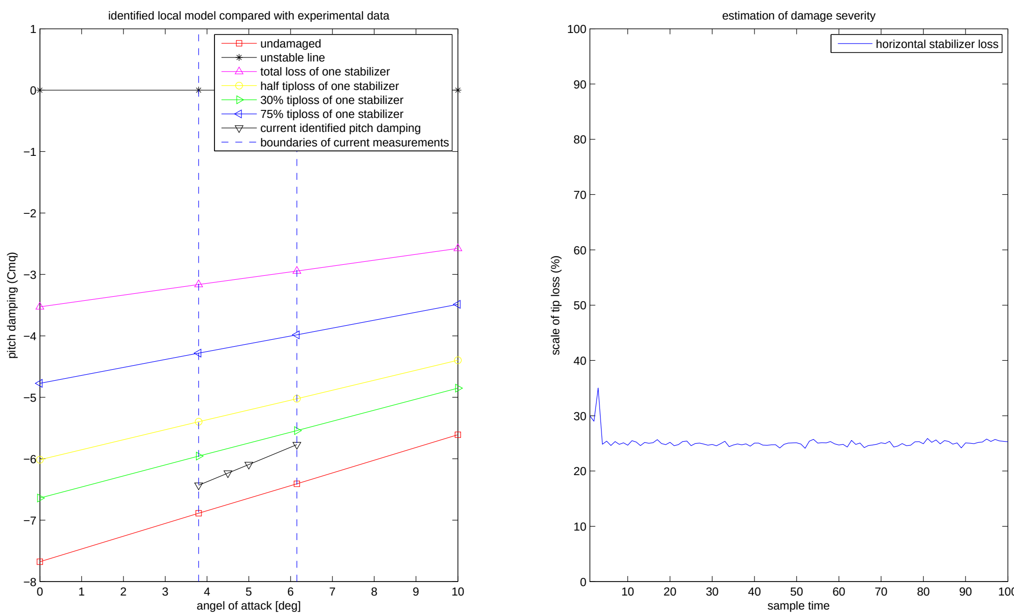


Figure 8. estimation of damgae scale based on experiment data

IV. Application in Database-driven System

The physical model identification method described above is developed to satisfy the need of linking online retrieval of safe flight envelopes to offline generated database of separate abnormal cases. When damages happen to the aircraft, the safe flight region has rapidly shrunken so that only local measurements can be obtained. However, the safe flight envelope cannot be calculated without a global model, which requires measurements from a large operational region. The contradicts between global model and local model give rise to the necessity of this method. As is illustrated in figure 9, the identification results will be used directly as the primary key to the database[21] so that the system can get the right safe flight envelope corresponding to each damage case and flight condition with only a small region of flight data.

V. Conclusion and Future Work

This paper presents an efficient way of online identifying the physical model of the damaged aircraft, which is an indispensable part of the online database-driven safe flight envelope prediction system. The detection, isolation and diagnosis of structural damages are generally based on the different aerodynamic effects of each damage case. Preliminary simulation results have shown that the method proposed in this

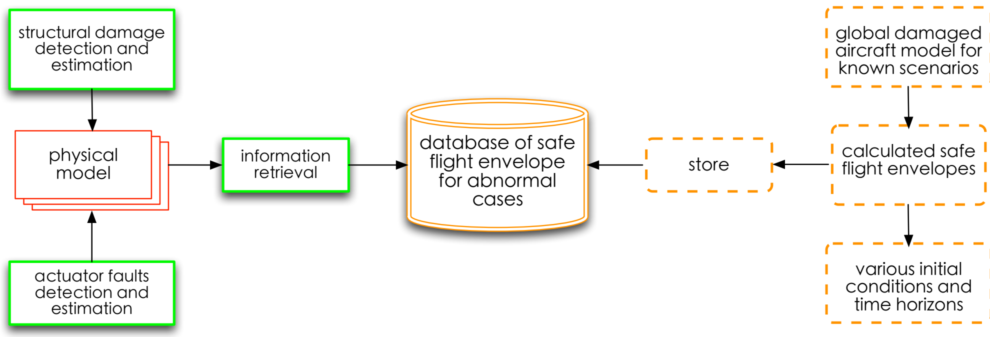


Figure 9. application in database-driven system of safe flight envelopes

paper can successfully isolate each damage case from others and estimate the approximate percentage of tip loss with respect to the whole span at each sample time. Within the established architecture, further studies will be focused on dealing with more complicated issues:

1. It is extremely important to make sure that the effects of abrupt or step change in mass properties after damage are appropriately modeled and accounted for. Besides, the general equations of motion need to be reconsidered when the assumption of symmetry are no longer valid due to asymmetric damages to the aircraft.

2. To enhance the accuracy and confidence of the detection and isolation of damage cases from other system failures, an expansion to a wider range of factors that may also result in similar changes of flight dynamics like actuator and sensor faults and environmental hazards like icing and bird strikes is necessary.

3. Mathematical models of representative aerodynamic coefficients of each damage case are needed. Probability theories may be used to solve for the problem of uncertainties that will cause infeasibilities, false alarms and imprecise estimation if not correctly taken into considerations.

4. The determination of an adaptive threshold should be investigated to improve the robustness of the system

References

[1]Shah, G., "Aerodynamic Effects and Modeling of Damage to Transport Aircraft," *AIAA Guidance, Navigation and Control Conference and Exhibit*, 2008.

[2]Belcastro, C. M., "Validation of Safety-Critical Systems for Aircraft Loss-of-Control Prevention and Recovery," *AIAA Guidance Navigation and Control Conference*, 2012.

[3]Wilborn, J. and Foster, J., "Defining Commercial Transport Loss-of-control: A Quantitative Approach," *AIAA Atmospheric Flight Mechanics Conference*, 2004.

[4]Chongvisal, J. and Talleur, D., "Loss-of-control Prediction and Prevention for NASA's Transport Class Model," *AIAA Guidance, Navigation, and Control Conference*, 2014.

[5]Lombaerts, T., Schuet, S., and Wheeler, K., "Safe Maneuvering Envelope Estimation Based on A Physical Approach," *Guidance, Navigation, and Control and Co-located Conferences*, 2013.

[6]Lygeros, J., "On Reachability and Minimum Cost Optimal Control," *Automatica*, Vol. 40, 2004, pp. 917–927.

[7]Mitchell, I. M., *Application of Levle Set Methods to Control and Reachability Problems in Continuous and Hybrid Systems*, Ph.D. thesis, 2002.

[8]Mcdonough, K., Kolmanovsky, I., Atkins, E., and Arbor, A., "Recoverable Sets of Initial Conditions and Their Use for Aircraft Flight Planning After a Loss of Control Event," *AIAA Guidance Navigation and Control Conference*, 2014.

[9]L. Tang, M. Roemer, J. Ge, J. Prasad, C. B., "Methodologies for Adaptive Flight Envelope Estimation and Protection," *AIAA Guidance, Navigation and Control Conference*, 2009.

[10]Moncayo, H., Perhinschi, M. G., and Davis, J., "Artificial-Immune-System-Based Aircraft Failure Evaluation over Extended Flight Envelope," *Journal of Guidance, Control, and Dynamics*, Vol. 34, No. 4, July 2011, pp. 989–1001.

[11]Lombaerts, T., Huisman, H., Chu, P., Mulder, J. a., and Joosten, D., "Nonlinear Reconfiguring Flight Control Based on Online Physical Model Identification," *Journal of Guidance, Control, and Dynamics*, Vol. 32, No. 3, 2009, pp. 727–748.

[12]Lopez, I. and Sarigul-Klijn, N., "A Review of Uncertainty in Flight Vehicle Structural Damage Monitoring, Diagnosis and Control: Challenges and Opportunities," *Progress in Aerospace Sciences*, Vol. 46, No. 7, 2010, pp. 247–273.

[13]Hayes, C., “Effects of Simulated Wing Damage on the aerodynamic characteristics of a swept-wing airplane model,” 1968.

[14]Frink, N., Pirzadeh, S., and Atkins, H., “CFD Assessment of Aerodynamic Degradation of a Subsonic Transport Due to Airframe Damage,” *48th AIAA Aerospace Sciences Meeting*, Orlando, 2010.

[15]Jordan, T., Langford, W., Belcastro, C. M., Foster, J., Shah, G., Howland, G., and Kidd, R., “Development of a Dynamically Scaled Generic Transport Model Testbed for Flight Research Experiments,” *AUVSI’s Unmanned Systems North America 2004 Symposium and Exhibition*, 2004, pp. 16.

[16]Shah, G., Foster, J., and Cunningham, K., “Simulation Modeling for Off-Norminal Conditions-Where Are We Now?” *AIAA Modeling and Simulation Technologies Conference*, Toronto, 2010.

[17]Shah, G. and Hill, M., “Flight Dynamics Modeling and Simulation of a Damaged Transport Aircraft,” *AIAA Modeling and Simulation Technologies*, 2012.

[18]de Visser, C. C., Chu, Q. P., and Mulder, J. a., “A New Approach to Linear Regression with Multivariate Splines,” *Automatica*, Vol. 45, No. 12, 2009, pp. 2903–2909.

[19]de Visser, C., Mulder, J., and Chu, Q., “A Multidimensional Spline-Based Global Nonlinear Aerodynamic Model for the Cessna Citation II,” *AIAA Atmospheric Flight Mechanics Conference*, 2010.

[20]Tol, H. J., de Visser, C. C., van Kampen, E., and Chu, Q. P., “Nonlinear Multivariate Spline-Based Control Allocation for High-Performance Aircraft,” *Journal of Guidance, Control, and Dynamics*, Vol. 37, No. 6, 2014, pp. 1840–1862.

[21]Elmasri, R. and Navathe, S. B., *Fundamentals of Database Systems*, Addison-Wesley, 6th ed., 2010.