Delay-compensating strategy to enhance string stability of adaptive cruise controlled vehicles

Wang, Meng; Hoogendoorn, Serge; Daamen, Winnie; van Arem, Bart; Shyrokau, Barys; Happee, Riender

DOI
10.1080/21680566.2016.1266973

Publication date
2016

Document Version
Accepted author manuscript

Published in
Transportmetrica B: Transport Dynamics

Citation (APA)
Wang, M., Hoogendoorn, S., Daamen, W., van Arem, B., Shyrokau, B., & Happee, R. (2016). Delay-compensating strategy to enhance string stability of adaptive cruise controlled vehicles. Transportmetrica B: Transport Dynamics, 6 (2018)(3), 211-229. https://doi.org/10.1080/21680566.2016.1266973

post print of: Delay-compensating strategy to enhance string stability of adaptive cruise controlled vehicles
Published online: 21 Dec 2016 http://dx.doi.org/10.1080/21680566.2016.1266973

Delay-compensating strategy to enhance string stability of adaptive cruise controlled vehicles

M. Wang[a,b*], S.P. Hoogendoorn[a], W. Daamen[a], B. van Arem[a], B. Shyrokau[b] and R. Happee[a,b]

[a] *Department of Transport & Planning, Delft University of Technology, the Netherlands*;
[b] *Department of BioMechanical Engineering, Delft University of Technology, the Netherlands*

(*Preprint*)

Abstract

A novel strategy to enhance string stability of autonomous vehicles with sensor delay and actuator lag is proposed based on a model predictive control framework. To compensate sensor delay, the approach entails estimating the (unknown) system state at the current time using the system state in a previous time, the applied control history and a system dynamics model. The actuator lag is compensated by including the lag in the state prediction model. The mathematical framework shows that without the anticipation strategy, sensor delay leads to a worse estimate of the initial condition for the optimal control problem and actuator lag increases the mismatch between the system state prediction model and the actual system behaviour. Simulation verified that sensor delay and actuator lag degrade string stability of platoons. The proposed anticipatory control strategy shows clear benefits in improving autonomous vehicle string stability and hence has potential to enhance traffic flow stability.

Keywords: Adaptive cruise control; string stability; system delay; model predictive control; anticipatory control

1. Introduction

Automated vehicles take over part or all of the driving tasks, depending on the level of automation. Based on the use of communication technology, they can be classified as autonomous and connected/cooperative vehicle systems. Autonomous vehicles rely solely on on-board sensors based on cyber-physical sensing technologies, such as radar and lidar (VanderWerf et al. 2002; Kesting et al. 2008; Wang et al. 2014a). Connected/cooperative vehicles exchange (state and control) information with each other via Vehicle-to-Vehicle (V2V) communication or with road infrastructure via Vehicle-to-Infrastructure (V2I) communication to improve situation awareness and/or to manoeuvre together under a common goal (VanderWerf et al. 2002; Monteil et al. 2013; Wang et al. 2014b; Ge and Orosz 2014; Wang et al. 2015; Milanés and Shladover 2014; Jia and Ngoduy 2016a,b). In this paper, we focus on advanced control strategies for autonomous systems that do not rely on wireless communications.

*Corresponding author, m.wang@tudelft.nl

Adaptive Cruise Control (ACC) is the earliest autonomous vehicle system, which specifically aims at enhancing driving comfort (VanderWerf et al. 2001; Van Arem et al. 2006). The most widely used ACC controller is a proportional-derivative controller, where the vehicle acceleration is proportional to the gap and the derivative of the gap (relative speed with respect to the preceding vehicle) in car-following conditions. This type of controller has been extensively studied (VanderWerf et al. 2001; Van Arem et al. 2006; Rajamani and Shladover 2001; Ploeg et al. 2014).

However, there is no safety mechanism in this type of controller. A separate collision avoidance system has to be designed avoid rear-end collision (Godbole et al. 1999). Nonlinear control methods for ACC systems have been proposed as well. ACC systems with variable time gap policies have been proposed (Wang and Rajamani 2004). Car-following models were used as nonlinear state feedback algorithms for ACC systems, such as the Optimal Velocity Model (Hasebe et al. 2003; Ngoduy 2015a) and the Intelligent Driver Model (Kesting et al. 2008). Artificial intelligence (AI) techniques are also used to design ACC systems (Bifulco et al. 2013). Model predictive control, also called receding horizon control, is used in the design of ACC controllers (Wang et al. 2014a,b; Li et al. 2014) recently. An advantage of the model predictive ACC controller is that it is flexible in dealing with constraints in state and control variables and it can avoid rear-end collision with the predecessor at safety-critical conditions even without the need of a separate collision avoidance system (Wang et al. 2014a). A Model Predictive Full speed Range ACC (MP-FR-ACC) controller is proposed and tested in simulation (Wang et al. 2014a).

Stability properties of autonomous systems and platoons are of great importance since they determine the impacts of automated vehicles on traffic flow stability to a great extent (Tampère 2004; Pueboobpaphan and van Arem 2010; Sau et al. 2014; van Hinsbergen et al. 2015; Zheng et al. 2016; He et al. 2016). Two stability concepts are relevant to vehicle platoons, being local stability and string stability (Treiber and Kesting 2013; Wilson 2008). *Local stability* refers to the stability property of a single vehicle, whether a disturbance in the speed profile can be attenuated. Local stability is a necessary feature for any ACC controller. *String stability* involves more vehicles compared to local stability. A vehicle platoon is string stable if an initial disturbance decays with the increase of vehicle indices in the platoon and is said to be string unstable otherwise. String stability is a more restrictive concept compared to local stability, i.e. even if every vehicle exhibits locally stable behaviour, the platoon can still display string instability.

Among many influencing factors, delays in sensor and actuator play an important role in local and string stability of automated vehicles. They deteriorate the controller performance, particularly local and string stability (Orosz et al. 2010; Ploeg et al. 2014; Ngoduy 2015b; Zhang and Orosz 2016; Redhu and Gupta 2015). However, much work on controller design and performance assessment of ACC systems simplifies or ignores the combination of sensor delay and actuator lag in the control loop, which may results in over-optimistic evaluations of the controller performance and the impacts of automated vehicles on traffic flow. More importantly, control strategies for autonomous ACC vehicles to improve the stability of platoons under explicit sensor delay and actuator lag have not been found. Such strategies, if developed, have great potential in enhancing the string performance as well as the stability of traffic flow.

The aforementioned problems lead to the objectives of this contribution: first, to get new insights into the negative impacts of sensor delay and actuator lag on the stability of full range model predictive ACC vehicle platoons; second, to propose a new anticipatory delay-compensating strategy to compensate the negative impacts of sensor delay and actuator lag. To achieve the objectives, we adopt a model predictive control framework that is proposed in our earlier work (Wang et al. 2014a). We generalize the previous work for MP-FR-ACC controller design by including sensor delay and actuator lag in the mathematical framework. This allows us to test and compare the negative impacts of sensor delay and actuator lag separately. An integrated strategy is proposed to compensate the negative impacts and enhance string stability under the influence of sensor delay and actuator lag. The sensor delay compensation strategy uses the system state in a previous time, the control history and the system dynamics model to get a better estimate of the current system state. The actuator lag is anticipated directly in the system state prediction model. The negative impacts of sensor delay and actuator lag and the benefits of the delay compensation strategy on platoon performance are tested systematically with simulation and a set of indicators are used to assess the string stability performance.

It is worth mentioning that we focus on numerical investigation of delay impact on string stability and delay compensation strategy for homogeneous platoons. Recent studies also point to flow stability of mixed traffic (Ngoduy 2012), which is beyond the scope of this paper.

In the remainder, we first present the longitudinal control architecture for ACC systems and main assumptions, after which the mathematical framework for ACC system is described with inclusion of sensor delay and actuator lag. Then, we formulate the anticipatory control approach and show the setup of the simulation experiments. Impacts of sensor delay and actuator lag and benefits of the anticipatory strategy on string stability are analyzed and we complete the paper by summarizing the findings and conclusions.

2. Vehicle longitudinal control

2.1. *Longitudinal control architecture*

A hierarchical control architecture is usually considered for ACC systems (Fancher and Bareket 1994; Rajamani 2011), where the upper level controller determines the desired acceleration for an ACC vehicle based on a simplified vehicle dynamics model and the lower level controller regulates the throttle and/or the brake commands to track the desired acceleration of the upper level controller. Nonlinear vehicle dynamics modelling and control techniques are usually employed at the lower level to calculate the brake and throttle inputs for the controlled vehicle (Fancher and Bareket 1994; Rajamani 2011). In this paper, we focus on the upper level controller design and performance assessment under explicit sensor delay and actuator lag.

2.2. *Design assumptions*

We make the following assumptions regarding the operations of ACC systems.

- On-board sensors measure vehicle speed, gap (relative distance) and relative speed with respect to the preceding vehicle on regular time intervals.
- The direct measurements from on-board sensors are prone to noise. On-board filters are able to filter out the noise and give an accurate estimate of the system state.
- The process of sensing and filtering is subject to delay, due to the discrete sampling of on-board sensors, the lag due to the radar or lidar filter and the bandwidth of low pass filters used for other sensors such as wheel speed sensor (Ploeg et al. 2014; Rajamani 2011; Xiao and Gao 2011). The overall delay in this process is referred to as sensor delay denoted with τ^{S} hereinafter.
- The ACC controller operates at real time, i.e. there is no computational delay in the controller. Such a delay could however be added.
- The lower level controller executes the desired acceleration command from the upper level controller subject to time delay, which is the time delay in the generation

of traction/brake wheel torques due to the lag of the powertrain actuator or the lag of brake actuator when braking (Ploeg et al. 2014; Rajamani 2011; Xiao and Gao 2011). The overall delay in this process is referred to as actuator lag denoted by τ^{A} hereinafter.

- At each control cycle, the ACC controller knows the sensor delay and vehicle actuator lag in the current control cycle. This can be realized by adding time stamps to the sensor measurements and actuator inputs. In practice, this is a reasonable assumption because the actuator lag can be measured offline from real world experiments in the form of a look-up table for online application or be directly estimated online by a delay observer (Chang and Lee 1995).

The assumptions are based on the operational characteristics of real vehicles and thus are not restrictive (Ploeg et al. 2014; Rajamani 2011; Xiao and Gao 2011).

2.3. *Model predictive full speed range ACC (MP-FR-ACC)*

We formulate the vehicle following control problem as a model predictive control problem, where an open-loop optimal control problem is recursively solved online (Wang et al. 2014a). Figure 1 shows an abstract representation of an upper level model predictive ACC controller (the gray rectangle). Let us consider the *system* from the perspective of an ACC vehicle i, of which the state $\mathbf{x}$ can be described by the gap to the predecessor and its own speed. At time instant t_0, the controller receives the system state of a previous time $t_0 - \tau^{\mathrm{S}}$ from on-board sensors and filters. Based on this information, the controller estimates the current state of the system and predicts the future state in a time horizon T_p, with the current system state as the initial condition. The optimal control input/variable $\mathbf{u}$, which is the desired acceleration of an ACC vehicle, is determined by optimizing a performance index or cost function reflecting undesirable situations. In a *receding horizon* implementation, only the first sample of the control input is realized by the lower level controllers and actuators in a retarded time $t_0 + \tau^{\mathrm{A}}$ (Morari and Lee 1999; Mayne et al. 2000). As the vehicle manoeuvres, the *system* state changes, and the optimal control signal $\mathbf{u}$ is recalculated with the newest information regarding the system state at the next control cycle , e.g. $t = t_0 + \delta t$ with δt being the sampling step of the controller.

3. Mathematical formulation for MP-FR-ACC

In this section, we present the mathematical formulation for autonomous vehicle-following control systems. We first present the benchmark ACC controller without any delay. After that, we formulate sensor delay and actuator lag in the control framework.

3.1. *Benchmark MP-FR-ACC controller without delay*

3.1.1. Benchmark system dynamics model

The system state from the perspective of an ACC vehicle i can be described by $\mathbf{x} = (s_i, v_i)^T$. s_i denotes the (distance) gap to the preceding vehicle $i-1$, i.e. $s_i = x_{i-1} - x_i - l$, with l representing the vehicle length. The longitudinal dynamics model of the system used for predicting future evolution of the system is simplified by the following differential

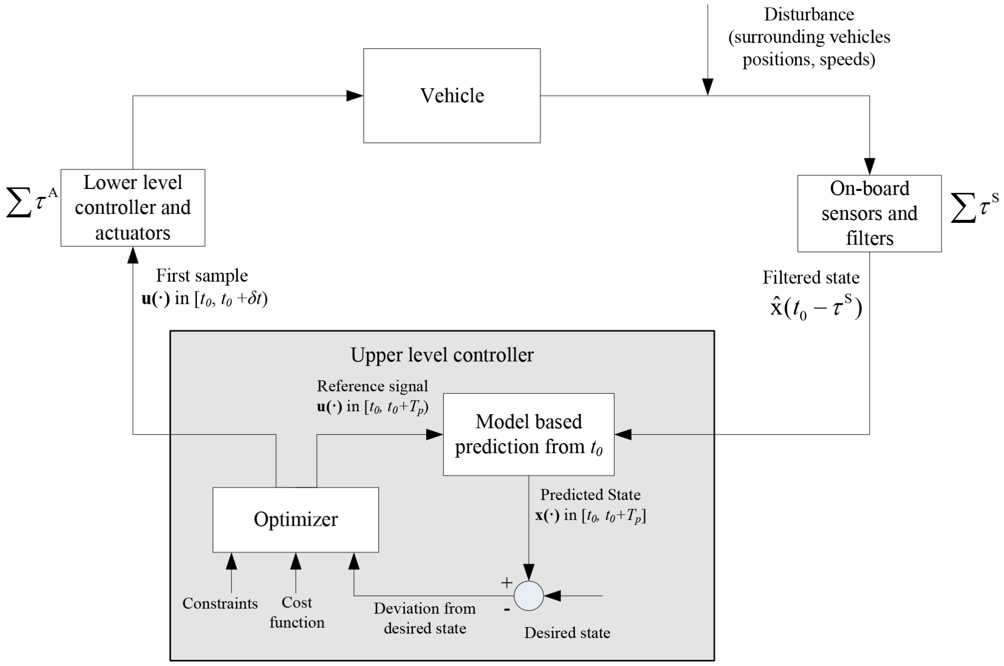


Figure 1. Abstract representation of model predictive ACC system.

equation:

$$\frac{d}{dt}\mathbf{x} = \frac{d}{dt}\begin{pmatrix} s_i \\ v_i \end{pmatrix} = \begin{pmatrix} v_{i-1} - v_i \\ u_i \end{pmatrix} = \begin{pmatrix} \Delta v_i \\ u_i \end{pmatrix} = \bar{\mathbf{f}}(\mathbf{x}, \mathbf{u}, \mathbf{d}) \tag{1}$$

$\Delta v_i = v_{i-1} - v_i$ denotes relative speed to the preceding vehicle.

Note that the benchmark system dynamics model (1) ignores the actuator lag τ^{A} in the vehicle system. This is the first source of the mismatch between the open-loop prediction and the closed-loop system behaviour, i.e. the mismatch between model and plant (Morari and Lee 1999; Mayne et al. 2000).

Here, $\mathbf{d}$ denotes the exogenous disturbance, which is the preceding vehicle speed for ACC controllers:

$$\mathbf{d} = v_{i-1} \tag{2}$$

The disturbance can be modelled by using *constant speed heuristics* (Wang et al. 2014a), i.e. the predecessor travels at the measured speed in the predicted future. For autonomous systems without V2V communication, this heuristics has been proven very effective (Wang et al. 2014a, 2013). *Constant acceleration heuristics* is also possible for autonomous ACC, but it entails differentiating the distance measurements to the second order and will suffer from noise of on-board sensors. In the closed-loop system, the preceding vehicle is not necessarily travelling at constant speed. The predecessor speed variation is the second source of the mismatch between the open-loop prediction model and the closed-loop system behaviour.

The MP-FR-ACC controller should cope with the mismatch between the model and the system and generate locally stable behaviour. As we will show with simulation in the ensuing, the MP-FR-ACC controller is quite robust to the mismatch due to the feedback nature of the receding horizon process.

3.1.2. Optimal control of the benchmark system

The MP-FR-ACC system seeks an optimal control trajectory $\mathbf{u}(\cdot)$ in the finite prediction horizon $[t_0, t_0 + T_p)$ that minimizes a cost function J (Wang et al. 2014a). It can be formulated as the following mathematical programme:

$$\min_{\mathbf{u}_{[t_0,t_0+T_p)}} J(\mathbf{x}(t), \mathbf{u}(t)) = \min_{\mathbf{u}_{[t_0,t_0+T_p)}} \left(\int_{t_0}^{t_0+T_P} \mathcal{L}(\mathbf{x}(t), \mathbf{u}(t))dt + \mathcal{G}(\mathbf{x}(t_0 + T_p)) \right) \tag{3}$$

where J denotes the cost function to be minimized. $\mathcal{L}$ denotes the so-called running cost and $\mathcal{G}$ denotes the so-called terminal cost. We set the terminal cost to be zero in this study.

The optimization is subject to:

(1) the system dynamics model of Eq. (1)
(2) the initial condition:

$$\mathbf{x}(t_0) = \hat{\mathbf{x}}(t_0) \tag{4}$$

(3) the constraints on state and control variables:

$$\mathbf{x}(t) \in \mathcal{X}\ ,\ \mathbf{u}(t) \in \mathcal{U} \tag{5}$$

In Eq. (4), the hat denotes the filtered state from the sensors and filter of the benchmark system without sensor delay.

3.1.3. Running cost specification

Here, we define the running cost function as (Wang et al. 2016):

$$L = \underbrace{\frac{c_1}{s}\Delta v^2}_{\text{safety}} + \underbrace{c_2(v^e(s) - v)^2}_{\text{equilibrium}} + \underbrace{c_3 u^2}_{\text{control}} \tag{6}$$

In Eq. (6), the *safety* cost term guarantees that the ACC vehicle will get a large penalty when approaching the preceding vehicle at small gaps, since these situations entail high risks of rear-end collisions. When there is no preceding vehicle detected by the on-board sensor, the safety cost term vanishes.

The *equilibrium* cost implies that the vehicle tends to relax to the local equilibrium speed determined by the distance to the predecessor. The equilibrium and safety costs are the proximity cost terms reflecting the interaction with the preceding vehicle. The local equilibrium speed v^e is determined by gap or local density:

$$v^e(s) = \begin{cases} v^d & \text{if } s > s_f \\ \frac{s-s^0}{t^d} & \text{if } s \le s_f \end{cases} \tag{7}$$

t^d denotes the user-defined target time gap and $s_f = v^d t^d + s^0$ is a gap threshold defining the change of slope in the equilibrium speed function.

The *comfort* cost is represented by penalizing large acceleration or deceleration. It also reflects control efforts.

3.1.4. Constraints specification

The constraint on the state variable $\mathcal{X}$ is specified as:

$$\mathcal{X} := \left\{ s > s^0; v \in [0, v_{\max}] \right\} \tag{8}$$

The constraint implies that the gap with respect to the preceding vehicle should be no less than the minimum distance s^0 and the controlled vehicle drives within a speed range of $[0, v_{\max}]$. The controlled inputs are limited to some admissible bounds:

$$\mathcal{U} := \{ u_i \in [a_{\min}, a_{\max}] \} \tag{9}$$

3.1.5. Derivation of optimal control input

This section shows how to solve the optimal control problem based on Pontrygain's Principle (Pontryagin et al. 1962; Wang et al. 2012). Without going too much into details, the solution approach entails defining the Hamiltonian $\mathcal{H}$ as follows:

$$\mathcal{H}(\mathbf{x}, \mathbf{u}, \boldsymbol{\lambda}) = \mathcal{L}(\mathbf{x}, \mathbf{u}) + \boldsymbol{\lambda}^T \cdot \mathbf{f}(\mathbf{x}, \mathbf{u}) \tag{10}$$

where $\boldsymbol{\lambda}$ denotes the so-called *co-state* or *marginal cost* of the state $\mathbf{x}$, which reflects the relative extra cost of J incurred due to making a small change $\delta\mathbf{x}$ on the state $\mathbf{x}$.

Using the Hamiltonian, we can derive the following necessary condition for the optimal control $\mathbf{u}^*$:

$$\mathbf{u}^* = \min \mathcal{H}(\mathbf{x}, \mathbf{u}, \boldsymbol{\lambda}) \text{ , s.t. } \mathbf{u} \in \mathcal{U}, \;\; \mathbf{x} \in \mathcal{X} \tag{11}$$

$$\mathbf{u}^* = \frac{-\lambda^v}{2c_3} \tag{12}$$

Equation (11) shows that the optimal control $\mathbf{u}^*$ minimises the Hamiltonian in the admissible range $\mathcal{U}$ and in the bounded set $\mathcal{X}$. In nearly all cases, this requirement will enable expressing the optimal control input $\mathbf{u}^*$ as a function of the state $\mathbf{x}$ and the co-state $\boldsymbol{\lambda}$.

The co-state are derived as:

$$-\frac{d\lambda^s}{dt} = \frac{\partial \mathcal{H}}{\partial s} = -\frac{c_1}{s^2}\Delta v^2 + \frac{2c_2}{t_d}\left(v^e(s) - v\right) \tag{13}$$

$$-\frac{d\lambda^v}{dt} = \frac{\partial \mathcal{H}}{\partial v} = -\frac{2c_1}{s}\Delta v - 2c_2(v^e(s) - v) - \lambda^s \tag{14}$$

The state dynamics equation (1) and the co-state dynamics equation of (13, 14) constitute a set of ordinary differential equations with initial conditions and terminal conditions. An iterative solution algorithm has been proposed to solve the two-point boundary value problem in (Wang et al. 2012) and the performance of the MP-FR-ACC has been tested in closed-loop simulations (Wang et al. 2014a).

3.2. *Inclusion of sensor delay*

A sensor delay τ^S changes the initial condition used by the controller for solving the state dynamics from the state in current time t_0 to the state in the previous time $t_0 - \tau^{\mathrm{S}}$:

$$\mathbf{x}(t_0) = \hat{\mathbf{x}}(t_0 - \tau^{\mathrm{S}}) \tag{15}$$

In other words, the controller will use the state in a previous time instant as the initial condition to predict state evolution. Compared to the initial condition for the state prediction in the benchmark case without sensor delay, i.e. Eq. (4), using Eq. (15) for state prediction will increase the mismatch between the open-loop prediction and the closed-loop behaviour of the system. This is the *third source* of mismatch between the prediction model and real system in addition to the other two sources as discussed earlier and is the mechanism of the performance deterioration of the model predictive ACC controller due to sensor delay. When $\tau^{\mathrm{S}} = 0$, Eq. (15) relaxes to Eq. (4).

3.3. *Inclusion of actuator lag*

The very existence of the actuator lag τ^{A} implies *retarded* behaviour of the vehicle in executing the desired acceleration of the upper level controller, i.e. the desired acceleration cannot be realized immediately, but with a delay. The system dynamics with actuator lag is then described by the following differential equation:

$$\frac{d}{dt}\mathbf{x} = \frac{d}{dt}\begin{pmatrix} s_i(t) \\ v_i(t) \\ a_i(t) \end{pmatrix} = \begin{pmatrix} \Delta v_i(t) \\ a_i(t) \\ \frac{u_i(t-\tau^S)-a_i(t)}{\tau^A} \end{pmatrix} = \mathbf{f}(\mathbf{x}, \mathbf{u}, \mathbf{d}) \tag{16}$$

where a_i denotes the *actual* acceleration while the control input (or signal) $\mathbf{u} = u_i$ can be interpreted as the *desired* acceleration of vehicle i. Note that the model (16) is based on an exact linearization approach of a nonlinear longitudinal vehicle dynamics model. We refer interested readers to (Sheikholeslam and Desoer 1993) for details of the linearization approach. When $\tau^{\mathrm{A}} = 0$, the third order model (16) relaxes to the second order model (1).

Note that the linear model may not adequately describe vehicle dynamics near conditions of physical limits, e.g., tire force saturation due to emergency braking or severe steering. We focus on operations at normal driving conditions and exam how disturbance from steady-state conditions evolves in the platoon. Therefore, the linear model suffices.

3.4. *Receding horizon implementation*

In the receding horizon implementation of the optimal control input, only the first sample of the optimal control input $\mathbf{u}^*(t) \in [t_0, t_0 + \delta t)$ is executed by the actuator and hence the *closed-loop system* can be represented by:

$$\mathbf{u}(t; \mathbf{x}(t_0)) = \mathbf{u}^*(t; \mathbf{x}(t_0)) \ , \ t \in [t_0, t_0 + \delta t) \tag{17a}$$

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t; \mathbf{x}(t_0)), \tilde{\mathbf{d}}(t)) \tag{17b}$$

where actuator lag is included in the closed-loop evolution of the system driven by Eqs. (16) and (17). Note that we use a different notation of the disturbance $\tilde{\mathbf{d}}$ in the closed-loop system (17b) since it may be different from the predicted $\mathbf{d}$ as in Eq. (1)

when the preceding vehicle is accelerating or decelerating, i.e. the second source of the model and system mismatch as discussed earlier.

4. Anticipatory control strategy to compensate delay

The receding horizon control approach entails prediction of the future evolution of the system. This feature can be explored to compensate delay in the control loop (Morari and Lee 1999; Mayne et al. 2000; Treiber et al. 2006). Computational delay in model predictive control has been addressed in (Findeisen and Allgöwer 2004; Liu et al. 2006), but the schemes do not distinguish sensor delay from actuator lag and thus are not directly applicable to separate sensor and actuator delays. We propose an integrated anticipatory strategy to compensate both sensor delay and actuator lag in the closed loop. The strategy to compensate sensor delay is based on a model-based estimation of the current system state based on the previous system state and applied control input series, while actuator lag can be included by using the higher order system dynamics model instead of the second order model. The strategy does not entail changes to the derivation of the optimal control input compared to the benchmark controller and the integrated anticipatory control strategy is formulated as an implementable algorithm, which is presented at the end of the section.

4.1. *Sensor delay compensation strategy*

The key of the sensor delay compensation strategy is that with a known delay τ^{S} in the knowledge of the system state information, the controller can estimate the current system state $\mathbf{x}(t_0)$ based on:

- the newest system state information from sensor and filter at the time instant $t_0 - \tau^{\mathrm{S}} : \hat{\mathbf{x}}(t_0 - \tau^{\mathrm{S}})$;
- the known applied control input history from the previous time to current time $\mathbf{u}(t) \in [t_0 - \tau^{\mathrm{S}}, t_0)$;
- the system dynamics model.

Mathematically, the current system state can be estimated by solving the integral:

$$\mathbf{x}(t_0) = \int_{t_0-\tau^{\mathrm{S}}}^{t_0} \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t), \mathbf{d}(t))dt + \mathbf{x}(t_0 - \tau^{\mathrm{S}}) \tag{18a}$$

$$\mathbf{x}(t_0 - \tau^{\mathrm{S}}) = \hat{\mathbf{x}}(t_0 - \tau^{\mathrm{S}}) \tag{18b}$$

$$\mathbf{u}(t) \in [t_0 - \tau, t_0) \tag{18c}$$

Note that when $\tau^{\mathrm{S}} = 0$, Eq. (18) relaxes to Eq. (4). The estimated system state is expected to be more accurate compared to the simple strategy using the previous system state as the current system state, i.e. Eq. (15). Similarly as the previous section, we use the *constant speed heuristics* to estimate the disturbance $\mathbf{d}(t)$ in Eq. (18).

4.2. *Actuator lag compensation strategy*

If the actuator lag is known, we can include it in the system dynamics prediction for each online optimal control problem, i.e. using the third order system dynamics model (16) for prediction instead of the second order model (1). This will reduce the first source of mismatch between the predicted system state and the realized system state. As we will show in the ensuing, this has clear benefits in improving the platoon performance. Note that the anticipation of actuator lag mitigates the mismatch, but does not lead to perfect prediction of the system evolution due to the presence of disturbance $\mathbf{d}$.

4.3. *Algorithm for the integrated delay compensation strategy*

The anticipatory control strategy to compensate sensor delay and actuator lag is formulated as an implementable algorithm as shown in Algorithm 1.

Data: Sensor delay τ^{S}, actuator lag τ^{A}, (delayed) system state $\mathbf{x}(t_0 - \tau^{\mathrm{S}})$, applied control input history $u_{[t_0-\tau^{\mathrm{S}},t_0)}$
Result: Optimal control input u^* in the horizon $[t_0, t_0 + T_p)$
initialization;
if *do benchmark controller* **then**
    Solve optimal control problem of Eqs. (3, 1, 4, 5) using the iterative numerical algorithm in (Wang et al. 2012)
end
if *do simple strategy without delay compensation* **then**
    Solve optimal control problem of Eqs. (3, 1, 15, 5) using the iterative numerical algorithm in (Wang et al. 2012)
end
if *do anticipatory strategy with delay compensation* **then**
    if $\tau^A = 0$ **then**
        Solve optimal control problem of Eqs. (3, 1, 18, 5) using the iterative numerical algorithm in (Wang et al. 2012)
    else
        Solve optimal control problem of Eqs. (3, 16, 18, 5) using the iterative numerical algorithm in (Wang et al. 2012)
    end
end

Algorithm 1: Integrated control algorithms with delay compensation strategy

We emphasize that the sensor delay and actuator lag need not to be constant. Stochastic delays can easily be included in the algorithm as long as they are known to the controller at time of solving the online optimal control problem. This can be realized by assigning a time stamp to the measurements and applied control inputs.

Here we complete the formulation of the control problem with explicit sensor delay and actuator lag. In the following section, we verify the impacts of sensor delay and actuator lag on the controller performance and demonstrate the effectiveness of the delay compensation strategy via simulation.

5. Simulation experimental design

In this section, we describe the simulation experiments that are used to test the impacts of sensor delay and actuator lag on the platoon performance and the benefits of the proposed delay-compensating strategy.

Table 1. Simulation scenarios

τ^{S} (s)	τ^{A} (s)	Control strategy
0	0	Benchmark control
0.2/0.3/0.4/0.5	0	Simple strategy without delay compensation
0	0.2/0.3/0.4/0.5	Simple strategy without delay compensation
0.3	0.2	Simple strategy without delay compensation
0.2/0.3/0.4/0.5	0	Anticipatory strategy with delay compensation
0	0.2/0.3/0.4/0.5	Anticipatory strategy with delay compensation
0.3	0.2	Anticipatory strategy with delay compensation

5.1. *Experimental setup*

To demonstrate the influence of delay and the benefits of the anticipatory strategy on controller stability, we conduct simulations with a platoon of 8 vehicles, of which 7 *autonomous* ACC vehicles follow a standard *exogenous* leader. A step function of acceleration is imposed to the leader behaviour and all followers are homogeneous ACC vehicles. The step function includes a decelerating phase where the leader brakes hard, representing a situation when vehicles drive into the tail of a jam on highway, and an accelerating phase where the leader accelerates sharply, representing a situation when vehicles move out of a jam on highway. In a simulation period of 50 seconds, the platoon starts from equilibrium conditions meaning they are travelling at the same speed as the platoon leader and at their desired gaps. The step function of the leader acceleration profile is set as follows:

- *Decelerating phase*: the leader starts with an initial speed of 25 m/s, decelerates with -4 m/s^2 from 3 to 5 seconds to 17 m/s and maintains this speed until the accelerating phase starts at 40 seconds.
- *Accelerating phase*: the leader accelerates with 1 m/s^2 from 40 to 48 seconds back to 20 m/s and maintains this speed for the rest of the simulation.

Note that in the numerical simulation, the control inputs are sampled at discrete time steps and there is one step delay in the control for the benchmark scenario.

5.2. *Simulation scenarios and parameter setting*

Literature showed that the sensor delay is between 0.1-0.3 s, and the actuator lag is in the order of 0.1-0.2 s (Ploeg et al. 2014; Rajamani 2011; Xiao and Gao 2011). We test different values of the time delays including those higher than the typical values to generate string stable, meta-stable and unstable platoons and thus to gain insights into the performance of the proposed anticipatory controllers in a wider range of conditions. The full setting of simulation scenarios with different sensor delay and actuator lag values and control strategies is listed below: To the working of the anticipation algorithm, the delay values are kept constant within each simulation scenario.

Controller parameters are set as follows: $t^d = 1$ s, $T_p = 5$ s, $s^0 = 2$ m, $c_1 = 10$, $c_2 = 0.1$, $c_3 = 0.5$, $a_{\max} = 1.5$ m/s^2, $a_{\min} = -8$ m/s^2. Simulation time step is 0.1 s. The sensor sampling step size is set at 0.1 s, which is achievable by modern ACC sensors.

The controller parameters have been manually tuned that the controller tracks the leader speed responsively, results in small overshoots and no oscillation in the benchmark case without sensor delay and actuator lag (Wang et al. 2014a). An overview of tuning methods for MPC can be found in (Garriga and Soroush 2010).

5.3. *Performance assessment indicators*

The performance of different autonomous ACC platoons will be assessed by simulations. The most important indicator for individual vehicle and collective platoon performance is the accumulation of running cost, since it reflects the multiple objectives of the controller. Particularly, we compare the cost of the first controlled follower and the last controlled follower (the 7th follower) in different scenarios. The maximum absolute relative speed and acceleration for the first and last followers in the platoon are also used as indicators for checking attenuation or amplification of speed disturbance of the leader.

Temporal evolution of applied acceleration, speed, relative speed and gap of all vehicles are plotted and compared. String stability and instability are (loosely) identified by the changes in the indicators and the plots.

6. Performance comparison

6.1. *Benchmark ACC controller*

Figure 2(a)(b)(c) shows the temporal evolution of the actual acceleration, speed and gap in the ideal case with $\tau^{\mathrm{S}} = 0$ s and $\tau^{\mathrm{A}} = 0$ s. Table 2 shows the main indicators of the platoon performance. As we can see clearly in the benchmark case, the first controlled follower of the platoon decelerates as a reaction to the platoon leader behaviour. The subsequent followers in the platoon decelerate consecutively as a reaction to the behaviour of their predecessors. All the controlled vehicles settle down to the new equilibrium speed at 17 m/s after about 17 seconds. In the acceleration phase, all the controlled followers accelerate from 17 m/s back to 25 m/s and maintain the same speed as the platoon leader and their desired gaps. The behaviours are expected from the mathematical formulation of the cost function, i.e. the minimum running cost is that the controlled vehicle travels at the same speed of the predecessor and with a time gap of t^d.

It can be seen clearly from Table 2 and Figure 2 that the disturbance in the leader acceleration is attenuated in the benchmark platoon, i.e. the cost, minimum relative speed, minimum acceleration, maximum relative speed, maximum acceleration, of the last controlled follower are all smaller than that of the first follower. The total cost for all the controlled followers is 165.93.

6.2. *Negative impacts of sensor delay*

Sensor delay has negative impacts on system performance, particularly on stability (Xiao and Gao 2011). Figure 2 shows the temporal evolution of acceleration, speed and gap of the ACC platoon with sensor delay of τ^{S} of 0.2, 0.3, 0.4 and 0.5 s respectively. It can be seen from the figure and Table 2 that the platoon performance deteriorates with sensor delay, i.e. the cost of the platoon increases with the increase of delay. The performance of the first controlled follower and that of the last follower also deteriorates with the increase of delay.

As we can see from Figure 2, we do not see a clear difference in the speed profiles with delay of 0.2 s compared to the benchmark case. Some local oscillations in the acceleration profiles are observed when the delay increases to 0.3 s, but there is no overshoot in the speed and gap profiles in deceleration and acceleration transitions. Overshoot in speed and gap profile is observed when sensor delay increases to 0.4 s. However, the maximum deceleration of the followers is not amplified with the increase of vehicle indices as shown in Figure 2(j) and the overshoot in speed profile only increases marginally as shown in Figure 2(k). The platoon settles down to the new equilibrium state in both deceleration and acceleration transitions within 20 seconds. This seems like a meta string stable state, given the fact that the step function of the leader acceleration will be smoother in reality.

As the sensor delay increases to 0.5 s, the maximum deceleration of controlled followers increases with the index of vehicles in the platoon. The overshoot in speed and gap profiles increases significantly and string instability is clearly seen in the figure. As a result, the cost of the first follower, last follower and the platoon increase substantially compared to the reference case.

6.3. *Negative impacts of actuator lag*

Figure 3 shows the temporal evolution of acceleration, speed and gap under different actuator lag values of 0.2, 0.3, 0.4 and 0.5 s. Similar to the sensor delay, the negative impacts of actuator lag are clearly seen in Figure 3. The cost of the platoon increases with the increase of the actuator lag. Some local oscillations in the acceleration and speed profiles are observed when the actuator lag increases to 0.3 s, cf. Figure 3(g) and (h). Overshoot is observed in the speed profile of Figure 3(k) when the actuator lag increases to 0.4 s. Nevertheless, the overshoot increases marginally with the increase of vehicle index, representing a meta string stable state for the platoon. When the delay increases to 0.5 second, string instability is clearly seen.

Interestingly, at the same value of delay, i.e. $\tau^{\mathrm{A}} = \tau^{\mathrm{S}}$, platoon cost, cost of the first follower and the last follower, maximum and minimum relative speed and acceleration of the platoon with the actuator lag are better than that of the sensor delay, as shown in Table 2. This is clearly visible when comparing Figure 3(m)(n)(o) with Figure 2(m)(n)(o).

6.4. *Benefits of the delay-compensating strategy*

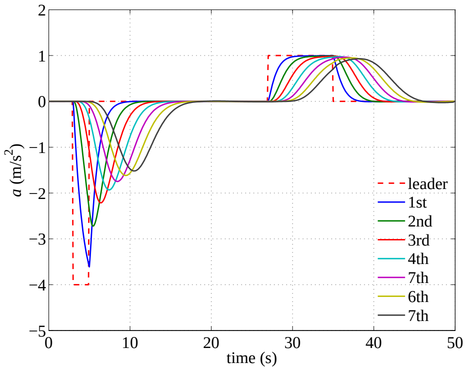
Sensor delay compensation
In this section, we show the vehicle and platoon performance when we include anticipation strategies for sensor and actuator lag in Figure 4(a)(b)(c)(d) and and Table 2. As depicted in Figure 4(a)(b), with a 0.4 s sensor delay, the local oscillation in the acceleration and speed profiles becomes much less when the controller employs the anticipatory control strategy. The platoon cost, cost of the first and the last follower are smaller than that of the scenario without delay compensation.

The benefits of sensor delay compensation become more pronounced when the platoon is string-unstable. When the sensor delay increases to 0.5 s, the delay-compensating strategy leads to significantly better platoon performance, i.e. the platoon cost decreases from 678.25 to 334.15.

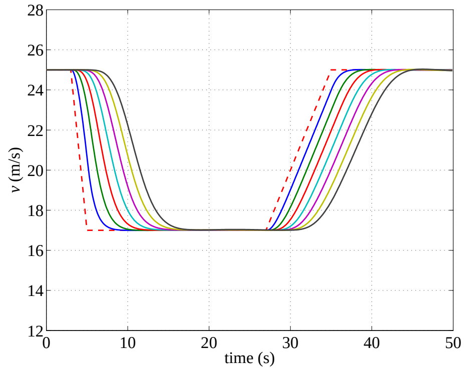
Actuator lag compensation
The effects of the actuator compensation strategy are shown in Figure 4(e)(f)(g)(h) and Table 2. When employing the actuator lag compensation strategy, the platoon performance enhanced substantially. The platoon cost goes down from 258.88 and 359.61 to 225.56 and 245.28 respective when $\tau^{\mathrm{A}} = 0.4$ s and $\tau^{\mathrm{A}} = 0.5$ s respectively.

Integrated delay compensation
In the last scenario, we tested the effectiveness of the integrated sensor and actuator lag compensation strategy. The integrated sensor delay and actuator lag compensation strategy lead to improved performance of the platoon. The platoon cost reduced substantially from 588.89 to 286.54. This clearly shows the benefits of the delay-compensating strategy in improving ACC controller performance and the string stability of ACC vehicles.

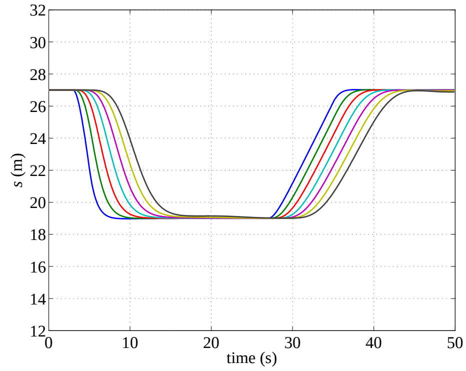
It is noticeable from Figure 4(c)(i) that without the delay compensation strategy, the vehicle platoon is string unstable, i.e. the speed error is amplified in the platoon. Figure 4 (d)(j) shows that the delay-compensating strategy brings the vehicle platoon



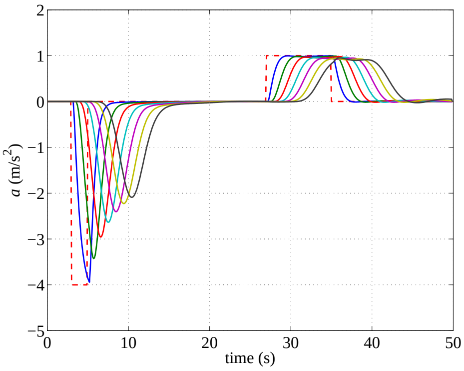
(a) Benchmark acceleration



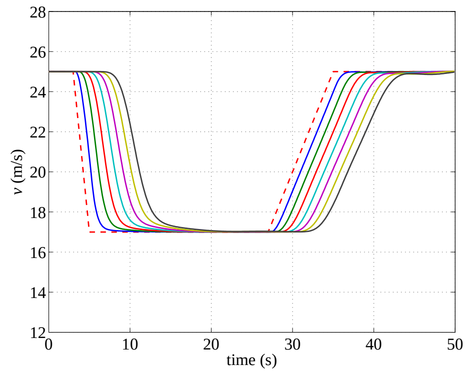
(b) Benchmark speed



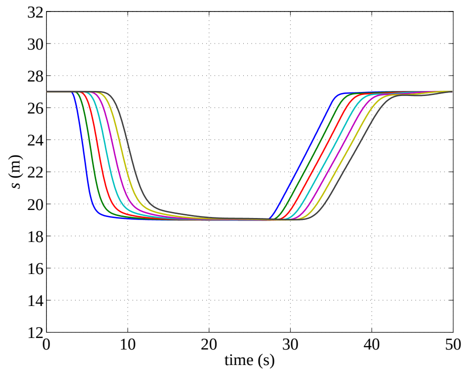
(c) Benchmark gap



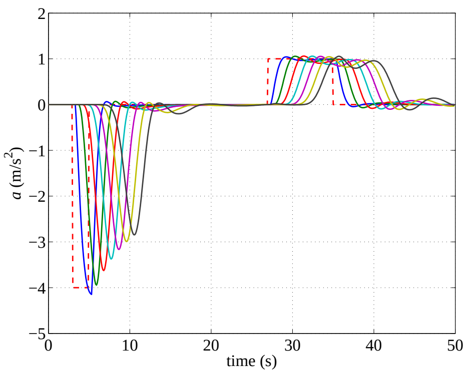
(d) Acceleration, $\tau^{\mathrm{S}} = 0.2$ s



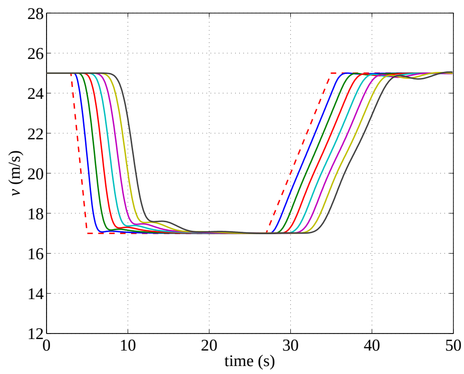
(e) Speed, $\tau^{\mathrm{S}} = 0.2$ s



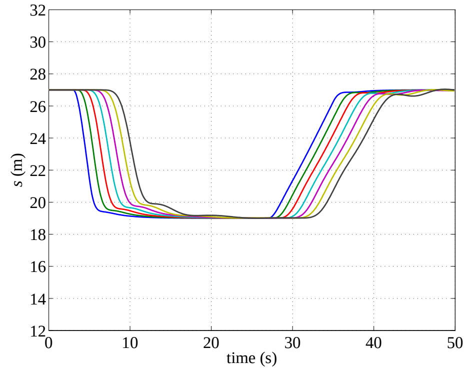
(f) Gap, $\tau^{\mathrm{S}} = 0.2$ s



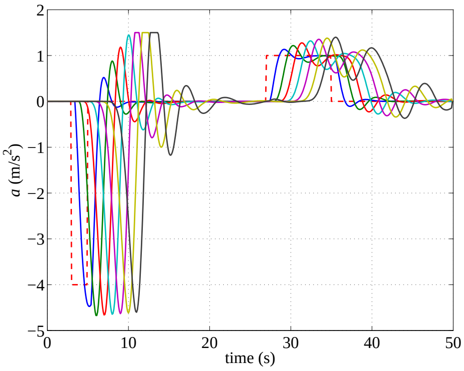
(g) Acceleration, $\tau^{\mathrm{S}} = 0.3$ s



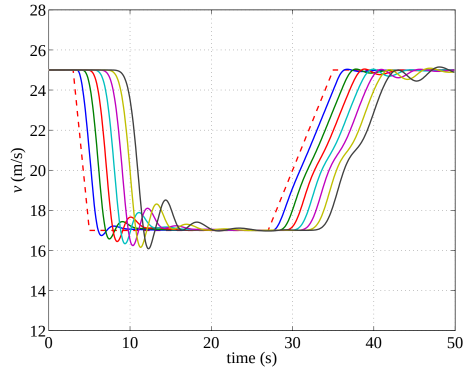
(h) Speed, $\tau^{\mathrm{S}} = 0.3$ s



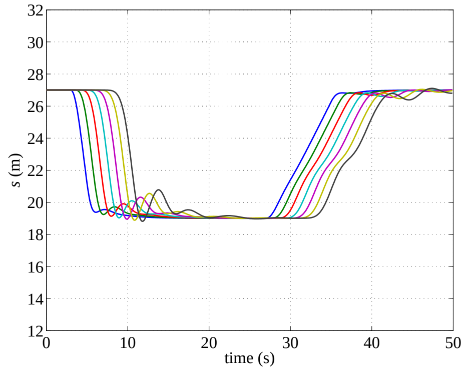
(i) Gap, $\tau^{\mathrm{S}} = 0.3$ s



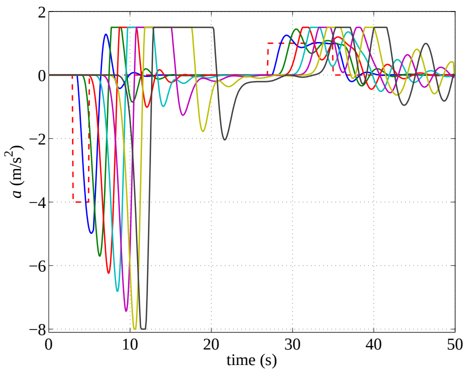
(j) Acceleration, $\tau^{\mathrm{S}} = 0.4$ s



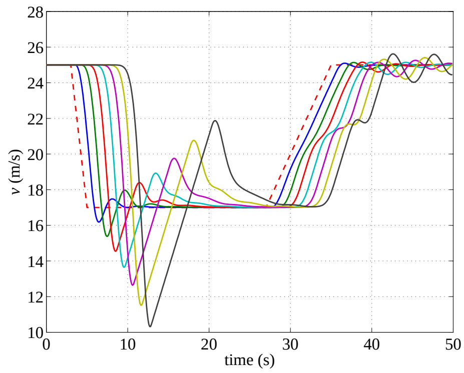
(k) Speed, $\tau^{\mathrm{S}} = 0.4$ s



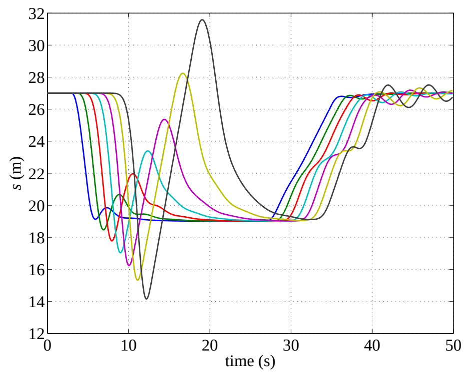
(l) Gap, $\tau^{\mathrm{S}} = 0.4$ s



(m) Acceleration, $\tau^{\mathrm{S}} = 0.5$ s

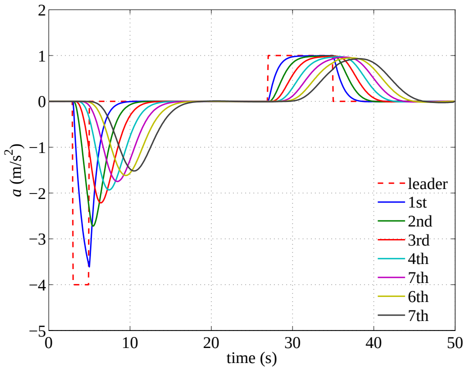


(n) Speed, $\tau^{\mathrm{S}} = 0.5$ s

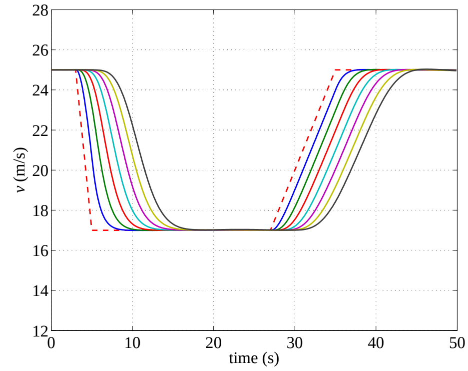


(o) Gap, $\tau^{\mathrm{S}} = 0.5$ s

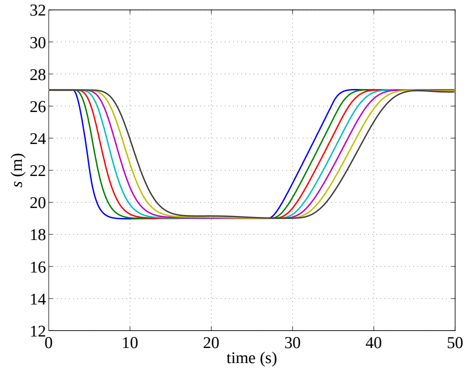
Figure 2. Negative impacts of sensor delay on MP-FR-ACC performance with $\tau^{\mathrm{A}} = 0$ s.



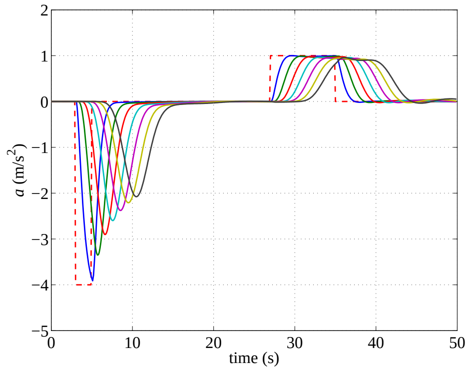
(a) Benchmark acceleration



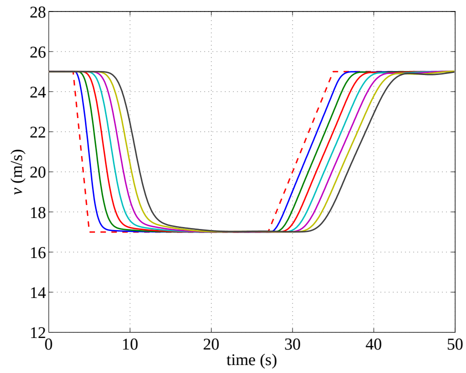
(b) Benchmark speed



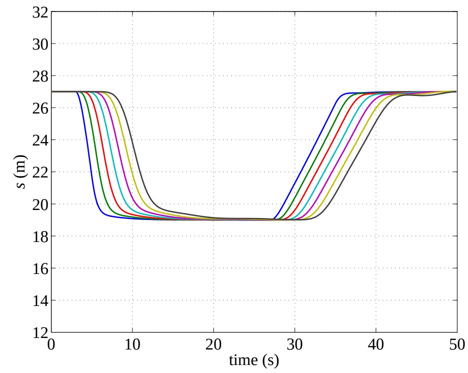
(c) Benchmark gap



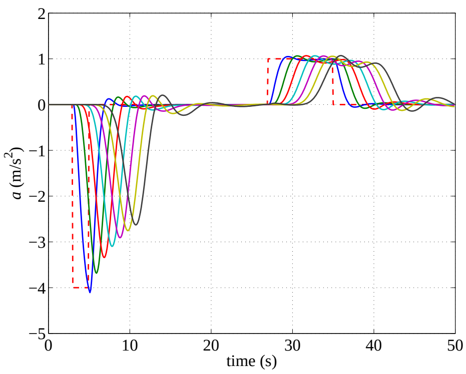
(d) Acceleration, $\tau^{\mathrm{A}} = 0.2$ s



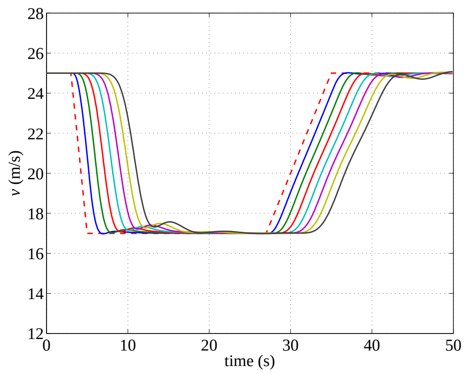
(e) Speed, $\tau^{\mathrm{A}} = 0.2$ s



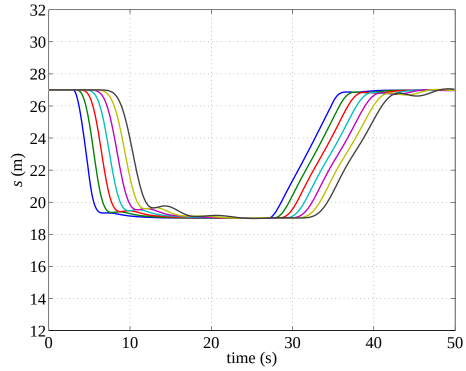
(f) Gap, $\tau^{\mathrm{A}} = 0.2$ s



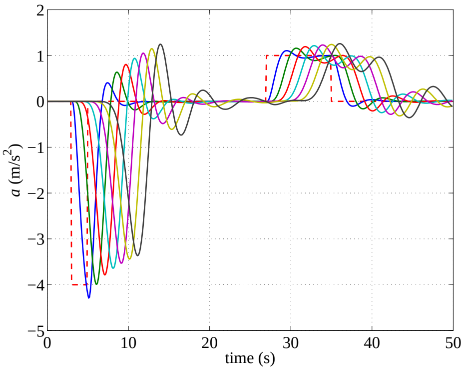
(g) Acceleration, $\tau^{\mathrm{A}} = 0.3$ s



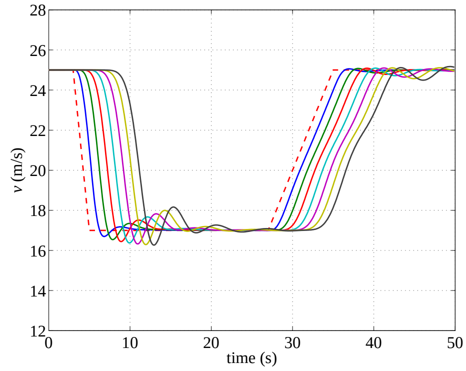
(h) Speed, $\tau^{\mathrm{A}} = 0.3$ s



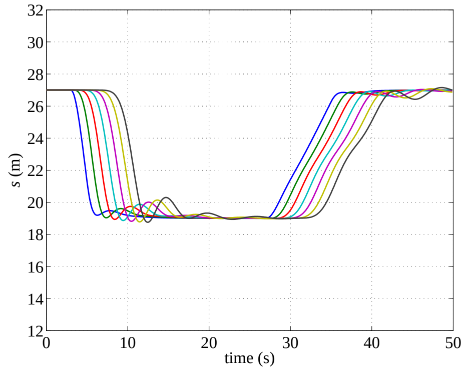
(i) Gap, $\tau^{\mathrm{A}} = 0.3$ s



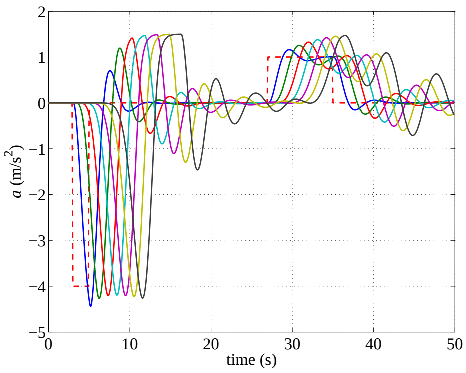
(j) Acceleration, $\tau^{\mathrm{A}} = 0.4$ s



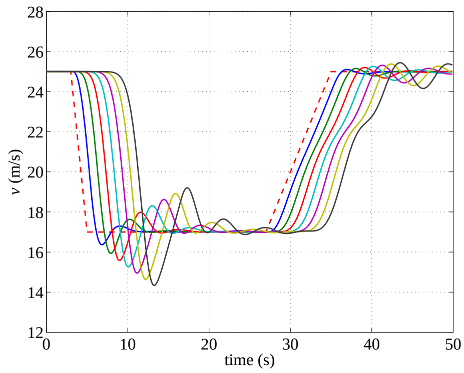
(k) Speed, $\tau^{\mathrm{A}} = 0.4$ s



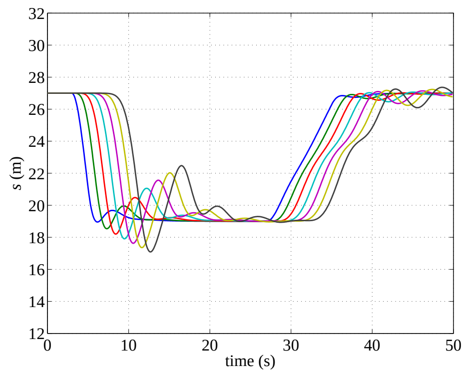
(l) Gap, $\tau^{\mathrm{A}} = 0.4$ s



(m) Acceleration, $\tau^{\mathrm{A}} = 0.5$ s

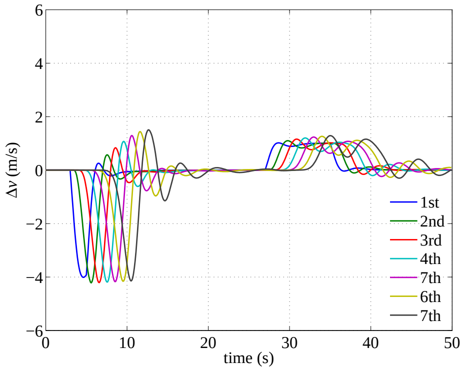


(n) Speed, $\tau^{\mathrm{A}} = 0.5$ s

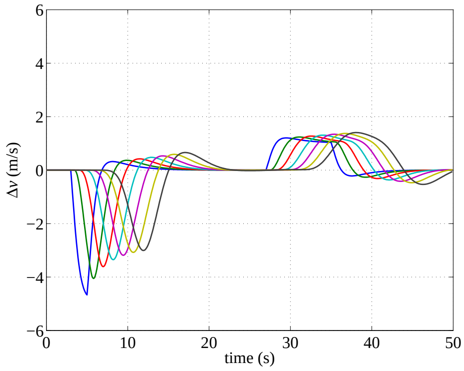


(o) Gap, $\tau^{\mathrm{A}} = 0.5$ s

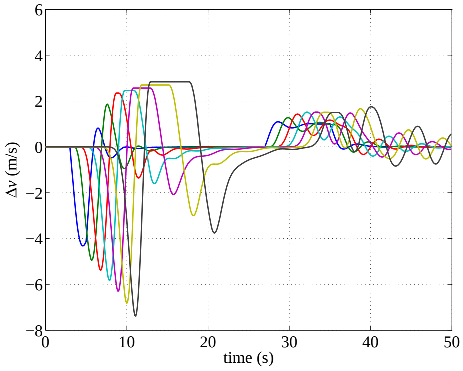
Figure 3. Negative impacts of actuator lag on MP-FR-ACC performance with $\tau^{\mathrm{S}} = 0$ s.



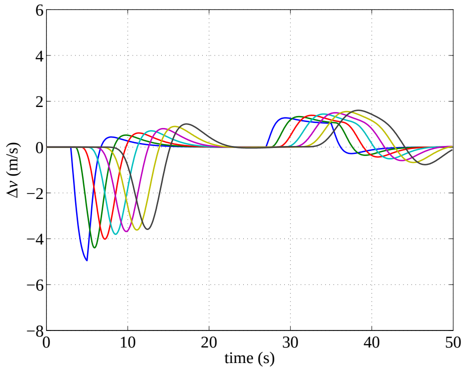
(a) $\tau^{\mathrm{S}} = 0.4$ s without compensation



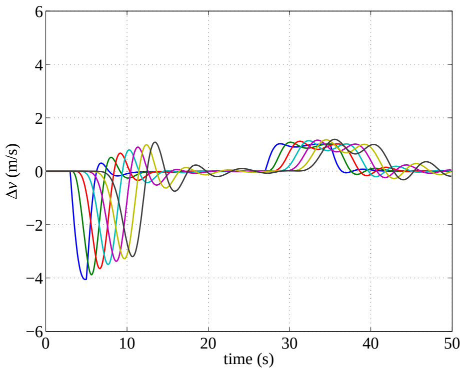
(b) $\tau^{\mathrm{S}} = 0.4$ s with compensation



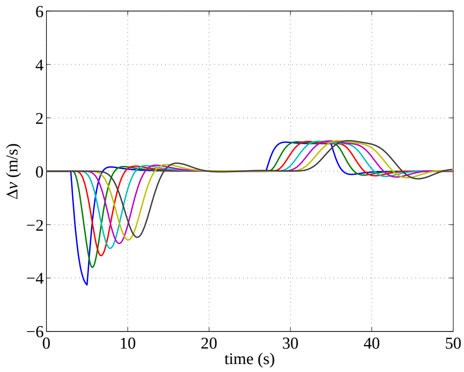
(c) $\tau^{\mathrm{S}} = 0.5$ s without compensation



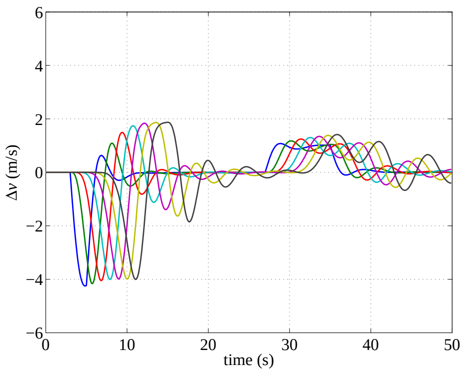
(d) $\tau^{\mathrm{S}} = 0.5$ s with compensation



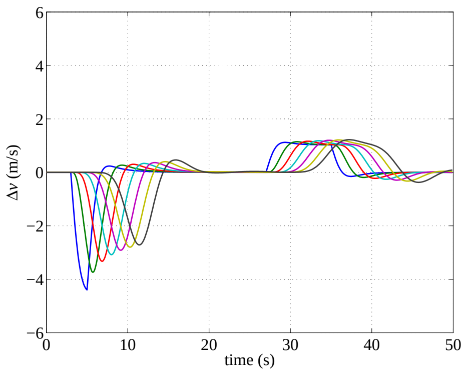
(e) $\tau^{\mathrm{A}} = 0.4$ s without compensation



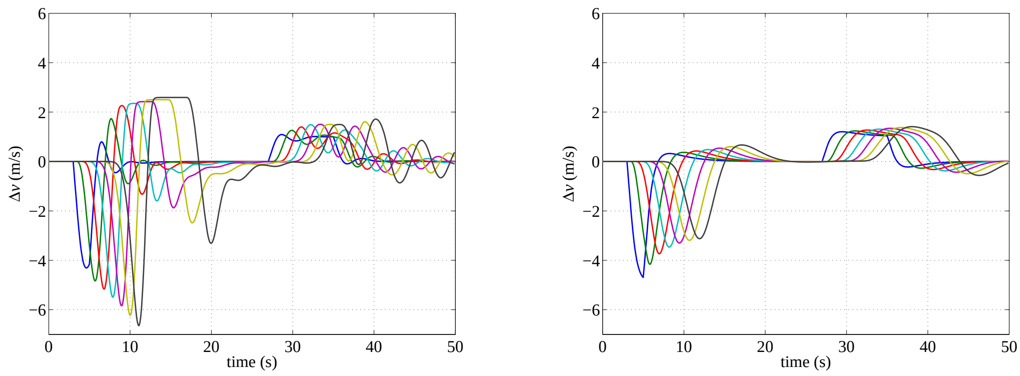
(f) $\tau^{\mathrm{A}} = 0.4$ s with compensation



(g) $\tau^{\mathrm{A}} = 0.5$ s without compensation



(h) $\tau^{\mathrm{A}} = 0.5$ s with compensation



(i) τ^{S}=0.3s, τ^{A}=0.2s without compensation

(j) τ^{S}=0.3s, τ^{A}=0.2s with compensation

Figure 4. Relative speed evolution of MP-FR-ACC platoons without and with delay compensation strategy

from unstable mode to stable mode, which leads to substantial improvement in platoon performance as shown in Table 2.

The simulation experiments presented so far are based on constant sensor delay and actuator lag. It is possible to include stochasticity in the sensor delay. Additional simulation experiments show that the proposed delay anticipation strategy results in improved platoon performance and string stability even in case of bounded stochastic sensor delay.

Table 2. Overview of MP-FR-ACC platoons performance indicators

Indicators	Benchmark	Simple strategy without delay compensation					Anticipatory strategy with delay compensation				
	τ^{S}=0s τ^{A}=0s	τ^{S}=0.4s τ^{A}=0s	τ^{S}=0.5s τ^{A}=0s	τ^{S}=0s τ^{A}=0.4s	τ^{S}=0s τ^{A}=0.5s	τ^{S}=0.3s τ^{A}=0.2s	τ^{S}=0.4s τ^{A}=0s	τ^{S}=0.5s τ^{A}=0s	τ^{S}=0s τ^{A}=0.4s	τ^{S}=0s τ^{A}=0.5s	τ^{S}=0.3s τ^{A}=0.2s
$\sum_{i=1}^{7} J_i$	165.93	290.45	678.25	258.88	359.61	588.19	278.23	334.15	225.56	245.28	286.54
J_1	31.36	39.12	44.50	39.18	42.99	44.36	42.85	47.10	38.50	40.63	43.75
J_7	19.73	44.63	189.15	36.50	62.36	148.39	40.22	52.44	29.44	33.04	41.77
max $\lvert\Delta v_1\rvert^{\mathrm{dcc}}$ (m/s)	3.65	4.01	4.32	4.06	4.25	4.31	4.67	4.96	4.26	4.40	4.69
max $\lvert\Delta v_7\rvert^{\mathrm{dcc}}$ (m/s)	1.53	4.15	7.39	3.20	4.01	6.64	3.01	3.59	2.48	2.71	3.12
max $\lvert a_1\rvert^{\mathrm{dcc}}$ (m/s^2)	3.61	4.48	4.98	4.29	4.44	4.93	4.04	4.15	3.87	3.90	4.05
max $\lvert a_7\rvert^{\mathrm{dcc}}$ (m/s^2)	1.52	4.60	8.00	3.36	4.26	7.81	2.64	3.05	2.33	2.52	2.73
max $\lvert\Delta v_1\rvert^{\mathrm{acc}}$ (m/s)	1.02	1.02	1.09	1.03	1.08	1.09	1.20	1.28	1.09	1.12	1.20
max $\lvert\Delta v_7\rvert^{\mathrm{acc}}$ (m/s)	0.94	1.29	1.75	1.19	1.41	1.72	1.40	1.61	1.14	1.22	1.41
max $\lvert a_1\rvert^{\mathrm{acc}}$ (m/s^2)	1.00	1.14	1.25	1.11	1.16	1.24	1.05	1.07	1.03	1.04	1.05
max $\lvert a_7\rvert^{\mathrm{acc}}$ (m/s^2)	0.93	1.40	1.50	1.26	1.47	1.50	1.26	1.40	1.09	1.15	1.26

7. Concluding remarks

In this paper, we generalized previous work on receding horizon control approach for ACC systems by including sensor delay and actuator lag in the framework. The mathematical framework gives insights regarding the performance deterioration mechanism of delay. Without the anticipation strategy, sensor delay leads to a reduction in the quality of the estimate of the initial condition for the optimal control problem and actuator lag increases the mismatch between the system state prediction model and the actual system behaviour.

We proposed an anticipatory control strategy to compensate sensor delay and actuator lag for MP-FR-ACC platoons. To compensate sensor delay, the approach entails estimating the (unknown) system state at the current time using the system state in a previous time, the applied control history and the system dynamics model. The actuator lag is compensated by including the lag in the state prediction model.

Systematic simulation experiments are carried out to test the impacts of sensor delay and actuator lag on the ACC platoon performance. Results verify that sensor delay and actuator lag reduce string stability of the platoon. Ignoring delay when assessing the impacts of automated vehicles on traffic flow may lead to over-optimistic results.

Interestingly, we are able to separate the impacts of sensor delay and actuator lag in the same framework and simulation results show that string stability deteriorates more with sensor delay than with the same value of actuator lag. No significant string instability of the ACC controller is observed for sensor delay and actuator lag up 0.4 s, which implies that the model predictive ACC is fairly robust to small delays in the control loop.

The anticipatory control strategy clearly improves the platoon performance and enhances string stability. It has the potential to be implemented in real-time. Future research is directed to the analytic analysis on the string stability criterion using Lyapunov method with the presence of the anticipatory strategy and simulation test in combination with lower-level vehicle controllers and nonlinear vehicle dynamic models. In the next stage, we will investigate the human interaction with the ACC system and combine the proposed anticipatory control strategy with data fusion methods to deal with noise in on-board measurements.

Acknowledgement

The authors would like to thank Dr. Jeroen Ploeg at TNO, The Netherlands, for the discussion on sensor delay and actuator lag of automated vehicles.

References

Bifulco, G. N., Pariota, L., Simonelli, F., and Pace, R. D. (2013). Development and testing of a fully adaptive cruise control system. *Transportation Research Part C: Emerging Technologies*, 29:156–170.

Chang, P. and Lee, J. W. (1995). Time delay observer: a robust observer for nonlinear plants using time-delayed signals. In *American Control Conference, Proceedings of the 1995*, volume 3, pages 1638–1642 vol.3.

Fancher, P. and Bareket, Z. (1994). Evaluating headway control using range versus range-rate relationships. *Vehicle System Dynamics*, 23(1):575–596.

Findeisen, R. and Allgöwer, F. (2004). Computational delay in nonlinear model predictive control. In *Proc. Int. Symp. Adv. Contr. of Chem. Proc*, pages 427–432.

Garriga, J. L. and Soroush, M. (2010). Model predictive control tuning methods: A review. *Industrial & Engineering Chemistry Research*, 49(8):3505–3515.

Ge, J. I. and Orosz, G. (2014). Dynamics of connected vehicle systems with delayed acceleration feedback. *Transportation Research Part C: Emerging Technologies*, 46(0):46 – 64.

Godbole, D., N., K., Sengupta, R., and Zandonadi, M. (1999). Breaking the highway capacity barrier: Adaptive cruise control-based concept. *Transportation Research Record: Journal of the Transportation Research Board*, 1679:148–157.

Hasebe, K., Nakayama, A., and Sugiyama, Y. (2003). Dynamical model of a cooperative driving system for freeway traffic. *Physical Review E - Statistical, Nonlinear, and Soft Matter Physics*, 68(2):026102/1–026102/6.

He, Z., Wang, Y., Sun, W., Huang, P., Zhang, L., and Zhong, R. (2016). Modelling car-following behaviour with lateral separation and overtaking expectation. *Transportmetrica B: Transport Dynamics*, 4(3):223–239.

Jia, D. and Ngoduy, D. (2016a). Enhanced cooperative car-following traffic model with the combination of {V2V} and {V2I} communication. *Transportation Research Part B: Methodological*, 90:172 – 191.

Jia, D. and Ngoduy, D. (2016b). Platoon based cooperative driving model with consideration of realistic inter-vehicle communication. *Transportation Research Part C: Emerging Technologies*, 68:245 – 264.

Kesting, A., Treiber, M., Schonhof, M., and Helbing, D. (2008). Adaptive cruise control design for active congestion avoidance. *Transportation Research Part C: Emerging Technologies*, 16(6):668–683.

Li, S., Jia, Z., Li, K., and Cheng, B. (2014). Fast online computation of a model predictive controller and its application to fuel economy-oriented adaptive cruise control. *Intelligent Transportation Systems, IEEE Transactions on*, PP(99):1–11.

Liu, G. P., Mu, J. X., Rees, D., and Chai, S. C. (2006). Design and stability analysis of networked control systems with random communication time delay using the modified MPC. *International Journal of Control*, 79(4):288–297.

Mayne, D. Q., Rawlings, J. B., Rao, C. V., and Scokaert, P. O. M. (2000). Constrained model predictive control: Stability and optimality. *Automatica*, 36(6):789–814.

Milanés, V. and Shladover, S. E. (2014). Modeling cooperative and autonomous adaptive cruise control dynamic responses using experimental data. *Transportation Research Part C: Emerging Technologies*, 48(0):285 – 300.

Monteil, J., Billot, R., Sau, J., Armetta, F., Hassas, S., and Faouzi, N.-E. E. (2013). Cooperative highway traffic : multi-agent modeling and robustness assessment to local perturbations. In *92nd Annual Meeting of Transportation Research Board, Washington, D.C.*

Morari, M. and Lee, J. H. (1999). Model predictive control: past, present and future. *Computers & Chemical Engineering*, 23(4-5):667 – 682.

Ngoduy, D. (2012). Application of gas-kinetic theory to modelling mixed traffic of manual and ACC vehicles. *Transportmetrica*, 8(1):43–60.

Ngoduy, D. (2015a). Effect of the car-following combinations on the instability of heterogeneous traffic flow. *Transportmetrica B: Transport Dynamics*, 3(1):44–58.

Ngoduy, D. (2015b). Linear stability of a generalized multi-anticipative car following model with time delays. *Communications in Nonlinear Science and Numerical Simulation*, 22(13):420 – 426.

Orosz, G., Wilson, R. E., and Stépán, G. (2010). Traffic jams: dynamics and control. *Philosophical Transactions of the Royal Society of London A: Mathematical, Physical and Engineering Sciences*, 368(1928):4455–4479.

Ploeg, J., van de Wouw, N., and Nijmeijer, H. (2014). L_p string stability of cascaded systems: Application to vehicle platooning. *IEEE Transactions on Control Systems Technology*, 22(2):786–793.

Pontryagin, L., Boltyanskii, V., Gamkrelidze, R., and Mishchenko, E. (1962). *The mathematical theory of optimal processes*. CRC Press.

Pueboobpaphan, R. and van Arem, B. (2010). Driver and vehicle characteristics and platoon and traffic flow stability. *Transportation Research Record: Journal of the Transportation Research Board*, 2189:89–97.

Rajamani, R. (2011). *Vehicle dynamics and control*. Springer.

Rajamani, R. and Shladover, S. E. (2001). An experimental comparative study of autonomous and co-operative vehicle-follower control systems. *Transportation Research Part C: Emerging Technologies*, 9(1):15–31.

Redhu, P. and Gupta, A. K. (2015). Delayed-feedback control in a lattice hydrodynamic model. *Communications in Nonlinear Science and Numerical Simulation*, 27(13):263 – 270.

Sau, J., Monteil, J., Billot, R., and Faouzi, N.-E. E. (2014). The root locus method: application to linear stability analysis and design of cooperative car-following models. *Transportmetrica B: Transport Dynamics*, 2(1):60–82.

Sheikholeslam, S. and Desoer, C. (1993). Longitudinal control of a platoon of vehicles with no communication of lead vehicle information: a system level study. *Vehicular Technology, IEEE Transactions on*, 42(4):546–554.

Tampère, C. M. J. (2004). *Human-Kinetic Multiclass Traffic Flow Theory and Modelling - With Application to Advanced Driver Assistance Systems in Congestion*. PhD thesis, Delft University of Technology, Delft.

Treiber, M. and Kesting, A. (2013). *Traffic Flow Dynamics - Data, Models and Simulation.*

Springer.

Treiber, M., Kesting, A., and Helbing, D. (2006). Delays, inaccuracies and anticipation in microscopic traffic models. *Physica A: Statistical Mechanics and its Applications*, 360(1):71–88.

Van Arem, B., van Driel, C. J. G., and Visser, R. (2006). The impact of cooperative adaptive cruise control on traffic-flow characteristics. *IEEE Transactions on Intelligent Transportation Systems*, 7(4):429–436.

van Hinsbergen, C., Schakel, W., Knoop, V., van Lint, J., and Hoogendoorn, S. (2015). A general framework for calibrating and comparing car-following models. *Transportmetrica A: Transport Science*, 11(5):420–440.

VanderWerf, J., Shladover, S. E., Kourjanskaia, N., Miller, M., and Krishnan, H. (2001). Modeling effects of driver control assistance systems on traffic. *Transportation Research Record: Journal of the Transportation Research Board*, 1748:167–174.

VanderWerf, J., Shladover, S. E., Miller, M., and Kourjanskaia, N. (2002). Effects of adaptive cruise control systems on highway traffic flow capacity. *Transportation Research Record: Journal of the Transportation Research Board*, 1800:78–84.

Wang, J. and Rajamani, R. (2004). Should adaptive cruise-control systems be designed to maintain a constant time gap between vehicles? *IEEE Transactions on Vehicular Technology*, 53(5):1480–1490.

Wang, M., Daamen, W., Hoogendoorn, S., and Van Arem, B. (2012). Driver assistance systems modeling by model predictive control. In *15th International IEEE Conference on Intelligent Transportation Systems*, pages 1543–1548, Alaska, USA.

Wang, M., Daamen, W., Hoogendoorn, S. P., and van Arem, B. (2014a). Rolling horizon control framework for driver assistance systems. Part I: Mathematical formulation and non-cooperative systems. *Transportation Research Part C: Emerging Technologies*, 40:271–289.

Wang, M., Daamen, W., Hoogendoorn, S. P., and van Arem, B. (2014b). Rolling horizon control framework for driver assistance systems. Part II: Cooperative sensing and cooperative control. *Transportation Research Part C: Emerging Technologies*, 40:290–311.

Wang, M., Daamen, W., Hoogendoorn, S. P., and van Arem, B. (2016). Cooperative car-following control: Distributed algorithm and impact on moving jam features. *IEEE Transactions on Intelligent Transportation Systems*, 17(5):1459–1471.

Wang, M., Hoogendoorn, S. P., Daamen, W., van Arem, B., and Happee, R. (2015). Game theoretic approach for predictive lane-changing and car-following control. *Transportation Research Part C: Emerging Technologies*, 58:73 – 92.

Wang, M., Treiber, M., Daamen, W., Hoogendoorn, S. P., and van Arem, B. (2013). Modelling supported driving as an optimal control cycle: Framework and model characteristics. *Transportation Research Part C: Emerging Technologies*, 36:547–563.

Wilson, R. E. (2008). Mechanisms for spatio-temporal pattern formation in highway traffic models. *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences*, 366(1872):2017–2032.

Xiao, L. and Gao, F. (2011). Practical string stability of platoon of adaptive cruise control vehicles. *Intelligent Transportation Systems, IEEE Transactions on*, 12(4):1184–1194.

Zhang, L. and Orosz, G. (2016). Motif-based design for connected vehicle systems in presence of heterogeneous connectivity structures and time delays. *IEEE Transactions on Intelligent Transportation Systems*, 17(6):1638–1651.

Zheng, L., Jin, P. J., Huang, H., Gao, M., and Ran, B. (2016). A vehicle type-dependent visual imaging model for analysing the heterogeneous car-following dynamics. *Transportmetrica B: Transport Dynamics*, 4(1):68–85.