Incentivizing Intelligent Customer Behavior in Smart-Grids: A Risk-Sharing Tariff & Optimal Strategies

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Abstract

Current electricity tariffs for retail rarely provide incentives for intelligent demand response of flexible customers. Such customers could otherwise contribute to balancing supply and demand in future smart grids. This paper proposes an innovative risk-sharing tariff to incentivize intelligent customer behavior. A two-step parameterized payment scheme is proposed, consisting of a prepayment based on the expected consumption, and a supplementary payment for any observed deviation from the anticipated consumption. Within a game-theoretical analysis, we capture the strategic conflict of interest between a retailer and a customer in a two-player game, and we present optimal, i.e., best response, strategies for both players in this game. We show analytically that the proposed tariff provides customers of varying flexibility with variable incentives to assume and alleviate a fraction of the balancing risk, contributing in this way to the uncertainty reduction in the envisioned smart-grid.

1 Introduction

Energy systems are in transition towards more sustainable generation portfolios, which must be matched with more flexible demand [Rohjans et al., 2010; Fang et al., 2012]. Many potential and existing problems that the main power grid is facing are connected to the need for continuous balance and the increasing peak demand, both essential in determining the resulting system efficiency and eventually the costs of electricity. Maintaining balance becomes more challenging in face of generation from natural resources such as the sun and wind, which are subjects to stochastic availability. Such stochastic fluctuations and especially deviations from predictions may be matched with expensive fast ramping conventional generators, e.g., gas turbines, which are otherwise only used for matching the peak demand. The balancing power of such quickly adjustable generators is traded on balancing markets. Due to the high marginal costs, the main strategy to control costs is to avoid the need to purchase balancing power by reducing deviations from energy demand and supply predictions, and thus reducing uncertainty.

In the current electricity system, retailers are the balancing responsible parties, pooling customers into larger portfolios. Electricity retailers are facing high risks of balancing market participation due to the volatility of reserve power prices for balancing. At the same time, the penetration of distributed renewable energy sources drives the increasing adoption of flexibility by retail customers, which may be micro-grids, energy cooperatives, prosumers, and plain consumers. Local renewable power generation implies higher risks of energy shortages or overproduction, since generation is volatile and locally highly correlated. Customers may use their flexibility, e.g., from storage, primarily to their own interest rather than in the interest of the retailers’ balancing needs [Vytlilmington et al., 2010].

Most existing electricity tariffs by electricity retailers, especially in Europe, do not provide incentives for intelligent behavior by customers, precluding flexible customers from assuming some of the high costs related to the participation in the balancing markets. Customers can only subscribe to tariffs given their amount of flexibility and local generation with flat or day-night tariffs that may exhibit different costs in relation to the capacity of their flexibility as well as their scale. Dynamic pricing is a means to encourage favorable changes in demand patterns by the customers [Borenstein et al., 2002; Roozbehani et al., 2010]. Time of use (ToU), critical peak price (CPP), and real-time pricing are some of the pricing schemes used to stimulate favorable customer behavior in different pilot studies [Owen and Ward, 2010], e.g., in Ontario or California. However, dynamic pricing approaches may introduce disruptive and unfavorable market behavior [Roozbehani et al., 2012; Herter and Wayland, 2010], and thus planning and ahead prices are required [Braithwait et al., 2007].

We consider a multi-agent system, where a buyer agent wants to purchase an uncertain quantity of a continuously divisible good from a seller agent. We refer to the buyer and the seller as the customer and the retailer respectively. We present the risk-sharing tariff, a novel approach to incentivize intelligent customer behavior (uncertainty reduction by the demand side) by giving customers the choice to assume balancing risk of the retailer. We consider settings where one customer has a direct or a representative influence on the balancing requirements of the retailer. This is the case in:

I. Service level agreements (SLAs) formally define an agreement between a service provider and the service
user, specifying the service and its characteristics, e.g., quality, risk. In the context of electricity markets we interpret SLAs as a direct extension of conventional electricity tariffs: While current electricity tariffs ensure delivery (100% quality) and a fixed kwh price (0% risk), SLAs may provide customer choice, such as assuming parts of the balancing risk, as discussed in this paper. Such SLAs may be better suited for decentralized trading of electricity between small-scale producers and individual customers.

**II. Highly correlated demand** can be the result of similar behavior of customers, influenced for instance by weather conditions in specific locations. The higher the correlation, the closer the deviations of one customer to the deviations of other customers, i.e., changes in the consumption behavior of one customer predicts the same change in the behavior of other customers. Therefore, the portfolio distribution may closely resemble the demand distribution of an individual customer for any specific location.

**III. Local balancing**: Current market-based balancing strategies do not consider the location of the customer, while it is in the retailer’s own interest to balance customers locally. This can lower the costs corresponding to energy losses, transportation costs, and network load.

We formalize the interaction between the retailer and the customer as a two-player game (Section 3), and we study optimal strategies for both players. We define a two-step payment scheme, where the customer pays for its expected demand and later pays for any imbalances (Section 3.2). We show that the proposed tariff provides variable incentives and elicits intelligent behavior by the customer (Section 3.3). We further demonstrate the existence of Nash equilibria in this game, assuming that the retailer has access to the private costs of the customer (Section 4). Last, we discuss the concept of bounded-rationality and show that retailers may offer larger incentives to bounded-rational customers (Section 4.1).

### 2 Related Work

Challenging problems are arising with the transition from the current energy system to the envisioned smart-grid [Fang et al., 2012]. These problems provide a fertile ground for tools like game-theory and multi-agent systems to study situations with more than one stakeholder [Fadlullah et al., 2011; Pipattanasomporn et al., 2009]. The conflict of interest between the retailer and the customer has been formalized as a non-cooperative game with regards to ToU tariffs [Oruc et al., 2012] and incentives provided to customers for load-shifting [Pettersen et al., 2005]. Similarly, we formalize a game between the retailer and the customer to study incentives for intelligent customer behavior. In line with related literature, which studies optimal procurement strategies for the retailer under the presence of either uncertain demand [Nair et al., 2014] or uncertain prices [Hoogland et al., 2015], we assume a two-step market setting, where the prices are fixed but the demand is uncertain. In the closest state-of-the-art work a “prediction-of-use” tariff is proposed, where customers are charged both on their predicted consumption, but also on their deviations from this prediction [Vinyals et al., 2014]. In the same fashion, we propose a two-step parameterized payment, where the customer precommits and prepays for its expected demand, and later pays for any deviation between the observed and the anticipated load.

To the best of our knowledge, this is the first game-theoretical study that considers incentives for intelligent customer behavior, giving customer the choice of how much risk to take from the retailer.

### 3 The Risk-Sharing Game

We capture the strategic interactions between the retailer and the customer in a two-player game. Figure 1 illustrates the extensive form representation of the risk-sharing game, showing the time sequencing of the actions. We consider a two-step market. The retailer first procures electricity in the ahead market with the unit price \( p \) and later pays for any absolute deviation, between the observed demand of the customer and the procured quantity in the ahead market, in the balancing market with the unit price \( p' > p \). The prices \( p, p' \) are determined by an exogenously process and cannot be influenced by the retailer (price-taker). Let \( x \) denote the random variable and \( f_x \) the probability distribution function (PDF) of the customer’s demand. We consider the distribution \( f_x \) as the default behavior by the customer. The distribution \( f_x \) is known to both players, since it can be observed, e.g., by smart-meters, and can be approximated given enough observations. Therefore, the proposed tariff requires the customer to precommit to and prepay the quantity \( b_c = E_f[x] \), which is equal to the anticipated consumption.

The retailer, based on the customer’s demand distribution \( f_x \), procures the quantity \( b_r \) in the ahead market. Any absolute deviation, between the quantity \( b_r \) and the observed consumption \( x \) of the customer, is balanced by the retailer in the balancing market. We consider the expected balancing costs as the balancing risk for the retailer [Ferguson, 1967], which is equal to \( E_f[b_r - x]p' \), assuming a direct influence of the customer to the balancing needs of the retailer (Cases I-III in Section 1).  

\[ b_r = \frac{E_f[b_r - x]}{p'} \]
Lemma 3.1 The first derivative of the expected utility of the retailer in (3) with respect to \(b_r\) is:

\[
\frac{d}{db_r} U_r^f = -p - 2p' F_x(b_r) + p',
\]

where \(F_x\) is the cumulative distribution function (CDF) of the random variable \(x\).

Theorem 3.2 The quantity \(b_r^*\) maximizes the expected utility of the retailer:

\[
b_r^* = F_x^{-1}\left(\frac{p' - p}{2p'}\right),
\]

where \(F_x^{-1}\) is the inverse cumulative distribution (ICDF) function.

Proof Equation 5 follows from \(\frac{d}{db_r} U_r^f = 0\). The expected utility of the retailer is a strictly concave function: \(\frac{d^2}{db_r^2} U_r^f = -2p'^2 f_x(b_r) < 0\). Therefore, \(b_r^*\) is indeed the unique optimum.

Note, for any given \(p' > p\), the quantity \(b_r^*\) is lower than the expected demand due to the absolute imbalance quantity.

3.2 Determining the Price for Risk-Sharing

In this section, we define the requirements and the properties of the risk-sharing tariff and we propose how to choose the price functions. An important requirement for the price functions \(p_c(\tau), p_r' (\tau)\) is that the expected utility of the retailer for any given \(\tau \in [0, 1]\) should be greater or equal to the expected utility when \(\tau = 1\). Analytically:

\[
U_r^f(\tau) \geq U_r^f(\tau = 1) = b_c \varphi, \quad \forall \tau \in [0, 1), \ \varphi \in \mathbb{R}^+,
\]

where \(\varphi\) denotes an extra profit for the retailer per expected unit of consumption. \(\varphi\) is approaching business costs in a perfect competition and arbitrarily large values in a monopoly.

Given the requirement in (6) and using (3), we derive the following inequality:

\[
p_c(\tau) \geq \frac{1}{b_c} (b_r^* p + \mathbb{E}_f([b_c - x][p' - \mathbb{E}_f([b_c - x][p_r'(\tau)]] + \varphi)\) \quad (7)
\]

To find functions \(p_c(\tau), p_r'(\tau)\) that satisfy the above inequality, we define the minimum imbalance price function:

\[
g_r^c(\tau) \triangleq (1 - \tau)p',
\]

which is equal to the price the customer would pay by participating in the balancing market for its share (1 - \(\tau\)) of balancing risk. Since \(p_c(\tau)\) is a free choice, we propose the minimum ahead price function that satisfies (7) when replacing \(p_r'(\tau)\) with (8):

\[
p_c(\tau) \geq \frac{1}{b_c} (b_r^* p + \mathbb{E}_f([b_c - x] + (\tau - 1)[b_c - x])p') + \varphi, \quad (9)
\]

We will proceed to show that this proposed price function guarantees the minimum profit margin \(\varphi\) for the retailer.

Theorem 3.3 Any tariff \((p_c(\tau), p_r'(\tau))\), using \(p_c(\tau)\) as defined in (9) and satisfying \(p_c(\tau) \geq g_r^c(\tau), \forall \tau \in [0, 1]\), and \(p_r'^c(1) = 0\), satisfies (6).

Proof Note that \(p_c(\tau)\) has been defined such that \(U_r^g(\tau) = U_r^f(1), \) if \(p_r'(\tau) = g_r(\tau).\) For any \(p_r'(\tau)\) that satisfies \(p_r'(\tau) \geq g_r(\tau), \forall \tau, U_r^g(\tau) \geq U_r^f(\tau),\) since only the profit from the term \(p_r'(\tau)\mathbb{E}_f([b_c - x] increases while all other terms are fixed. 

The function \(p_r'(\tau)\) refers to the price per unit for any absolute deviation of the customer’s consumption given the choice of \(\tau.\) We propose \(p_r'(\tau)\) to embrace some additional desired
properties with regards to the ability of the customer to reduce its demand uncertainty.

Consider a customer that can alter its demand distribution $f_x \to g_x$, such that $E_g[|b_c - x|] \leq E_f[|b_c - x|]$. We define $g_x$ as the demand response of the customer. We propose a tariff that additionally imposes the constraint $E_g[x] = E_f[x]$. Let $\tau^*(g_x)$ denote the risk that maximizes the utility of the customer under $g_x$. The following two properties are common sense conditions for demand response tariffs.

**Property 1.** No demand response, no risk incentive: If $E_u[|b_c - x|] = E_f[|b_c - x|]$ then $\tau^*(g_x) = 1$.

**Property 2.** Demand response proportional risk: If $E_g[|b_c - x|] < E_u[|b_c - x|] < E_f[|b_c - x|]$ then $0 \leq \tau^*(g_x) < \tau^*(u_x) < 1$.

Figure 2 illustrates the shape of the customer for any given $g_x$. The further improvement in (11) follows the same ranking, as it is the product of $X_{\theta}$ and $X_{\tau}$.

Theorem 3.6 For a customer with uncertain demand response $g_x$ that can at least reduce the uncertainty of its demand to the level of $u_x$, such that (14) holds, $U_c^p(\tau^*(u_x)) > U_c^p(\tau^*(u_x)) > U_c^p(\tau^*(u_x)) > U_c^p(\tau^*(u_x)) > U_c^p(\tau^*(u_x))$. We will proceed to show that a customer with uncertain demand response has incentives to participate in the risk-sharing tariff contributing its maximum available demand response. Consider the distribution $u_x$ such that:

$$E_g[|b_c - x|] < E_u[|b_c - x|] < E_f[|b_c - x|],$$

where $u_x$ provides a threshold ability of the customer to reduce the expected absolute deviation of the demand.

**Theorem 3.5** The quantity $\tau^*$ maximizes the expected utility of the customer for any given $g_x$ and $C_{\Delta}$.

$$\tau^*(g_x) = \frac{a_g - a_f}{2 a_a} + 1$$

where $|x|^h = \min(l, \max(h, x))$.

**Proof** Equation 13 follows from $\frac{\partial}{\partial \tau} U_c^p = 0$. The utility function of the customer with regards to the risk assumption $\tau$ is strictly concave, since $\frac{\partial^2}{\partial \tau^2} U_c^p = -2a_a < 0$. Therefore, $\tau^*$ is the unique optimum.

Under the assumption of a cost-free demand response model, i.e., $C_{\Delta} = 0$, we will proceed to show that a customer with uncertain demand response has incentives to participate in the risk-sharing tariff contributing its maximum available demand response. Consider the distribution $u_x$ such that:

$$E_g[|b_c - x|] < E_u[|b_c - x|] < E_f[|b_c - x|],$$

where $u_x$ provides a threshold ability of the customer to reduce the expected absolute deviation of the demand.
Theorem 3.6 implies that a customer with uncertain demand response \( g_x \), that is bounded by \( u_c \), can only benefit by contributing all available demand response. Furthermore, any choice of \( \tau \geq \tau^*(u_c) \) ensures a lower bound for the utility of the customer.

For the remainder of this paper, we assume that both \( g_x, f_x \) are normal distributions with \( \mu_g = \mu_f \), and standard deviations \( \sigma_f \) and \( \sigma_g \in (0, \sigma_f) \) respectively. For this restricted case, we will apply a simplified notation. The optimal strategy for the flexible customer is denoted by \( \tau^*(\sigma_g) \), similarly the cost by \( C_\Delta(\sigma_g) \). Figure 3 presents the function \( \tau^*(\sigma_g) \) for different values of \( \vartheta \) in (10). For \( \vartheta \approx 0 \), the utility of the customer becomes a linear function that is monotonically increasing in \( \tau \) when \( \sigma_g = \sigma_f \). Thus, the optimal choice of the customer becomes \( \tau^*(\sigma_f) = 1 \), and \( \tau^*(\sigma_f < \sigma_f) = 0 \).

For \( \vartheta \approx \infty \), the optimal choice of the customer is to assume no risk \( \tau^*(\sigma_g) = 1 \), \( \forall \sigma_g \in (0, \sigma_f) \), since the penalty term \( \Phi \) in (10) is infinitely scaled.

In this section, we derived the optimal strategy \( \tau^*(g_x) \) of the customer. We showed how the choice of the parameter \( \vartheta \) by the retailer can influence the optimal strategy of the customer. Furthermore, we demonstrated by Theorem 3.6 that the risk-sharing tariff is attractive to customers with uncertain demand response.

### 3.4 Comparison of the Utilities

We compare the expected utilities of both players, again under the assumption of a cost-free demand response model, i.e., \( C_\Delta(\sigma_g) = 0 \), \( \forall \sigma_g \in (0, \sigma_f) \). Figure 4 illustrates the expected utilities of both players. Let the tuple \( (U'_c, U'_r) \) illustrate the point in the utility space that represents the current flat tariff situation, i.e., \( \sigma_g = \sigma_f \) and \( \tau = 1 \). Each line segment in the figure represents the utility tuples given a specific demand response \( \sigma_g \) and varying \( \tau \). The empty circles represent the utility tuples when the customer chooses to assume no risk \( (\tau = 1) \). In such a case, demand response only yields benefits to the retailer. On the contrary, filled circles represent the utility tuples when the customer chooses to assume the full share of risk. Increasing the risk assumption (moving across the line segments from \( \tau = 1 \) to \( \tau = 0 \)) requires a certain level of demand response to be profitable for the customer. For high demand response (low \( \sigma_g \)), it results in the utility increase for the customer. For low demand response (high \( \sigma_g \)), only the retailer benefits from the decreasing uncertainty of the demand. Reduced uncertainty in the demand side can contribute to the improved social welfare (sum of the players’ utilities) through the risk-sharing tariff.

Theorem 3.6 can also be illustrated using Figure 4. Note that for normal distributions, \( \mathbb{E}[|x - \mu|] = \sigma_g \sqrt{2 \pi} / \sigma_f [\text{Geary, 1935}] \). Hence, \( \sigma_g < \sigma_f \) implies that the inequalities in (14) hold. According to Theorem 3.6, it follows that \( U'_r(\tau^*(\sigma_u)) > U'_r(\tau^*(\sigma_u)) > U'_c(1) \). Intuitively, the customer can increase its utility by switching from \( \sigma_u \) to \( \sigma_g \), or more generally by switching from \( u_c \) to \( g_x \).

### 4 Nash Equilibrium Strategies

In this section, we study the Nash equilibria (NE) of the risk-sharing game. Where necessary, we make the dependence of utilities on both strategies more explicit by using notation \( U_c(\pi_c, \pi_r) \) and \( U_r(\pi_r, \pi_c) \), where \( \pi_r, \pi_c \) denote the strategies of the retailer and the customer respectively. NE are pairs of strategies \( (\pi^*_c, \pi^*_r) \), such that \( U_c(\pi^*_c, \pi^*_r) \geq U_c(\pi_c, \pi_r), \forall \pi_c \) and \( U_r(\pi^*_c, \pi^*_r) \geq U_r(\pi_c, \pi^*_r), \forall \pi_r \). Let \( C_\Delta(\sigma_g) \geq 0, \forall \sigma_g \in (0, \sigma_f) \) be an arbitrary cost model for demand response and \( C_{\Delta}(\sigma_f) = 0 \), i.e., cost without demand response is zero. We assume that the demand response cost model is known by the retailer.

First, we define the strategies of the two players. For the retailer, the only free choice is the scalar \( \vartheta \) that parameterizes the proposed tariff in (10). The strategy of the retailer is denoted by \( \pi_r = \vartheta \). For the customer, the strategy \( \pi_c = (\tau, \sigma_g) \) refers to the choice of risk \( \tau \) and demand response \( \sigma_g \).

Furthermore, the strategy includes the credible threat of returning to the flat tariff without any demand response, \( \pi_{\text{athrtt}} = (1, \sigma_g) \), if the utility drops below the reference utility \( U'_c \).

The threat is possible due to the action sequence indicated in Figure 1 and credible since the threat strategy outperforms the protected equilibrium strategy. Given any \( C_\Delta(\sigma_g) \) we know from Theorem 3.5 that for any given strategy \( \pi_c \), there always exists a strategy \( \pi^*_c \) that maximizes the expected utility of the customer: \( \pi^*_c = (\tau^*(\sigma_g), \sigma_g) \), \( \sigma_g = \arg \max_{\sigma_g} \mathbb{E}[U'_c(\tau^*(\sigma_g))] \).

Figure 5 illustrates the utilities of the two players, computed using the quadratic demand response cost model \( C_\Delta(\sigma_g) = w(\sigma_f - \sigma_g)^2 \), for \( w = 10 \). Each curve corresponds to one of the following three retailer strategies: \( \pi^*_r = (\vartheta^*, \sigma^*_g) = (\vartheta \geq \vartheta^*) \), \( \pi^*_r = (\vartheta < \vartheta^*) \). The utility tuples along each curve correspond to the customer strategies \( \pi_c = (\tau^*(\sigma_g), \sigma_g) \). The curves start from the utility tuple...
Figure 6: Left: The choice $\nu^*$ by the retailer depending on the irrationality parameter $\lambda$. The value of $\nu^0$ corresponds to NE I. Right: The utility pairs starting from the rational NE pair (star) and ending at the open circle, where the customer acts almost randomly (the curve would continue in a straight zero-sum line to the upper left). The filled circle indicates $\nu^*$, which maximizes the utility of the customer.

$(U'_c, U'_g)$ denoted by the empty circle, where the customer does not assume any risk, and hence no demand response, i.e., $\pi_c = (1, \sigma_f)$. The curves end where $\pi_c = (0, 0)$ denoted by the filled circles. Note that the strategy $\pi^*_c = \nu^*$ that maximizes the utility of the retailer (solid curve), yields $U^*_g(\tau^*(\sigma^*_g), \sigma^*_g) = U^*_g(1, \sigma_f)$ for the customer. The utility of the customer of using demand response (star on solid curve) becomes equal to the utility without demand response (open circle).

**Theorem 4.1** The strategy pair $I$, $(\pi^*_r, \pi^*_c) = (\nu^*, \nu^*)$, and the set of pairs $II$, $(\pi^*_r, \pi^*_{\text{threat}} = (1, \sigma_f))$, are the only two types of NE in the risk-sharing game.

**Proof** Any positive change $\varepsilon$ in the strategy of the retailer, such that $\nu \leftarrow \nu^* + \varepsilon$ (e.g., $\pi^*_r$), will cause the customer to adopt the strategy $\pi^*_{\text{threat}}$, since $U^*_c(\pi^*_c, \nu) < U'_c$, leading both players to the utility pair $(U'_c, U'_g)$. On the other hand, any negative change $\varepsilon$ (e.g., $\pi^*_c$) will directly reduce the retailer’s income. Hence, the retailer has no incentive to deviate from I. In addition, the customer strategy is the best response by definition, and thus the customer has no incentive to deviate either, making I a NE. Now, consider any strategy pair $(\pi^*_r, \pi^*_c)$. The retailer can gain by deviating from this strategy pair by increasing $\nu$, making sure that no equilibrium containing $\pi^*_c$ exists. Finally, consider any pair $(\pi^*_r, \pi^*_{\text{threat}})$. Since $U'_c$ is unaffected by $\nu$ given $\tau^*(\sigma^*_c) = 1$, providing no incentive to apply demand response, no player can gain by deviating unilaterally. Hence, each pair of strategies in the set II is a NE.

We showed the existence of two types of NE within the risk-sharing game, where $(\pi^*_r, \pi^*_c) = (\nu^*, \nu^*)$ Pareto dominates $(\pi^*_r, \pi^*_{\text{threat}} = (1, \sigma_f))$ and therefore is favorable for both players.

4.1 Bounded-Rational Customer

The concept of bounded rationality [Simon, 1972] assumes that agents, automated or not, do not behave as perfectly rational decision-makers, bounded by imperfect information and their limited capacity.
References


