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Integrated dynamic modelling and multivariable control of HVAC components

Harish Satyavada, Robert Babuška and Simone Baldi[1]

Abstract— **The field of energy efficiency in buildings offers challenging opportunities from a control point of view. Heating, Ventilation and Air-Conditioning (HVAC) units in buildings must be accurately controlled so as to ensure the occupants' comfort and reduced energy consumption. While the existing HVAC models consist of only one or a few HVAC components, this work involves the development of a complete HVAC model for one thermal zone. Also, a novel multivariable control strategy based on PI auto-tuning is proposed by combining the aforementioned model with optimization of the desired (time-varying) equilibria. One of the advantages of the proposed PI strategy is the use of time-varying input equilibria and zone set-point temperature, which can lead to important energy savings. A comparison with a baseline control strategy with constant set-point temperature is presented: the comparison results show good tracking performance and improved energy efficiency in terms of HVAC energy consumption.**

I. INTRODUCTION

Heating, Ventilation and Air-Conditioning (HVAC) systems widely used in residential and commercial buildings, are responsible for a large part of the energy consumption worldwide. According to the EC's Joint Research Center, Institute for Energy (2009), HVAC systems in the European Union member states were estimated to account for approximately 313 TWh of electricity use in 2007, about 11% of the total 2800 TWh consumed in Europe that year [1]. Energy savings in HVAC systems was therefore identified as a key element to fulfill the target of reducing energy consumption by 20% by 2020 and increased attention has been focused on the reduction of HVAC energy costs while catering to comfort requirements [2], [3] in the form of more efficient equipment [4], [5], [6], novel approaches to HVAC energy storage [7], [8], [9] or supervisory control techniques [10], [11], [12].

To assess the performance of the control strategy on an HVAC system, it is essential to have a model of the system. A typical HVAC system is comprised of a boiler, an air handling unit (AHU), a radiator, a thermal zone, valves, dampers, fan, pump, pipes and ducts. Earlier works implementing control strategies for HVAC systems have developed models of HVAC systems consisting of a radiator along with a zone [13] or an AHU with a zone [14], [15], [16]. Three types of models can be used to construct an HVAC system namely, white-box, black-box and gray-box models [17], [18].

Most of the controllers commissioned in industrial areas are of Proportional-Integral-Derivative (PID) type. Tuning a PID controller requires an accurate model of the process and an effective controller design rule. The tuning procedure can be a time-consuming, expensive and difficult task. This is also true in HVAC systems [19], [20]. Work by Brandt [21] discusses the need for auto-tuning the PID parameters in an optimal way, over manual tuning. PID auto-tuning, aids in automatically determining PID parameters without human intervention, thereby relieving the pain of manually tuning the controller. Some of the earlier works have focused on automated tuning of PID control parameters, to achieve desired performance using either process identification method [19] or a computer based auto-tuning using the step test of Ziegler and Nichols [22]. In [2] the coefficients of a PID controller modulating the damper opening, were calibrated and updated repeatedly from a window of potential coefficient values to minimize the energy consumption at every iteration.

This work involves the development of a complete HVAC model for one thermal zone. In contrast with [13], [14], [15], [16] where single or a few HVAC components were considered, this work presents an integrated model of an HVAC system comprised of a boiler, a radiator, an AHU, a thermal zone, valve, damper, fan, pump, pipes and ducts. Furthermore, while in most literature a fixed return/fresh air mixing is considered in the AHU (c.f. [23], [24]) here, the recirculating damper is modelled so as to enable the proportion of mix of return air and fresh air to be varied according to the damper control signal. In addition, a multiple-input-multiple-output control approach is adopted in this work. Instead of decentralized controllers, we tune five PI controllers which cooperate towards the same task. The five PI controllers control the power supplied by the boiler, the velocity of water circulating through the system, the velocity of air circulating through the AHU, the damper and valve opening. Differently from [2] where the controller coefficients (proportional, integral, derivative) were tuned optimally one after the other, this work involves optimal tuning of all the controller parameters simultaneously. A novel multivariable control strategy based on PI auto-tuning is proposed by combining the aforementioned integrated model with optimization of the desired (time-varying) equilibria. One of the advantages of the proposed PI strategy is the use of time-varying input equilibria and zone set-point temperature which can lead to important energy savings. A comparison with a baseline control strategy with constant set-

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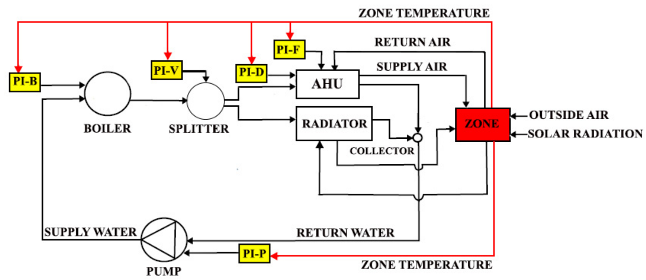


Fig. 1: HVAC system and control architecture

point temperature is presented: the comparison results show good tracking performance and improved energy efficiency in terms of HVAC energy consumption.

The paper is organized as follows: Section II describes the modelling of the HVAC system based on first principles. Section III describes the control oriented modelling of the system. Section IV presents the proposed control strategy. Section V presents the simulation results while section VI presents comparisons and analysis.

II. Modelling the HVAC system

This section presents the heat and mass transfer equations governing a complete HVAC system consisting of a boiler, a radiator, an AHU, pipes, a valve, a damper, a pump and a thermal zone. The AHU is comprised of ducts, a fan, a recirculating damper, a mixing zone and a heating coil. The boiler heats supply water, which is circulated throughout the system. The hot supply water, aids in warming the zone through the radiator and also interacts with the air in the AHU. Lumped capacitance models are chosen for radiator, boiler, pipes, ducts and heating coil. The resulting partial differential equations are for a linear spatial dimension (denoted by l).

A. Radiator

A simplifying assumption is made in the model of the radiator [13], assuming the same temperature for radiator surface as the water temperature inside the radiator. By this assumption, the heat from the radiator is transferred to the ambient only by convection. The indoor air in the zone acts as an ambient for the radiator.

B. Boiler

In a boiler, fuel is consumed and heat is produced which heats the return water. The boiler is modelled as in [25].

C. Pipes

Pipes transport water across various components in the HVAC system. An assumption is made that the pipe's surface temperature is the same as the water temperature inside the pipe. In the system under consideration all the pipes are located inside the walls. As a result, the ambient temperature for the pipes is the wall temperature.

D. Ducts

Ducts transport air across the sub-components of the AHU. It is assumed that the duct surface temperature is the same as the air temperature in the duct. As the ducts are located inside the walls, the ambient temperature for the ducts is the wall temperature.

E. Heating Coil

The heating coil is a sub-component of the AHU. In the heating coil, the air from mixing zone interacts with the hot water. The result of this interaction is modelled.

F. Valve and Splitter

The supply water is divided into two flows at the splitter. One flow goes to the heating coil in AHU and the other to the radiator. The valve dictates the mass flow rate of water in each of the flows.

G. Collector

The collector is a counterpart of the splitter. The collector accumulates water flows from the radiator and the AHU. Due to merging, the resulting mass flow rate changes.

H. Pump and Fan

Pumps control flow rate of water through the HVAC system. Analogous to the pump, the fan determines the flow rate of the air through the AHU.

I. Mixing zone

The mixing zone is a sub-component of the AHU. As a result of interaction between fresh air and return air through the recirculating damper, there is a change in the mass flow rate and temperature of the mixture. The dynamics of the interaction are similar to that of a collector.

J. Damper

In this model, a recirculating air damper is considered. The recirculating damper controls the percentage mix of the return and fresh air. The mass flow-rate of air leaving the recirculating damper is modelled as in [26].

K. Zone

In this model of the zone all walls, ceiling and glazing are assumed to have the same characteristics which are grouped together as envelop parameters. Two separate temperature layers for the floor are considered to make the model extendable to the situation where floor heating is available as another heat source. The evolution of temperature of wall, two layers of floor and indoor air temperature are modelled as in [13].

III. Control Oriented Modelling

The resulting partial differential equations of the lumped capacitance models of the radiator, boiler, pipes, ducts and heating coil are discretized to obtain a model suitable for simulation and control. These lumped models can be discretized into N elements. Temperature in every element T_n for $n \in 1,2 \ldots N$ can be modelled as a separate state. The state of each one of these elements represents the temperature of water or air in that particular element. Backward discretization in space has been used for the dicretization of these models [26]. For control purposes we use only one element in each of the discretized models of the aforementioned lumped models. Furthermore, for simulation purposes the radiator, boiler, pipes, ducts and heating coil have been discretized with three elements each. The corresponding equations which have been implemented in MATLAB®/Simulink®, are not shown due to lack of space. We thus come up with a model (finely discretized) used for simulation and analysis, and a relatively simpler model (coarsely discretized) used for control purposes.

IV. Controller Synthesis

The control objective is to maintain the zone temperature at a value guaranteeing desired comfort and indoor air quality, while at the same time minimizing energy consumption. Two control strategies with different desired zone temperature set-points and objective functions are proposed in this paper. The five PI controllers shown in Fig. 1 are used in each of the two control strategies to control the following:

- PI-B to control power supplied by the boiler.
- PI-P to control velocity of the water flow through the system.
- PI-F to control velocity of the air flow in the AHU.
- PI-V to provide control signal to the splitter, which in turn controls the opening/closing of the valve.
- PI-D to provide control signal to the recirculating damper which in turn controls the opening/closing of the re-circulating damper.

A. Baseline PI Strategy

For the baseline PI control strategy we consider a desired temperature of 23 ^{o}C. The objective function F chosen for the baseline PI strategy is as follows:

$$F = \sum_{t=1}^{T} (T_z(t) - T_{z_{des}}(t))^2 + 0.01(Q_b(t))^2 + 0.01 v_a((t))^2 + 0.01(v_w(t))^2), \qquad (1)$$

where $T_{z_{des}}$ is the desired zone temperature (23 ^{o}C) and T represents the total duration of the simulation. The power supplied by the boiler, air and water velocity are scaled by the same weights so as to have them in the same order of magnitude and contribute fairly to the total score. The objective function in (1) is chosen such that while we keep the zone temperature at the desired set-point we also minimize power consumed by the boiler, water circulation velocity and air circulation velocity. In the baseline PI strategy the PI parameters of the five PI controllers are tuned in such a way that the objective function (1) is minimized.

B. Optimal PI strategy

The dynamics of the various HVAC components in the system together represent a nonlinear model of the complete system. The nonlinear model can be written in the form of:

$$\dot{x}(t) = f(x(t)) + g(x(t), u(t), d(t)), \qquad (2)$$

where $x \in \mathbb{R}^{19}$, represents all the states, $u \in \mathbb{R}^5$ represents the inputs to this system and d represents the disturbances that act on the system. The states, inputs and disturbances of the system are listed in Table I. The set of equilibrium points of the nonlinear model in (2) are found using the MATLAB® command `lsqnonlin` by equating $\dot{x} = 0$. For a period of ten days, at every time instant all sets of equilibria vectors $(x_e, u_e) \in \mathbb{R}^{19} \times \mathbb{R}^5$ which satisfy $\dot{x_e} = 0$ are found. Some of the resulting equilibria profiles are shown in Fig. 2 together with some of the external disturbances. Differently from the baseline PI strategy, the optimal PI control strategy is designed in such a way that the set-point of the indoor air temperature in the zone is not rigidly fixed. The set-point for desired zone temperature in the optimal PI strategy is the equilibria zone temperature profile shown in Fig. 2a. The objective function F_{opt} chosen for the optimal PI strategy is:

$$F_{opt} = \sum_{t=1}^{T} \Big[\sum_{i=1}^{19} (x_i(t) - x_{i_{eq}}(t))^T (x_i(t) - x_{i_{eq}}(t)) + \sum_{j=1}^{5} (u_j(t) - u_{j_{eq}}(t))^T 0.01 (u_j(t) - u_{j_{eq}}(t)) \Big], \qquad (3)$$

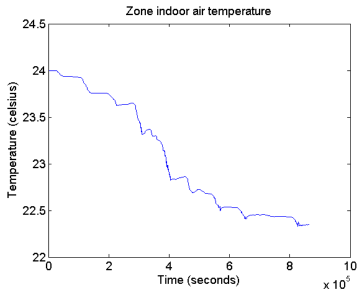
where x_{eq} represents the state equilibrium value, $]eq$ represents the equilibrium input value and 0.01 represents the scaling factor corresponding to the inputs of the system. The PI parameters of the optimal PI strategy are tuned in such a way that the objective function in (3) is minimized.

V. SIMULATION RESULTS

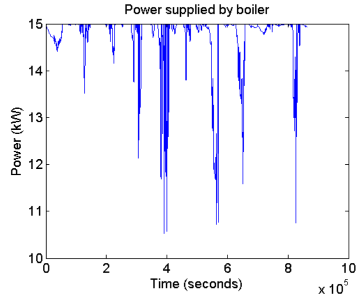
In this section, the simulation results upon implementing both control strategies (baseline and optimal PI) for the HVAC system are presented.

A. Baseline PI strategy results

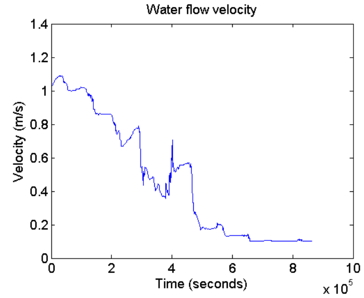
One of the goals of this work is to auto-tune the parameters of the five PI controllers based on an optimization routine. The starting conditions of all the states in the system are found optimally using `lsqnonlin` by equating $\dot{x} = 0$ in (2) for the first time instant. Using MATLAB® command



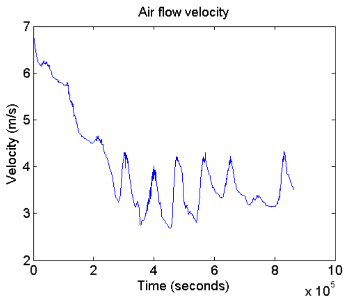
(a) Equilibria profile of zone temperature



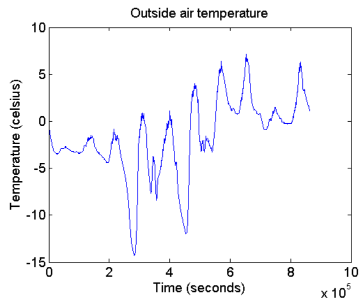
(b) Equilibria profile of power supplied by boiler



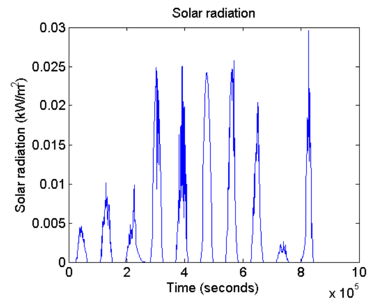
(c) Equilibria profile of water circulation velocity



(d) Equilibria profile of air circulation velocity



(e) Outside air temperature



(f) Solar radiation

Fig. 2: (a)-(b)-(c)-(d): Equilibria profile of selected states and inputs. Fig. 2: (e)-(f): Few disturbances acting on the system

TABLE I: States, inputs and disturbances of the system

State	State Description	State	State Description
x_1	Temperature of water in boiler	x_{17}	Temperature of air entering zone from AHU
x_2	Temperature of pipe from pump and boiler	x_{18}	Temperature of first layer of floor
x_3	Temperature of pipe from boiler to splitter	x_{19}	Temperature of second layer of floor
x_4	Temperature of wall		
x_5	Temperature of pipe from splitter to radiator	Input	Input Description
x_6	Temperature of radiator	u_1	Power supplied by boiler
x_7	Temperature of indoor air of zone	u_2	Velocity of water flow
x_8	Temperature of pipe from radiator to collector	u_3	Velocity of air flow
x_9	Temperature of pipe from heating coil to collector	u_4	Control signal to splitter
x_{10}	Temperature of inlet water at the pump	u_5	Control signal to recirculating damper
x_{11}	Temperature of pipe from splitter to heating coil		
x_{12}	Temperature of water in heating coil	Disturbance	Disturbance Description
x_{13}	Temperature of duct to mixing zone	d_1	Temperature of outside air
x_{14}	Temperature of air in heating coil	d_2	Solar radiation
x_{15}	Temperature of duct from zone to recirculating damper	d_3	Ground temperature
x_{16}	Temperature of duct from heating coil to fan		

`fmincon` the ten proportional and integral gains of the five PI controllers are tuned such that the objective function in (1) is minimized. The optimization is run over three simulation days. The gains obtained after completion of the optimization routine are summarized in Table II. Further, using the optimally found PI gains for the controllers, anti-windup scheme is introduced. Back calculation method is used for the anti-windup scheme. Back calculation coefficients for the five PI controllers are tuned by using MATLAB® command `fmincon` such that the objective function in (1) is minimized. The back calculation coefficients after completion of the optimization routine are also summarized in Table II.

TABLE II: PI parameters and back calculation coefficients

Gain	Starting value	Value after optimization
Kp_b	$5.2701 \cdot 10^{-5}$	$6.0016 \cdot 10^{-5}$
Ki_b	0.0895	0.0018
Kp_p	$1.5115 \cdot 10^{-4}$	$1.5019 \cdot 10^{-4}$
Ki_p	0.1431	0.0015
Kp_f	$3.2024 \cdot 10^{-7}$	$5.0586 \cdot 10^{-6}$
Ki_f	0.0769	$4.1156 \cdot 10^{-4}$
Kp_v	$7.3344 \cdot 10^{-7}$	$5.1753 \cdot 10^{-6}$
Ki_v	0.0128	$2.8436 \cdot 10^{-4}$
Kp_d	$1.3338 \cdot 10^{-7}$	$5.0004 \cdot 10^{-6}$
Ki_d	$1.6807 \cdot 10^{-4}$	$1.1171 \cdot 10^{-4}$
Kb_b	0.8	0.0435
Kb_p	0.8	0.0435
Kb_f	0.8	0.0373
Kb_v	0.8	0.0373
Kb_d	0.8	$1.1000 \cdot 10^{-06}$

In Table II Kp_b and Ki_b are proportional and integral gains of PI-B, Kp_p and Ki_p are proportional and integral gains of PI-P, Kp_f and Ki_f are proportional and integral gains of PI-F, Kp_v and Ki_v are proportional and integral gains of PI-V, Kp_d and Ki_d are proportional and integral gains of PI-D, Kb_b represents back calculation coefficient of PI-B, Kb_p represents back calculation coefficient of PI-P, Kb_f represents back calculation coefficient of PI-F, Kb_v represents back calculation coefficient of PI-V and Kb_d represents back calculation coefficient of PI-D. The cost comparison after optimizing the controller parameters and back calculation coefficients is given in Table III.

TABLE III: Total cost comparison after optimizing controller parameters and back calculation coefficients

Total Cost	Period of 3 days	Period of 10 days
Initial conditions	$1.3836 \cdot 10^{11}$	$3.7303 \cdot 10^{12}$
After optimization	$7.2500 \cdot 10^{5}$	$3.0408 \cdot 10^{6}$

B. Optimal PI strategy results

Similar to the baseline PI strategy, using MATLAB® command `fmincon`, the ten proportional-integral gains of the five PI controllers are tuned. The starting conditions of all the states in the system are found optimally using `lsqnonlin` by equating $\dot{x} = 0$ in (2) for the first time instant. The controller parameters are tuned optimally such that the objective function in (3) is minimized. The gains obtained after completion of the optimization routine are summarized in Table IV. As mentioned in section V, the desired zone temperature set-point varies according to the equilibria profile for the zone temperature found in section IV. Using the optimally found PI gains for the controllers, anti-windup scheme using back calculation method is introduced is introduced. Back calculation coefficients are tuned using MATLAB® command `fmincon` to minimize the objective function (3) and results are listed in Table IV.

TABLE IV: Optimal PI control parameters and back calculation coefficients

Gain	Starting value	Value after optimization
Kp_b	$20 \cdot 10^{-5}$	$5.2701 \cdot 10^{-5}$
Ki_b	1	0.0895
Kp_p	$20 \cdot 10^{-5}$	$1.5115 \cdot 10^{-4}$
Ki_p	1	0.1431
Kp_f	$20 \cdot 10^{-5}$	$3.2024 \cdot 10^{-7}$
Ki_f	1	0.0769
Kp_v	$20 \cdot 10^{-5}$	$7.3344 \cdot 10^{-7}$
Ki_v	1	0.0128
Kp_d	$20 \cdot 10^{-5}$	$1.3338 \cdot 10^{-7}$
Ki_d	1	$1.6807 \cdot 10^{-4}$
Kb_b	0.8	7.9675
Kb_p	0.8	0.2145
Kb_f	0.8	$1.3957 \cdot 10^{-5}$
Kb_v	0.8	5.2940
Kb_d	0.8	0.1583

Cost comparison after optimizing the controller parameters and back calculation coefficients is given in Table V. The zone temperature with optimal PI control is shown in Fig. 3.

TABLE V: Total cost comparison after optimizing controller parameters and back calculation coefficients

Total Cost	Period of 3 days	Period of 10 days
Initial conditions	$3.1147 \cdot 10^{9}$	$5.1912 \cdot 10^{12}$
After optimization	$2.6074 \cdot 10^{7}$	$8.1516 \cdot 10^{7}$

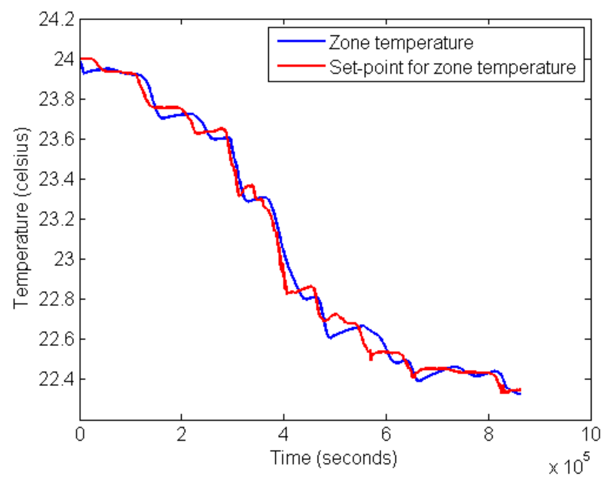


Fig. 3: Zone temperature using optimal PI

VI. COMPARISON AND ANALYSIS

In this section, the baseline PI strategy is compared with the optimal PI strategy in terms of performance and power consumption. The energy consumed by each of the boiler, pump and fan for the baseline and optimal PI strategy for a simulation period of three days is given in Table VI. The reduction in energy consumption is also mentioned in the table.

TABLE VI: Total energy consumption in [kWh] for a simulation length of three days

Energy consumption in [kWh]	Baseline PI	Optimal PI
Boiler	1035.1	973.53
Fan	160.19	172.94
Pump	190.99	113.47
Total	1386.28	1259.94
Reduction wrt Baseline PI	-	126.34 (9.11%)

From Table VI it is evident that the energy consumption is minimized for the optimal PI strategy when compared to the baseline PI strategy. Also, the optimal PI strategy has better performance than the baseline strategy in terms of set-point tracking. This validates that the optimal PI strategy performs better than the baseline PI strategy in the view point of performance and energy efficiency. The reason behind the better performance of the optimal PI strategy is that, at each and every time instant we find the minimum energy condition of each and every state and input by solving the non linear system to find the equilibrium points.

VII. CONCLUSIONS AND FUTURE WORK

This paper presented the modelling of a complete HVAC system with one zone. The model was developed in MATLAB®/SIMULINK® and was used for implementation of control, for comparisons and analysis. Two control strategies (baseline and optimal PI) based on auto-tuning of multiple controller parameters using optimization technique were proposed. The objective function to be minimized was chosen differently for the two strategies. The controller parameters were tuned for a three day simulation length and validated over a ten day period. It was observed that the optimal PI strategy outperforms the baseline PI strategy in terms of both performance and reduced energy consumption.

Future work will include considering more elements in the discrete model of the radiator, boiler, pump, duct and the heating coil in order to better account for transport delays. Heat exchange at every component can be modelled as a function of mean of output and ambient temperature. Fresh and exhaust air dampers can be added apart from the recirculating damper in the AHU. Valve and damper closing/opening delays can be considered. A more complicated model of the thermal zone can be considered.

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