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Robust unit commitment with dispatchable wind power

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ABSTRACT

The increasing penetration of uncertain generation such as wind and solar in power systems imposes new challenges to the unit commitment (UC) problem, one of the most critical tasks in power systems operations. The two most common approaches to address these challenges — stochastic and robust optimization — have drawbacks that restrict their application to real-world systems. This paper demonstrates that, by considering dispatchable wind and a box uncertainty set for wind availability, a fully adaptive two-stage robust UC formulation, which is typically a bi-level problem with outer mixed-integer program (MIP) and inner bilinear program, can be translated into an equivalent single-level MIP. Experiments on the IEEE 118-bus test system show that computation time, wind curtailment, and operational costs can be significantly reduced in the proposed unified stochastic–robust approach compared to both pure stochastic approach and pure robust approach, including budget of uncertainty.

1. Introduction

In recent years, higher penetration of variable and uncertain generation (e.g., wind and solar power) has challenged independent system operators (ISOs) to maintain a reliable and still economical operation of power systems. To achieve this and to be prepared for future demand, ISOs decide about startup and shutdown schedules of generating units some time (typically a day) up front by solving the so-called unit commitment (UC) problem, whose main objective is to minimize operational costs while meeting power system constraints. These operational costs are commitment-related costs, and dispatch costs of both thermal and wind generating units, where the latter can present negative cost (or bids) [1]. High levels of variable and uncertain generation significantly increase the uncertainty in the net forecasted future demand, increasing the difference between a fail-safe solution and an economical solution, and this dilemma thereby increases the complexity of the UC optimization problem [2].

The two main approaches for dealing with the uncertainty in UC problems are stochastic and robust optimization. Stochastic optimization (SO) [2–5] typically consists of minimizing expected costs over a set of possible scenarios for uncertain parameters. However, SO can become impractical under high-dimensional problems mainly because of a heavy computational burden [2]. Additionally, the main goal for ISOs is to ensure a safe operation of the system, and SO does not give sufficient guarantees on meeting the constraints in realizations of the uncertainty. Moreover, SO requires a large number of scenarios to be reliable and their associated probability distribution is hard to obtain.

In robust optimization (RO) [6–10] the costs are minimized maintaining feasibility under *all* possible realizations of uncertain parameters within some specified uncertainty set. Consequently, the resulting schedules could turn out to be over-conservative under a large uncertainty set: although the probability of the worst-case event is virtually nil, the chosen schedule is robust for this event, and hence much more costly than what is actually required. One way to reduce over-conservatism is to use a budget of uncertainty, which models a smaller uncertainty set in a flexible way [6]. A robust UC typically requires solving a bilevel optimization problem, where the outer level is a mixed-integer linear program (MIP), and the inner level is usually a *bilinear program*, which is non-deterministic polynomial-time hard (NP-hard) [6]. Finding optimal robust solutions in time for large-scale systems is still a major challenge; in particular, solving the respective bilinear problems usually requires sub-optimal heuristic methods or computationally expensive exact methods [6,11–14]. Further, this bilinear problem is typically solved multiple times as part of an iterative algorithm such as column-and-constraint generation [11], which also requires solving a difficult master problem multi-

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ple times. This computational difficulty highlights the challenge of creating efficient methods to find effective robust UC solutions.

Recently, the so-called unified stochastic–robust (SR) optimization approach has been proposed to strike a very good balance between robustness and efficiency [7]. This approach increases safe operation of the system while mitigating the over-conservatism of robust optimization. However, since this formulation combines the stochastic formulation (which requires optimization over many scenarios) with the bilinear robust formulation, the computational challenge is even stronger.

This paper considers wind dispatch flexibility by allowing curtailment in the UC formulation. In various previous works, wind power has also been modeled as dispatchable by allowing wind curtailment in UC or expansion planning models [1,12,15–19]. In this paper we will exploit some consequences that are obtained in the robust UC problem when wind power is dispatchable, which will lead to very computationally efficient formulations.

This paper rises up to various current challenges in the UC problem by providing the following contributions:

1. Under the case in which wind power availability is modeled through a box uncertainty set, and assuming that wind power can be curtailed, we prove that the usually non-linear and NP-hard [6] second stage of a fully adaptive robust UC problem has an equivalent linear program (LP) representation, which solves in polynomial time. Consequently, in this case the fully adaptive two-stage robust UC formulation can be translated into an equivalent single-level MIP problem. This allows solving realistically-sized problem instances very close to global optimality significantly faster than the traditional bi-level robust UC. When compared with a typical robust formulation based on budget of uncertainty, the proposed formulation provides similar results but solves significantly faster.
2. We also show that considering dispatchable wind can contribute to efficiently solving a unified stochastic–robust optimization problem [7] by linking the wind dispatch between the stochastic and the robust parts. Thus, the proposed SR formulation provides a cheaper operation, higher robustness, less wind curtailment, while simultaneously having lower computational burden than a typical RO formulation based on budget of uncertainty. Also, the SR needs very few scenarios to provide very similar results to a scenario-rich SO, hence naturally avoiding the high computational burden associated to considering a large number of scenarios.

The remainder of this paper is organized as follows. Section 2 details the proposed robust UC reformulation with dispatchable wind, and shows how to complement stochastic UC by incorporating the robust part. Section 3 provides and discusses results from several experiments, where a comparison between robust, stochastic and unified UC formulations is made. Finally, main conclusions are drawn in Section 4.

2. Mathematical models and structural results

This section formulates the mathematical models and presents a set of results that exploit the structure of the robust UC with dispatchable wind. Section 2.1 defines this problem, and Section 2.2 presents a general structural result that characterizes the subset of elements of the uncertainty set that can achieve the *worst case*. Section 2.3 studies this structural result under the widely used budget and box uncertainty sets. Finally, Section 2.4 studies the consequences of these structural results in the unified stochastic–robust UC.

2.1. Robust UC with dispatchable wind

We extend the 3-binary setting UC formulation [20] to a robust UC with dispatchable wind. The compact form of this robust UC is expressed as

$$\min_{\boldsymbol{x}\in X}\left\{\boldsymbol{b}^\top\boldsymbol{x}+\max_{\boldsymbol{\xi}\in\Xi}\ \min_{(\boldsymbol{p},\boldsymbol{w})\in\Omega(\boldsymbol{x},\boldsymbol{\xi})}\left(\boldsymbol{c}^\top\boldsymbol{p}+\boldsymbol{d}^\top\boldsymbol{w}\right)\right\} \tag{1}$$

where

$$X=\left\{\boldsymbol{x}\in\{0,1\}^{3|\mathcal{G}||\mathcal{T}|}:\ \boldsymbol{A}\boldsymbol{x}\le\boldsymbol{a}\right\} \tag{2}$$

and

$$\Omega(\boldsymbol{x},\boldsymbol{\xi})=\{(\boldsymbol{p},\boldsymbol{w}):\quad \boldsymbol{E}\boldsymbol{p}+\boldsymbol{F}\boldsymbol{w}\le\boldsymbol{g}+\boldsymbol{G}\boldsymbol{x} \tag{3a}$$

$$\boldsymbol{w}\le\boldsymbol{\xi}\ \}. \tag{3b}$$

Here, $\boldsymbol{x}$ is a vector of *first-stage* decisions including the binary on/off, start-up and shut-down decisions of conventional generators. These decisions are constrained through set X defined in (2), which includes the logical relations between on/off, start-up and shut-down variables, as well as minimum up and down times. In (2), $\mathcal{G}$ is the set of conventional generators and $\mathcal{T}$ is the set of time periods. Vector $\boldsymbol{\xi}$ contains all uncertain parameters in the problem, corresponding to the availability of wind power at all wind farms and time periods, i.e., $\boldsymbol{\xi}=\left(\xi_{it}:i\in\mathcal{W},t\in\mathcal{T}\right)$, where ξ_{it} is the available wind power at bus i and time t, and $\mathcal{W}$ is the set of buses containing wind production. The closed set Ξ is an *uncertainty set* that describes the realizations of $\boldsymbol{\xi}$. Vectors $\boldsymbol{p},\boldsymbol{w}$ are *second-stage* power dispatch decisions for conventional generators and wind farms, respectively, i.e., $\boldsymbol{p}=\left(p_{gt}:g\in\mathcal{G},t\in\mathcal{T}\right)$ and $\boldsymbol{w}=(w_{it}:i\in\mathcal{W},t\in\mathcal{T})$, where p_{gt} and w_{it} are the power output of conventional generator g and of wind farms at bus i, at time t, respectively. These power dispatch decisions are constrained through set $\Omega(\boldsymbol{x},\boldsymbol{\xi})$ defined in (3). In $\Omega(\boldsymbol{x},\boldsymbol{\xi})$, Eq. (3a) involves dispatch-related constraints such as power output bounds for conventional generators, nonnegativity of power output at wind farms, ramping constraints, transmission line capacity constraints and energy balance constraints. Eq. (3b) represents the upper bound for power output at wind farms, depending on available wind power, that is, $w_{it}\le\xi_{it}$ for all i,t. Finally, the objective function of problem (1) consists of minimizing the sum of no-load, start-up and shut-down costs, given by $\boldsymbol{b}^\top\boldsymbol{x}$, and worst-case dispatch costs, given by the inner max-min problem, where $\boldsymbol{c}$ contains the production costs of conventional generators and where $\boldsymbol{d}$ is the production costs of wind farms, which is usually zero or negative representing negative bids (resulting in wind curtailment penalization) [1].

The robust UC problem (1) is an *adaptive robust optimization* problem [21]. In this problem, $\boldsymbol{p}$ and $\boldsymbol{w}$ are adaptive decision variables whose values can depend on the realization of the vector of uncertain parameters $\boldsymbol{\xi}$, while $\boldsymbol{x}$ is a "here-and-now" decision that is taken before $\boldsymbol{\xi}$ is realized. This adaptive robust framework for the UC problem was first proposed in [6,22,23]. This section focuses on studying the consequences of considering wind power to be dispatchable, that is, that wind power output w_{it} can take any value between 0 MW and its availability ξ_{it}. In what follows, we provide a general result that characterizes a subset of the uncertainty set that necessarily contains the worst-case realization of $\boldsymbol{\xi}$.

2.2. Worst-case is achieved in the set of minimal elements

The inner max–min problem in the robust UC (1)

$$\max_{\boldsymbol{\xi}\in\Xi}\ \min_{(\boldsymbol{p},\boldsymbol{w})\in\Omega(\boldsymbol{x},\boldsymbol{\xi})}\left(\boldsymbol{c}^\top\boldsymbol{p}+\boldsymbol{d}^\top\boldsymbol{w}\right) \tag{4}$$

has a special structure: the only dependence of $\Omega(\boldsymbol{x},\boldsymbol{\xi})$ on $\boldsymbol{\xi}$ is through constraint (3b), namely, $\boldsymbol{w}\le\boldsymbol{\xi}$. We can use this structure

to characterize a subset of Ξ that includes the worst-case. We need the following definition. The *set of minimal elements* of Ξ is given by

$$\mathrm{ME}(\Xi) = \{\boldsymbol{\xi} \in \Xi : \ \nexists \boldsymbol{\varphi} \in \Xi \ \text{ s.t. } \ \boldsymbol{\varphi} \le \boldsymbol{\xi} \text{ and } \boldsymbol{\varphi} \ne \boldsymbol{\xi}\} \tag{5}$$

that is, $\mathrm{ME}(\Xi)$ corresponds to the elements $\boldsymbol{\xi}$ of Ξ for which there is no other element in Ξ that is less than or equal to $\boldsymbol{\xi}$ in all components of the vector, and strictly less in at least one.

An important property of $\mathrm{ME}(\Xi)$ that we need is given in the following lemma:

Lemma 1. *Suppose that $\boldsymbol{\xi}^* \in \Xi \setminus \mathrm{ME}(\Xi)$, then there exists $\hat{\boldsymbol{\xi}} \in \mathrm{ME}(\Xi)$ such that $\hat{\boldsymbol{\xi}} \le \boldsymbol{\xi}^*$.*

Proof. Define the subset S of all elements $\boldsymbol{\varphi} \in \Xi$ suchthat $\boldsymbol{\varphi} \le \boldsymbol{\xi}^*$ and $\boldsymbol{\varphi} \ne \boldsymbol{\xi}^*$. This set is nonempty by definition of $\boldsymbol{\xi}^*$. Since Ξ is closed, S is also closed, thus $\mathrm{ME}(\Xi)$ is also nonempty. Next, consider an element $\hat{\boldsymbol{\xi}} \in \mathrm{ME}(S)$. It thus holds that $\hat{\boldsymbol{\xi}} \le \boldsymbol{\xi}^*$. To show that $\hat{\boldsymbol{\xi}} \in \mathrm{ME}(\Xi)$, suppose there is an element $\boldsymbol{\varphi} \in \Xi$ such that $\boldsymbol{\varphi} \le \hat{\boldsymbol{\xi}}$ and $\boldsymbol{\varphi} \ne \hat{\boldsymbol{\xi}}$. Then $\boldsymbol{\varphi} \in S$, because $\boldsymbol{\varphi} \le \hat{\boldsymbol{\xi}} \le \boldsymbol{\xi}^*$. However, this contradicts that $\hat{\boldsymbol{\xi}} \in \mathrm{ME}(S)$. Therefore, there is no such $\boldsymbol{\varphi}$ and thus $\hat{\boldsymbol{\xi}} \in \mathrm{ME}(\Xi)$. □

One important result that leads to significant computational savings is that the worst-case in the robust UC with dispatchable wind (1) has to be within the set of minimal elements of the uncertainty set. Below we prove this main structural result:

Proposition 2. *The following equality holds:*

$$\begin{aligned} &\max_{\boldsymbol{\xi} \in \Xi} \ \min_{(\boldsymbol{p},\boldsymbol{w}) \in \Omega(\boldsymbol{x},\boldsymbol{\xi})} \left(\boldsymbol{c}^\top \boldsymbol{p} + \boldsymbol{d}^\top \boldsymbol{w}\right) \\ &= \max_{\boldsymbol{\xi} \in \mathrm{ME}(\Xi)} \ \min_{(\boldsymbol{p},\boldsymbol{w}) \in \Omega(\boldsymbol{x},\boldsymbol{\xi})} \left(\boldsymbol{c}^\top \boldsymbol{p} + \boldsymbol{d}^\top \boldsymbol{w}\right). \end{aligned} \tag{6}$$

Proof. Define here

$$f(\boldsymbol{\xi}) = \min_{(\boldsymbol{p},\boldsymbol{w}) \in \Omega(\boldsymbol{x},\boldsymbol{\xi})} \left(\boldsymbol{c}^\top \boldsymbol{p} + \boldsymbol{d}^\top \boldsymbol{w}\right). \tag{7a}$$

Since $\mathrm{ME}(\Xi) \subset \Xi$ it follows that

$$\max_{\boldsymbol{\xi} \in \Xi} f(\boldsymbol{\xi}) \ge \max_{\boldsymbol{\xi} \in \mathrm{ME}(\Xi)} f(\boldsymbol{\xi}). \tag{7b}$$

Thus, we only need to show the reverse inequality. Let $\boldsymbol{\xi}^* \in \Xi$ be the optimal solution of

$$f(\boldsymbol{\xi}^*) = \max_{\boldsymbol{\xi} \in \Xi} f(\boldsymbol{\xi}). \tag{7c}$$

If $\boldsymbol{\xi}^* \in \mathrm{ME}(\Xi)$ then we have

$$\max_{\boldsymbol{\xi} \in \Xi} f(\boldsymbol{\xi}) = f(\boldsymbol{\xi}^*) \le \max_{\boldsymbol{\xi} \in \mathrm{ME}(\Xi)} f(\boldsymbol{\xi}). \tag{7d}$$

Otherwise, if $\boldsymbol{\xi}^* \notin \mathrm{ME}(\Xi)$, by Lemma 1 there exists $\hat{\boldsymbol{\xi}} \in \mathrm{ME}(\Xi)$ such that $\hat{\boldsymbol{\xi}} \le \boldsymbol{\xi}^*$. This directly implies that $\Omega(\boldsymbol{x}, \hat{\boldsymbol{\xi}}) \subset \Omega(\boldsymbol{x}, \boldsymbol{\xi}^*)$ due to the definition of $\Omega(\boldsymbol{x}, \cdot)$. Consequently given the definition of $f(\cdot)$ we have that $f(\hat{\boldsymbol{\xi}}) \ge f(\boldsymbol{\xi}^*)$. Thus

$$\max_{\boldsymbol{\xi} \in \Xi} f(\boldsymbol{\xi}) = f(\boldsymbol{\xi}^*) \le f(\hat{\boldsymbol{\xi}}) \le \max_{\boldsymbol{\xi} \in \mathrm{ME}(\Xi)} f(\boldsymbol{\xi}) \tag{7e}$$

which completes the proof. □

2.3. Results for the budget and box uncertainty sets

The structural result in Proposition 2 is not very useful if we cannot characterize $\mathrm{ME}(\Xi)$ in explicit form. Fortunately, for the most widely used uncertainty set, the *budget* uncertainty set, this is possible. The budget uncertainty set is defined as

$$\begin{aligned} \Xi^{\mathrm{bud}}\left(\boldsymbol{\Delta}\right) = \ & \Big\{\boldsymbol{\xi} : \tilde{\xi}_{it} - \hat{\xi}_{it} \le \xi_{it} \le \tilde{\xi}_{it} + \hat{\xi}_{it} \ \ \forall \ i \in \mathcal{W}, t \in \mathcal{T} \\ & \sum_{i \in \mathcal{W}} \frac{|\xi_{it} - \tilde{\xi}_{it}|}{\hat{\xi}_{it}} \le \Delta_t \ \ \forall \ \ t \in \mathcal{T}\Big\} \end{aligned} \tag{8}$$

where $\tilde{\xi}_{it}$ is the nominal available wind power at bus i, time t, $\hat{\xi}_{it}$ is the allowed deviation of available wind power from its nominal value, and $\Delta_t \in [0, |\mathcal{W}|]$ is the budget of uncertainty at time $t \in \mathcal{T}$, determining the size of the uncertainty set and thus the conservativeness of the robust UC (1).

Now, let us consider the following set $\hat{\Xi}^{\mathrm{bud}}$, which we show is exactly the set of minimal elements of Ξ^{bud}:

$$\begin{aligned} \hat{\Xi}^{\mathrm{bud}}\left(\boldsymbol{\Delta}\right) = \ & \Big\{\boldsymbol{\xi} : \tilde{\xi}_{it} - \hat{\xi}_{it} \le \xi_{it} \le \tilde{\xi}_{it} \ \ \forall \ i \in \mathcal{W}, \ t \in \mathcal{T} \\ & \sum_{i \in \mathcal{W}} \frac{\xi_{it} - \tilde{\xi}_{it}}{\hat{\xi}_{it}} = -\Delta_t \ \forall \ t \in \mathcal{T}\Big\}. \end{aligned} \tag{9}$$

This is formally shown as follows.

Proposition 3. *The following holds:* $\mathrm{ME}(\Xi^{\mathrm{bud}}) = \hat{\Xi}^{\mathrm{bud}}$.

Proof. Let us first show that $\hat{\Xi}^{\mathrm{bud}} \subseteq \mathrm{ME}(\Xi^{\mathrm{bud}})$. Consider $\boldsymbol{\xi} \in \hat{\Xi}^{\mathrm{bud}}$. Suppose that there exists $\boldsymbol{\varphi} \in \Xi^{\mathrm{bud}}$ such that $\boldsymbol{\varphi} \le \boldsymbol{\xi}$ and $\boldsymbol{\varphi} \ne \boldsymbol{\xi}$. That is, there must be some i', t' such that $\varphi_{i't'} < \xi_{i't'}$. Then, we must have

$$\sum_{i \in \mathcal{W}} \frac{\varphi_{it'} - \tilde{\xi}_{it'}}{\hat{\xi}_{it'}} < \sum_{i \in \mathcal{W}} \frac{\xi_{it'} - \tilde{\xi}_{it'}}{\hat{\xi}_{it'}} = -\Delta_{t'} \tag{10a}$$

which implies that

$$\sum_{i \in \mathcal{W}} \frac{|\varphi_{it'} - \tilde{\xi}_{it'}|}{\hat{\xi}_{it'}} > \Delta_{t'} \tag{10b}$$

which is a contradiction, since $\boldsymbol{\varphi} \in \Xi^{\mathrm{bud}}$. Therefore, there is no $\boldsymbol{\varphi} \in \Xi^{\mathrm{bud}}$ such that $\boldsymbol{\varphi} \le \boldsymbol{\xi}$ and $\boldsymbol{\varphi} \ne \boldsymbol{\xi}$, which means that $\boldsymbol{\xi} \in \mathrm{ME}(\Xi^{\mathrm{bud}})$.

$$\sum_{i \in \mathcal{W}} \frac{\xi_{it'} - \tilde{\xi}_{it'}}{\hat{\xi}_{it'}} > -\Delta_{t'} \tag{10c}$$

- Case 1: $\boldsymbol{\xi} \le \tilde{\boldsymbol{\xi}}$. Select i' such that $\xi_{i't'} > \tilde{\xi}_{i't'} - \hat{\xi}_{i't'}$ and define

$$\nu = \max\left\{\tilde{\xi}_{i't'} - \hat{\xi}_{i't'}, \tilde{\xi}_{i't'} - \hat{\xi}_{i't'}\left(\Delta_{t'} + \sum_{i \ne i'} \frac{\xi_{it'} - \tilde{\xi}_{it'}}{\hat{\xi}_{it'}}\right)\right\}. \tag{10d}$$

- Case 2: $\boldsymbol{\xi} \ne \tilde{\boldsymbol{\xi}}$. Select i' such that $\xi_{i't'} > \tilde{\xi}_{i't'}$ and define $\nu = \tilde{\xi}_{i't'}$.

Lemma 4. *The following equality holds:*

$$\begin{aligned} &\max_{\boldsymbol{\xi} \in \Xi^{\mathrm{bud}}} \ \min_{(\boldsymbol{p},\boldsymbol{w}) \in \Omega(\boldsymbol{x},\boldsymbol{\xi})} \left(\boldsymbol{c}^\top \boldsymbol{p} + \boldsymbol{d}^\top \boldsymbol{w}\right) \\ &= \max_{\boldsymbol{\xi} \in \hat{\Xi}^{\mathrm{bud}}} \ \min_{(\boldsymbol{p},\boldsymbol{w}) \in \Omega(\boldsymbol{x},\boldsymbol{\xi})} \left(\boldsymbol{c}^\top \boldsymbol{p} + \boldsymbol{d}^\top \boldsymbol{w}\right). \end{aligned} \tag{11}$$

At this point, it is important to understand the computational consequences of the above Lemma. How much "simpler" is $\hat{\Xi}^{\mathrm{bud}}$ than Ξ^{bud}? One way to look at this is through the number of variables and constraints required to represent these polyhedral sets. We can first observe that to formulate $\hat{\Xi}^{\mathrm{bud}}$ we require $|\mathcal{W}|T$ variables, $2|\mathcal{W}|T$ inequality constraints, and T equality constraints. By removing equality constraints, we then require $|\mathcal{W}|T - T$ variables, and $2|\mathcal{W}|T$ inequality constraints. Now, to represent the budget

uncertainty set Ξ^{bud} we can use an extended formulation to handle the absolute values. In fact, Ξ^{bud} is the projection of a set with $2|\mathcal{W}|T$ variables and $4|\mathcal{W}|T+T$ inequality constraints. In short, Ξ^{bud} requires a total of $6|\mathcal{W}|T+T$ variables and inequality constraints, while $\hat{\Xi}^{\text{bud}}$ only requires a total of $3|\mathcal{W}|T-T$.

Finally, an important special case arises when the budget uncertainty set corresponds to a "box" set, i.e., $\Delta_t = |\mathcal{W}|$ for all t. In this case, the set of minimal elements of the budget uncertainty set contains a single element and the overall max-min problem consequently collapses to a single minimization problem. The next theorem summarizes this result.

Theorem 5. *If $\Delta_t = |\mathcal{W}|$ for all t, then $\hat{\Xi}^{\text{bud}} = \left\{\tilde{\boldsymbol{\xi}} - \hat{\boldsymbol{\xi}}\right\}$ and the robust UC with dispatchable wind (1) becomes*

$$\min_{\boldsymbol{x}\in X}\left\{\boldsymbol{b}^\top\boldsymbol{x} + \max_{\boldsymbol{\xi}\in\Xi^{\text{bud}}(|\mathcal{W}|)}\ \min_{(\boldsymbol{p},\boldsymbol{w})\in\Omega(\boldsymbol{x},\boldsymbol{\xi})}\left(\boldsymbol{c}^\top\boldsymbol{p}+\boldsymbol{d}^\top\boldsymbol{w}\right)\right\} = \min_{\boldsymbol{x}\in X,\ (\boldsymbol{p},\boldsymbol{w})\in\Omega(\boldsymbol{x},\tilde{\boldsymbol{\xi}}-\hat{\boldsymbol{\xi}})}\boldsymbol{b}^\top\boldsymbol{x}+\boldsymbol{c}^\top\boldsymbol{p}+\boldsymbol{d}^\top\boldsymbol{w}. \tag{12}$$

Proof. The only $\boldsymbol{\xi} \in \hat{\Xi}^{\text{bud}}$ satisfying (9) when $\Delta_t = |\mathcal{W}|$ for all t is given by $\boldsymbol{\xi} = \tilde{\boldsymbol{\xi}} - \hat{\boldsymbol{\xi}}$. The rest follows directly from Lemma 4. □

The importance of this result lies in the fact that a very difficult min-max-min problem in the general case becomes a much simpler single-level MIP problem under the assumptions that (i) wind power is dispatchable, and that (ii) the uncertainty set is a box set. This means that under these assumptions, it is not necessary to implement computationally difficult algorithms to solve the robust UC, since it can be directly solved by MIP solvers.

2.4. Consequences for the unified stochastic–robust UC

The unified stochastic–robust UC approach was first proposed in [7]. In this paper, we explore the consequences of the special structure of wind dispatchability in the problem, to show that under this setting a very powerful and efficient stochastic–robust UC approach is obtained with just a few scenarios, exploiting the structure of the max-min problem representing worst-case dispatch costs. We then propose a constraint that relates the stochastic and robust parts creating a formulation where the robust part provides robustness to the dispatch scenarios of the stochastic part. Although this constraint can increase the robustness of a unified UC it can also increase operational costs. Section 3.3.2 discusses the effects of this extra constraint in the unified UC formulations.

The unified stochastic–robust UC is defined here as

$$\min_{\boldsymbol{x}\in X}\{\boldsymbol{b}^\top\boldsymbol{x} + (1-\alpha)\max_{\boldsymbol{\xi}\in\Xi}\ \min_{(\boldsymbol{p},\boldsymbol{w})\in\Omega(\boldsymbol{x},\boldsymbol{\xi})}\left(\boldsymbol{c}^\top\boldsymbol{p}+\boldsymbol{d}^\top\boldsymbol{w}\right) + \alpha\sum_{s=1}^{S}\pi_s\min_{(\boldsymbol{p}_s,\boldsymbol{w}_s)\in\Omega(\boldsymbol{x}_s,\boldsymbol{\xi}_s)}\left(\boldsymbol{c}^\top\boldsymbol{p}_s+\boldsymbol{d}^\top\boldsymbol{w}_s\right)\} \tag{13}$$

where there are two characterizations of uncertainty for available wind power $\boldsymbol{\xi}$: the uncertainty set Ξ, as in problem (1), and S scenarios $\boldsymbol{\xi}^s$, with respective probabilities π_s. Now, for the stochastic part, $\boldsymbol{p}_s$ and $\boldsymbol{w}_s$ are the dispatch decisions of thermal and wind units for scenario s, respectively. The objective is to minimize the sum of commitment costs $\boldsymbol{b}^\top\boldsymbol{x}$, and a weighted combination of worst-case dispatch cost (with weight $1-\alpha$) and expected cost (with weight α), where parameter $\alpha \in [0, 1]$.

Motivated by the fact that the worst-case wind dispatch is in the set of minimal elements, in this paper we propose to relate the dispatch of the stochastic and worst-case scenarios as

$$\boldsymbol{w}_s \geq \boldsymbol{w}\ \forall s \in \mathcal{S} \tag{14}$$

which guarantees that all the wind dispatch scenarios for the stochastic part are greater than or equal to the worst-case wind dispatch of the robust part. The purpose of combining the robust and stochastic components of the model in this way is ensuring that the stochastic solution is indeed "protected" by the robust solution. With this approach, any uncertain wind realization above $\boldsymbol{w}$ is protected since, in the worst-case, it can be curtailed to $\boldsymbol{w}$. This relation between the stochastic and robust parts through (14) can significantly improve the robustness of the model, as shown in Section 3.3.2.

3. Computational experiments

To validate the proposed formulations, we compare the performance of the following network-constrained UC models:

RO: The robust UC formulation under box uncertainty set (see (12) in Theorem 5) where its level of conservatism is adjusted by shrinking the uncertainty set. For this, we introduce the parameter $\boldsymbol{\pi}$ to control the level of conservatism: $[\tilde{\boldsymbol{\xi}} - (\hat{\boldsymbol{\xi}})\boldsymbol{\pi}, \tilde{\boldsymbol{\xi}} + (\hat{\boldsymbol{\xi}})\boldsymbol{\pi}]$. Since for *RO*, the worst-case scenario lies on the lower bound of the uncertainty set, then by changing $\boldsymbol{\pi}$ from 0 to 1 the worst-case scenario changes from $\tilde{\boldsymbol{\xi}}$ to $(\tilde{\boldsymbol{\xi}} - \hat{\boldsymbol{\xi}})$.

ROB: The robust UC formulation (1) including budget of uncertainty (8). Similarly to *RO*, the level of conservatism is controlled by $\pi \in [0, 1]$, thus expressing the budget of uncertainty as $\Delta_t = \pi|\mathcal{W}|$. The two-level optimization problem is solved using a column-and-constraint generation algorithm [11] and the alternating direction method [13,14]. See [13,11] for implementation details of these algorithms.

SO: The stochastic UC formulation (equivalent to the second part of (13)).

SR: The proposed unified stochastic–robust UC formulation (13) and (14), where *RO* is used for the robust part.

SRB: The unified stochastic–robust UC formulation (13) and (14), where *ROB* is used for the robust part.

In this section, *SR* and *SRB* include (14) since it improves their robustness, as further discussed in Section 3.3.2.

After detailing the experimental setup, this section studies the impact of different scenarios on the stochastic and unified formulations. For *SR* and *SRB*, we then establish an optimal weight α in the objective function. Also we analyze the effect of changing the penalty for wind curtailment. Finally, we compare the performance of the robust and unified formulations when changing their level of conservatism.

3.1. Experimental setup

As a case study, we use the IEEE 118-bus test system, which was adapted to consider startup and shutdown power trajectories [24]. This system has 186 transmission lines, 54 thermal units, 91 loads, and three buses with wind production. The penalty costs for demand-balance and transmission-limits violations are set to 10000 \$/MWh and 5000 \$/MWh [25], respectively. In addition, wind bids are set to $\boldsymbol{d}$=−300\$/MWh, about ten times the average wind bid in some markets [26,1], to simulate cases where curtailment is undesired. Although the optimization problems are solved using $\boldsymbol{d}^\top\boldsymbol{w}$, the final values shown here are obtained using $\boldsymbol{d}^\top(\boldsymbol{w}-\boldsymbol{\xi})$ to only reflect curtailment (penalization) costs [1].

All experiments are solved using CPLEX 12.6.3 with default parameters and are run on an Intel-Xeon 3.7-GHz personal computer with 16 GB of RAM memory. All instances are solved until they reach an optimality tolerance of 5×10^{-4}.

To compare the performance of the different UC models, we make a clear distinction between the scheduling and (out-of-sample) evaluation stages. The scheduling stage solves the different UC models and obtains their commitment policy using a small representative number of wind scenarios (up to 50) for the stochas-

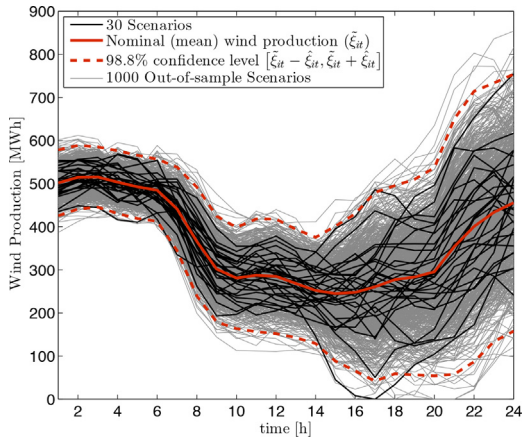

Fig. 1. Wind production on bus 69. Example of 30 generated scenarios for the stochastic formulations. Uncertainty set defined by 98.8% confidence level. The 1000 out-of-sample scenarios used for the evaluation stage are shown on the background.

tic formulations, and uncertainty sets for the robust formulations. The evaluation stage, for each fixed commitment policy, solves a network-constrained economic dispatch problem repetitively for a set of 1000 out-of-sample wind scenarios (see Fig. 1), thus obtaining an accurate estimate of the expected performance of each UC policy.

To generate the scenarios, we assume that wind production follows a multivariate normal distribution, which is truncated for nonnegative values. For the robust formulations *RO*, *ROB*, *SR* and *SRB*, the uncertainty set is defined by a nominal value $\tilde{\boldsymbol{\xi}}$ and a deviation $\hat{\boldsymbol{\xi}}$ corresponding to 98.8% confidence level (2.5 standard deviations). For the stochastic formulations *SO*, *SR* and *SRB*, the scenarios are generated following a predicted nominal value and volatility matrix. To optimally distribute the samples and explore the whole experimental region, we employ Latin Hypercube Sampling (LHS) [18]. That is, a given number of scenarios is generated with minimal correlation and maximal mutual distance [27]. Fig. 1 shows an example of such a set of the scenarios generated for one of the three buses containing wind production.

3.2. Robust, stochastic, and unified

Table 1 shows an overview of the problem size for *RO*, and for *SO* and *SR* for 1, 5 and 30 scenarios. The sizes of *ROB* and *SRB* are not listed in Table 1 since they are not constant. They start with an initial size similar to *RO* and *SR*, respectively, but their master problems increase through the iterations of the column-and-constraint generation algorithm [11,13]. Note that for a given number of scenarios, *SR* is slightly larger than *SO* because *SR* adds an extra scenario representing the worst-case scenario (since the worst-case scenario of *SR* is unique, as shown in Theorem 5). Despite the number of scenarios, all the UCs have the same number of binary variables, because they all obtain the same number of binary (first-stage) decisions.

In this section, we set the level of conservatism (π) of *RO*, *ROB*, *SR* and *SRB* at 0.6, 0.5, 0.6 and 0.4, respectively. Because the formulations perform the best at these values of π, as discussed in Section 3.3. Similarly, we set the weight α of *SR* and *SRB* at 0.9, as discussed in Section 3.2.3.

3.2.1. Stochastic (SO) vs. robust (RO and ROB) formulations

In Table 2, we assess the performance of the different UC policies on size aspects. For the scheduling stage: (1) the fixed production costs (FxdCost [k$]), which includes non-load, startup and shutdown costs, and (2) the time required to solve the UC problem (CPU Time [s]). For the evaluation stage we record (3) the average of the total production costs including the wind curtailment penalization (AvgTC [k$]); (4) the maximum total cost of the 1000 out-of-sample scenarios, representing the worst-case scenario (WorstTC); (5) the total accumulated number of violations in both demand-balance and transmission-limits constraints (# Viol); and (6) the average percentage of wind that was curtailed (% WCurt).

We can clearly observe that, first, the higher the number of scenarios, the better the *SO* performance (lower AvgTC and Viol), as expected. Second, on the one hand, the stochastic formulation *SO* using 30 scenarios guarantees robustness (Viol = 0), and a *SO* with a higher number of scenarios presents a small improvement at the expense of higher computational cost: from 30 to 50 scenarios, *SO* improves AvgTC in less than 0.01% and takes more than 2× longer to solve. On the other hand, the robust formulations *RO* and *ROB* guarantee robustness by only optimizing for the worst-case scenario, but they schedule too few reserves (lower FxdCost) hence not ensuring that higher wind production levels could be dispatched (WCurt was around 10× larger than *SO*). The robust formulations ignore the possibility of optimistic (high wind) scenarios hence not taking advantage of them. However, compared with *SO* with 30 scenarios and *ROB*, the formulation *RO* proposed in this paper solves more than an order of magnitude faster (above 71× and 13×, respectively). Furthermore, since *RO* only considers one scenario, it solves in similar time as *SO* with one scenario, but *RO* reduces the violations to zero, lowers the AvgTC and WorstTC by 20.5% and 83.4%, respectively, although increase WCurt by more than twice.

3.2.2. Stochastic (SO) vs. unified robust–stochastic (SR) formulations

Table 2 shows how the robust part of *SR* drastically improves the performance of the stochastic formulations *SO* regardless of the number of scenarios used. Even when very few scenarios are considered, *SR* presents a significantly better performance than *SO*: for the case of one and five scenarios, *SR* presents no violations, instead of 698 and 191, a cost reduction of more than 24% and 4%, and a worst-case (WorstTC) reduction of more than 84% and 71%, respectively.

The most important part, however, is that, compared with *SO*, adding the worst-case scenario into *SR* comes at almost no extra costs in terms of run time (even lower in some cases).

3.2.3. Unified robust–stochastic formulations, SR vs. SRB

For five or more scenarios, *SR* and *SRB* perform similarly in the out-of-sample evaluation. *SRB* presents violations when only one (the nominal) scenario is considered, because the level of $\pi = 0.4$ is too low to provide robustness for this one-scenario case; however, for a $\pi = 0.5$, *SRB* with one scenario achieves 0 violations, an AvgTC of 769.39 k$, a WorstTC of 890.41 k$, and a WCurt of 0.34%. Although *SR* and *SRB* perform similarly in the out-of-sample evaluation, *SR* solves more than 6.8× faster because it can be solved directly as single-level MIP problem, instead of requiring ad-hoc algorithms to solve the two-level MIP problem including a bilinear inner problem.

Number of scenarios needed by SR and SRB. Unlike the stochastic *SO* formulation, which needs a large number of scenarios to guarantee robustness, the stochastic–robust formulations *SR* and *SRB* are robust and few scenarios can be used to obtain a good performance. Although *SR* and *SRB* using one scenario present already a better performance than *SO* using 25 scenarios, henceforth, we use 5 scenarios for the stochastic part of *SR* and *SRB* since it further decreases AvgTC, WorstTC and WCurt, and they still solve in less than one and four minutes, respectively. Moreover, considering more than 5 scenarios adds a very little performance improvement at a high computational cost, e.g., using 10, took more than 3× and 2.7× longer, respectively.

Objective weights for SR and SRB. We aim to establish the optimal balance between the costs of the worst-case component and

Table 1
Problem size comparison of UC formulations.

UC	SC #	Constraints	Continuous variables	Binary variables	Nonzero elements
SO	1	15855	7128	6390	305938
	5	56367	35640	6390	1413190
	30	309567	213833	6390	8332689
RO	1	15855	7128	6390	305938
SR	1	26055	14256	6390	582985
	5	66855	42761	6390	1689430
	30	321848	220954	6390	8614565

Table 2
Comparison between different UC formulations.

UC	SC #	Scheduling		Out-of-sample evaluation			
		FxdCost [k$]	CPU Time [s]	AvgTC [k$]	WorstTC [k$]	Viol #	WCurt %
SO	1	54.02	14.9	1023.88	5793.01	698	0.40
	5	65.09	38.0	805.63	3075.12	171	0.22
	10	63.20	184.8	782.10	1971.05	86	0.23
	15	62.89	277.0	782.16	1972.61	86	0.25
	20	66.33	266.8	780.13	1973.04	86	0.16
	25	65.04	460.8	772.68	1577.55	50	0.20
	30	64.52	980.6	763.82	875.10	0	0.16
	50	64.56	1971.2	763.79	875.05	0	0.16
RO	1	59.22	13.8	813.16	961.93	0	1.05
ROB	1	63.22	184.4	816.78	959.83	0	1.11
SR	1	60.13	20.2	769.76	891.64	0	0.34
	5	66.48	50.9	765.86	883.49	0	0.21
	10	62.08	161.9	765.41	884.51	0	0.21
	15	62.91	139.0	764.88	882.15	0	0.22
	20	65.65	289.6	764.47	876.23	0	0.14
	25	64.73	554.8	764.43	880.02	0	0.15
	30	68.82	570.9	764.42	879.2	0	0.14
	50	66.98	1932.5	764.17	879.71	0	0.17
SRB	1	62.95	239.4	771.95	1190.16	18	0.36
	5	65.94	233.9	765.34	883.30	0	0.17
	10	63.74	654.8	765.23	882.41	0	0.17
	15	65.01	940.9	765.04	880.61	0	0.14
	20	65.61	1388.4	764.76	880.08	0	0.15
	25	66.64	2557.4	764.56	877.53	0	0.14
	30	66.34	3359.3	764.17	880.55	0	0.17
	50	66.47	8705.4	763.92	880.33	0	0.17

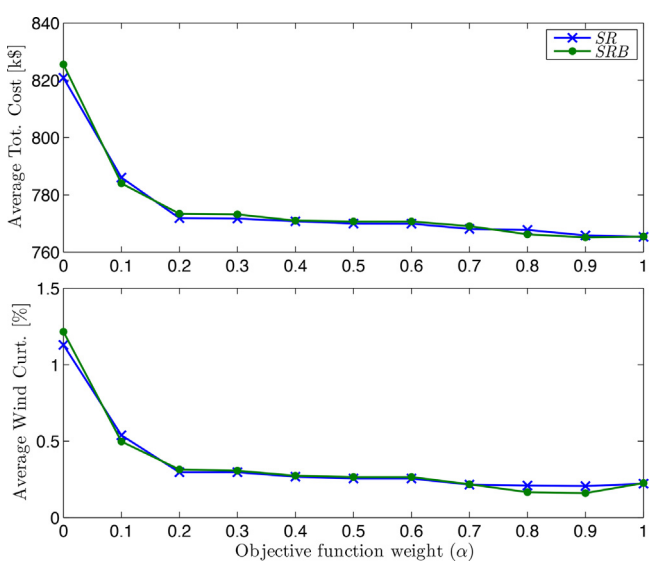

Fig. 2. Different levels of α for *SR* and *SRB* using 5 scenarios. Upper graph: Average total costs over the 1000 out-of-sample scenarios. Lower graph: Average wind curtailment of the 1000 scenarios.

the other scenarios in the objective function of the *SR* and *SRB* formulations. Fig. 2 presents the results of *SR* and *SRB* using 5 scenarios for objective weight α ranging from 0 (all weight to robust) to 1 (all weight to stochastic).

We observe that as α increases, the average total costs decreases. This is because the problem becomes less conservative when the robust part of *SR* has a smaller weight. The same behavior was previously observed in [7]. Similarly, wind curtailment decreases as α increases, and both *SR* and *SRB* achieve their lowest value when $\alpha = 0.9$. We also observe that results are not very sensitive to α: in the range [0.2, 1] we see a difference in total costs lower than 1.5%.

It is important to highlight that none of the *SR* and *SRB* cases present any violations, even when the robust part is ignored in the objective function ($\alpha = 1$). This is because the worst-case scenario is still in the set of constraints, hence guaranteeing robustness.

3.2.4. Wind penalization

The approach we put forward in this paper is enabled by wind curtailment. In some power systems, wind curtailment may be undesirable; however, violations of the demand balance and of transmission-capacity limits are even worse. The objective of this experiment is to study the effect of different penalties on wind curtailment, which is equivalent to different negative values of wind bids [1].

Fig. 3 shows the performance of the following formulations for different penalties on wind curtailment: the robust formulations*RO* and*ROB*, the stochastic–robust formulations *RS* and *RSB* with 5 scenarios, and the stochastic formulations *SO*. In general for a given UC formulation, as the wind curtailment penalization increases, wind

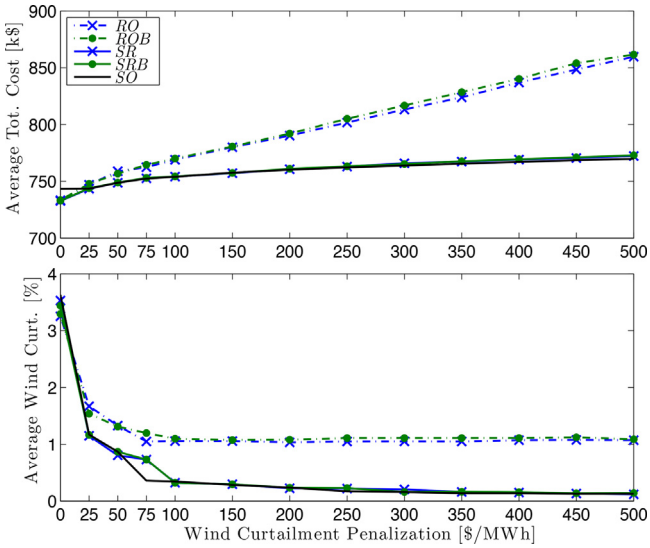

Fig. 3. Effect of penalizations of wind curtailment.

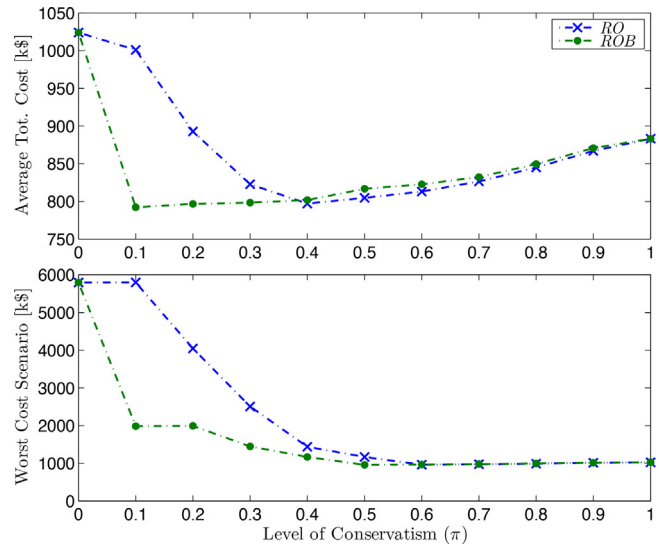

Fig. 4. Comparison of robust formulations.

curtailment decreases. Consequently, we need to schedule more resources (up and down reserves) to better accommodate different wind realizations. This, in turn, also increases the average total costs.

When wind curtailment is not penalized (zero penalization) the UC approaches accommodate the least quantity of wind, hence they also carry the least quantity of downwards reserves. Consequently, *SO* with 30 scenarios reported violations for the case in which the penalization of wind curtailment was set to zero. Therefore, the least quantity of reserves was insufficient to avoid violations in the out-of-sample evaluation stage. Surprisingly, when wind curtailment penalty is equal to zero, *RO* and *ROB* obtain curtailment and average costs similar to the stochastic and stochastic–robust formulations.

The robust formulations *RO* and *ROB* present very similar performance, but *RO* solves 17× faster. The stochastic and stochastic–robust formulations also present similar performance to each other, where *SO* present slightly lower costs for penalties different than zero; however, *SR* solves the problems more than 7× faster than *SRB*, and more than an order of magnitude faster (20×) than *SO*.

For non-zero wind-curtailment penalties, *RO* and *ROB* present the highest curtailment. This is a result of their conservative policy, which avoids infeasible solutions but cannot guarantee that a high wind production will be dispatched. This also leads *RO* and *ROB* to have higher average total costs compared to *SR*, *SRB* and *SO*.

3.3. Level of conservatism: budget and box of uncertainty

3.3.1. Robust formulations, RO vs. ROB

As detailed at the beginning of this section, the level of conservatism of the robust formulations *RO* and *ROB* can be controlled through the parameter $\pi \in [0, 1]$. Fig. 4 shows the average total costs and the worst-case scenario of the 1000 out-of-sample scenarios for *RO* and *ROB* when changing the level of conservatism from 0 to 1. Bear in mind that both formulations are exactly equivalent when the budget of uncertainty is 0 (nominal scenario) or 1 (complete box). *RO* presents violations (non-served energy & transmission limits) for levels of conservatism (box-size) lower and equal to 0.5, while *ROB* presents violations for levels of conservatism (budget of uncertainty) below 0.4.

For low values of π, below 0.4, the robustness of *ROB* dominates presenting fewer violations, hence lower average costs and also lower worst-case scenario (see lower graph in Fig. 4). This is the main advantage of *ROB* which is protecting against a worst-case scenario even when π is very low, considering the full limits of the box of uncertainty, instead of reducing the box completely as *RO* does.

On the other hand, for levels of conservatism above 0.4, both formulations present similar average and worst-case costs. It is important to highlight that the ideal value of conservatism should be low to avoid expensive operation costs due to an over-conservative policy but high enough to avoid violations completely. That is why we set $\pi = 0.6$ for *RO* and $\pi = 0.5$ for *ROB*, which are the values where the average costs are the lowest while avoiding any possible violation.

3.3.2. Unified stochastic-robust formulations

Apart from the previous two stochastic–robust formulations *SR* and *SRB*, the following two formulations are also implemented:

SRI: The same as *SR* but disregarding constraint (14), hence making the set of constraints of the stochastic part completely independent from those of the robust part.

SRBI: The same as SRB but disregarding constraint (14).

Fig. 5 compares the four unified robust–stochastic formulations. The highest average cost of the four stochastic–robust formulations (782.29 k$ for *SRI* and *SRBI* at $\pi = 0.3$, see upper part of Fig. 5) is lower than the lowest average cost of *RO* and *ROB* (792.12 k$ for RO at $\pi = 0.1$, see upper part of Fig. 4). This is mainly because the stochastic part of the unified formulations helps to minimize expected cost by also accommodating different scenarios of wind (and not just minimal elements of the uncertainty set), which at the end result in lower wind curtailment reducing operating costs, as shown in Fig. 6. That is, pure robust planning leads to considerably more costly operation, even when including budget of uncertainty, compared with stochastic–robust. Also the robustness of the solution is considerably improved when adding the robust part to the stochastic part: notice that for any given level of conservatism, the highest worst-cost scenario of the unified formulations (lower part Fig. 5) is always lower than those of the pure robust formulations (lower part Fig. 4).

Similarly to the pure robust formulations, all unified formulations *SR*, *SRB*, *SRI* and *SRBI* present similar average costs for levels of conservatism above 0.5; however, between 0.4 and 0.5, the budget of uncertainty make *SRB* and *SRBI* more robust. On the other hand,

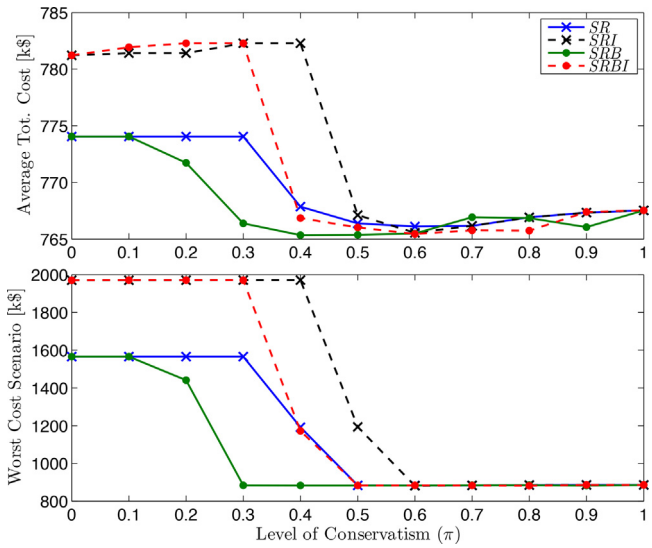

Fig. 5. Comparison of unified robust–stochastic formulations.

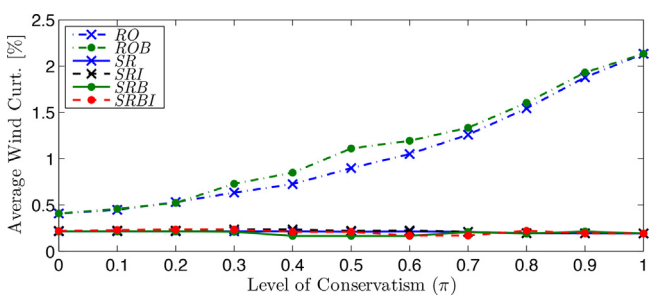

Fig. 6. Curtailment comparison of different formulations.

for values below 0.4, *SR* and *SRB* present fewer violations and lower worst-case cost (lower part Fig. 5) than *SRI* and *SRBI*.

When adding (14) to the unified formulations, the robustness improves considerably for low values of π. For higher values of π (above 0.5), *SR* and *SRB* are very similar to *SRI* and *SRBI*, respectively, because (14) becomes inactive. However, for values between 0.4 and 0.5, (14) starts to dominate forcing that the worst-case dispatch wind scenario remains below all the stochastic scenarios even though this worst-case scenario could be very similar to the mean wind value. For values below 0.4, the lower elements of the stochastic scenarios dominate increasing the robustness of the solution despite the low values of π. That is, the unified formulation protects itself using the lower envelope of the wind scenarios of its own stochastic part.

We set the levels of conservatism to $\pi = 0.6$ for *SR* and to $\pi = 0.4$ *for SRB*. These are the values where the average costs are the lowest while avoiding any possible violations.

3.3.3. Computational performance

Although the two formulations *RO* and *ROB* solve a robust UC, *RO* is a single level MIP formulation compared with the two-level MIP of *ROB*, which also include a bilinear term, hence *RO* solves more than 6× faster than *ROB* on average. Moreover, the slowest case of *RO* ($\pi = 0.5$) solves in 39.8 s, is faster than the fastest case of *ROB* ($\pi = 0.3$), 43.3 s. These computational comparisons are made for π different than 0, since *RO* and *RS* are computationally equivalent to *ROB* and *RSB*, respectively, when $\pi = 0$. Similarly, although *SR* and *SRB* present similar performance in average costs, wind curtailment and robustness, *SR* solves 3.9× faster where its slowest case ($\pi = 0.5$), 85.9 s, is faster than the fastest case of *SRB* ($\pi = 0.2$), 181.2 s.

As mentioned above, the robustness of the stochastic–robust formulations improves by linking the stochastic and robust parts using (14). However, this extra constraint also brings an extra computational burden, resulting, on average, in 60% extra computational burden for *SR* and *SRB*.

It is interesting to note that *SR* outperforms *ROB* in all aspects, average costs, robustness and wind curtailment, despite the value chosen for π, while simultaneously *SR* solves the problems always faster (1.6× on average).

4. Conclusions and future work

This paper presents a single-level mixed-integer linear programming formulation (MIP) for fully adaptive robust unit commitment (UC) with dispatchable wind. We show that this is possible by allowing wind curtailment and considering a box uncertainty set for wind availability. Consequently, the proposed formulation solves considerably faster than traditional robust formulations, which usually require to solve a MIP bi-level and bilinear problem.

Moreover, the level of conservatism can be controlled by shrinking the box of uncertainty, providing similar robustness to a formulation considering a budget of uncertainty, as shown in the numerical experiments.

We also show how to link the wind dispatch constraints between the stochastic and the robust parts further increasing the robustness of the unified stochastic–robust UC formulation. This unified formulation overcomes the disadvantages of both the stochastic and the robust UCs. It reduces the over-conservatism of pure robust UC because an expected value is now optimized on a set of scenarios, and it does not require the large number of scenarios to guarantee feasibility as pure stochastic UC does. More importantly, the computational burden of the unified approach remains low, since the proposed robust part just adds a single extra scenario to the stochastic UC. Moreover, the proposed unified UC outperforms a traditional pure robust UC, including budget of uncertainty, in all aspects, average costs, robustness and wind curtailment, despite the level of conservatism chosen, while also solving significantly faster.

In short, by considering dispatchable wind, solving a stochastic–robust UC becomes computationally feasible and has a good performance with just a few scenarios.

A straightforward application of the results in this paper would be to incorporate the worst-case solution to any deterministic UC formulation, e.g., based on reserves, thereby greatly improving its robustness without significantly affecting its computational burden. Another interesting direction for further research is to obtain computationally efficient robust formulations considering other types of uncertainty sets, in particular those modeling spacial and temporal correlations.

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