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Dynamic Airline Booking Forecasting

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Air Transport Research Society World Conference

This paper proposes a model for dynamic booking forecasting using a time-inhomogeneous Markov process. The transition probabilities are estimated based on a combination of an empirical and a parametric distribution. This model is applied for flight booking forecasting, where flight forecasts are updated on a daily basis over a time horizon of up to 300 days before the day of departure. The distribution of flight bookings over this time horizon, as well as the expected average flight bookings are determined. Historical data of two years of flights is used in our numerical analysis. The performance of our model is compared with two classical forecasting methods: the additive pick-up method and the historical average. We show that our proposed model is up to 8% more accurate than the two classical methods mentioned above. Moreover, by determining the distribution of the flight bookings over a horizon of 300 days before departure, we provide additional information about the uncertainty around the flight bookings.

Airline Booking Forecasting | Markov Processes |

Abbreviations: DBD, days-before-departure; BH, Bookings Horizon; KQ, Kenya Airways; MC, Markov Chains; PU, Pick-up; HA, Historical Average; SSE,Error Sum of Squares; LF, Load Factor; A/C, Aircraft

1. Introduction

Accurate forecasting of demand for a flight is key for revenue management. The higher the accuracy of the booking forecast, the higher the revenue that the airline can achieve. Studies on the impact of forecasting accuracy on passenger revenues [1] have shown that a 10% improvement in forecasting accuracy can lead to a 0.5% to 3% increase in annual passenger revenue.

Forecasting for revenue management has, however, proven to be a challenging task because of the dynamics and complexity of the bookings process. Bookings are influenced by many factors such as seasonality, group bookings, pricing discounts [2]. Capturing such factors is essential for the accuracy of the forecast. Moreover, most existing work on forecasting for airlines provide only a single point forecast. Yet, expressing demand only as a single point (e.g., expected value of bookings for a specific day/month) was shown to provide insufficient information [3] on the degree of uncertainty around the forecast that could provide enhanced support for decision-making and pricing schemes. Our work differs from this in that we determine the distribution of bookings. As an application, the model is used for dynamic flight booking forecasting for a horizon of up to one year before the day of departure. In doing so, we consider daily updates for the bookings forecasts. We show that our proposed model outperforms two classical forecasting methods: the additive pick-up method [9] and the historical average [8]. Moreover, we determine the distribution of flight bookings for various time horizons.

Existing forecasting methods can be divided into three groups [1]: historical bookings methods, advanced bookings methods and combined bookings methods.

Historical bookings methods use only historical data to determine forecasts. Within this framework, in [4] the exponential smoothing approach was employed to forecasts flight bookings. In [5], ARIMA time series and regression models were developed. The regression models were shown to outperform the ARIMA models. Neural networks and *Q-forecasting* were employed in [6] and [7], respectively.

Advanced bookings methods determine forecasts based on current data regarding flights that have not yet departed. The pick-up method is an advanced bookings method that determines the number of bookings to be picked up from a given point in time to another time-point in the bookings process. The classical pick-up method [8] is used to determine the number of bookings at the day of departure. The additive pick-up method incorporates the most recent data by estimating the pick-up in smaller increments [9]. The multiplicative pick-up method [10] assumes future bookings to be proportional to current bookings. The regression method [11] is also defined to be an advanced bookings method. A regression model that included the current bookings, external cause factors and random error has been proposed [1].

The combined bookings methods use historical and current data. A model combining a non-causative regression model with a time-series model is introduced [12]. A full-information model that combines final bookings in historical data and current bookings data in advanced bookings data is employed in [1], [13]. A weighted average forecast combining a short-term with a long-term forecast for hotel reservations is proposed [14].

When comparing the forecasting methods above, it is shown that the performance of the analyzed methods depends on the specific forecasting problem and the dataset used. However, the advanced pick-up method is shown to have an overall good performance and performs at least as well as the best method found in most of the comparisons [10, 15–20].

The forecasting methods mentioned above determine the expected number of bookings at various future time moments. However, the distribution of the bookings is not determined. This provides additional insight into the spread of the future bookings and, most importantly, enables the stochastic optimization of seat prices, seat allocation and booking limits, which, in turn, provides support for enhanced profitability.

In the application of the airline industry the interval forecast is the most used method to provide insight in the uncertainty of bookings [21]. However interval forecasts yield less information about the uncertainty of bookings than distributions [3]. The Markov process conditional probabilities can be used to determine forecast distributions [22–24].

In this paper, we propose a dynamic booking forecasting method that determines both the distribution and the expected number of bookings at various time points. Our model is based on the theory of time-inhomogeneous Markov processes. The model can be classified as an advanced bookings method. As an application, we consider flight bookings forecasting, with a forecasting horizon of 300 days prior to flight departure until the day of the departure. The forecast is updated daily, i.e., as time passes, the distribution and expected value of the bookings is updated for the remaining time horizon. We compare the performance of our proposed model with two classical forecasting methods, advanced pick-up method and the historical average method. We compute the average error of the forecast for several points along the horizon and compare the relative performances of each method. Furthermore, we assess the accuracy of the distributions obtained by our proposed model using interval scores.

The remainder of the paper is structured as follows. In Section 2 we define our model on dynamic booking forecasting. In Section 3 we introduce the metrics employed to evaluate the performance of our model. In Section 4 we briefly explain two classical forecasting methods, the performance of which will be compared with our proposed model. In Section 5 we explain the case study considered. In Section 6 we show and discuss the results. Conclusions and future work are discussed in Section 7.

2. Methodology

In this section we first formulate the dynamic booking forecasting problem as a time-inhomogeneous Markov chain. We then describe our approach to estimating the transition probability matrices of the Markov chain.

2.1. Model formulation. Let the Markov chain $\{X_t\}$ denote the number of net booking at t days before the flight departure, $t \in \{N, N-1, ..., 0\}$, with N the horizon over which we forecast. The state space of $\{X_t\}$ is $\mathbb{C} = \{0, 1, ..., K\}$, where K is the maximum number of bookings. Let π_t determine the distribution of bookings at time t, i.e., $\pi_t = [\mathbb{P}(X_t = 0), \mathbb{P}(X_t = 1), \dots, \mathbb{P}(X_t = K)]^T$.

Let $P_{t,t-1}$ denote the transition probability matrix from time t to time $t-1$, with $P_{t,t-1} = [P^{i,j}_{t,t-1}]$, $i, j \in \{0, \dots, K\}$, where

$$P^{i,j}_{t,t-1} = \mathbb{P}\left(X_{t-1} = j \mid X_t = i\right).$$

Following Chapman-Kolmogorov equations, the distribution of net bookings at time $t-\beta$ is

$$\pi_{t-\beta} = \prod_{l=1}^{\beta} P_{t-l+1} \cdot \pi_t,$$

where $\beta \in \{1, ..., N-t\}$.

The expected number of net bookings at time t days before flight departure is:

$$E[X_t] = \sum_{i=0}^{K} \pi_t(i) \cdot i,$$

where $\pi_t(i)$ denotes the i-th entry of the vector π_t.

2.2. Transition probability matrices $P_{t,t-1}$. The transition probability matrices $P_{t,t-1}$ of the Markov process $\{X_t\}$ are estimated based on a combination of i) a direct estimation of each subsequent net bookings as its relative frequency and ii) estimating the increments (changes) in net bookings. In both cases, to estimate $P_{t,t-1}$ we make use of all available data regarding the bookings up to time t.

i) Firstly, we determine $\bar{P}^{i,j}_{t,t-1}$ by directly estimating the probability of each subsequent net bookings as its relative frequency. This approach captures net bookings dependent transitions.

However, this approach requires a very large dataset of observations of the net bookings to accurately estimate the transition probabilities. Moreover, in practice, this approach leads to a large number of zero entries for $P_{t,t-1}$. To address this, we next estimate the increments (changes) in net bookings.

ii) Secondly, we estimate the increments in the net bookings $X_{t-1} - X_t$ observed in the dataset from time t to $t-1$, following the approach in [22]. Different from the direct estimation of the transition probabilities, this approach assumes that the increments $X_{t-1} - X_t$ are independent of the net bookings at time t.

In contrast with [22], where the distribution of the increments is fitted to a normal distribution and a shifted Poisson distribution, we maintained the initial distribution of the increments since we numerically observed that this approach leads to the lowest forecasting errors.

$P^{i,j}_{t,t-1}$ is now estimated by combining $\bar{P}^{i,j}_{t,t-1}$ and $\tilde{P}^{i,j}_{t,t-1}$ (see Fig. 1). Here, we use a weighting factor $\alpha \in [0, 1]$ as follows,

$$P^{i,j}_{t,t-1} = \begin{cases} \alpha \cdot \bar{P}^{i,j}_{t,t-1} + (1-\alpha) \cdot \tilde{P}^{i,j}_{t,t-1}, & \sum_{j=0}^{K} \bar{P}^{i,j}_{t,t-1} \neq 0, \\ \tilde{P}^{i,j}_{t,t-1}, & \text{otherwise.} \end{cases} \tag{1}$$

for any $i, j \in \{0, ..., K\}$.

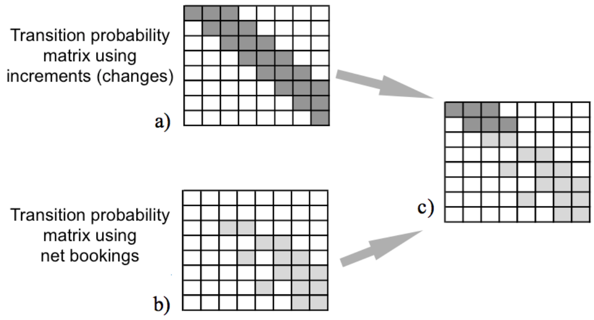

Fig. 1. Determining $P_{t,t-1}$ by combining the direct estimation of transition probabilities for net bookings with the estimation of changes in the net bookings [22].

3. Performance measures

To assess the performance of our model, we consider the Mean Average Error (MAE) and Interval Scores.

a) Mean Average Error is the absolute average difference between the bookings forecast and the actual bookings. Let $MAE(t)$ denote the average absolute error at time t over m forecasts [10], with

$$MAE(t) = \frac{1}{m} \sum_{k=1}^{m} \left| E\left[X_t^k\right] - X_t^k \right|. \tag{2}$$

where X_t^k denotes the $k-$th bookings forecast at time t.

b) Interval Scores [25] quantify the quality of an interval prediction. Let $S_\alpha(p_u, p_l, X_t)$ denote the prediction interval score at time t for the $(1-\alpha) \cdot 100\%$ prediction interval, $\alpha \in (0, 1)$. Then,

$$S_\alpha(p_u, p_l, X_t) = (p_u - p_l) + \frac{2}{\alpha}(p_l - X_t)\mathbb{1}_{X_t < p_l} + \frac{2}{\alpha}(X_t - p_u)\mathbb{1}_{X_t > p_u} \tag{3}$$

where p_u and p_l are the upper and lower limit of the interval.

The lower the value of the interval score, the better the forecast is defined. Since the interval score is non-dimensional, it only be used as a relative and not as an absolute measure to assess the performance of our proposed forecasting method in varying configurations.

4. Other forecasting methods

In this section, we describe two classical forecasting methods: the additive advanced pick-up method and the historical average, which we will use for performance comparisons with our forecasting method, introduced in Section 2.

4.1 Additive pick-up model. The additive pickup method [9] determines the average of reservations picked up between different particular days to forecast the future pickup between the same particular days for the same flight number in the future.

Let $U_{t,t-1}$ denote the booking pick-up from time t to $t-1$. Let X_t^k denote the net number of bookings for flight k at time t. Then,

$$U_{t,t-1} = \frac{1}{n}\sum_{k=1}^{n}\left(X_{t-1}^k - X_t^k\right), \tag{4}$$

where n is the number of flights used from the dataset to obtain the forecast.

The forecast of the final number of bookings, denoted by $\hat{X}_0$, is:

$$\hat{X}_0 = X_t + U_{t,0}, \tag{5}$$

where X_t is the most recent known net number of bookings for the flight.

4.2 Historical average. Let $\hat{X}_t$ denote the forecast of the net number of bookings at time t and X_t^k the net number of bookings for flight k in the historical bookings dataset at time t [8]. Then

$$\hat{X}_t = \frac{1}{n}\sum_{k=1}^{n} X_t^k, \tag{6}$$

where n represents the number of flights in the historical dataset.

5. Case Study

We consider data on the number of bookings on a time horizon of 336 days before departure for 2 flights (KQ101 and KQ512). One flight operates daily to Europe, while the other flight flies three times a week to Western-Africa. The Europe route has 204 seats in economy class and the Africa route 116 seats. A summary of the characteristics of these two flights can be seen in Table 1.

Table 1. Subset of flights used in the study to test the model

Flight	A/C Type	Market	LF	Flights in dataset
KQ101	B787-800	Europe	High	731
KQ512	B737-700	West Africa	Low	457

We consider the bookings data recorded in 2014, 2015, 2016. Moreover, we consider only flights that operated with the same aircraft type as the currently deployed aircraft.

Analyzing the data, the variance of the number of bookings is significant. As an example, Fig. 2 shows that the number of bookings for KQ101 at the day of departure varies between 90 to 220 bookings (mean 176, standard deviation 28.5). Similarly for KQ512, the mean number of bookings is 66 with a standard deviation of 14.5.

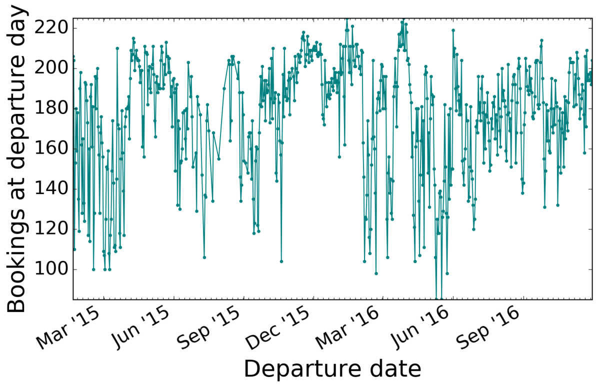

Fig. 2. The variance in the bookings data of KQ101 at the day of departure.

6. Numerical Results

In this section the numerical results using the model proposed in Section 2 are shown. Bookings forecasts are made for 92 KQ101 flights that are planned to departure in the period 1st of October 2016 - 31st of December 2016, and for 64 KQ512 flight planned to departure in the period 1st of August 2016 - 31st of December 2016. Firstly, the performance of the model both in terms of accuracy and uncertainty modeling is addressed when considering various weighting factors α. We also analyze seasonality trends in the flight bookings. In addition, the results of a comparative study with traditional forecasting methods are outlined.

6.1 Weighting factor α. The weighting factor α determines the relative weight that is given to the direct and increment distributions used to calculate $P_{t,t-1}^{i,j}$ in (1).

Table 2 shows the results for various α that yield the best performance with respect to MAE and interval scores. A more detailed analysis is provided in Table 3.

For KQ101, $\alpha = 0.8$ leads to be the best performance with respect to MAE and interval score, except for the short-term forecast of 7 days where $\alpha = 0.7$ resulted in the lowest MAE (see also Fig. 3).

For KQ512, Table 2 shows that larger values of α, i.e., $\alpha \in \{0.5, 0.8\}$, leads to the best performance with respect to MAE for extended horizons of 300 and 100 days before departure. Lower values $\alpha < 0.5$ lead to the best performance with respect to MAE for shorter time horizons of less than 40 days before departure.

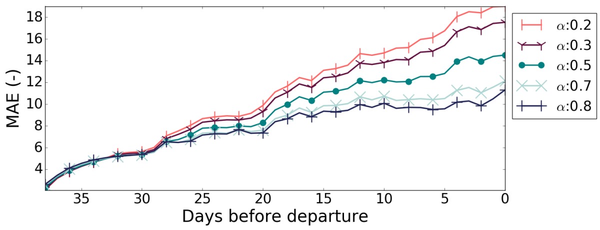

Fig. 3. MAE for flight *KQ101* with a bookings horizon of 39 days.

Table 2. Summary of model calibration results

Horizon	Model setting	Setting yielding lowest MAE across all observation points: KQ101	Setting yielding lowest MAE across all observation points: KQ512	Setting yielding lowest int. score across all observation points: KQ101	Setting yielding lowest int. score across all observation points: KQ512
300	α	0.8	0.8	0.8	0.8
	Dataset	Seasonal	DOW	Seasonal	Month & DOW
100	α	0.8	0.5	0.8	0.8
	Dataset	Seasonal	DOW	Seasonal	DOW
39	α	0.8	0.2	0.8	0.8
	Dataset	Total set	DOW	Total set	DOW
7	α	0.7	0.3	0.8	0.7
	Dataset	Total set	Total set	Seasonal	DOW

Table 3. Results for various model settings expressed as a sum of MAE and interval score across the observation points.

			MAE results across all observation points				Interval score results across all observation points			
			KQ101		KQ512		KQ101		KQ512	
Horizon	Parameter	Setting	Sum	% compared with lowest	Sum	% compared with lowest	Sum	% compared with lowest	Sum	% compared with lowest
300	α	0.2	235.2	+6.4	44.4	+3.5	3792.4	+30.8	853.7	+13.4
		0.3	230.9	+4.5	44.0	+2.5	3464.3	+19.5	829.7	+10.2
		0.5	225.0	+1.8	43.4	+1.1	3111.9	+7.3	791.8	+5.2
		0.7	221.7	+0.3	43.0	+0.3	2948.1	+1.7	767.4	+2.0
		0.8	221.0	-	42.9	-	2898.9	-	752.7	-
	Dataset	Total Set	221.0	+6.3	43.4	+4.9	2898.9	+13.8	791.8	+7.2
		DOW	219.6	+5.6	41.3	-	2865.2	+12.4	780.0	+5.6
		Seasonal	208.0	-	42.4	+2.6	2548.2	-	777.6	+5.3
		Month-DOW	211.5	+1.7	45.0	+8.9	2825.2	+10.9	738.6	-
100	α	0.2	149.4	+31.9	71.0	+0.1	1888.1	+13.6	1156.8	+12.2
		0.3	142.4	+25.6	71.0	+0.1	1821.3	+9.6	1117.7	+8.4
		0.5	129.4	+14.1	70.9	-	1740.9	+4.7	1054.3	+2.3
		0.7	118.0	+4.1	71.1	+0.3	1681.9	+1.2	1039.7	+0.9
		0.8	113.3	-	71.3	+0.5	1662.1	-	1030.7	-
	Dataset	Total Set	113.3	+0.6	70.9	+9.1	1662.1	+1.8	1054.3	+0.5
		DOW	131.1	+16.4	65.0	-	1736.8	+6.4	1048.7	-
		Seasonal	112.6	-	71.9	+10.7	1632.2	-	1081.2	+3.1
		Month-DOW	144.0	+27.9	76.2	+17.3	1933.0	+18.4	1200.8	+14.5
39	α	0.2	120.5	+42.1	93.7	-	1460.9	+21.3	1317.5	+4.8
		0.3	113.7	+34.0	93.9	+0.1	1398.1	+16.1	1301.4	+3.5
		0.5	100.4	+18.4	94.3	+0.6	1297.9	+7.8	1278.5	+1.7
		0.7	89.0	+5.0	94.7	+1.1	1232.6	+2.4	1259.8	+0.2
		0.8	84.8	-	94.9	+1.2	1204.0	-	1257.4	-
	Dataset	Total Set	84.8	-	94.3	+9.7	1204.0	-	1278.5	+3.0
		DOW	101.0	+19.1	85.9	-	1288.2	+7.0	1241.1	-
		Seasonal	90.7	+7.0	96.3	+12.0	1243.2	+3.3	1365.3	+10.0
		Month-DOW	109.6	+29.3	91.5	+6.5	1380.2	+14.6	1353.4	+9.0
7	α	0.2	37.6	+3.6	38.6	+0.5	507.6	+2.2	563.7	+0.7
		0.3	37.0	+2.0	38.4	-	503.0	+1.2	564.6	+0.9
		0.5	36.3	+0.1	38.5	+0.1	496.9	-	561.2	+0.3
		0.7	36.3	-	39.2	+2.0	502.4	+1.1	559.8	-
		0.8	36.5	+0.6	39.8	+3.7	510.7	+2.8	562.6	+0.5
	Dataset	Total Set	36.5	-	38.5	-	510.7	+0.7	561.2	+5.8
		DOW	36.7	+0.6	38.5	+0.1	507.4	+0.1	530.6	-
		Seasonal	36.6	+0.3	42.1	+9.3	507.0	-	583.8	+10.0
		Month-DOW	40.3	+10.4	50.1	+30.1	536.1	+5.7	647.9	+22.1

Table 2 also shows that both for KQ101 and KQ512 the weighting factor α leading to the lowest MAE does not necessarily also lead to the lowest interval score.

Based on the results above, for the remaining analysis, for KQ101, we consider $\alpha = 0.8$. For KQ512, we chose $\alpha = 0.5$ since resulted in a good performance in terms of MAE and for the interval scores the setting is performing well both in the first as in the second half of the horizon.

6.2 Seasonality. In this section we consider monthly seasonal datasets, data of the same day-of-the-week (DOW) datasets and datasets where the data has to fulfill both these characteristics.

The performance of our proposed model for these different datasets isdisplayed in Table 2.

For KQ101, Table 2 shows that using either monthly seasonal datasets or the total set of data leads to the best performance with respect to MAE and interval score. Table 3 also shows that these two datasets lead to a better performance than the DOW and *Monthly & DOW* combined datasets with respect to MAE and interval score.

For KQ512, Table 2 shows that the DOW dataset eads to the best performance with respect to MAE and interval score. Table 3 also supports these findings. This finding is in line with the knowledge that KQ101 is more dependent on the seasons over the year because of holiday tourists and KQ512 more dependent on day of week because of business passengers within Africa that show not much variation over the year.

6.3 Comparative performance analysis. The performance of our proposed model (MC) outlined in Section 2 is compared with two classical forecasting methods, historical average (HA) and additive pick-up method (PU), which are outlined in Section 4.

A summary of the results of the three methods (MC, HA, PU) over all observation points is shown in Table 5. For each time horizon, we considered 11 uniformly spread time points $V_0, V_1, \ldots, V_{10}$ to show the performance of the models. For example, for a horizon of 300 days, we consider 11 time points, uniformly distributed acrross the remaining 300 days before departure. Table 4 shows the actual MAE's of the considered forecasting methods for each observation point.

For KQ101, our proposed method underperforms in comparison with the pick-up method in long-term forecasts of 200 and 300 days. However, for the remaining horizons our proposed model yields the lowest MAE for most of the observation points, as indicated in Table 5. Especially from 60 days or less before departure, MAE for our proposed model improves significantly in comparison with the pick-up method (see Fig.4 for an example). Nevertheless, in the first days of the forecast horizon the pick-up method slightly outperforms our proposed model as can be seen in Table 4.

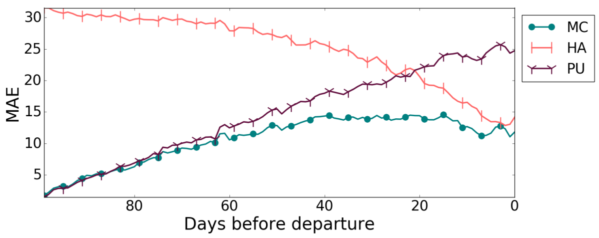

Fig. 4. MAE for a forecast for flight *KQ101* with a bookings horizon of 100 days.

For KQ512 the differences in performance between the three methods are much smaller. In the short-term, our proposed model outperforms the other two methods. Exceptions are the first days of the forecast horizon where the pick-up method yields the lowest forecasting error (see Fig.5 for an example).

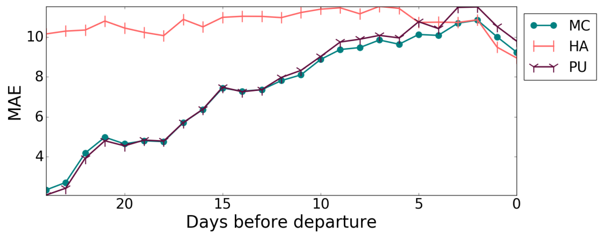

Fig. 5. MAE for flight *KQ512* with a bookings horizon of 25 days.

A Kruskal-Wallis test [26] is used to test if the difference in performance of the three forecasting methods is significant enough to state that one method is more accurate than the other. The Null-hypothesis in this test is that the medians of all the groups are equal. The significance level α of *0.05* is assumed.

For KQ101, Table 5 shows that a substantial number of times our proposed model significantly yields the lowest MAE. Especially for the medium term forecasts the hypothesis is rejected. For flight KQ512 the difference in forecasting performance between the methods is not significant enough. For the long-term forecasts the hypothesis is only rejected for one of the 11 observation points for our proposed model and the pick-up.

Table 4. MAE for various forecasting horizons (bold numbers indicat the significant differences)

		MAE per observation point and per flight																					
		KQ101											KQ512										
Horizon	**Method**	V_0	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8	V_9	V_{10}	V_0	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8	V_9	V_{10}
300	MC	0.8	8.4	13.4	16.8	22.1	25.0	29.5	28.3	26.3	22.8	14.5	**0.1**	1.5	1.3	1.4	1.3	1.7	3.2	5.9	4.6	11.1	9.2
	HA	13.3	16.4	18.6	21.3	24.8	27.8	31.9	31.2	28.7	24.1	14.5	1.2	1.2	1.0	1.2	1.1	1.6	3.1	5.8	4.5	11.3	9.3
	PU	0.4	7.6	12.5	16.4	21.8	24.6	29.2	27.4	26.0	23.7	18.7	**0.1**	1.7	1.6	2.0	1.9	2.2	3.4	6.1	4.5	10.8	8.8
200	MC	1.4	7.3	10.2	12.4	15.5	17.4	16.1	16.2	18.4	16.4	**11.3**	**0.0**	0.7	1.1	1.3	2.8	4.7	4.8	4.6	8.7	10.7	8.9
	HA	23.1	24.2	26.5	28.6	31.1	31.7	29.7	27.9	25.7	20.6	14.2	1.1	1.1	1.5	1.5	2.9	4.7	4.8	4.6	8.7	10.5	8.7
	PU	1.2	7.4	10.1	12.4	15.4	16.8	15.5	15.6	18.9	19.4	18.5	0.0	**0.7**	1.1	1.3	2.9	4.8	4.9	4.6	8.9	11.1	9.1
100	MC	1.7	4.9	6.3	9.2	10.5	12.8	**14.3**	**14.0**	**14.4**	**12.7**	**11.8**	0.6	1.3	2.6	4.1	4.5	5.4	7.1	8.9	9.5	10.8	10.2
	HA	31.5	30.5	29.7	29.8	27.9	27.0	25.7	23.3	20.6	16.5	14.2	4.4	4.7	4.6	4.4	4.5	5.9	8.2	9.8	10.4	11.4	8.9
	PU	1.5	4.4	6.9	10.0	12.5	15.5	18.0	19.3	21.9	23.9	24.7	0.5	1.2	2.6	4.1	4.4	5.2	7.0	9.0	9.6	11.0	10.8
60	MC	2.2	4.8	7.7	9.0	9.7	10.4	**10.9**	**11.7**	**11.9**	**10.6**	**11.8**	0.7	2.2	4.8	5.8	7.5	7.7	8.8	8.8	10.2	11.2	9.2
	HA	27.9	27.7	27.2	26.3	25.0	23.3	21.2	19.2	16.7	13.9	14.2	4.6	5.1	7.8	8.2	9.7	9.8	10.1	10.1	11.0	11.4	8.9
	PU	1.8	4.8	7.9	9.6	10.5	12.5	14.3	16.8	18.6	19.1	20.6	0.8	2.2	4.6	5.5	7.1	7.5	8.6	8.7	10.0	11.4	10.0
39	MC	2.8	4.7	5.6	6.8	7.9	**9.2**	**10.3**	**10.6**	**10.2**	**10.7**	**11.9**	1.5	3.0	4.5	6.0	7.5	8.3	9.2	10.4	12.0	12.8	10.7
	HA	24.9	24.8	22.9	22.4	21.7	19.5	18.8	16.4	14.4	13.3	14.2	8.9	9.4	9.4	9.4	10.3	10.2	11.0	11.2	11.5	10.7	8.9
	PU	2.1	4.3	5.7	7.6	9.1	11.7	13.5	15.3	15.5	17.4	18.6	1.5	3.0	4.6	6.2	7.7	8.5	9.7	10.9	13.0	14.0	12.0
25	MC	3.0	5.3	6.4	8.0	8.5	9.1	8.8	**9.2**	**10.8**			2.3	4.2	4.8	6.4	7.4	8.9	9.8	10.1	9.2		
	HA	21.2	21.9	19.5	18.9	17.2	16.5	14.4	13.4	14.2			10.1	10.3	10.2	10.5	11.0	11.4	11.5	10.7	8.9		
	PU	2.7	4.9	6.4	8.1	9.6	11.0	11.2	13.5	14.8			2.1	3.9	4.8	6.4	7.4	9.0	10.1	10.4	9.8		
14	MC	3.9	5.3	6.7	6.1	6.5	7.6	8.3	10.0				2.9	4.3	5.5	7.1	7.2	8.6	9.6	8.9			
	HA	17.2	16.7	16.5	15.5	13.9	13.4	12.8	14.2				11.0	11.0	11.4	11.2	11.4	10.7	10.9	8.9			
	PU	3.5	5.4	6.8	6.7	7.9	9.5	10.2	12.1				2.6	4.0	5.6	7.3	7.8	9.3	10.8	10.4			
7	MC	3.2	4.1	4.8	5.2	5.5	5.8	8.1					3.8	4.4	4.5	5.3	6.0	6.9	7.7				
	HA	13.9	13.4	13.4	13.3	12.8	13.0	14.2					11.4	10.7	10.7	10.7	10.9	9.5	8.9				
	PU	2.7	4.1	4.8	5.4	5.5	6.1	8.9					3.7	4.4	4.4	5.3	6.0	7.4	8.5				

From the Kruskal-Wallis test, for KQ101 the MAE obtained using our proposed method were found to be significantly different from the MAE obtained using the other two methods. The hypothesis that our proposed model performs better than those methods is accepted for most of the horizons considered. Overall, our model leads to an improvement in MAE of 8% than the pick-up method. For KQ512, however, the significancy test indicated that the hypothesis is rejected, i.e., the differences in the MAE obtained using the three different forecasting methods are not significantly different.

Table 5. Summary of the results of the comparative study for MC, HA, PU.

		Method with lowest MAE over observation points				MAE improvement compared with other methods (%)	
		Nr. times lowest		Nr. times p<0.05			
Horizon	Method	KQ101	KQ512	KQ101	KQ512	KQ101	KQ512
300	MC	2	1	0	1	-	-
	HA	0	7	0	1	+22.1	-0.8
	PU	9	4	0	1	-9.3	+10.4
200	MC	4	6	1	1	-	-
	HA	0	5	0	0	+48.6	+16.0
	PU	7	2	0	1	+2.6	+1.4
100	MC	9	3	5	0	-	-
	HA	0	1	0	0	+54.2	+21.8
	PU	2	7	1	0	+18.8	-2.4
60	MC	9	3	5	0	-	-
	HA	0	1	0	0	+53.3	+25.9
	PU	2	7	1	0	+17.1	-0.1
39	MC	9	7	6	0	-	-
	HA	0	3	0	0	+52.4	+24.1
	PU	2	1	1	0	+15.2	+4.3
25	MC	6	6	2	0	-	-
	HA	0	1	0	0	+52.9	+33.3
	PU	3	3	0	0	+10.1	-0.5
14	MC	7	6	0	0	-	-
	HA	0	0	0	0	+53.4	+36.4
	PU	1	2	0	0	+9.3	+3.3
7	MC	4	5	0	0	-	-
	HA	0	0	0	0	+61.0	+45.7
	PU	3	3	0	0	-0.1	+1.6

7. Conclusions

In this paper, we have developed a forecasting model based on a time-inhomogeneous Markov process. We used the model to investigate the performance relative to two classical forecasting methods, additive pick-up method and historical mean. Moreover, we obtained the probability distribution of the flight bookings for a time horizon of up to a year defore departure. The analysis was conducted using data of 2 years of departures of 2 different flights.

Using our proposed model, the forecasting error was reduced, compared with other 2 classical forecasting models, both in the case of long-term and in short-term forecasts. Analysis per flight revealed that our proposed model outperformed the other two methods in the case of flights with a high load factor (up to 8% lower forecasting errors). In the case of flights with a low factor load, there was no significant reduction in the forecasting error between our model and the other two classical approaches considered. Moreover, the forecasting errors, when considering a time horizon of up to 100 days before the day of departure, were significantly smaller when using our proposed model.

An analysis of the model parameters indicated that improvement in the model performance might be achieved when the optimal parameter value is determined for every day along the forecast horizon. A similar analysis of the model should be conducted over multiple flights and horizons to verify this hypothesis. Other future research should include a study on the performance of the model when unconstrained data is used as an input. This may have a significant impact on flights with a high load factor.

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Bibliography

1. A.O. Lee, Airline reservations forecasting: probabilistic and statistical models of the booking process, PhD dissertation, Department Of Civil Engineering, Massachusetts Institute Of Technology, 1990.
2. P. Guo, B. Xiao and J. Li, Unconstraining methods in revenue management systems: Research overview and prospects,Advances in Operations Research, 2012.
3. C. Chatfield, Calculating Interval Forecasts, Journal of Business & Economic Statistics 11 (1993), pp. 121–135.
4. A.V. Duncanson, Short-Term Traffic Forecasting, Proceedings of the 14th Annual AGIFORS Symposium, 1974.
5. J. Sa, Reservations Forecasting in Airline Yield Management, Massachusetts Institute of Technology, 1987.
6. L.R. Weatherford, T.W. Gentry and B. Wilamowski,Neural network forecasting for airlines: A comparative analysis, Journal of Revenue & Pricing Management, 1 (2003), pp. 319–331.
7. P.P. Belobaba and C. Hopperstad,Presented at the Meeting of the INFORMS Section on Revenue Management, Massachusetts Institute of Technology, 2004.
8. K. Littlewood, Forecasting and control of passenger bookings, Journal of Revenue & Pricing Management, 4 (1974).
9. E. L'Heureux, A New Twist in Forecasting Short-Term Passenger Pickup, AGIFORS Symposium Proceedings, 1986.
10. A.G. Zakhary, E. Neamat and A.F. Atiya, A Comparative Study of the Pickup Method and its Variations Using a Simulated Hotel Reservation Data, ICGST-AOML, 8 (2008), pp. 15–21.
11. P. Harris and G. Marucci, A Short Term Forecasting Model, AGIFORS Symposium Proceedings, 1983.
12. M. Ben-Akiva, Improving airline passenger forecasts using reservation data, Fall ORSA/TIMS Conference, 1987.
13. C. Hopperstad,Efficient Forecasting: 1/92 Equation Simplifications, Boeing Internal Report, 1991.
14. M. Rajopadhye, M. Ben Ghalia, P.P. Wang, T. Baker and C.V. Eister, Forecasting uncertain hotel room demand, Information Sciences, 132 (2001), pp. 1–11.
15. W. Stikvoort and F. Van der Zwan, Revenue optimization through demand forecasting,International Air Transport and Operations Symposium, 2010, pp. 193–216.
16. S. Karmarkar, D. Goutam, B. Tathagata, Revenue impacts of demand unconstraining and accounting for dependency, Journal of Revenue and Pricing Management, 10 (2011), pp. 367–381.
17. R.R. Wickham, Evaluation of Forecasting Techniques for Short-Term Demand of Air Transportation, Massachusetts Institute Of Technology, 1995.
18. D.K. Skwarek, Competitive Impacts Of Yield Management System Components: Forecasting And Sell-Up Models, Massachusetts Institute of Technology, 1996.
19. L.R. Weatherford, Forecasting Issues in Revenue Management, Spring INFORMS Conference, 1998.
20. L.R. Weatherford, S.E. Kimes,A comparison of forecasting methods for hotel revenue management, International Journal of Forecasting, 19 (2003), pp. 401–415.
21. S.K. Wan, H. Song and D. Ko, Density forecasting for tourism demand, Annals of Tourism Research, 60 (2016), pp. 27–30.
22. J.H. Goto, M.E. Lewis and M.L. Puterman,Coffee, Tea, or ...?: A Markov Decision Process Model for Airline Meal Provisioning An MDP Model for Airline Meal Provisioning,Transportation Science, 38 (2004), pp. 107–118.
23. C. Hsu and S. Liu, Predicting City-Pair Air Passenger Traffic Using Grey Topological Forecasting Model, Journal of the Eastern Asia Society for Transportation Studies, 5 (2003), pp. 453–470.
24. L.M. Bobb and E. Veral, Open issues and future directions in revenue management, Journal of Revenue and Pricing Management, 7 (2008), pp. 291–301.
25. T. Gneiting and A.E. Raftery, Strictly Proper Scoring Rules, Prediction, and Estimation, Journal of the American Statistical Association, 102 (2007), pp. 359–378.
26. Y. Liu and W. Chen, A SAS Macro for Testing Differences among Three or More Independent Groups Using Kruskal-Wallis and Nemenyi Tests, Huazhong University of Science and Technology and Springer, 32 (2012), pp. 130–134.