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# CONTROLLED ISLANDING OF POWER NETWORKS BASED ON GRAPH REDUCTION AND SPECTRAL CLUSTERING

*Ilya Tyuryukanov<sup>1</sup>, Jairo Quirós-Tortós<sup>2</sup>, Matija Naglic<sup>1</sup>, Marjan Popov<sup>1</sup>,  
Mart A.M.M. van der Meijden<sup>1,3</sup>, Vladimir Terzija<sup>4</sup>*

<sup>1</sup>*Delft University of Technology, Faculty of EEMCS, Delft, Netherlands*

<sup>2</sup>*EPER-Lab, School of Electrical Engineering, University of Costa Rica*

<sup>3</sup>*TenneT TSO B.V., Arnhem, Netherlands*

<sup>4</sup>*The University of Manchester, School of Electrical and Electronic Engineering*

*\*Ilya.Tyuryukanov.2016@ieee.org*

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## Abstract

Intentional controlled islanding aims to split the power system into self-sustainable islands after a severe disturbance, but prior the uncontrolled network separation. Given its nature (i.e. last resort for blackout prevention), this emergency control technique must be adopted as quickly as possible. This paper proposes a computationally efficient method based on graph reduction and spectral clustering. The paper contributes by describing important details of the graph reduction process in the context of controlled islanding and by the formalisation of this process. Furthermore, it demonstrates how to adopt embedded graphs to enhance the Multiway Spectral Clustering graph partitioning. Finally, it is shown how to explicitly incorporate important cannot-link constraints between coherent generator groups into the islanding problem. The proposed method is detailed using the IEEE 39-bus test case. To evaluate the algorithm performance, the method is applied to realistically-sized PEGASE test networks.

## 1 Introduction

Intentional controlled islanding (ICI) is an adaptive, corrective measure that aims to limit the spread of disturbances across the grid by separating it into self-sustainable islands [1]-[6]. This emergency control action is used after large disturbances, and should be adopted before an uncontrolled grid separation takes place. It can be used to isolate different kinds of adverse scenarios in power systems, e.g. cascading trips, voltage collapse or undamped oscillations. Although two aspects are important in islanding: when to island, and where to island, this paper focuses on the latter.

In order to ensure the stability of the islands, multiple constraints should be satisfied. Among those, load-generation balance, generator coherency, transmission line availability, thermal limits, voltage stability, and transient stability are important [2], [6]. However, to

produce a set of feasible candidate solutions within a short period of time (a few seconds in practice), it is possible to consider only a subset of the aforementioned constraints [5]-[6]. Excluding some constraints, which largely reduces the initial complexity of the problem, means that islanding solutions must be coordinated with other corrective measures (e.g. post-islanding load shedding) to ensure the adequate operation of the electrical islands. In this paper, the first three constraints from the previous list will be considered.

It was shown in [2] that explicit load-generation imbalance minimization among the islands is an NP-hard problem which makes it very computationally restrictive for power network models with more than about 40 nodes [5]. A related, yet different, objective of minimal power flow disruption allows for efficient determination of islanding solutions since well-established graph theoretic techniques can be used. Moreover, it has significant benefits from the power system point of view [5], including the reduction of MW interdependency between islands [7]. However, existing approaches for minimal power flow disruption controlled islanding are often slow when applied to large power systems [2] or do not include some important constraints like generator coherency grouping [7].

Graph reduction was used in [3]-[4] in order to increase the computational speed and simultaneously incorporate constraints into the network separation problem. Compared to the previous work in [3]-[4], the methodology proposed in this paper, which is based on [8], guarantees the equivalence of the initial and reduced constrained problems in terms of graph cut by showing how the edge weights of the reduced graph should be assigned. One important advantage of the graph reduction method is that a large number of constraints becomes beneficial from a computational point of view as it results in a smaller network; this, in turn, significantly accelerates the identification of the optimized solution.

After the incorporation of constraints on transmission line availability and generator coherency by using graph reduction, a solution for the minimal power flow disruption objective should be obtained for

the reduced network [3]-[4]. The well-established normalized spectral clustering algorithm [9] is adopted here to solve this problem. Two improvements to this standard algorithm are proposed in this paper in order to make it more suitable for ICI. First, the edge connectivity information is taken into account as proposed in [10]. The idea suggested in [10] is explained in detail and, based on it, the notion of embedded graph is introduced in this paper. Second, a computationally efficient approach to ensure that each group of coherent generators remains in its own island is proposed here. While references [3]-[4] do not consider this problem and only show how to keep coherent generators in each group together, reference [5] limits itself to the case of sequential bipartitioning which is computationally demanding and often less optimal as compared to the multiway partitioning [11] which is implemented in this paper.

The rest of the paper is organized as follows: Section 2 briefly introduces the necessary theoretical information (fundamentals of spectral clustering, islanding constraints representation and islanding quality metrics). Section 3 shows the implementation of constrained spectral clustering. In particular, it explains the version of graph reduction algorithm used in this paper. Section 4 provides benchmark results and discusses the efficiency of the algorithm and Section 5 concludes the paper.

## 2 Theoretical Preliminaries

### 2.1 Spectral Clustering

Spectral Clustering is an efficient graph-based partitioning technique widely used in many domains. It aims to find the best set of lines that minimizes the power flow (i.e. cut) between islands. The theoretical foundations and properties of spectral clustering are thoroughly explained in [9]. Its application in power systems has been presented in [5], [6], [10].

A power system with  $n$  buses can be represented as a weighted undirected graph  $G = (V, E, \omega)$ . The nodes and edges of  $G$  can be denoted as  $v_i \in V$ ,  $i = 1, 2, \dots, n$  and  $e_{ij} \in E \subset V \times V$ ,  $i, j = 1, 2, \dots, n$  respectively. The number  $w_{ij} = \omega(e_{ij})$ ,  $i, j = 1, 2, \dots, n$  represents the weight of the edge  $e_{ij} \in E$  and is equal to the average active branch power flow between the nodes  $i$  and  $j$ . Then the *normalized* Laplacian  $\mathbf{L}_n$  can be computed as follows [6], [10]:

$$[\mathbf{L}_n]_{ij} = \begin{cases} 1, & \text{if } i = j \\ \frac{-w_{ij}}{\sqrt{d_i} \sqrt{d_j}}, & \text{if } i \neq j \text{ and } (i, j) \in E \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

where  $d_i = \sum_{j=1}^n w_{ij}$  is the weighted degree of the node  $v_i$ .

**2.1.1 Spectral Embedding** Having  $\mathbf{L}_n$  computed, the next step is to calculate its first  $r$  smallest eigenvalues and their corresponding eigenvectors, where  $r$  is the assumed number of islands. The rows of the resulting eigenvector matrix  $\mathbf{X} \in \mathbb{R}^{n \times r}$  should be normalized to have length one. This results in the matrix  $\mathbf{Y} \in \mathbb{R}^{n \times r}$ , the rows of which can be considered as the  $n$  point coordinate vectors in the  $r$ -dimensional Euclidian space. This so-called spectral  $r$ -embedding [10] maps each of the  $n$  network buses to a point in  $\mathbb{R}^r$ , and it should reveal the clustering structure of the grid with respect to minimal active power flow disruption.

**2.1.2 Embedded graph** Spectral embedding does not consider the actual interconnections between the buses. To overcome this issue, it is recommended in [10] to basically reconstruct the topological structure of the initial power network in spectral embedding. It is possible to create a new graph  $G^{SC}$  with the same set of edges and nodes as in the graph which is used for the computation of  $\mathbf{L}_n$ . In this section, in order to be able to explain by an example, it is assumed that the graph which is used to compute (1) is the initial graph  $G$ . Then the new set of edge weights  $w_{ij}^{SC} = \omega^{SC}(e_{ij})$ ,  $i, j = 1, 2, \dots, n$  is assigned by setting  $w_{ij}^{SC} = \|y_i - y_j\|$  if  $e_{ij} \in E$ , where  $y_i$  and  $y_j$  are the coordinates of the two points in the spectral embedding corresponding to the vertices  $v_i$  and  $v_j$  of the graph  $G$  (i.e. the  $i$  and  $j$  rows of the normalized eigenvector matrix  $\mathbf{Y}$ ) and  $\|\cdot\|$  is the Euclidean norm. The distance between any two points in spectral embedding is supposed to be measured as the shortest path distance between the corresponding vertices of  $G^{SC}$ , which takes into account the topological structure of the original graph.

### 2.2 Representation of islanding constraints

It is common in graph partitioning to describe the relationships between nodes of graph  $G$  as must-link (ML) or cannot-link (CL) constraints. Must-link implies that the nodes should be in the same partition, and cannot-link implies the nodes cannot be in the same partition. Since spectral clustering finds a cut with a small active power flow disruption, the task is to encode the generator coherency and transmission line availability constraints into the solution. The process of doing this will be explained in Section 3, and only some preliminary knowledge is given here.

In the context of ICI, the generator coherency constraint is used as a practical substitution of the true transient stability constraint, as it enhances the transient stability of the islands [5], [6]. The neglect of this constraint may lead to unstable islands [5], [6]. The transmission line availability constraint is related to the fact that certain power network branches (e.g. transformers or lines without synchro-check relays) should not be the part of the islanding cutset [6], [12].

Since transformers and transmission lines without synchro-check relays cannot be a part of the islanding cutset, their terminals should belong to one single island, i.e. a ML constraint should be introduced between the corresponding vertices of the graph. This kind of must-link constraint is referred to as pairwise must-links, as it covers the case of pairs of adjacent vertices that should be kept together.

A coherent generator group should not be split as well, which introduces a ML constraint on the graph nodes corresponding to the coherent generator buses. In addition, it is highly desirable to be able to ensure that each island resulting from the algorithm contains only one coherent generator group. This would produce the maximal reasonable number of islands, and if a smaller number is desirable, some islands could be merged in a deliberate manner by re-establishing the connections between them. Therefore, CL constraints are also introduced between generators belonging to different coherent generator groups.

### 2.3 Islanding quality metrics

The cut of an island with a node set  $V_k \subset V$  can be expressed as  $cut(V_k, \bar{V}_k) = \sum_{v_i \in V_k, v_j \in \bar{V}_k} w_{ij}$  and it represents the total weight of the island's external connections. The volume of an island with the node set  $V_k$  is the sum of weighted degrees of its nodes  $vol(V_k) = \sum_{v_i \in V_k} d_i$ . Now the expansion ratio of an island can be defined similarly to [6], [10] as

$$\phi(V_k) = \frac{cut(V_k, \bar{V}_k)}{vol(V_k)} \quad (2)$$

The value of  $\phi(V_k)$  ranges from zero to one, with smaller values corresponding to better islands. From the power system perspective, if an island obtained from the graph  $G$  (as defined in Section 2.1) has a small expansion, it has a low interaction with the rest of the network in terms of active power flow, which in turn corresponds to a small power flow disruption for this island.

## 3 Constrained spectral clustering for ICI

### 3.1 Graph reduction and must-link constraints

The basic approach to incorporate the must-link constraints of Section 2.2 into the ICI problem is as in [3], however it is augmented with several valuable ideas from [8]. First, the proposed reduction algorithm is presented as bullet points, and further an explanation is given about its salient features. The graph reduction steps can be summarized as follows.

0. Check the constraints on coherency and branch availability for consistency. There should be no must-link branch connecting two generator buses belonging to different coherent generator groups

(such a situation should be relatively rare for practical power network models).

1. Create a simple unweighted graph  $G^{pML} = (V, E^{pML})$  representing the unavailable branches (i.e. the pairwise ML). The total number of buses connected by unavailable branches is denoted by  $n_{pML}$ .
2. Find the connected components of  $G^{pML}$ . If several unavailable branches have any buses in common, they will be merged into a larger bus group at this step. Thus, the identified connected components of  $G^{pML}$  reveal the  $g_{pML}$  aggregated node groups.
3. Initialize the reduced graph  $G^R = (V^R, E^R, \omega^R)$  with  $v_p^R \in V^R$ ,  $p = 1, 2, \dots, n_R$ , and  $e_{pq}^R \in E^R \subset V^R \times V^R$ ,  $p, q = 1, 2, \dots, n_R$  where  $n_R = n - n_{pML} + g_{pML}$ . Copy the nodes of  $G$  which are not subject to pairwise ML (these nodes get the name *free nodes*) and the edges between them into  $G^R$ . The free nodes get new indices  $p = 1, 2, \dots, n - n_{pML}$  in  $G^R$ . The last  $g_{pML}$  nodes of  $G^R$  (these nodes get the name *merged nodes*) are all isolated at this point and represent the aggregated ML node groups with indices  $p = n - n_{pML} + 1, n - n_{pML} + 2, \dots, n_R$ . Keep track of the coherency status of the merged nodes: if at least one original bus in a  $p$ th *merged node* was a generator bus, the  $p$ th merged node inherits its coherency group.
4. For the merged nodes do the following
  - a. Add an edge  $e_{pq}^R$  between a merged node  $p$  and any other node  $q$  in  $G^R$  if there are edges between the two groups of nodes in the initial graph  $G$  to which the current pair of nodes can be mapped back. The weight of the edge is  $w_{pq}^R = \sum_{i \in p} \sum_{j \in q} w_{ij}$  [8].
  - b. Repeat the previous operation for every merged node.
5. Identify subnetworks for coherent generator nodes in  $G^R$ , e.g. using a shortest path algorithm with subsequent tree trimming, see [3] for the details. Similar to Step 2, the resulting trees represent aggregated node groups which are associated with pairwise must-link constraints.
6. Create a final reduced graph  $G^F = (V^F, E^F, \omega^F)$  by repeating Steps 3 and 4 with  $G^R$  as the input graph and the coherent generator spanning trees as the aggregated ML node groups. The number of nodes in  $G^F$  is denoted as  $n_F$ .

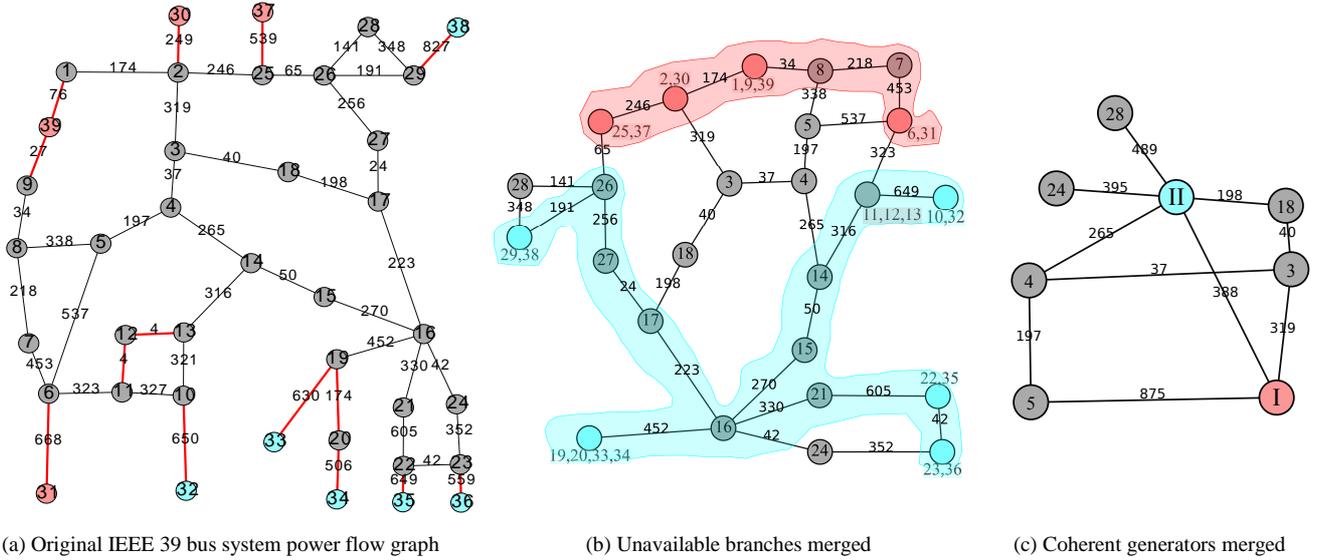


Figure 1. Graph reduction process of the IEEE 39 bus test system active power flow graph

The decision to first reduce unavailable branches is a simple means to ensure that there will be no such branch between any two coherent generator subnetworks at Step 5, which would lead to the merging of the two subnetworks into one. However, as two reduced graph representations are actually utilized due to this decision (i.e.  $G^R$  and  $G^F$ , see Fig. 1b – 1c), it is necessary to keep track of the coherency status of the merged nodes while producing the intermediate reduced graph  $G^R$ . In fact, [3] proposes some additional graph reduction techniques relevant for ICI (e.g. removal of closed loops), but only the two techniques essential for the ICI constraints integration are detailed here.

The idea to use the graph  $G^{pML}$  in Steps 1 and 2 stems from [8] and allows to merge the nodes connected by any kind of pairwise must-link constraints (e.g. unavailable branches) more efficiently.

The second enhancement of the graph reduction process is about how to assign the weights of the reduced graph based on the initial graph weights. It was demonstrated in [8] that the weights assignments according to Step 4 result in the equivalence of the initial and reduced problems in terms of graph cut.

The above algorithm is illustrated using the IEEE 39 bus network. Fig. 1a illustrates the initial active power flow graph. Edge labels represent edge weights (rounded to integers) which are the active power flows in MW. The unavailable branches are shown in red. In addition to all transformers, the lines 1-39 and 1-9 are shown as unavailable as their eventual disconnection would create an island only consisting of the largest generator in the network. This may also serve as an example of deliberate must-link constraints. For the purpose of illustration it is assumed that generator buses

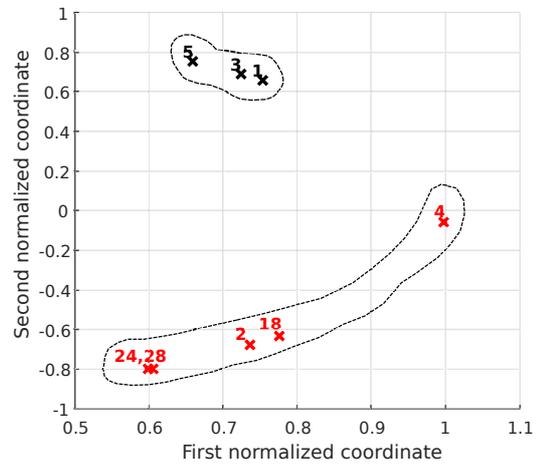


Figure 2. Spectral embedding of the reduced IEEE 39 bus test system power flow graph

{30, 31, 37, 39} and {32, 33, 34, 35, 36, 38} form two coherent groups.

After the execution of Steps 1-4, the reduced network graph  $G^R$  is as in Fig. 1b. The identified spanning trees for coherent generators (Step 5) are highlighted here as well. The final reduced graph  $G^F$  is shown in Fig. 1c.

### 3.2 Cannot-link constraints in spectral embedding

The spectral clustering algorithm described in Section 2.1 is used to calculate the normalized Laplacian of the reduced graph  $G^F$  according to (1) and to find the first 2 eigenvectors of  $\mathbf{L}_n$  (since it is desired to obtain two islands, equal to the number of coherent generator groups). The resulting spectral embedding is shown in Fig. 2 and it has two coordinates (equal to the number of used eigenvectors).

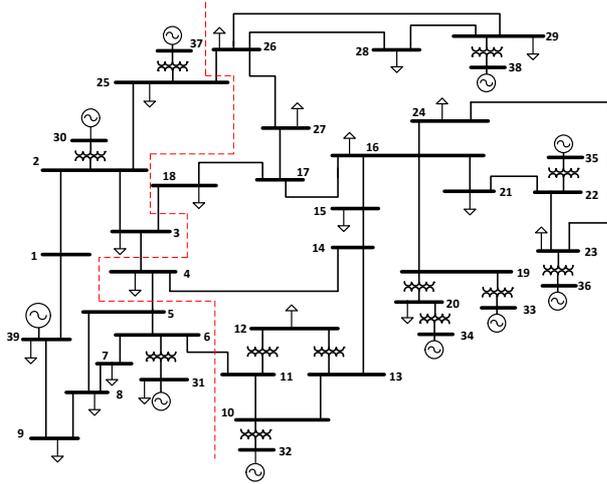


Figure 3. Final islanding cutset for the IEEE 39 bus test system

Cannot-link constraints between the coherent generator groups can be realized by selecting the last  $n_{coh}$  rows of the normalized eigenvector matrix  $\mathbf{Y}$  as centroids and assigning all other points in spectral embedding to the nearest centroid. The idea of using generator centroids with spectral clustering for ICI is proposed in [6], but in this paper the coherent generators are first grouped into  $n_{coh}$  merged nodes. Besides speeding up the eigendecomposition of  $\mathbf{L}_n$ , this results in a significantly lower number of centroids. Clustering around centroids is implemented by computing shortest path distances (e.g. with Dijkstra's algorithm) in the graph  $G^{SC}$  built based on  $G^F$  and the spectral embedding (see Section 2.1 and 3.1) from each centroid to the remaining  $n_F - n_{coh}$  points, which is computationally efficient even for large networks since the number of centroids is usually small.

After node clusters have been found in the reduced graph, the vertices of each cluster are mapped back to the vertices of the original graph, and the cutset is defined as the set of edges between the buses belonging to different clusters. The final islanding solution for the IEEE 39 bus network can be seen in Fig. 3.

## 4 Numerical results

The provided numerical results were obtained on MATLAB R2015a (64-bit) on a PC with an Intel® Xeon® E5 3.70 GHz CPU and 8 Gb of RAM on a single core.

The section aims to demonstrate the efficiency of the proposed cutset identification method for large power systems. Power network models from MATPOWER [13] were utilized for tests. Coherent generator groups were identified with the Power System Toolbox (PST) [15], for which purpose the static data from MATPOWER models was augmented with the dynamic generator data. The generator data was taken from [16] according to the nearest active power rating. This substitution usually results in reasonably looking coherent generator groups which is enough for the algorithm testing purposes.

A comparison of the proposed method with an existing spectral islanding controlled islanding (SCCI) method [5] has been performed for the IEEE 39 and 300 bus test networks ('case39' and 'case300' respectively) as well as for PEGASE test networks [14]. The results of tests are summarized in Table 1 and 2.

Table 1 ICI test results with the SCCI method [5]

Test system	$n_{coh}$	Run time, [ms]	Expansion ratios of islands	Total MW cut, [MW]
'case39'	2	17	[0.0039, 0.025]	90
'case39'	4	40	[0.011, 0.012, 0.025, 0.051]	246
'case89pegase'	2	21	[0.044, 0.045]	2062
'case89pegase'	3	34	[0.045, 0.064, 0.265]	3016
'case300'	2	41	[0.0017, 0.0076]	153
'case300'	3	54	[0.0054, 0.008, 0.063]	670
'case1354pegase'	2	218	[0.014, 0.034]	7607
'case2869pegase'	3	1231	[0.0052, 0.0134, 0.029]	5829
'case2869pegase'	4	1971	[0.0073, 0.013, 0.03, 0.202]	11909

Table 2 ICI test results with the proposed method

Test system	$n_{coh}$	Run time, [ms]	Expansion ratios of islands	Total MW cut, [MW]
'case39'	2	10	[0.0039, 0.025]	90
'case39'	4	14	[0.011, 0.013, 0.025, 0.052]	252
'case89pegase'	2	16	[0.056, 0.057]	1629
'case89pegase'	3	19	[0.058, 0.069, 0.25]	3648
'case300'	2	28	[0.0017, 0.0076]	153
'case300'	3	29	[0.0054, 0.0067, 0.048]	578
'case1354pegase'	2	325	[0.011, 0.029]	6080
'case2869pegase'	3	735	[0.0054, 0.015, 0.046]	6400
'case2869pegase'	4	745	[0.0083, 0.017, 0.049, 0.135]	9433

Run time is measured from the time instance when the adjacency matrix of the initial active power flow graph and the initial bus groupings are retrieved (e.g. from a SCADA system) to the time instance when the final cutset is returned. It was also assumed that each identified coherent generator group should remain in its own island, i.e. the number of islands for each test case is equal to the number of identified coherent generator groups.

As it may be observed, the proposed algorithm clearly outperforms the existing one in terms of execution time. The difference is especially pronounced for a larger number of coherent generator groups. The quality of the obtained cutset (both in terms of the total power flow disruption and the expansion ratios of islands) is comparable between the two methods. It should be noted here that the performance of the

proposed islanding method also depends on the kind of spanning tree building algorithm utilized in Step 5 in Section 3.1. An advanced spanning tree building algorithm could possibly help to reap more benefits of the multiway spectral graph partitioning, which (as it was mentioned in the introduction) should usually result in better graph cuts as compared to the sequential bipartitioning presented in [5].

## 5 Conclusion

This paper has proposed an improved computationally efficient algorithm for the controlled separation of large power networks based on graph reductions and spectral clustering which is intended for an offline planning environment. The algorithm has important improvements compared to the earlier references: high computational speed, demonstrated equivalence of the reduced problem to the initial one, possibility to incorporate important must-link and cannot-link constraints. Its effectiveness and computational performance have been demonstrated on MATPOWER test networks, including the two realistically-sized networks.

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