Quantification of Shading Tolerability for Photovoltaic Modules

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Abstract—Despite several decades of research in the field of photovoltaic (PV) systems, shading tolerance has still not been properly addressed. PV modules are influenced by shading concerning many factors, such as number and configuration of cells in the module, electrical and thermal characteristics of the cells, number and type of bypass circuits, electrical characteristics of bypass elements, and shading profile features. Along with the random nature of shading profile over the lifetime of a PV system, it is difficult to choose the best module for a location which is most of the time sunny, partly cloudy, or cloudy. This paper suggests a measurable parameter, the so-called Shading Tolerability (ST), to classify PV modules regarding the ability to oppose shading effects. Based on mathematical and probability analysis, the ST parameter is extracted and then measured using a Large Area Steady State Solar (LASSS) simulator. Finally, the results of on-field experiments are presented as a proof for shading quantification method and its significant contribution to Performance Ratio (PR) improvement.

Index Terms—Photovoltaic (PV) technology, partial shading, performance ratio (PR), maximum power point tracking (MPPT), bypass diode

I. INTRODUCTION

The annual growth for photovoltaic installation has been found to be a stunning rate of 44% in the years between 2000 and 2014 [1]. However, quality of the installed PV systems in terms of Performance Ratio (PR) can be further increased. In the 1990’s, typical PR of a PV system was about 70% while in 2010’s it has touched 90% [2]. The main reason preventing PR from reaching higher-than-90% values is that PV systems are normally designed and evaluated indoors whereas they should work outdoors for years. One of the difficult-to-predict outdoor circumstances is the partial shading, which is responsible of up to 25% PR and, depending on system design and equipment selection, of substantial output energy yield reduction [3-5].

Non-uniform irradiation on PV module surface means shading, which could cause (i) disproportional power loss [6], (ii) hotspot and thermal instability [7], (iii) module aging [8], and even (iv) overcurrent or nuisance trip [9, 10]. The tremendous increase of Building Integrated PV (BIPV) systems and solar roads [11, 12] makes it practically impossible to eliminate the source of static shading (side buildings, trees, etc.). Besides, there will always be dynamic shading (moving clouds, birds, etc.). The study of shading and its effects started in 1960’s and since then several shading tolerability approaches have been proposed by researchers [13-16]. Some of them are now being utilized in PV industry, such as silicon p-n and Schottky bypass diodes, cell integrated bypass diode, cool bypass switch, and IntegraBus technology [17-21]. The issue of shading tolerability of a photovoltaic system can be addressed at photovoltaic level and subsequently at power electronics level.

Cell-, module-, and array-based approaches are categorized in the photovoltaic level, as they aim to reduce negative effects of shading [22-24]. In other words, approaches at photovoltaic level try to harness the produced but unavailable power by providing alternative passes for the blocked current to flow at shading condition. Photovoltaic level approaches influence the current-voltage (I-V) curve of PV array. Then, power electronic converters should track the maximum available power [25]. This is normally done by maximum power point tracking (MPPT) techniques which could be module-based, string-based, or array-based [26]. Approaches aimed to improve hardware and algorithms of MPPT for fast, efficient and accurate MPP tracking are classified in the power electronics level [27, 28]. Photovoltaic and power electronics approaches work in series in a PV plant, as power electronic converters can only track the MPP provided by the approaches at photovoltaic level.

In this context, the proper selection of PV modules is of dominant importance in the PV system design. The right choice is made more challenging when the location of the installation is prone to shading. In modules datasheet, the ability of the modules to oppose shading effects is normally expressed qualitatively. General statements such as: better shading response [29], outstanding low light behavior [30], patented bypass circuit [31], excellent performance even when partially shaded [32], and shade tolerant [33] may not help the designer to select the most suitable module for a specific location. On the other hand, a quantified parameter, a number, which classifies PV modules in terms of shading tolerability, can be more meaningful. The establishment of such a parameter is the goal of this contribution.

The rest of the paper is organized as follows. In section II, theoretical framework is illustrated and mathematical study is performed to precisely formulate the concept of partial shading. Section III proves the mathematics in two measurement stages and confirms the correlation between the proposed Shading Tolerability (ST) parameter and the PR of a PV system. Last section provides an outlook for the usage of proposed ST method and summarizes the results of this paper.

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II. MATHEMATICAL MODELLING OF SHADING PROBLEM

Probability laws provide the proper tools for handling the design of systems that involve randomness [34]. Probability has many applications within electrical engineering (e.g., reliability and device failure rate, noise effect minimization, etc.). Besides, weather forecast is frequently presented in terms of probabilistic variables (e.g., a 30% chance, or probability, of rain). Therefore, shading on PV modules which involves both weather and electrical systems can be seen and studied as a random process in the mathematical framework of probability laws.

A. Sample space development for PV module shading trial

In probability theory, the sample space of a random trial is the collection of all possible events [34]. For PV module shading trial, a major obstacle is the infinite possible shading profiles\(^1\) resulting in an infinitely large sample space. To simplify our shading trial, two sensible assumptions are adopted for a PV module:

1) On the surface of a PV cell (encapsulated in a module), irradiation is homogenous and can have any value between 0 and 1 kW/m\(^2\), and all values of irradiation have an equal chance to occur.

\(\delta s\) is the difference value between two consecutive irradiation levels. Since the sample module in the figure has 12 cells, then \(\epsilon = 12\). The outer layer shows a PV module containing 12 cells with possible shading profiles. Since \(\delta s \to 0\), then there will be infinite possible shading profile for the module. Each shading profile has the \(\lim_{i \to \infty} (1/i^c)\) chance to occur.

2) The chance of shading for different cells of a module is equal and independent from their location in the module or in the array where their module is mounted.

Thus, for a PV module with \(c\) cells and \(i\) possible irradiation levels, the total number of possible shading profiles is equal to \(i^c\). Since there are infinite numbers of irradiation levels between 0 to 1 kW/m\(^2\), each unique shading profile has the \(\lim_{i \to \infty} (1/i^c)\) occurring possibility. Fig. 1 shows a circular-tree diagram as graphical illustration of the sample space for shading probability trial. Although the aforementioned assumptions do not reduce the possible number of shading profiles to a finite value, the sample space is now carefully determined\(^2\).

B. Mathematical expectation of power production at shading

According to probability theory, decision making strictly concerns with mathematical expectation\(^3\) [35]. For a PV module, higher mathematical expectation of power production at shading, or higher shading tolerance, persuades designers to select that module for, e.g., a cloudy location. Thus, a

\(^1\) Each unique shadow profile makes unique influence on PV module electrical characteristics (I-V).

\(^2\) The assumptions ignore irradiance levels above 1 kW/m\(^2\). This will not affect the model and makes the formulation more understandable.

\(^3\) Mathematical expectation, also known as the expected value or expectation is the integration of possible values from a random variable. In other words, it is the product of the probability of an event occurring and the value corresponding with the actual observed occurrence of the event.
mathematical expectation value for PV modules is developed in this paper as a benchmark to rate, compare, and select the best module for a specific installation location in terms of cloudiness/shading.

Expected value of a random variable $x$ with the occurring chance of $p(x)$ is obtained by [35]:

$$E(x)=\sum_{k=1}^{\infty} x_k p(x_k)$$

(1)

Using (1) the Shading Tolerability of a PV module is defined as:

$$ST_{(i,c)}=\frac{1}{P_{mod\_mpp}} \sum_{k=1}^{n-b} P_k \left( \frac{1}{i} \right)$$

(2)

where $ST_{(i,c)}$ stands for shading tolerability, $c$ and $i$ are the total number of PV cells (within the module) and irradiation levels, respectively. $P_k$ corresponds to the MPP at each shading profile (in W), while $P_{mod\_mpp}$ is the maximum power of PV module (in W). $P_{mod\_mpp}$ normalizes the value of mathematical expectation and makes it possible to compare PV modules with different rated powers. So far, the PV module which gains higher value from equation (2), acts better at shading. However, the value of equation (2) is not measurable experimentally, because of the infinite possible irradiation levels between 0 and 1 kW/m$^2$.

C. How to make ST practically measurable

Although equation (2) is not practically measurable for $i \rightarrow \infty$, it is indeed measurable for $i = 2$. If we prove that the module which provides higher $ST$ at $i = 2$ it will also give higher $ST$ at $i \rightarrow \infty$, then $ST_{(i,c)}$ can be measured instead of $ST_{(i\rightarrow\infty,c)}$ as a standard for PV module’s ability to withstand shading. Fig. 2 illustrates the probability distribution $p(s)$ of irradiance levels ($s$) from discrete binary distribution ($i = 2$) to uniform continuous distribution ($i \rightarrow \infty$).

![Fig. 2. Probability distributions $p(s)$ of PV module shading problem for different number of irradiation levels. $\delta s$ is the difference value between two consecutive irradiation levels.](image)

To find a general equation for $ST_{(i,c)}$, one can obtain $ST_{(i=2,c)}$ (irradiance level is either 0 or 1 kW/m$^2$), $ST_{(i=3,c)}$ (irradiance level is either 0, 0.5, or 1 kW/m$^2$), and continue this procedure to obtain the equation for $ST_{(i,c)}$. By means of mathematical permutation, the general equation for shading tolerability of PV modules is as follows:

$$ST_{(i,c)} = \left( \frac{m}{c} \right) \left( \frac{1}{i} \right) \left[ \sum_{k=1}^{n-b} P_k \left( \frac{1}{j} \right) \right] \left[ \sum_{j=1}^{n} \frac{1}{j} \sum_{b=1}^{b} \left( \frac{a+b}{b} \right)^{n-b} \right]$$

(3)

where $n$ is the number of series-connected PV cells, $m$ is the number of PV cell strings in a module ($c = n \times m$), and $j = i - 1$. In equation (3), where its mathematical demonstration can be found in Appendix A, the first series term corresponds to the shading profiles in which all cells receive the same amount of irradiation, while the second term stands for the shading profiles with non-uniform irradiation. Equation (3) shows that the shading tolerability of a PV module is independent from the number of PV cell strings (i.e. independent from $m$). In fact, using point-wise numerical calculation method [36], one can demonstrate that as $i \rightarrow \infty$, equation (3) converges to $1 / (n+1)$. Remarkably, this means that the shading tolerability of a PV module is inversely proportional to the factor of $(n+1)$. Such result can also be extended to array level, $ST_{array} = ST_{Module} / (q+1)$, where $q$ indicates the number of series connected PV modules ($q > 1$) in an array (see Appendix B). For instance, a PV array formed by 8 x 3 PV modules is 28.6% more vulnerable to shading than a 6 x 4 PV array configured with the same PV modules ($8 + 1 / (6 + 1) = 1.286$).

$ST_{(i=2,c)}$ is a special case of equation (3). Simply, by substituting $i = 2$ in equation (3):

$$ST_{(i=2,c)} = \left( \frac{1}{2^{n-1}} \right)$$

(4)

Considering both equations (3) and (4), it is easy to comprehend that when $ST_{(i=2,c)}$ is higher for module$1$ than for module$2$, then $n_1 < n_2$ which results in higher $ST_{(i\rightarrow\infty,c)}$ for module$1$ than module$2$. In other words:

$$ST_{(i=2,c)}^{(module_1)} > ST_{(i=2,c)}^{(module_2)} \Rightarrow ST_{(i\rightarrow\infty,c)}^{(module_1)} > ST_{(i\rightarrow\infty,c)}^{(module_2)}$$

(5)

Equation (5) shows that $ST$ can be measured for $i = 2$ and used instead of $ST$ for $i \rightarrow \infty$. However, equation (3) may not fully guarantee that testing PV modules for $i = 2$ condition in laboratory can stand for shading tolerability for $i \rightarrow \infty$ in real outdoor circumstances. The reason is, different modules come with different approaches to oppose shading (such as number and type of bypass circuits) while equation (3) only considers PV cells and their series-parallel configuration in a module. All the approaches which contribute to shading tolerability enhance the value of $ST$. Therefore, a coefficient, $\lambda_{(i,c)}$, is defined for equation (3) to model the facilities that the manufacturer has used to make the module more tolerable to shade. Since $\lambda$ may vary by number of cells in a module and number of possible irradiation levels on the surface of a cell, it is defined as function of $i$ and $c$. By considering this coefficient, the final general equation for shading tolerability of a PV module is written as follows:

$$ST_{(i\rightarrow\infty,c)} = \lambda_{(i\rightarrow\infty,c)} \left( \frac{1}{n+1} \right)$$

(6)

where $\lambda$ depends on the PV module’s design and manufacturing. Obtaining a general equation for $\lambda_{(i,c)}$ is
difficult because each module has its own way to oppose shading effects and an approach’s effectiveness may vary by irradiation distribution and number of cells. However, it is possible to find the boundaries of $\lambda_{i(c)}$. Its minimum value is 1, meaning that the adopted shading tolerability approach has no influence on the PV module performance. The maximum of $\lambda_{i(c)}$ means that the shaded cells in a module have no effects on the performance of sunny cells. Simply stated, the cells can produce energy independently. Now, the maximum $ST_{(i \rightarrow \infty)}$ for a single cell is equal to 1/2 because average irradiation on a cell is 0.5 kW/m² (at any uniform probability distribution depicted in Fig. 2). Hence, for a PV module in which solar cells work independently, the maximum $ST_{(i \rightarrow \infty)}$ is also equal to 1/2. For example, when probability distribution of irradiation matches the subplot $i=3$ in Fig. 2, for a single PV cell the maximum $ST$ is equal to: $ST_{(i=3,c=1,2)} = (1/P_{cell}) \times (1/3^2) \times (0+0.5+1) \times P_{cell} = 0.5$, and for a module with two PV cells the maximum $ST$ is also equal to: $ST_{(i=3,c=2,3)} = (1/(2 \times P_{cell})) \times (1/3^2) \times (0+0.5+1+1+1.5+1.5+2) \times P_{cell} = 0.5$. By substituting $ST_{(i \rightarrow \infty)} = 1/2$ in equation (6), the boundaries of $\lambda_{i(c)}$ are obtained as:

$$1 \leq \lambda_{(i \rightarrow \infty)} = \frac{n+1}{2} . \quad (7)$$

When the function of $\lambda_{i(c)}$ is determined, then it is possible to mathematically investigate whether it is correct to measure $ST_{(i=2,c)}$ instead of $ST_{(i \rightarrow \infty)}$ for all type of modules or not. Since there is no immediate way to model $\lambda_{i(c)}$ mathematically for different commercial PV modules, this paper investigates the correctness of equation (5) through experiments in the next section.

III. EXPERIMENTAL WORK

The experiments measure the value of $ST_{(i=2,c)}$ for various commercial PV modules through indoor tests. After extracting the quantified value of shading tolerability for each module, some of them are chosen to be tested under real outdoor condition (as a circumstance in which $i \rightarrow \infty$). Afterwards, based on gathered experimental data, correctness of equation (5) is investigated.

A. Indoor Measurements

To cover a wide range of PV markets, various PV modules with different technologies, number of cells and bypass techniques, were selected. In the experiments, $c$ is six for all modules. It means that the active area of each PV module has been divided into six parts, proportional to the size of that PV module. The reason for selection of $c = 6$ is that for higher values of $c$, the number of required tests (and subsequently required measurement time and energy) for each single module increases exponentially (number of tests = $2^c$) and reduces the chance of industrial application of $ST$. Besides, six is an even number, which makes it easy to divide module’s length and width into three times two sections. Note that although PV modules come in a variety of shapes, the most common is the rectangular one [37]. All $2^c = 64$ shading profiles, as shown in Fig. 3, have been applied to each selected module and I-V characteristics of the modules have been measured for each case using an EternalSun Large Area Steady State Solar (LASSS) AAA-class simulator. Every single test has been performed at 1 kW/m², AM 1.5, and 25 °C. It is worth pointing out that 25 °C is the imposed ambient temperature instead of the module temperature. The reason is

![Image](image-url)
that shading causes hotspot and power dissipation in the module, resulting in temperature rise. In such a condition, some modules perform better some worse. Therefore, in this special test, keeping the modules temperature fixed would cause inaccuracy. Moreover, because of various shading profiles, the temperature varies significantly within the area of the module which makes it difficult to keep the temperature of whole module stable and uniform.

Shading profiles are coded in a binary format as a representative for applied discrete irradiation. Since shadows in real condition are not perfectly dark (caused by diffuse irradiation), a dark material which passes 1/4 of the received irradiation (250 W/m²) has been chosen as shading object. It is worth mentioning that total indoor measurement time for all eleven modules was about 63.11 hours, average of 5.73 hours for each module (including data saving and exportation).

Datasheet information of the tested PV modules together with corresponding obtained STs and %STs are given in Table I. To obtain %ST, defined as the percentage value of ST, measured ST from equation (2) were divided by the

<table>
<thead>
<tr>
<th>Company/Commercial Name</th>
<th>Technology</th>
<th>Electrical specification</th>
<th>Mechanical size</th>
<th>Weight</th>
<th>Notes on module’s datasheet regarding shading tolerance</th>
<th>Measured ST(♯)</th>
<th>Percentage value of ST(♯)</th>
<th>Suggested Shading Class Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Neste/ Module PV A12</td>
<td>a-Si</td>
<td>Voc=22 V</td>
<td>MPP=7.5 W</td>
<td>3.0 kg</td>
<td>None</td>
<td>0.36</td>
<td>58%</td>
<td>(Flexible)</td>
</tr>
<tr>
<td>2 Victron Energy/ SPM30-12</td>
<td>Mono c-Si</td>
<td>Voc=22.5 V</td>
<td>MPP=30 W</td>
<td>2.5 kg</td>
<td>None</td>
<td>0.24</td>
<td>38%</td>
<td>None</td>
</tr>
<tr>
<td>3 Wurth Solar/ GeneCIS module 80W</td>
<td>CIS</td>
<td>Voc=44 V</td>
<td>MPP=200 W</td>
<td>12.7 kg</td>
<td>None</td>
<td>0.57</td>
<td>91%</td>
<td>(Flexible)</td>
</tr>
<tr>
<td>4 Schueten/ Multisol P6-54 series 200</td>
<td>Poly c-Si</td>
<td>Voc=33 V</td>
<td>MPP=20 W</td>
<td>20.0 kg</td>
<td>None</td>
<td>0.22</td>
<td>35%</td>
<td>None</td>
</tr>
<tr>
<td>5 Calyxo/ CX3-77 Thin film solar module</td>
<td>CdTe/CdS</td>
<td>Voc=62.5 V</td>
<td>MPP=77.5 W</td>
<td>12.0 kg</td>
<td>None</td>
<td>0.39</td>
<td>63%</td>
<td>None</td>
</tr>
<tr>
<td>6 SunPower/ SPR X20 327-BLK</td>
<td>Mono c-Si</td>
<td>Voc=67.6 V</td>
<td>MPP=327 W</td>
<td>18.6 kg</td>
<td>None</td>
<td>0.21</td>
<td>33%</td>
<td>None</td>
</tr>
<tr>
<td>7 Masdar PV/ MPV-T</td>
<td>Tandem a-Si/a-Si</td>
<td>Voc=137.54 V</td>
<td>MPP=109.81 W</td>
<td>29.5 kg</td>
<td>None</td>
<td>0.25</td>
<td>40%</td>
<td>None</td>
</tr>
<tr>
<td>8 IKS Photovoltaik/ STA14 SolarTrainer 10W module</td>
<td>Poly c-Si</td>
<td>Voc=22 V</td>
<td>MPP=10 W</td>
<td>Not specified</td>
<td>None</td>
<td>0.25</td>
<td>40%</td>
<td>None</td>
</tr>
<tr>
<td>9 Solland/ SunWeb module- 235 W</td>
<td>Poly c-Si</td>
<td>Voc=36.97 V</td>
<td>MPP=235 W</td>
<td>22 kg</td>
<td>None</td>
<td>0.24</td>
<td>39%</td>
<td>None</td>
</tr>
<tr>
<td>10 Hanergy/ PowerFlex 90W</td>
<td>CIGS</td>
<td>Voc=22 V</td>
<td>MPP=27.4 W</td>
<td>3.3 kg</td>
<td>Shade tolerant</td>
<td>0.31</td>
<td>50%</td>
<td>(Flexible)</td>
</tr>
<tr>
<td>11 Uni-Solar/ PowerBond ePVL</td>
<td>Multi-junction a-Si</td>
<td>Voc=10.44 V</td>
<td>MPP=7.8 W</td>
<td>1.8kg</td>
<td>Excellent performance even when partially shaded</td>
<td>0.37</td>
<td>59%</td>
<td>(Flexible)</td>
</tr>
</tbody>
</table>

(♯) ST and %ST values are rounded to the closest integer.
maximum theoretical value of $ST$. Note that, since shading objects pass 1/4 of the received irradiation, in this case the maximum theoretical value of $ST$ is 0.625 instead of 0.5. Inspired by meteorology [38], Table I suggests three shading tolerability classes/symbols for PV modules: sunny ($\%ST < 50\%$), partly-cloudy ($50\% \leq \%ST < 80\%$), and cloudy ($80\% \leq \%ST$). The boundaries, 50% and 80%, are selected based on Linear Support Vector Machine (LSVM) algorithm, which maximizes the distance between boundaries and closest data [39]. Fig. 4 shows the $\%ST$ data set together with calculated hyperplanes and proposed boundaries. Since industry needs straightforward and effective boundaries, the calculated hyperplanes have been displaced a little to obtain proposed boundaries.

\[
PR = \frac{\sum_{t} P_{out}}{\sum_{t} G_{module}}
\]

where $G_{a} = 1\ kW/m^{2}$ is the reference irradiance and $G_{module}$ is the in-plane irradiance received at module surface (in W/m²). $P_{0}$ is nominal watt-peak ($W_{p}$) on the datasheet of the PV module and $P_{out}$ is the in-field output power of the PV module (in W). Note that, same output cables with negligible ohmic resistance were used for all modules. Also, power electronic interface was removed to eliminate the influence of converters efficiency on PR values. Therefore, only modules performance is compared.

<table>
<thead>
<tr>
<th>Sunny</th>
<th>Partly-Cloudy</th>
<th>Cloudy</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>50°</td>
<td>90°</td>
</tr>
<tr>
<td>0°</td>
<td>45°</td>
<td>100°</td>
</tr>
<tr>
<td>Suggested Boundary</td>
<td>45%</td>
<td>50%</td>
</tr>
<tr>
<td>ST (%)</td>
<td>33%</td>
<td>50%</td>
</tr>
</tbody>
</table>

Fig. 4. Proposed shading classification boundaries for PV modules

Table I reveals that modules #3 and #5, which consist of long and narrow cells, perform better at $ST$ measurement than modules #10 and #11, despite using one bypass diode per cell. Therefore, the number of bypass diodes in a module is not always a valid benchmark for shading tolerability comparison. If modules which have performed better at indoor $ST$ test keep on providing higher output at on-field outdoor tests, then it would be rational to say that the $ST$ parameter and its measurement procedure are valid.

B. Outdoor Measurements

For outdoor tests, three types of PV modules (two identical modules of each type, for a total of six modules) were selected from different shading tolerability classes (PV modules #3, #6, and #10 from Table I). Modules were separated into two identical groups and placed at two locations on the same roof, as illustrated in Fig. 5. To obtain more valid results, both groups should experience similar circumstances, except the shading condition. Hence, the two groups of modules were installed on two locations as close as possible while one location is mostly sunny during day-time and the other one is frequently shaded by side trees, a chimney, and a fence. For all modules, tilt and azimuth angles were selected to be 0° and 100° (for easy installation and safety reasons), respectively. Since the aim of this test is performance comparison of PV modules, there is no need for title and azimuth optimization.

For twelve days, the electrical output characteristics of all PV modules, irradiation, ambient temperature, and wind speed were measured from 8.00 to 17.00 using a portable 1-V curve tracer (at predetermined time intervals). Position of the modules were exchanged within each same group every day, therefore all the three modules in shading group experienced similar random shading scenarios.

Performance ratio, as a figure of merit for PV system comparison [40], is calculated using the following equation:

![Fig. 5. Outdoor measurements location: installation location of two groups of PV modules (blue rectangles). Black x indicates the location in which outdoor experiments took place. The orange curve is the sun trajectory in June 2016, and the yellow area around (between black and orange curves) is the variation of sun trajectories during the year. According to sun path during 8.00 to 17.00, PV modules in group 2 experience shading most of the time while group 1 modules are exposed to sun. Average local times of sunset (22.00), sunrise (05.00), and solar noon (13:45) during June 2016 are also depicted to provide better prospective of the sun position during outdoor measurements.](image-url)
more energy. While the PR of modules at sunny location are relatively close to each other, there is a huge gap between PR values at shading location. Thus, as the PR difference between best and worst PV modules at shading is more than 20% (93.75-73.31), improper selection of PV modules may lead to considerable yield reduction of the PV system (see Appendix B for an example of ST application in PV system design).

IV. CONCLUSION

This paper has introduced the shading tolerability, ST, as a measurable parameter to accurately classify the capability of a PV module to withstand shading. It was mathematically proven that the shading tolerability of a PV module can be modelled by the function of $\lambda/(n+1)$. Experimental results showed that accurate selection of PV modules (based on ST), can boost the performance ratio of a PV system by over 20 percentage points. For each tested PV module, shading tolerability was determined in less than 6 hours. Consequently, it is industrially feasible to perform ST test on a single or couple of modules which are randomly selected from an identical group of modules. In this way, for a small amount of energy consumed within six hours, a huge extra energy will be extracted from the sun during the PV system lifetime by selecting correct PV modules. Therefore, it is suggested to add ST on photovoltaic modules datasheet as a benchmark to distinguish PV modules regarding shading tolerability.

Measured data has also ignited the idea of a possible linear correlation between ST and PR. If such a formula is found, then it is even possible to accurately calculate the output energy of a PV system which is exposed to random shading profiles. This is one of our future research goals together with mathematical modelling of $\lambda$. Extracting the mathematical function of $\lambda$ helps to comprehend how each physical feature of a PV module contributes to the module’s performance at shading.
APPENDIX

A. Demonstration of the general equation for shading tolerability of PV modules, \( (ST_{(i,c)}) \)

An ideal PV cell can be modelled by a current source with an anti-parallel diode. Output power of the PV cell at a constant temperature is almost linearly proportional to received irradiation and each cell provides \( P_{cell} \) watts at 1 kW/m\(^2\). Consider a hypothetical PV module consisting of 2 series-connected solar cells. Assume that irradiation has only two possible values at each PV cell’s surface, either 0 or 1 kW/m\(^2\) (uniform binary distribution, as depicted in Fig. 2). Then, there would be 4 working conditions in which the output power of the module is equal to \(2 \times P_{cell}, 0 \times P_{cell}, 0 \times P_{cell}, \) and \( 0 \times P_{cell} \) (since cells are modelled as ideal current source, the power of the module is determined by the power of the cell which receives the lowest amount of irradiation). Therefore, shading tolerability value is equal to \( ST_{(i=2,c=2)} = (1/2^3)(2+0+0+0) \times P_{cell} \). Fig. A.1 (a) shows the four working conditions for hypothetical 2-cell PV module with two irradiation levels. Further, keeping the number of cells to 2 but increasing the possible irradiation levels to three (0, 0.5, and 1 kW/m\(^2\)), \( ST_{(i=3,c=2)} = (1/3^3)(2+1+1+0+0+0+0) \times P_{cell} \). (See Fig. A.1(b)). By following this pattern, increasing the possible irradiation levels and keeping the number of cells constant, the formula of shading tolerability \( ST \) for a module with 2 series-connected cells is obtained as:

\[
ST_{(i,c=2)} = \frac{1}{2 \times P_{cell}} \left( \frac{1}{1^3} \right) P_{cell} \left[ \sum_{k=1}^{k=1} \left( \frac{2}{j} \right) \right] + \sum_{a=1}^{a=1} \left( \frac{3}{j} \right) \sum_{a=1}^{a=1} \left( \frac{2}{j} \right) \left( 2a \right)
\]

\[\text{(A.1)}\]

where \( c \) is the total number of PV cells in the module and \( j = i-1 \).

![Graphical demonstration of ST formulation procedure: (a) PV module with two series-connected cells and two possible irradiation levels (0 and 1 kW/m\(^2\)) which results in total 2\(^2\) working conditions, (b) PV module with two series-connected cells and three possible irradiation levels (0, 0.5, and 1 kW/m\(^2\)) which results in total 3\(^2\) working conditions. Output power of PV modules at each working condition is also depicted.]

Now, one can do the same procedure for a module with 3 series-connected cells and obtain the following formula:

\[
ST_{(i,c=3)} = \frac{1}{3 \times P_{cell}} \left( \frac{1}{1^3} \right) P_{cell} \left[ \sum_{k=1}^{k=1} \left( \frac{3}{j} \right) \right] + \sum_{a=1}^{a=1} \left( \frac{3}{j} \right) \sum_{a=1}^{a=1} \left( \frac{3}{j} \right) \left( 3a + 3a^2 \right)
\]

\[\text{(A.2)}\]

Or extend it further to a module with 4 series-connected cells:

\[
ST_{(i,c=4)} = \left( \frac{1}{4 \times P_{cell}} \right) \left( \frac{1}{1^4} \right) P_{cell} \left[ \sum_{k=1}^{k=1} \left( \frac{4}{j} \right) \right] + \sum_{a=1}^{a=1} \left( \frac{4}{j} \right) \sum_{a=1}^{a=1} \left( \frac{4}{j} \right) \left( 4a^3 + 6a^2 + 4a \right)
\]

\[\text{(A.3)}\]

Considering equations (A.1), (A.2), and (A.3), it is possible to come up with a general equation in which the number of cells is also a parameter:

\[
ST_{(i,c)} = \left( \frac{1}{n} \right) \left( \frac{1}{1^n} \right) P_{cell} \left[ \sum_{k=1}^{k=1} \left( \frac{n}{j} \right) \right] + \sum_{a=1}^{a=1} \left( \frac{n}{j} \right) \sum_{a=1}^{a=1} \left( \frac{n}{j} \right) \left( n^a b^a \right)
\]

\[\text{(A.4)}\]

As it can be seen, the term \( P_{cell} \) has been removed from both numerator and denominator of equation (A.4). Keep in mind that in equation (A.4), \( n \) is the number of series-connected solar cells and the PV module has only one string of series connected cells. To expand the formula for a PV module with more than one string of cells, we can consider \( m \) as the number of parallel strings (each string consists of \( n \times m \) identical probability trials (\( m \) as the number of PV cell strings) is equal to \( m \) times the expected value of each trial (each string), then the general normalized shading tolerability equation is obtained as:

\[
ST_{(i,c)} = \left( \frac{m}{n \times m} \right) \left( \frac{1}{1^n} \right) P_{cell} \left[ \sum_{k=1}^{k=1} \left( \frac{n}{j} \right) \right] + \sum_{a=1}^{a=1} \left( \frac{n}{j} \right) \sum_{a=1}^{a=1} \left( \frac{n}{j} \right) \left( n^a b^a \right)
\]

\[\text{(A.5)}\]

B. \( ST \) application in PV system calculation and design

In order to show how the \( ST \) number can help designers to select proper PV module type for a certain PV system, consider the following example: A local load of 3 kW requires a PV system to be installed in an area with occasional shades. A suitable PV inverter (2-string, 3 kW, 600 V, 5 A) is chosen for the system. There are three options for PV modules with the same efficiency and price:

- Module #1: 100 W, 40 V, 2.5 A, \( ST=65\% \rightarrow \text{Array #1: } 15 \times 2 \)
- Module #2: 125 W, 50 V, 2.5 A, \( ST=55\% \rightarrow \text{Array #2: } 12 \times 2 \)
- Module #3: 150 W, 60 V, 2.5 A, \( ST=45\% \rightarrow \text{Array #3: } 10 \times 2 \)

According to the obtained results in the paper, the PV array which provides higher value of \( ST_{array} = ST_{module} / (q+1) \) possibly produces more energy per \( W_{p} \) during its lifetime (assuming that all arrays have the same lifetime). Then:

- Array #1: \( 65 / (15+1) = 4.06 \)
- Array #2: \( 55 / (12+1) = 4.23 \)
- Array #3: \( 45 / (10+1) = 4.09 \)

Therefore, module #2 should be chosen for the PV system.

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Results presented in this work strictly concern the individual photovoltaic modules available and tested in the PV Laboratory of the PVMD group of TU Delft. The performance of such modules might not reflect that of similar or updated modules from the same brand and/or under different circumstances.

REFERENCES


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