The Observable Field for Antennas in Reception

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The Observable Field for Antennas in Reception

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Abstract— **In this paper the portion of the incident field that can be received by an antenna is investigated: *the observable field*. This field can be estimated relying only on the volume allocated to the antenna and thus independently from the specific antenna geometry. The observable field is composed by a single spherical wave that first converges into the origin and then diverges to infinity. The power associated to the converging wave is the power available to an antenna located within the defined volume. Previously, an estimation of this observable spherical wave was obtained by truncating the spectral spherical modal series representation of the incident field. Here, instead, we provide a more applicable approximation of the observable field, by truncating a spatial integral representation of the incident field that is based on the use of equivalent *ideal* currents. Eventually, for the vast majority of antennas, the estimation of the available power that can be obtained by approximating the observable field via the ideal currents is more accurate than the estimation that would be obtained via the spectral modal expansion. Moreover analytical expressions for the observable field are provided here. The ideas are set here considering the case of single plane wave incidence, but the extension to multiple plane waves is straightforward.**

Index Terms—**Antenna theory, receiving antennas, equivalence theorem, spherical modes**

I. INTRODUCTION

Despite the fact that, thanks to reciprocity, antennas in reception can be analyzed resorting to techniques developed for analyzing antennas in transmission, the scattering and absorption properties of antennas in reception have triggered dedicated attention in the recent literature [1]-[11]. More specifically, starting in the last decade or so, the study of the absorption efficiency of antennas has been exploited in [4]-[8], to derive guidelines for the numerical optimization of the currents on the optimal antennas, and also the main physical bounds for the gain and bandwidth of general antennas. Despite these advances, there are still aspects of antennas in reception that are not well understood. One that seems particularly important is related to the quantification of the power available to loads connected to antennas in reception. To this regard, less than ten years ago, Kwon and Pozar, in the corner stone paper [9], introduced the clarifying concept of available power for antennas localized in a given volume. This power represents the maximum power that an ideal (lossless) antenna can extract from the incident field. Specifically [9] used a field representation in terms of TE and TM spherical vector modes [12] to represent the incident field and the perturbation due to the antenna. Exploiting this representation, a procedure to evaluate the power available to a matched load connected to an ideal antenna was proposed. In [9] it was also shown that only a finite number of spherical waves should be used to evaluate the *available power*. This number would depend only on the radius of the sphere enclosing the volume allocated to the antenna. By identifying a truncation for the spectral spherical mode series of the incident field, [9] implicitly suggested the selection of those components (observable) of the incident field that can be received by an antenna located within a given spherical volume. This observable portion of the incident field was associated to the *N lower order modes* of the series representation. The suggestion in [9] was, however, limited to the use of the spherical mode representation of the incident field and, therefore, the results for the available power were quantized. Especially for spherical domains of radius in the order of a fraction of a wavelength, the available power is inadequately approximated when compared with experiments [10]-[11]. In the recent decades, the engineering branch of the antenna community has responded to this difficulty heuristically: the available power predicted by spherical modes for a single plane wave incidence has been interpolated to render it a continue function of the volume of the antenna [10], rather than a quantized function. Nevertheless, there is no proof (nor indication, actually) that the heuristic interpolation [10] constitutes a hard limit for the available power. Moreover, since the heuristic extension is based on an interpolation of the power associated to a single plane wave, this method cannot be applied for incident fields comprising the superposition of several plane waves. More recently, efforts to find more rigorous procedures to estimate the required number of modes are being investigated. In [11], a continuous expression for the available power was derived by introducing a weighting factor in the number of spherical modes, that is inversely proportional to the radiation Q of the individual modes. The curve presented in Fig.2 of [11] is slightly lower than the heuristic approximation in the region of moderate sized antennas.

In this paper we provide an alternative representation of the observable field that is more applicable for antenna problems than the one proposed in [9] and more intuitive than the one in [11]. The procedure is obtained by truncating a spatial integral representation of the incident field obtained by identifying a set of equivalent *ideal antenna* currents defined only on the physical domain of the antenna. Using these ideal antenna currents, we can provide an analytical expression of the observable field with an estimation of the available power without incurring in obvious quantization errors related to the physical domain of the antenna intrinsic to the spherical mode expansion. It will be shown that, when the ideal antenna current methodology is applied to estimate the maximum effective area of antennas for a given volume, the predicted results converge to the values provided by the procedure in [9] for very electrically large or very small antennas. However, for moderate sized antennas the results are closer to the ones proposed in [11] and appear to constitute an upper limit in comparison with the experimental data available in the open literature also provided in [11]. The limit to the antenna gain that emerges by using the present version of the ideal antenna currents procedure is of practical nature and not a hard theoretical limit. For instance, super directive array designs could potentially beat these limits, if realized with extremely low loss materials. The key advantage of the present formulation with respect to those in [10] and [11] is that the available power can be rigorously but, simply estimated also for a coherent superposition of plane waves.

The paper is structured as follows. In section II the main characteristics of the observable field, independently on how is derived, are described. Section III summarizes the spherical mode representation of the incident fields proposed in [9] and highlights the properties of the spherical mode representation of the observable field, which was contained implicitly in [9]. In Section IV the ideal equivalent currents method to estimate the observable field is described, and its range of usability in the case of plane wave incidence are discussed in Section V. Finally, the conclusions are discussed in Section VI

II.The Observable Field

Expressing the incident field as the superposition of an *observable* field and a remaining component has a wide applicability in the design of antennas in reception, independently from the specific field representation that is adopted to derive it, i.e. spectral (modal or integral), or spatial (integral or differential). Thus, the first suggestion of this paper is to systematically represent the incident field as superposition of an *observable* field component and a remaining component:

$$\vec{e}_{inc}(\vec{r}) = \vec{e}_{obs}(\vec{r}) + \vec{e}_{rem}(\vec{r}). \tag{1}$$

Fig.1 shows the problem of antennas in reception, highlighting the allocated antenna domain (taken as a spherical in this paper for comparison with a spherical mode representation), the source region (points far away from the antenna, $\vec{r} = \vec{r}_\infty$), the observable and remaining fields. As it will be clear after the following sections, both [9] and this paper propose to express the observable field in the source region as the superposition of an inward and an outward propagating spherical wave:

$$\vec{e}_{obs}(\vec{r}_\infty)=\vec{e}_{obs}^{\,inw}(\vec{r}_\infty) + \vec{e}_{obs}^{\,outw}(\vec{r}_\infty) \tag{2}$$

with

$$\vec{e}_{obs}^{\,(inw/outw)}(\vec{r}_\infty) = \vec{V}_{obs}^{(inw/outw)}(\theta,\phi)\frac{e^{(+/-)jkr_\infty}}{r_\infty} \tag{3}$$

where $\vec{V}_{obs}^{(inw/outw)}(\theta,\phi)$ represents the angular distribution of the (inward/outward) observable spherical wave. Alternative field representations will then differ in how they approximate the functions $\vec{V}_{obs}^{(inw/outw)}$.

The relationship between inward and outward spherical waves depends on the polarization, with fields proportional to the local unit vectors $\hat{\theta} = \hat{\theta}(\theta,\phi)$ and $\hat{\phi} = \hat{\phi}(\theta,\phi)$. Specifically, having set $\vec{V}_{obs-TM}^{outw}(\theta,\phi) = V_{obs-TM}^{outw}(\theta,\phi)\hat{\theta}$ and $\vec{V}_{obs-TE}^{outw}(\theta,\phi) = V_{obs-TE}^{outw}(\theta,\phi)\hat{\phi}$, in congruence with spherical modes, the converging and diverging field components are related by

$$V_{obs-TM}^{outw}(\theta,\phi)\hat{\theta} = V_{obs-TM}^{inw}(\pi-\theta,\pi+\phi)\,\hat{\theta} \tag{4}$$

$$V_{obs-TE}^{outw}(\theta,\phi)\hat{\phi} = -V_{obs-TE}^{inw}(\pi-\theta,\pi+\phi)\hat{\phi} \tag{5}$$

This notation expresses the fact that the observable field arriving from a given direction and for any polarization, eventually goes through the caustic in the origin of the reference system, where the representation (2) is not valid, but then emerges with polarization unperturbed, as highlighted in Fig. 1.

The letter V characterizing the complex vector amplitude $\vec{V}_{obs}^{inw/outw}$ is chosen to highlight that these amplitudes are dimensionally voltages. In the far field the spherical waves can be locally approximated as plane waves. Thus the magnetic field can be expressed as

$$\varsigma\vec{h}_{obs}(\vec{r}_\infty)\Big|_{r>\frac{2(2a)^2}{\lambda}} = \vec{V}_{obs}^{inw}(\theta,\phi)\times\hat{r}\frac{e^{jkr_\infty}}{r_\infty} + \hat{r}\times\vec{V}_{obs}^{outw}(\theta,\phi)\frac{e^{-jkr_\infty}}{r_\infty} \tag{6}$$

One should note that only if the observable field is defined in terms of spherical modes, one can be sure about its orthogonality to the remaining fields. Whenever the observable field is defined by means of a different procedure, as the one proposed in section IV, the orthogonality is not automatically guaranteed.

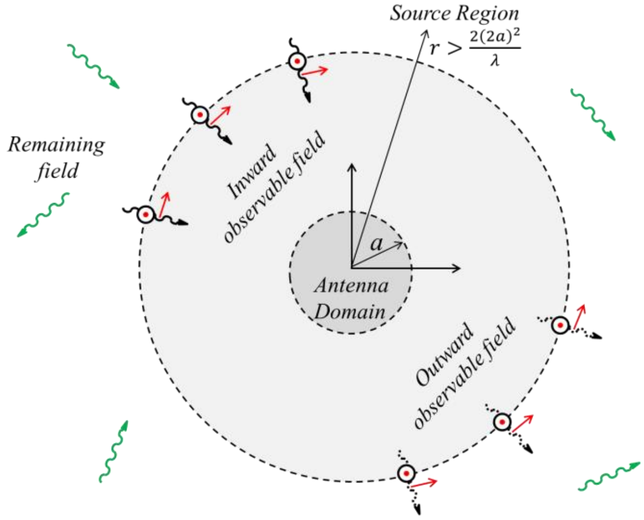

Fig.1 Source field representation as the sum of (1) the observable field (Black arrows) that interacts with the antenna domain and (2) a remaining field (Green arrows). This figure also shows the ray like vector nature of the observable field. One should note that both inward and outward components of the observable field are generally defined for all observation angles, and not in different angular domains.

III Spherical Mode Representation of the Incident Field

In a spherical mode representation, the incident field in any observation point $\vec{r}$ can be expressed, [12] as

$$\vec{e}_{inc}(\vec{r}) = \textstyle\sum_{n=0}^{\infty}\vec{E}_n(r,\theta,\phi), \tag{7}$$

where the spherical modal functions, $\vec{E}_n$, are defined with respect to the origin of a reference system chosen in the surrounding of the receiving antenna. The modal functions in (7) can be of TE and TM type, i.e. $\vec{E}_{nTE}(r,\theta,\phi)//\hat{\phi}\,(r,\theta,\phi)$, or $\vec{E}_{nTM}(r,\theta,\phi)//\hat{\theta}(r,\theta,\phi)$. The modal functions $\vec{E}_n(r,\theta,\phi)$ can always be expressed as superposition of inward and outward propagating waves; i.e. $\vec{E}_n(r,\theta,\phi) = \vec{E}_n^{inw}(r,\theta,\phi) + \vec{E}_n^{outw}(r,\theta,\phi)$.

III.A Modes involved in the field representation

The number of spherical wave harmonics needed to represent a field in (7), depends on the observation and the source points, $\vec{r},\vec{r}'$. In case of sources located far away from the origin it is well known that only a few harmonics are sufficient to represent the field for observation points close the origin. On the contrary, for observation points far away from the origin, a very large number of harmonics is required. This property can be easily appreciated by observing the radial dependence of the scalar free space Green's function, $g(\vec{r},\vec{r}')$. As shown in pag 699 of [13] the dependence from the source, $\vec{r}'$, and observation, $\vec{r}$, points of g can be expressed in terms of spherical functions as:

$$g(\vec{r},\vec{r}') = \frac{e^{-jk|\vec{r}-\vec{r}'|}}{4\pi|\vec{r}-\vec{r}'|} = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} C_{mn} L_{m,n}(\theta,\theta',\phi,\phi') d_n(r,r'), \quad (8)$$

where C_{mn} is a function only of the indices (n, m), $L_{m,n}$ are Legendre polynomials which depend on the angular coordinates, and only $d_n(r,r')$ accounts for the radial dependences. Assuming sources farther way from the origin than the observation point, it results:

$$d_n(r,r') = \frac{j_n(kr) h_n^2(kr')}{krr'} \quad (9)$$

where j_n and h_n^2 are the spherical Bessel and Hankel functions of integer order, respectively. For observation points very close to the origin ($r \to 0$), it is simple to show that only one spherical mode is required, and thus:

$$\lim_{r\to 0} d_n(r,r') = \begin{cases} \frac{h_0^2(kr')}{r'} & for\ n = 0 \\ 0 & \forall\ n > 0 \end{cases} \quad (10)$$

For observation point farther from the origin, more spherical modes will be needed to represent the Green's function (and accordingly the source field). The number of spherical modes is independent from the specific location of the sources as long as $r \ll r'$.

III.B Low order and high order fields

Separating the modes in (7) in higher and lower order modes was the key step used in [9] for defining the observable field and the available power. With reference again to Fig.1, we can distinguish two main, not confining, regions of interest: the antenna domain (sphere of radius a), and the region containing the sources (zone external to a sphere of radius[1] $\vec{r}_\infty \frac{2(2a)^2}{\lambda}$). The portion of the incident field that interacts with the antenna, that in the introduction was named the observable field, here will be indicated as the low order modes field, "LO", and it is different from zero in the antenna region. The remaining field instead can be indicated as the high order modes field, *"HO":*

$$\vec{e}_{inc}(\vec{r}) = \vec{e}_{LO}(\vec{r}) + \vec{e}_{HO}(\vec{r}) \quad (11)$$

[1] In case of electrically small domains, the limit for the far field is given by $\vec{r}_\infty \gg 20a$ and $\vec{r}_\infty \gg \lambda/2\pi$

where

$$\vec{e}_{LO}(\vec{r}) = \sum_{n=0}^{N} \vec{E}_n(r,\theta,\phi) \quad (12)$$

$$\vec{e}_{HO}(\theta,\phi) = \sum_{n=N+1}^{\infty} \vec{E}_n(r,\theta,\phi) \quad (13)$$

N is chosen such that the higher order fields are negligible, for all possible observation points within a sphere of radius a ($N = ka$). The incident field close to the antenna, is well approximated by the lower order (LO) field

$$\vec{e}_{inc}(\vec{r}) \approx \vec{e}_{LO}(\vec{r})\ \forall\ r \le a \quad (14)$$

In the case of a plane wave incident from broad side, the spatial dependence of the incident fields can be expressed as

$$\vec{e}_{inc}^{pw}(\vec{r}) = \vec{E}_{inc}^{pw} e^{jkz}, \vec{h}_{inc}^{pw}(\vec{r}) = \vec{H}_{inc}^{pw} e^{jkz}\ \forall\ \vec{r} \quad (15)$$

Fig. 2 shows the amplitude of the "LO" electric field in the vicinity of the antenna domain due to a plane wave, x-polarized, incident from the negative broadside, ($\theta_{inc} = 180^o$) with amplitude $E_{inc}^{pw} = 1\ V/m$. The graph highlights how de facto, the number of modes in the expansion constitute a spatial filter that selects only the central portion of the observation space. Three different antenna domain are considered, $a_1 = 0.01\ \lambda, a_2 = \lambda$, and $a_3 = 10\ \lambda$. The observation points are scanned for z=0 on the E-plane. It is apparent that the LO fields represent a very good approximation of the incident field in the antenna domain for large and relatively large antennas. Outside the antenna domain the field decreases relatively fast.

III.C LO Fields in the source region

The LO fields, however, are significantly different from zero also at a large distance from the antenna domain, they are in fact defined in the entire space. For observation points far from the origin, the radial dependence of each spherical mode and thus of the LO fields tends to the spherical spreading function, $\frac{e^{\pm jkR}}{R}$. Moreover, the electric and the magnetic fields also tend to be orthogonal one the other and transversely polarized. Accordingly when the LO field is observed at large distance from the origin, all vector spherical functions can be summed in amplitude and phase to obtain a unique angular function a single inward and single outward propagating spherical wave. This leads to a much simpler possible expression for the LO field which is valid only at large distances, $\vec{r}_\infty$, from the antenna. The expressions are equal to the ones presented in section I for the observable field, eq. (2)-(6) by just replacing the pedix *"obs"* with the pedix *"LO"*.

Fig. 3 shows the normalized amplitude of the outward propagating LO fields in the far region of the antenna due to a plane wave, x-polarized, incident from the negative broadside ($\theta_{inc} = 180^o$) with amplitude $E_{inc}^{pw} = 1\ V/m$. Again, three different antenna domains are considered, $a_1 = 0.01\ \lambda, a_2 = \lambda$, and $a_3 = 10\ \lambda$. The observation points are scanned for $\theta \in (-\pi,\pi)$ and $\phi = 0$. The maxima are in $\theta = 0$ direction since they are in the direction of propagation of the outward component of the incident fields. Notice that the observable field associated to the smallest antenna domain shown in Fig.3 resembles the Huygens' source pattern, [14].

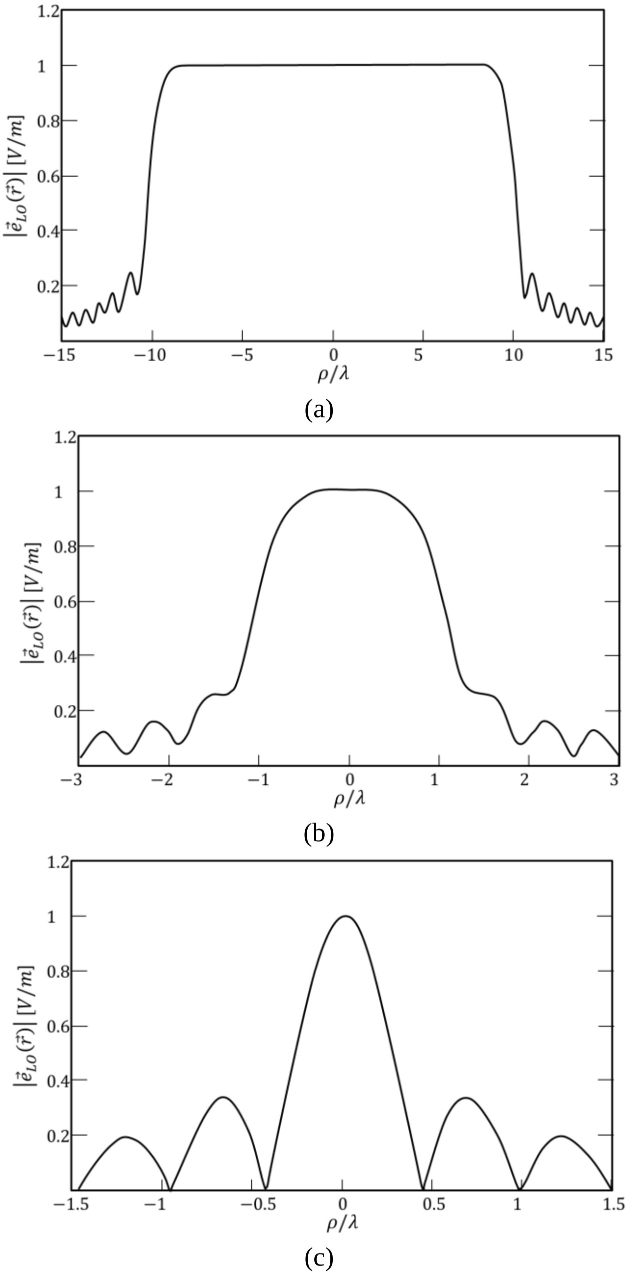

Fig. 2 LO electric fields in the antenna region (E-plane), due to a plane wave incident from broad side for three different antenna domains of radii (a) 10 λ (N=62), (b) 1 λ (N=6), and (c) 0.01 λ (N=1).

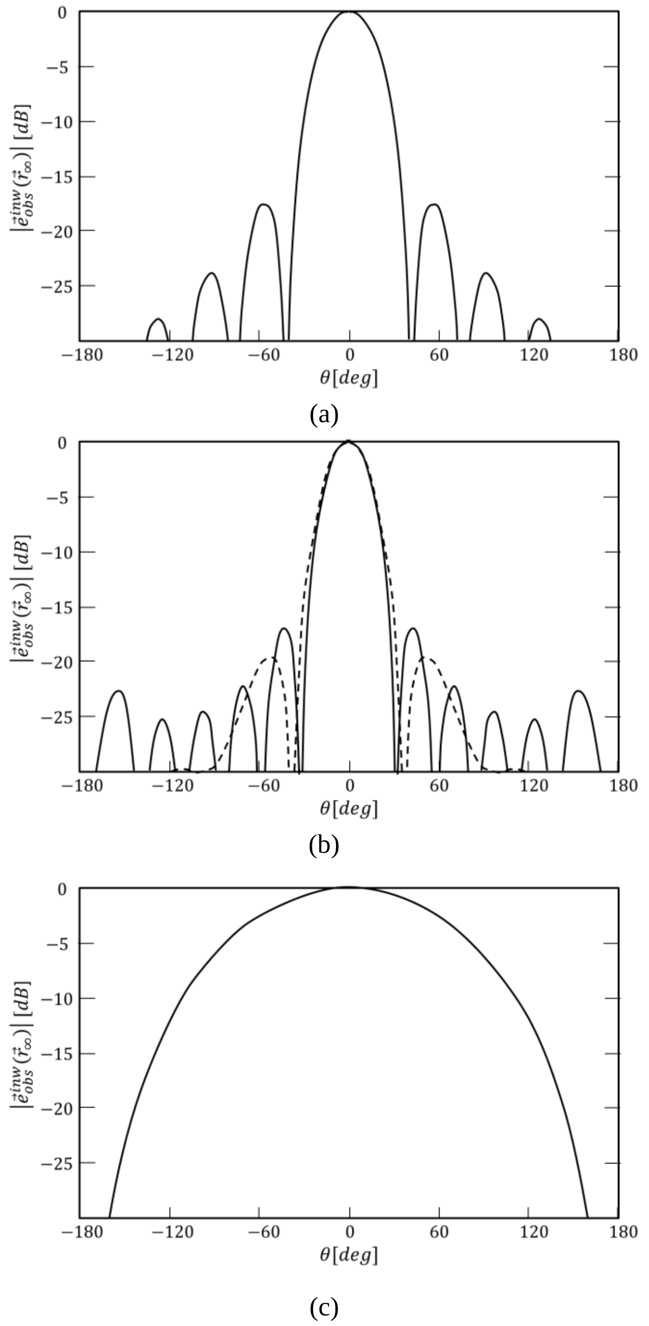

Fig. 3 Observable field patterns in the source region (E-plane), due to a plane wave incident from broad side, calculated by using the LO fields (continuous lines) or the ideal current method (dashed lines), for three different antenna domains of radii: (a) 10 λ, (b) 1 λ, and (c) 0.01 λ.

We should stress at this point that the higher order field is not negligible outside the antenna region:

$$\vec{e}_{HO}(\vec{r}_\infty) \neq 0 \quad \forall\, r_\infty > a \tag{16}$$

Accordingly, the LO fields alone do not provide a reasonable approximation of the incident field at large distance from the antenna since the incident field does not necessarily resemble a spherical wave. Thus:

$$\vec{e}_{inc}(\vec{r}_\infty) \neq \vec{e}_{LO}(\vec{r}_\infty) \ \forall\, r_\infty > a \tag{17}$$

III.D Estimation of the available power via spherical modes

The total flux of Poynting vector associated to any incident field on a sphere surrounding the antenna is zero. This is because all the inward propagating spherical waves that contribute to a positive flux then continue to propagate outward from the origin, as in Fig.1, and a contribute to a negative flux. However [9] clarified that if one calculates such flux retaining only the inward propagating components of the LO field, one would estimate the power that could be received by the ideal antenna contained within the sphere: the *available power.* Accordingly

$$P^{LO}_{avail} \equiv P_{inw} = \iint_{S_\infty} \frac{1}{2\varsigma} \left| e^{inw}_{LO}(\vec{r}) \right|^2 d\vec{r}, \tag{18}$$

where S_∞ indicates the surface of a sphere at large distance from the origin. The available power in (18) is not dependent from radius of the sphere on which the flux is evaluated, but on only on the angular distribution of the inward field and, therefore, it can be calculated on the source region.

For the case of a single plane wave incidence one can evaluate the effective area of an ideal antenna that receives all the available power as:

$$A_{eff}^{LO} = \frac{P_{avail}^{LO}}{\frac{1}{2\zeta}\left|\vec{E}_{inc}^{pw}\right|^2} \tag{19}$$

Fig. 4 shows the effective area, A_{eff}^{LO} for an antenna contained in a sphere of radius a, due to a plane wave, x-polarized, incident from broadside. The data is plotted as a function of the cross section in terms of the wavelength. The dots represent the effective areas of antenna designs available in the literature reported in [11]. One should note that only measured data pertinent to antennas of finite ground planes are considered relevant here.

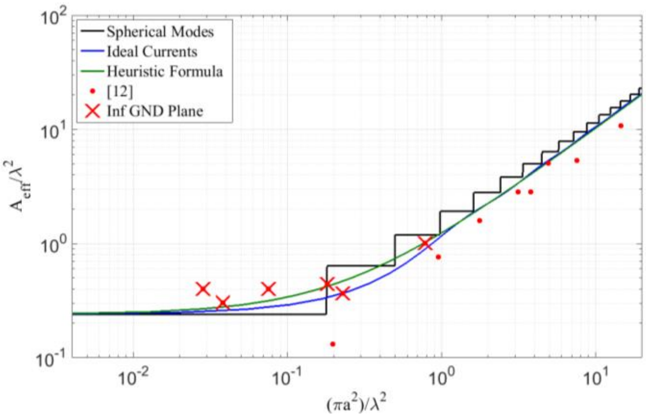

Fig. 4 Effective area of an antenna that receives the available power as a function of the radius of the containing sphere, normalized to the squared wavelength. Three curves are presented: the prediction due to the LO fields, the heuristic interpolation [10], and the curves based on the ideal current method proposed in section IV. Some points pertinent to best performing designs available in open literature, [11], are also reported .

The LO curve is a stepped function whose discontinuities are very noticeable for small cross sections. This is because the lower order number of modes ($N = ka$) that defines the LO field in (12) is a quantized integer. This leaves a margin of uncertainty: for a given a should one take $N-1$, N, or $N+1$ modes? This indetermination is such that (18) cannot be accurately used to estimating the power available to small antennas. In fact, let us assume that an antenna is small in terms of the wavelength. One would need to decide whether to retain, $n = 0$ or also $n = 1$, in the summation (8), depending on the values of $d_{1/0}(r, r')$ in (9). If the antenna domain is so small that for all observation points $d_1(r, r') \ll d_0(r, r')$ one can simply decide to neglect d_1. However for antennas that are small with respect to the wavelength, but not infinitesimal d_1 could be not negligible:

$$\lim_{r\to a} d_1(r, r') = \frac{\pi a}{\lambda}\frac{h_1^2(kr')}{r'} \tag{20}$$

Taking the safe bet, and retaining both the modes $n = 0$ and $n = 1$, would imply an overestimation of the observable field and consequently of the available power.

III.E Heuristic Extensions

For very large radii, the LO approximation of the effective area, (19), converges to the physical area of the antennas, which is a well-accepted limit in the antenna community. However in the intermediate cases, that account for 90% of all antennas, resorting to the LO fields to estimate the available power, is not useful. The antenna community has responded to this difficulty heuristically: a simple formula has been proposed by [10] that prescribes the maximum effective area, A_{eff}^{heu}. Despite the deeper analysis that they perform eventually even the authors of [11], suggest to use the heuristic formula from [10] for practical applications. In the heuristic formula the effective area is expressed as to the sum of the antenna domain physical area, A_{ph} plus an Huygens' source effective area:

$$A_{eff}^{heu} = A_{ph} + \frac{3}{4\pi}\lambda^2 \tag{21}$$

The reasoning being that A_{ph} is well known to be a good approximation of the effective area for large antennas, and that the term, $\frac{3}{4\pi}\lambda^2$, is equal to the Huygens' source effective area which is predicted by the LO modes for extremely small antennas, so that the sum of the two could give an approximation valid for every antenna domain. Fig. 4 also shows the comparison of the effective area prescribed by the LO modes compared with the heuristic formula. This heuristic procedure has been shown in [11] to be more accurate than the stepped LO curve when compared to experiments for single plane wave incidence. Despite the fact that the lack of jumps in the effective area provides more confidence to a designer, there is no reason to think that the heuristic formula predicts well the maximum power that an antenna within the given volume can receive. Nor there is any way to extend the heuristic formula to the realistic cases in which there are multiple coherent plane waves incidences.

IV The Ideal Equivalent Current Method

In this section a novel procedure to estimate the observable field is devised. To facilitate the presentation, the observable field is discussed first for the simple case of an ideal focusing system excited by an external plane wave, as in Fig.5. Later in this section, the procedure is extended to the more complex case of plane wave incidence.

IV.A The ideal antenna in a focusing system

It is well known that the incident field within a focusing system, when excited by an external plane wave, Fig. 5a, can be represented as a single spherical wave that first converges to the focus (origin of the selected reference system), and then diverges. Therefore the incident field, can be appropriately represented as

$$\vec{e}_{inc} = \vec{e}_{inc}^{inw} + \vec{e}_{inc}^{outw} \tag{22}$$

The *ideal antenna* located in the focus of the system will convert the inward propagating portion of the incident field, $\vec{e}_{inc}^{inw}$ into a guided wave, so that the field below the antenna will be essentially zero, Fig. 5b. This conversion can be mathematically represented, for observation points outside the antenna domain, using the equivalence theorem as in Fig.

5c: scattering currents equivalent to the ideal antenna are defined on an equivalent surface and these radiate in the far field of the antenna an outward propagating scattered field, $\vec{e}_{scat}^{ideal}$ that is equal and opposite to the outward component of the incident field:

$$\vec{e}_{scat}^{ideal} = -\vec{e}_{inc}^{outw} \tag{23}$$

So that the outward component of the total field, which is the sum of the incident and scattered field, is zero

$$\vec{e}_{tot}^{outw} = \vec{e}_{inc}^{outw} + \vec{e}_{scat}^{ideal} = 0 \tag{24}$$

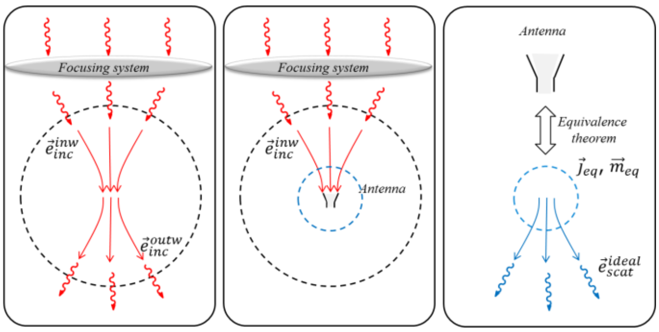

Fig. 5 Field picture in an ideal focusing system, a) Incident field as sum of inward and outward propagating waves. b) the ideal antenna captures all the inward incident field. c) The scattered field radiated by the currents equivalent to the ideal antenna is equal and opposite to the outward incident field.

In $\vec{e}_{scat}^{ideal}$, the subscript "outw" is omitted since scattered waves are always outward propagating, with respect to the antenna domain. In this situation the available power is the one received by this ideal antenna and corresponds to the power carried by the inward component of the incident field. Therefore it can be simply quantified integrating over any surface surrounding the antenna of the inward component of the Poynting vector associated to the incident field:

$$P_{avail} = P_{rec}^{ideal} = P_{inc}^{inw} \tag{25}$$

Moreover, since the powers associated to the outgoing and ingoing components of the incident field are equal $P_{inc}^{outw} = P_{inc}^{inw}$, the power scattered by the antenna is also known

$$P_{scat}^{ideal} = P_{inc}^{outw} \tag{26}$$

Note that (25) and (26) are congruent with an absorption efficiency of ½, as discussed in [7].

It is immediate to identify, in this physical picture of the focusing system, the roles of the observable field representation in eq. (1): $\vec{e}_{rem} = 0$, while $\vec{e}_{obs}(\vec{r}) = \vec{e}_{inc}$, and thus $\vec{e}_{obs}^{inw} = \vec{e}_{inc}^{inw}$, $\vec{e}_{obs}^{outw} = \vec{e}_{inc}^{outw}$. Before proceeding one can note that the ideal antenna here has been introduced as the one capable of converting all the power associated to the incident field into guided waves. This implicitly means that there are no significant losses involved in the reception mechanism neither ohmic nor due to impedance or polarization mismatching (i.e. gain is equivalent to directivity).

IV.B The ideal antenna in a general configuration

The intuitive procedure to estimate the observable field used for focusing systems cannot easily be extended to cases of antennas under a plane wave incidence, $\vec{E}_{inc}^{pw}(\vec{r}) = \vec{E}_{inc}^{pw} e^{j\vec{k}_{in}\cdot\vec{r}}$ as shown in Fig6a. In order to obtain a field picture according to the observable field definition in (1) also for this case, one needs to devise a procedure to define $\vec{e}_{obs}^{outw}$, $\vec{e}_{obs}^{inw}$ and eventually $\vec{e}_{rem}$. Our proposal is to define the outward spherical wave component of the observable field as *the opposite of the field scattered by the ideal antenna for the given incident plane wave and antenna domain*:

$$\vec{e}_{obs}^{outw}(\vec{r}) \equiv -\vec{e}_{scat}^{ideal}(\vec{r}) \tag{27}$$

The terms "ideal" refers to an antenna that scatters power equal to the one it receives, just as in the focusing system example. The power scattered by an ideal antenna is thus the maximum possible in the given volume for the considered incident field. In the next section the case of plane wave incidence on a finite antenna domain is discussed.

V Observable Field for a Single Plane Wave

Since for plane wave incidence the power received by an antenna can be related to its effective area, the power received by the ideal antenna can be expressed as

$$P_{rec}^{ideal} = \frac{1}{2\zeta}\left|\vec{E}_{inc}^{pw}\right|^2 A_{eff}^{ideal} = P_{scat}^{ideal} \tag{28}$$

where A_{eff}^{ideal} is the effective area that characterizes the ideal antenna contained in the given volume. Accordingly $P_{scat}^{ideal} = P_{rec}^{ideal}$ is the power scattered by the ideal antenna. For the ideal, lossless, reciprocal antenna, $A_{eff}^{ideal} = \frac{\lambda^2}{4\pi} D^{ideal}$, i.e. the effective area can be related to the maximum directivity of the ideal antenna. So *the original problem of defining the outward observable field is transformed in the problem of estimating the maximum possible directivity for an antenna in a given domain*. In the following we will assume that the domain is identified by a sphere of radius *a*, *Fig. 6a*. To estimate this maximum directivity we will rely on an agreeable hypothesis:

Hypothesis[2]: the maximum directivity associated to an ideal antenna contained within a sphere of radius *a* is associated to the fields radiated by an uniform distribution of equivalent electric and magnetic currents within a planar cross section S_{in} of the sphere.

Recalling that, from eq. (27) the outward observable field in regions far from the antenna domain is the opposite of the

[2] This hypothesis excludes the cases of super-gain antennas, but includes antennas that are characterized not only by electric but also by magnetic current distributions, which could be unrealistic to realize with low loss implementations in very small domains. The reason to include magnetic currents at this time is to obtain a good agreement between the results obtained via the equivalent currents with those typically obtained via LO spherical modes. Conceptually equivalent procedures relying only on electric currents (distributed on volumes or surfaces) would have the advantage of explicitly allowing the analysis of losses and thus provide useful information on the practical limits of small antennas. These alternative approaches will be considered elsewhere.

field radiated by the ideal currents. The ideal currents can be derived from the unique set of equivalent currents, a combination of uniform electric and magnetic currents over the cross section of the antenna domain, necessary to radiate the incident field in the outward region. Fig. 6b shows the equivalent currents defined over an infinite domain that will radiate the incident field in the outward region. These currents can be defined over two different domains as depicted in Fig6b and c: one defined over the top boundary of the antenna sphere and one along the sphere cross section. The currents in these two domains would be different but would necessarily generate the same field in the outward region. Except for an amplification factor, the ideal currents, $\vec{\imath}_{ideal}, \vec{m}_{ideal}$ can be derived by assuming a spatial truncation of these infinite domains either over the top sphere boundary or over its cross section. Both truncations lead to the same outward propagating field since the remaining field radiated by the currents in the domain outside the antenna region would also be the same for both cases.

The retained ideal currents are both electric and magnetic, and corresponds to using both TE and TM modes in the spherical mode expansion.

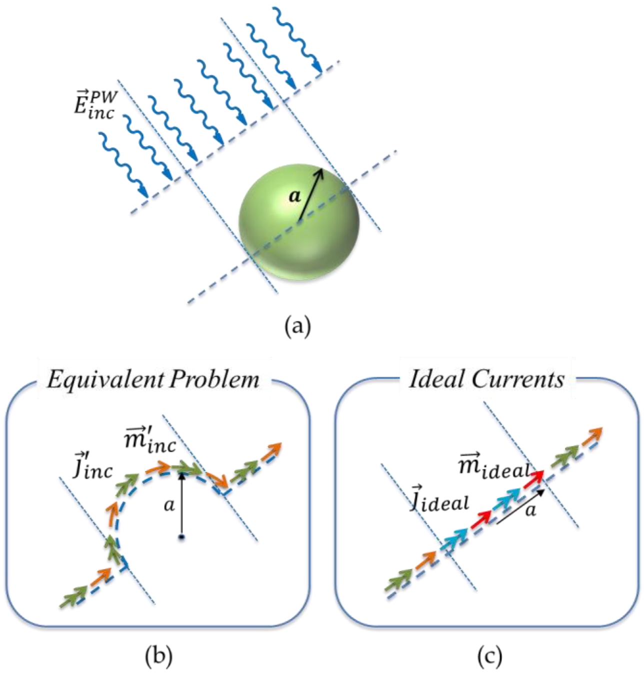

Fig.6 Definition of the ideal currents for a single plane wave incidence: (a) geometry showing the antenna domain and the plane wave incidence; (b) Equivalent currents, defined over an infinite surface, that radiate the incident field in the outward region (c) Spatial truncation of the currents (ideal antenna currents) radiating the spherical outward propagating component of the observable field.

For a plane wave incidence, the ideal currents are defined over a planar cross section orthogonal to the incident plane wave, $\hat{k}_{in}$. We will indicate as A_{ph} the area of the surface S_{in}, $A_{ph} = \pi a^2$, see Fig 7. The explicit expression for the ideal equivalent currents which radiate the ideal scattered field, $\vec{e}^{ideal}_{scat}$, is assumed to be:

$$\begin{aligned} \vec{m}_{ideal}(\hat{k}_{in},\vec{r}) &= C\, \vec{m}_{po}(\hat{k}_{in},\vec{r}) \\ \vec{J}_{ideal}(\hat{k}_{in},\vec{r}) &= C\, \vec{J}_{po}(\hat{k}_{in},\vec{r}) \end{aligned} \quad (29)$$

Except for the multiplying factor C, the ideal equivalent currents, $\vec{J}_{idel}, \vec{m}_{ideal}$ in Fig.6 are the same as the Physical Optics (PO) current approximation, $\vec{J}_{po}, \vec{m}_{po}$ which are related to the incident field and the antenna domain as follows:

$$\begin{aligned} \vec{m}_{po}(\hat{k}_{in},\vec{r}) &= [\vec{E}^{pw}_{i}(\vec{r}) \times \hat{k}_{in}] \\ \vec{J}_{po}(\hat{k}_{in},\vec{r}) &= [\hat{k}_{in} \times \vec{H}^{pw}_{i}(\vec{r})] = -\vec{E}^{pw}_{i}/\zeta_0 \end{aligned} \quad \forall \vec{r} \in S_{in}(\hat{k}_{in}) \quad (30)$$

A set of currents as in (30) or in (29), with the electric and magnetic currents orthogonal one to the other and in ratio $\frac{m_{eq}}{\zeta} = j_{eq}$, can be recognized as a Huygens' source surface distribution, Fig.8.

The amplification factor C is the one that guarantees that the powers scattered and received by the ideal antenna coincide. The analytic evaluation of C is presented in the appendix B and results in the following expression

$$C = \frac{A_{PO}}{A_{Ph}} \quad (31)$$

where A_{PO} is the effective area associated to uniform equivalent currents defined as the PO ones and A_{Ph} is the area of the circular cross section where the currents are confined. This is an amplification factor that accounts for the fact that small antennas have effective area much larger than their physical area.

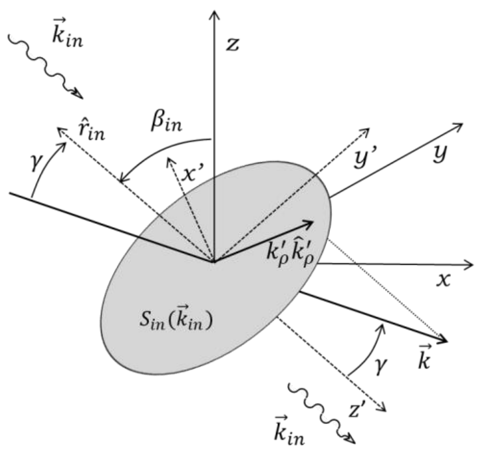

Fig.7 The equivalent disc surface where the ideal currents are defined for plane waves impinging from directions different from broadside.

V.A General representation of the observable field

Equation (27) defines the outward observable field in regions far from the antenna domain as the opposite of the field radiated by the ideal currents in (29). Proceeding as in appendix A it is then immediate to verify that the outward observable field, the negative of the incident field in the outward region, is a spherical wave

$$\vec{e}^{outw}_{obs}(\vec{r}_\infty, \hat{k}_{in}) = -\frac{A_{PO}}{A_{Ph}} \vec{V}^{outw}_{PO}(a, \hat{k}_{in}, \vec{k}) \frac{e^{-jkr_\infty}}{r_\infty}, \quad (32)$$

with angular distribution that can be expressed as the product of two terms, as follows:

$$\vec{V}^{outw}_{PO}(a, \hat{k}_{in}, \vec{k}) = Airy_{in}(\hat{k}_{in}, \vec{k}) \vec{H}(\hat{k}_{in}, \vec{k}) \quad (33)$$

Where the term $\vec{H}(\hat{k}_{in}, \vec{k})$ has the spectral signature of the Huygens' source pointing in the $\hat{k}_{in}$ direction, and can be expressed as

$$\vec{H}(\hat{k}_{in}, \vec{k}) = \frac{jk}{4\pi}\hat{k} \times [\vec{E}^{pw}_{inc} \times (\hat{k} + \hat{k}_{in})] \quad (34)$$

This term is characterized by a null in the direction of the sources generating the incident plane wave, and a maximum in the direction of the plane wave. The term $Airy_{in}(\vec{k}^{in}, \vec{k})$ (see Appendix A) represents an Airy distribution, symmetric with respect to the cross section S_{in}, and with directivity that increases for larger values of a. The observable field, in (32) is also complemented by an inward propagating wave component that can be simply inferred from the outward component as in equations (2)-(6).

V.B Range of validity of ideal currents hypothesis

The hypothesis that the maximum directivity for a given antenna volume can be obtained with the ideal currents (29) is justified in two very important cases.

For large and directive antennas, $a \gg \lambda$, excited by a plane waves, it is well understood that the effective area of the antenna can be designed to be comparable to the physical cross section of the antenna, i.e. $\frac{A_{PO}}{A_{Ph}} \to 1$. Also it is well known that the scattered fields can be accurately represented in far regions as the fields radiated by the PO currents. The angular distribution, $\vec{V}^{outw}_{obs}(a, \hat{k}_{in}, \theta, \phi)$ of the outgoing observable field calculated via the ideal antennas can be observed in Fig. 3a where is it compared with the observable field evaluated resorting to the LO spherical modes. It is clear that the two representations are equivalent down to -30 dB for apertures characterized by radius of $a = 10\lambda$, and for this reason the two curves cannot be distinguished.

For extremely small antenna domains, $a \ll \lambda$, the LO field can be approximated resorting to only the $n = 0$ mode in the expansion (8). Named $\vec{E}^{outw}_{n=0}(\vec{r})$ this term, [14] showed that $\vec{E}^{outw}_{n=0}(\vec{r})$, can be rigorously represented as the field radiated in free space by an single Huygens' source located in the origin of the reference system. The Huygens' source in [14] can be expressed as

$$\begin{aligned} \vec{m}_{ele}(\hat{k}_{in}, \vec{r}) &= A^{eff}_{Huy}[\vec{E}^{pw}_{i}(\vec{r} = 0) \times \hat{k}_{in}]\delta(\vec{r}) \\ \vec{J}_{ele}(\hat{k}_{in}, \vec{r}) &= A^{eff}_{Huy}[\hat{k}_{in} \times \vec{H}^{pw}_{i}(\vec{r} = 0)]\delta(\vec{r}) \end{aligned} \quad (35)$$

where $A^{eff}_{Huy} = \frac{3}{4\pi}\lambda^2$ is the effective area of the Huygens'source and $A^{eff}_{Huy}|\vec{E}^{pw}_{inc}|$ is its amplitude. The subscript, '*ele*' is to indicate that these are elementary sources. Thus for very small antenna domains the outgoing spherical mode expansion representation of the observable field is obtained as radiation of a single Huygens' source in the origin that reradiates in a wide beam aligned along the incident direction of the considered plane wave.

However, it is simple to recognize that the field radiated by this single Huygens' source (35) can also be represented as the field radiated by an *equivalent surface source distribution* of Huygens' sources, extended over a very small surface orthogonal to the incidence plane wave, $S_{in}(\vec{k}^{in})$, as in Fig. 7. Fig. 8, explicitly shows a single Huygens' source in the origin and the equivalent surface distribution obtained dividing the amplitude by the physical area A_{ph}. It is sufficient to let $\frac{A_{PO}}{A_{ph}} \to \frac{\frac{3}{4\pi}\lambda^2}{A_{ph}}$ and the fields, $\vec{e}^{outw}_{obs}(\vec{r}_\infty)$ in (32) due to the ideal currents in (29), can be shown to be comparable to $\vec{E}^{outw}_{n=0}(\vec{r}_\infty)$:

$$\lim_{a\to 0} \vec{e}^{outw}_{obs}(\vec{r}_\infty, a) = \vec{E}^{outw}_{n=0}(\vec{r}_\infty) \quad (36)$$

The explicit comparison between the angular amplitude distribution, $\vec{V}^{outw}_{obs}(a, \hat{k}_{in}, \theta, \phi)$ obtained resorting to the ideal current procedure and the one obtained resorting to $\vec{E}^{outw}_{n=0}(\vec{r}_\infty)$ can be observed in Fig. 3c. The curves are apparently overlapping for extremely small domains ($a \to 0$). One should however note that the equivalence between $\vec{E}^{outw}_{n=0}(\vec{r}_\infty)$ and $\vec{e}^{outw}_{obs}(\vec{r}_\infty, a)$ is not exact. For any small but finite radii a, for which one would resort to a single spherical mode to represent the scattered fields, there is always a difference in amplitude between the LO field approximation of the observable field, which depends from the actual radius a.

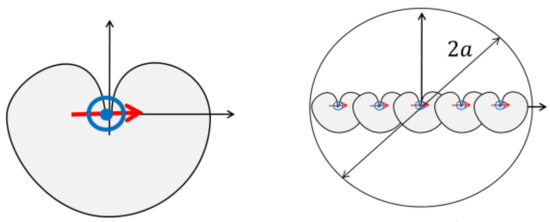

Fig.8 An equivalent source distribution over a small disk of radius a, radiates in the far region the same field as the single Huygens' source, which is equivalent to the n=0 outgoing spherical modes.

V.C Power received for single plane wave incidence

An incident plane wave has been represented in (1) as the superposition of an observable wave component by an ideal antenna that first converges toward the origin and then diverges from the origin, plus a remaining field (that is essentially zero in the antenna domain). The power associated to the converging and diverging waves ($P^{inw}_{obs} = P^{outw}_{obs}$) are exactly the same, since in absence of an antenna, the same wave first converges to the origin and then then diverges. The *ideal antenna* is the one that captures (transforms into guided waves) all the fields associated to the converging observable wave, $P^{ideal}_{rec} = P^{inw}_{obs}$, and cancels the diverging waves. The cancellation occurs with the ideal antenna scattering a field, that matches with opposite sign the diverging portion of the observable field, $P^{ideal}_{scat} = P^{outw}_{obs}$. So that, overall one can define the available power as the one received by the ideal antenna,

$$P_{avail} \equiv P^{ideal}_{rec} = P^{inw}_{obs} = P^{outw}_{obs} = P^{ideal}_{scat} \quad (37)$$

Specifically the available power can be calculated by integrating the angular distribution of the inward (or outward) observable field as follows:

$$P_{avail}(a, \hat{k}^{in}) \equiv \frac{1}{2\varsigma}\iint_{4\pi} |\vec{V}^{out}_{obs}(a, \hat{k}^{in}, \theta, \phi)|^2 sin\theta d\theta d\phi \quad (38)$$

Depending on which field representation is adopted for the incident field, the result of (38) gives different results.

The numerical procedure to evaluate, according to the ideal currents procedure, the outward propagating voltage component of the observable field associated to an antenna contained within a sphere of radius a for a plane wave incidence, $\vec{E}_{inc}^{pw}$, along $\vec{k}^{in}$, can be summarized as follows:

$$\vec{V}_{obs}^{out}(a,\hat{k}_{in},\theta,\phi) = -\frac{A_{PO}}{A_{Ph}} Airy_{in}(\hat{k}_{in},\vec{k})\vec{H}(\hat{k}_{in},\vec{k}) \quad (39)$$

where

$$A_{Ph} = \pi a^2$$

$$A_{PO} = \lambda^2 \frac{\left|Airy_{in}(\hat{k}_{in},\vec{k})\vec{H}(\hat{k}_{in},\vec{k})\right|^2}{\iint_{4\pi} \left|Airy_{in}(\hat{k}_{in},\vec{k})\vec{H}(\hat{k}_{in},\vec{k})\right|^2 sin\theta d\theta d\phi}$$

$$Airy_{in}(\hat{k}_{in},\vec{k}) = 2\pi a^2 \frac{J_1(kasin\gamma)}{kasin\gamma}$$

$$\gamma = \cos^{-1}[\hat{k}\cdot(\hat{k}_{in})]$$

$$\hat{k} = \frac{\vec{k}}{k} = sin\theta cos\phi\hat{x} + sin\theta sin\phi\hat{y} + cos\theta\hat{z}$$

$$\hat{k}_{in} = \frac{\vec{k}_{in}}{k} = sin\beta_{in}cos\alpha_{in}\hat{x} + sin\alpha_{in}sin\phi_{in}\hat{y} + cos\beta_{in}\hat{z}$$

$$\vec{H}(\hat{k}_{in},\vec{k}) = \frac{jk}{4\pi}\hat{k}\times[\vec{E}_{inc}^{pw}\times(\hat{k}+\hat{k}_{in})]$$

Using this methodology the available power changes continuously with respect to the antenna radius a (i.e. it is not quantized) as shown in Fig. 4. It is apparent that the effective area predicted by the ideal currents for very small antenna domains is at about the same value of the single Huygens' source, predicted by the LO spherical mode representation. For larger antenna domains the effective area varies smoothly and converges to the physical area such as in the case of the LO fields since both procedures predict essentially the same far field distribution.

For moderate size antenna domain, where the modal quantization of the available power is very significant (see Fig.4), the observable field derived with the spherical modes or the antenna domain is different as shown in Fig.3b. In this region the available power derived with the ideal currents arrives to be roughly 30% lower than the heuristic expression [10] would predict. The present results are actually closer to the theoretical results predicted in [11]. Some of the measured results presented in [11] would suggest that for the intermediate region, the most debated one, effective areas larger than the one predicted by the ideal currents could be realized. However, all those cases include infinite ground planes to avoid the use of baluns. This implies that those results would be very difficult to achieve with a practical feeding structure in an electrically small antenna domain.

VII Conclusions

In this paper a novel representation for the incident field is proposed that is particularly useful to study antennas in reception. Specifically an incident plane wave is represented as an observable wave that first converges toward the origin and then diverges from the origin. The difference between the incident field and the observable field is simply indicated as the remaining field and is essentially zero in the antenna domain. The *ideal antenna* currents in reception generate a scattered field that cancels the diverging component of the observable field**.** A procedure to construct the ideal currents and an analytical expression of the observable field is proposed by truncating a spatial integral representation of the incident field. This procedure differs from the spherical mode procedure where the truncation is performed on the modal spectrum. The power available to an antenna in reception can be taken as quality parameter to assess the usefulness of the procedure to construct the observable field. When the spherical modes provide accurate estimation of the available power, for very small and very large antenna domains, also the procedure based on the ideal currents does. In those intermediate antenna domains for which the spherical modes are known to provide inaccurate estimation of the available power, the ideal currents provide more accurate results which appear to provide an upper bound with respect to experimental results available in literature. The ideal current procedure can also been extended to generalized field incidence cases by means of a plane wave expansion.

Appendix A

The scattered filed (outward) due to an equivalent source distribution, $\vec{m}_{PO}(\hat{k}_{in},\vec{r}'),\vec{J}_{PO}(\hat{k}_{in},\vec{r}')$ defined in a disc $S_{in}(\hat{k}_{in})$ can be indicated as $\vec{e}_{PO}^{outw}(\hat{k}_{in},a,\vec{m}_{eq},\vec{J}_{eq},\vec{r}_\infty)$. This field can be expressed as a standard radiation integral

$$\vec{e}_{PO}^{outw}(\hat{k}_{in},a,\vec{m}_{PO},\vec{J}_{PO},\vec{r}_\infty) = \iint_{S_{in}(\hat{k}_{in})} [\bar{\bar{g}}^{ej}(\vec{r}_\infty,\vec{r}')\vec{J}_{PO}(\hat{r}^{in},\vec{r}') + \bar{\bar{g}}^{em}(\vec{r}_\infty,\vec{r}')\vec{m}_{PO}(\hat{r}^{in},\vec{r}')]\, d\vec{r}' \quad (A1)$$

where the primed points span the disc $S_{in}(\hat{k}_{in})$ orthogonal to the incident plane wave direction $\vec{k}^{in}$, $[\bar{\bar{g}}^{ej}, \bar{\bar{g}}^{em}]$ represent the Green's functions providing the electric field radiated in free space by equivalent electric and magnetic currents and $\vec{r}_\infty$ indicates observation points in the far field. For observation points in the far field of the current distribution, the scattered field, $\vec{e}_{PO}^{outw}(\hat{k}_{in},a,\vec{m}_{eq},\vec{J}_{eq})$ can be evaluated analytically very simply. The non-uniform asymptotic evaluation of (A1) leads to

$$\vec{e}_{PO}^{outw}(\vec{r}_\infty,\hat{k}_{in}) = \vec{V}_{PO}^{outw}(a,\hat{k}_{in},\vec{k})\frac{e^{-jkr_\infty}}{r_\infty} \quad (A2)$$

Note that in the arguments $\vec{e}_{PO}^{outw}(..)$ the explicit dependence from the equivalent currents has been omitted to simplify the notation. The amplitude, $\vec{V}_{PO}^{outw}(a,\hat{k}_{in},\vec{k})$ of the outward propagating component of the scattered field is expressed explicitly as a function of the direction of propagation of the incident plane wave, $\vec{k}^{in}$, as follows

$$\vec{V}_{PO}^{outw}(a,\hat{k}_{in},\vec{k}) = -\frac{jk}{4\pi}\{\zeta(\bar{\bar{I}}-\hat{k}\hat{k})\vec{J}_{PO}(\hat{k}_{in},\vec{k}) - \hat{k}\times\vec{M}_{PO}(\hat{k}_{in},\vec{k})\} \quad (A3)$$

where $\hat{k} = \frac{\vec{k}}{k} = sin\theta cos\phi\hat{x} + sin\theta sin\phi\hat{y} + cos\theta\hat{z}$ and $\vec{J}_{eq}$ and $\vec{M}_{PO}$ represent the Fourier Transform (FT) of the PO equivalent electric and magnetic currents, (30),:

$$\vec{J}_{PO}(\hat{k}_{in}, \vec{k}) = \iint_{S_{in}} \vec{j}_{PO}(\hat{k}_{in}, \vec{r}) e^{j\vec{k}\cdot\vec{r}} d\vec{r} \quad \text{(A4)}$$

$$\vec{M}_{PO}(\hat{k}_{in}, \vec{k}) = \iint_{S_{in}} \vec{m}_{PO}(\hat{k}_{in}, \vec{r}) e^{j\vec{k}\cdot\vec{r}} d\vec{r} \quad \text{(A5)}$$

However, since the integration is performed over planes in which the plane wave presents no spatial variation, the FT can be evaluated analytically as follows

$$\vec{J}_{PO}(\hat{k}_{in}, \vec{k}) = -\vec{E}_i^{pw}/\zeta_0 Airy_{in}(\vec{k}^{in}, \vec{k}) \quad \text{(A6)}$$

$$\vec{M}_{PO}(\hat{k}_{in}, \vec{k}) = \vec{E}_i^{pw} \times \hat{k}_{in}\, Airy_{in}(\vec{k}^{in}, \vec{k}) \quad \text{(A7)}$$

where $Airy_{in}(\vec{k}^{in}, \vec{k})$ is the FT of a circular domain orthogonal to $\vec{k}^{in}$. It is simple to show that

$$Airy_{in}(\hat{k}_{in}, \vec{k}) = 2\pi a^2 \frac{J_1(ksin\gamma a)}{ksin\gamma a} \quad \text{(A8)}$$

is a distribution factor which depends on radius of the sphere a and the angle between the direction of incidence of the plane wave and the observation direction, $\gamma = \cos^{-1}[\hat{k} \cdot (\hat{k}_{in})]$. Substituting (A6-A8) into (A3), we can express $\vec{V}_{PO}^{outw}(a, \hat{k}_{in}, \vec{k})$ as the product of an Huygens' source like term and the Airy distribution as follows

$$\vec{V}_{PO}^{outw}(a, \hat{k}_{in}, \vec{k}) = Airy_{in}(\hat{k}_{in}, \vec{k}) \vec{H}(\hat{k}_{in}, \vec{k}) \quad \text{(A9)}$$

where

$$\vec{H}(\hat{k}_{in}, \vec{k}) = \frac{jk}{4\pi} \hat{k} \times [\vec{E}_{inc}^{pw} \times (\hat{k} + \hat{k}_{in})] \quad \text{(A10)}$$

since $(\bar{\bar{I}} - \hat{k}\hat{k})\, \vec{E}_{inc}^{pw} = \vec{E}_{inc}^{pw} - \hat{k}(\vec{E}_{inc}^{pw} \cdot \hat{k}) = \hat{k} \times (\vec{E}_{inc}^{pw} \times \hat{k})$

The advantage of this second representation is that the dependence from the incident plane wave is concentrated in a single term, $\vec{H}_{in}(\hat{k}_{in}, \vec{k})$. This term is the spectrum of the field radiated by an elementary Huygen source, of unitary amplitude, i.e. defined only by the amplitude of the electric field characterizing the plane wave that defines it, in the origin of the reference system.

APPENDIX B

The power, P_{PO}, radiated by the PO currents in (30) can be related to the effective area via the definition of the directivity. The directivity in the direction of maximum radiation, $\vec{k} = \vec{k}_{in}$, of the scattered field by the PO currents can be expressed as

$$D_{PO} = 4\pi \frac{\frac{1}{2\zeta} \left|\vec{V}_{PO}^{outw}(a, \hat{k}_{in}, \vec{k}_{in})\right|^2}{P_{PO}} \quad \text{(B1)}$$

and correspondingly the effective area, assuming no losses associated to the same currents is

$$A_{PO} = \frac{\lambda^2}{4\pi} D_{PO} = \lambda^2 \frac{\frac{1}{2\zeta} \left|\vec{V}_{PO}^{outw}(a, \hat{k}_{in}, \vec{k}_{in})\right|^2}{P_{PO}} \quad \text{(B2)}$$

Inverting (B2) the radiated power can be expressed as

$$P_{PO} = \lambda^2 \frac{\frac{1}{2\zeta} \left|\vec{V}_{PO}^{outw}(a, \hat{k}_{in}, \vec{k}_{in})\right|^2}{A_{PO}} \quad \text{(B3)}$$

From (A9), we can derive the value of the amplitude of the outward scattered field in the direction of maximum radiation:

$$\vec{V}_{PO}^{outw}(a, \vec{k}^{in}, \vec{k}^{in}) = \frac{-j}{\lambda} \vec{E}_{inc}^{pw} A_{ph} \quad \text{(B4)}$$

where A_{ph} is the cross section of the spherical antenna domain. And accordingly:

$$P_{PO} = \frac{A_{ph}}{A_{PO}} \frac{1}{2\zeta} \left|\vec{E}_{inc}^{pw}\right|^2 A_{ph} \quad \text{(B5)}$$

The ideal antenna currents are defined in (30) to be the PO currents, (A6-7), times an amplification factor C. The power scattered by these currents can therefore be expressed as $P_{scat}^{ideal} = C^2 P_{PO}$. Accordingly,

$$P_{scat}^{ideal} = C^2 \frac{A_{ph}}{A_{PO}} \frac{1}{2\varsigma} \left|\vec{E}_{inc}^{pw}\right|^2 A_{ph} \quad \text{(B6)}$$

By equating the scattered power in (B6) to the received power as defined in (29), we can find the value of the amplification factor to be

$$C = \frac{A_{PO}}{A_{Ph}} \quad \text{(B7)}$$

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Nuria Llombart (S'06–M'07–SM'13) received the Electrical Engineering and Ph.D. degrees from the Polytechnic University of Valencia, Spain, in 2002 and 2006, respectively. During her Master's degree studies she spent one year at the Friedrich-Alexander University of Erlangen-Nuremberg, Germany, and worked at the Fraunhofer Institute for Integrated Circuits, Erlangen, Germany. From 2002 until 2007, she was with the Antenna Group, TNO Defence, Security and Safety Institute, The Hague, The Netherlands, working as a Ph.D. student and afterwards as a Researcher. From 2007 until 2010, she was a Postdoctoral Fellow at the California Institute of Technology, working for the Submillimeter Wave Advance Technology Group, Jet Propulsion Laboratory, Pasadena, USA. She was a Ramón y Cajal Fellow at the Optics Department of the Complutense University of Madrid, Spain, from 2010 to 2012. In September 2012, she joined the THz Sensing Group at the Technical University of Delft, The Netherlands, where she is currently an Associate Professor.

She has co-authored over 150 journal and international conference contributions. Her research interests include the analysis and design of integrated antennas and quasi-optical systems, with emphasis in periodic structures, leaky wave antennas, and, lens antennas operating in the THz domain. Dr. Llombart was co-recipient of the H.A. Wheeler Award for the Best Applications Paper of the year 2008 in the IEEE Transactions on Antennas and Propagation, the 2014 THz Science and Technology Best Paper Award of the IEEE Microwave Theory and Techniques Society and several NASA awards. She also received the 2014 IEEE Antenna and Propagation Society Lot Shafai Mid-Career Distinguished Achievement Award. She has served as associate editor for the AP and MTT Societies, she currently serves as a Board Member of the IRMMW-THz International Society since 2010. In 2015, she was awarded with an European Research Council (ERC) Starting Grant.

Angelo Freni (S'90-M'91-SM'03) received the Laurea (Doctors) degree in Electronics Engineering from the University of Florence in 1987. From 1990 to 2002 he was Assistant Professor of Electromagnetism at the Department of Electronic Engineering of the University of Florence, Italy. In 2002 he was appointed Associate Professor of Electromagnetism and in 2014 he obtained the Full Professor qualification. From 1995 to 1999 he has also been an Adjunct Professor at the University of Pisa, and from 2012 a Visiting Professor at the TU Delft University of Technology, Delft, The Netherlands. During 1994, he was involved in research at the Engineering Department of the University of Cambridge, UK, concerning the extension and the application of the finite element method to the electromagnetic scattering from periodic structures. Between 2009 and 2010 he also spent one year as a researcher at the TNO Defence, Security and Safety, The Hague, The Netherlands, working on the electromanetic modeling of kinetic inductance devices and their coupling with array of slots in THz range.

His research interests include meteorological radar systems, radiowave propagation, numerical and asymptotic methods in electromagnetic scattering and antenna problems, electromagnetic interaction with moving media and remote sensing. In particular, part of his research concerned numerical techniques based on the integral-equation, with focus on domain-decomposition and fast solution methods.