Robust Unit Commitment with Dispatchable Wind

Germán Morales-España, Álvaro Lorca, Laura Ramírez-Elizondo, and Mathijs M. de Weerdt

Abstract—The increasing penetration of uncertain generation such as wind and solar in power systems imposes new challenges to the Unit Commitment (UC) problem, one of the most critical tasks in power systems operations. The two most common approaches to address these challenges — stochastic and robust optimization — have drawbacks that prevent or restrict their application to real-world systems. This paper demonstrates that an adaptive robust UC in which, by considering wind dispatch flexibility, the second-stage problem, usually being non-convex, can be represented with an equivalent linear program (LP). Consequently, the full two-stage robust UC formulation, which is typically a bi-level problem, can be translated into an equivalent single-level mixed-integer program. Experiments on the IEEE 118-bus test system show that the computation time, and the number of scenarios and violations can be significantly reduced in the unified stochastic and robust approach compared to a pure stochastic approach. In this paper, the formulation is evaluated considering dispatchable wind (i.e., allowing wind curtailment), but it can be applied to any uncertain source with the possibility of being curtailed.

Index Terms—Stochastic optimization, robust optimization, dispatchable wind, unit commitment.

I. INTRODUCTION

In recent years, higher penetration of variable and uncertain generation (e.g., wind and solar power) has challenged independent system operators (ISOs) to keep a reliable and still economical operation of power systems. To achieve this and to be prepared for future demand, ISOs decide about start-up and shutdown schedules of power generating units some time (typically a day) up front by solving the so-called unit commitment (UC) problem, whose main objective is to minimize operational costs while meeting power system constraints. High levels of variable and uncertain generation significantly increases the uncertainty in the net forecasted future demand, and thereby also increases the complexity of the UC optimization problem [1].

The two main approaches for dealing with the uncertainty in UC problems are stochastic and robust optimization. Stochastic optimization (SO) is a generalization of standard optimization, but typically the objective is to minimize the expected costs over a set of possible future scenarios. However, SO is considered impractical by most ISOs, mainly because of its computational burden [1]. Additionally, the main goal for ISOs is to ensure a safe operation of the network, and SO does not give sufficient guarantees on meeting the related network constraints in all uncertainty realizations. Moreover, SO requires a large number of scenarios to be reliable and their associated probability distribution is also hard to obtain.

In robust optimization (RO) [2], [3] the costs are minimized considering the feasibility of all possible realizations of the modelled uncertainty. Typically the resulting schedules are over-conservative: although the probability of the worst-case event is virtually nil, the chosen schedule is robust for this event, and hence much more costly than what is actually required. Recent work on a budget of uncertainty aims to solve this over-conservatism by modelling a smaller uncertainty set in a flexible way [2]. However, the robust UC typically requires solving a bilevel optimization problem, where the outer level is a mixed-integer linear programming (MIP) problem, and the inner level is usually a bilinear program. Such a problem is typically solved using ad-hoc heuristics which can only guarantee local optimality [2], [4]. Although there are different alternatives to solve the robust UC [5], [6], finding optimal robust solutions in time is still a major challenge.

The principal contributions of this paper are as follows:

1) By allowing wind curtailment for a basic (box) uncertainty set for wind, we prove that the second stage of a fully adaptive robust UC problem which is typically non-linear and NP-hard [2] has an equivalent LP formulation, which can be solved in polynomial time.

2) Consequently, the fully adaptive two-stage robust formulation can be translated into an equivalent single-level MIP problem. This allows solving realistically-sized problem instances very close to global optimality significantly faster than the traditional bi-level robust UC, which relies on approximations using ad-hoc heuristics to solve a bilinear program and can only guarantee local optimality [2], [3], [6].

3) To deal with the RO over-conservatism, we use a unified stochastic-robust (SR) optimization approach [3], which dramatically improves the SO performance when considering few scenarios. Moreover, the results of this paper allow us to link the wind dispatch constraints between the stochastic and the robust parts, thus further increasing the robustness of the SR formulation. Unlike [3], the computational burden of the final SR formulation, which is single-level MIP, remains low since the proposed RO part only adds a single extra scenario to the SO part.

4) Experiments confirm that, compared with a traditional RO including budget of uncertainty, the proposed RO provide similar results but solves significantly faster; and the proposed SR provides a cheaper operation, higher robustness, less wind curtailment, while simultaneously having lower computational burden.

The remainder of this paper is organized as follows. Section II details the proposed single-level robust UC reformulation with dispatchable wind, and shows how to complement stochastic UC by incorporating the robust part. Section III provides and discusses results from several experiments, where a comparison between robust, stochastic and unified UC formulations is made. Finally, relevant conclusions and future works are drawn in Section IV.

II. MATHEMATICAL MODELS AND STRUCTURAL RESULTS

This section formulates the mathematical models and presents a set of results that exploit the structure of the
robust UC with dispatchable wind. Section II-A defines this problem, and Section II-B presents a general structural result that characterizes the subset of elements of the uncertainty set that can achieve the worst case. Section II-C studies this structural result under the widely used budget and box uncertainty sets. Finally, Section II-D studies the consequences of these structural results in the unified stochastic-robust UC.

A. Robust UC with Dispatchable Wind

We formulate the robust UC with dispatchable wind in compact form as the following min-max-min problem:

\[ \min_{x \in X} \left\{ b^\top x + \max_{\xi \in \Xi} \min_{(p, w) \in \Omega(x, \xi)} \left( c^\top p + d^\top w \right) \right\} \] (1)

where

\[ X = \left\{ x \in \{0, 1\}^{3|G||T|} : Ax \leq a \right\} \] (2)

and

\[ \Omega(x, \xi) = \left\{ (p, w) : E p + F w \leq g + G x, w \leq \xi \right\} \] (3a)

\[ \text{w} \leq \xi \] (3b)

Here, \( x \) is a vector of first-stage decisions including the binary on/off, start-up and shut-down decisions of conventional generators, here we follow the 3-binary setting UC formulation [7]. These decisions are constrained through set \( X \) defined in (2), which includes the logical relations between on/off, start-up and shut-down variables, as well as minimum up and down times. In (2), \( G \) is the set of conventional generators and \( T \) is the set of time periods. Vector \( \xi \) contains all uncertain parameters in the problem, corresponding to the availability of wind power at all wind farms and time periods, i.e., \( \xi = (\xi_{it} : i \in W, t \in T) \), where \( \xi_{it} \) is the available wind power at bus \( i \) and time \( t \), and \( W \) is the set of buses containing wind production. Set \( \Xi \) is an uncertainty set that determines the allowed realizations of \( \xi \). Vectors \( p, w \) are second-stage power dispatch decisions for conventional generators and wind farms, respectively, i.e., \( p = (p_{it} : g \in G, t \in T) \) and \( w = (w_{it} : i \in W, t \in T) \), where \( p_{it} \) and \( w_{it} \) are the power output of conventional generator \( g \) and of wind farms at bus \( i \), at time \( t \), respectively. These power dispatch decisions are constrained through set \( \Omega(x, \xi) \) defined in (3). In \( \Omega(x, \xi) \), eq. (3a) involves dispatch-related constraints such as power output bounds for conventional generators, nonnegativity of power output at wind farms, ramping constraints, transmission line capacity constraints and energy balance constraints. Eq. (3b) represents the upper bound for power output at wind farms, depending on available wind power, that is, \( w_{it} \leq \xi_{it} \) for all \( i, t \). Finally, the objective function of problem (1) consists of minimizing the sum of no-load, start-up and shut-down costs, given by \( b^\top x \), and worst-case dispatch costs, given by the inner max-min problem, where \( c \) and \( d \) contain the production costs of conventional generators and wind farms, respectively.

The Robust UC problem (1) is an adaptive robust optimization problem [8]. In this problem, \( p \) and \( w \) are adaptive decision variables whose values can depend on the realization of the vector of uncertain parameters \( \xi \), while \( x \) is a “here-and-now” decision that is taken before \( \xi \) is realized. This adaptive robust framework for the UC problem was first proposed in [2], [9], [10]. This section focuses on studying the consequences of considering wind power to be dispatchable, that is, that wind power output \( w_{it} \) can take any value between 0 MW and its availability \( \xi_{it} \), which is very important in power systems with a very large wind power presence. In what follows, we provide a general result that characterizes a subset of the uncertainty set that necessarily contains the worst-case realization of \( \xi \).

B. Worst-case is achieved in the set of minimal elements

The inner max-min problem in the robust UC (1)

\[ \max_{\xi \in \Xi} \min_{(p, w) \in \Omega(x, \xi)} \left( c^\top p + d^\top w \right) \] (4)

has a very special structure: the only dependence of \( \Omega(x, \xi) \) on \( \xi \) is through constraint (3b), namely, \( w \leq \xi \). We can use this structure to characterize a subset of \( \Xi \) where the worst-case must lie. We will need the following definition. The set of minimal elements of \( \Xi \) is given by:

\[ \text{ME}(\Xi) = \left\{ \xi \in \Xi : \hat{\varphi} \in \Xi \text{ s.t. } \varphi \leq \xi \text{ and } \varphi \neq \xi \right\} \] (5)

that is, \( \text{ME}(\Xi) \) corresponds to the elements \( \xi \) of \( \Xi \) for which there is no other element in \( \Xi \) that is less than or equal to \( \xi \) in all components, and strictly less in at least one.

An important property of \( \text{ME}(\Xi) \) that we will need is given in the following Lemma:

**Lemma 1.** Suppose that \( \xi^* \in \Xi \setminus \text{ME}(\Xi) \), then there exists \( \xi \in \text{ME}(\Xi) \) such that \( \xi \leq \xi^* \).

**Proof:** Define the subset \( S \) of all elements \( \varphi \in \Xi \) such that \( \varphi \leq \xi^* \) and \( \varphi \neq \xi^* \). This set is nonempty by definition of \( \xi^* \). Next, consider an element \( \xi \in \text{ME}(\Xi) \). It thus holds that \( \xi \leq \xi^* \). To show that \( \xi \in \text{ME}(\Xi) \), suppose there is an element \( \varphi \in \Xi \) such that \( \varphi \leq \xi \) and \( \varphi \neq \xi \). Then \( \varphi \in S \), because \( \varphi \leq \xi \leq \xi^* \). However, this contradicts that \( \xi \in \text{ME}(\Xi) \). Therefore, there is no such \( \varphi \) and thus \( \xi \in \text{ME}(\Xi) \).

One important result that leads to significant computational savings is that the worst-case in the robust UC with dispatchable wind (1) has to be within the set of minimal elements of the uncertainty set, below we prove this main structural result:

**Proposition 2.** The following equality holds:

\[ \max_{\xi \in \Xi} \min_{(p, w) \in \Omega(x, \xi)} \left( c^\top p + d^\top w \right) = \max_{\xi \in \text{ME}(\Xi)} \min_{(p, w) \in \Omega(x, \xi)} \left( c^\top p + d^\top w \right). \] (6)

**Proof:** Define here

\[ f(\xi) = \min_{(p, w) \in \Omega(x, \xi)} \left( c^\top p + d^\top w \right). \]

Since \( \text{ME}(\Xi) \subset \Xi \) it follows that

\[ \max_{\xi \in \Xi} f(\xi) \geq \max_{\xi \in \text{ME}(\Xi)} f(\xi). \]
Thus, we only need to show the reverse inequality. Let \( \xi^* \in \Xi \) be the optimal solution of
\[
f(\xi^*) = \max_{\xi \in \Xi} f(\xi).
\]
If \( \xi^* \in \text{ME}(\Xi) \) then we have
\[
\max_{\xi \in \Xi} f(\xi) = f(\xi^*) \leq \max_{\xi \in \text{ME}(\Xi)} f(\xi).
\]
Otherwise, if \( \xi^* \not\in \text{ME}(\Xi) \), by Lemma 1 there exists \( \hat{x} \in \text{ME}(\Xi) \) such that \( \xi \leq \xi^* \). This directly implies that \( \Omega(x, \xi^*) \subseteq \Omega(x, \xi) \) due to the definition of \( \Omega(x, \cdot) \). And consequently given the definition of \( f(\cdot) \) we have that \( f(\xi^*) \geq f(\xi) \). Thus
\[
\max_{\xi \in \Xi} f(\xi) = f(\xi^*) \leq f(\xi) \leq \max_{\xi \in \text{ME}(\Xi)} f(\xi)
\]
which completes the proof.

C. Results for the box and uncertainty sets

The structural result in Proposition 2 is not very useful if we cannot characterize \( \text{ME}(\Xi) \) in explicit form. Fortunately, for the most widely used uncertainty set, the budget uncertainty set, this is possible. The budget uncertainty set is defined as
\[
\Xi_{\text{bud}}(\Delta) = \left\{ \xi : \xi_{it} - \hat{\xi}_{it} \leq \xi_{it} \leq \hat{\xi}_{it} + \xi_{it} \quad \forall i \in W, t \in T \right\}
\]
where \( \hat{\xi}_{it} \) is the nominal (or forecast) available wind power at bus \( i \), time \( t \). \( \xi_{it} \) is the allowed deviation of available wind power from its nominal value, and \( \Delta_t \in [0, |W|] \) is the budget of uncertainty at time \( t \in T \), determining the size of the uncertainty set and thus the conservativeness of the robust UC (1).

Now, let us consider the following set:
\[
\Xi_{\text{bud}}(\Delta) = \left\{ \xi : \xi_{it} - \hat{\xi}_{it} \leq \xi_{it} \leq \hat{\xi}_{it} + \xi_{it} \quad \forall i \in W, t \in T \right\}
\]
\[
\sum_{i \in W} \xi_{it} - \hat{\xi}_{it} = -\Delta_t \quad \forall t \in T \right\}.
\]
(8)

The following Proposition gives a complete characterization of the set of minimal elements of the budget uncertainty set.

**Proposition 3.** The following holds: \( \text{ME}(\Xi_{\text{bud}}) = \Xi_{\text{bud}} \).

**Proof:** Let us first show that \( \Xi_{\text{bud}} \subseteq \text{ME}(\Xi_{\text{bud}}) \). Consider \( \xi \in \Xi_{\text{bud}} \). Suppose that there exists \( \varphi \in \Xi_{\text{bud}} \) such that \( \varphi \leq \xi \) and \( \varphi \neq \xi \). That is, there must be some \( i' \), \( t' \) such that \( \varphi_{i't'} < \xi_{i't'} \). Then, we must have
\[
\sum_{i \in W} \varphi_{it} - \hat{\xi}_{it} < \sum_{i \in W} \xi_{it} - \hat{\xi}_{it} = -\Delta_t
\]
which implies that
\[
\sum_{i \in W} \frac{\varphi_{it'} - \hat{\xi}_{it'}}{\xi_{it'}} > \Delta_t
\]
which is a contradiction, since \( \varphi \in \Xi_{\text{bud}} \). Therefore, there is no \( \varphi \in \Xi_{\text{bud}} \) such that \( \varphi \leq \xi \) and \( \varphi \neq \xi \), which means that \( \xi \in \text{ME}(\Xi_{\text{bud}}) \).

Let us now show that \( \text{ME}(\Xi_{\text{bud}}) \subseteq \Xi_{\text{bud}} \). Let us take \( \xi \in \Xi_{\text{bud}} \setminus \Xi_{\text{bud}} \); with this, we only need to prove that \( \xi \notin \text{ME}(\Xi_{\text{bud}}) \). Since \( \xi \notin \Xi_{\text{bud}} \) there must be a \( t' \) such that
\[
\sum_{i \in W} \xi_{it'} - \hat{\xi}_{it'} > -\Delta_{t'}
\]
which implies that there must be an \( i' \) such that \( \xi_{i't'} > \hat{\xi}_{i't'} - \xi_{i't'} \). Now, consider
\[
\nu = \max \left\{ \xi_{i't'} - \hat{\xi}_{i't'}, \xi_{i't'} - \xi_{i't'} \left( \Delta_{t'} + \sum_{\xi_{i't'} \notin \Xi_{\text{bud}}} \xi_{it'} - \hat{\xi}_{it'} \right) \right\}
\]
and with this, consider \( \varphi \) such that \( \varphi_{it} = \xi_{it} \) for all \( (i, t) \neq (i', t') \), and \( \varphi_{i't'} = \nu \). By construction we have that \( \varphi \in \Xi_{\text{bud}} \), \( \varphi \leq \xi \), and \( \varphi \neq \xi \). With this, we have \( \xi \notin \text{ME}(\Xi_{\text{bud}}) \).

We summarize our findings with the following Lemma that follows directly from Propositions 2 and 3:

**Lemma 4.** The following equality holds:
\[
\max_{\xi \in \Xi_{\text{bud}}(p, w) \in \Omega(x, \xi)} \left( c^T p + d^T w \right) = \max_{\xi \in \Xi_{\text{bud}}(p, w) \in \Omega(x, \xi)} \left( c^T p + d^T w \right).
\]
At this point, it is important to understand the computational consequences of the above Lemma. How much “simpler” is \( \Xi_{\text{bud}} \) than \( \Xi_{\text{bud}} \)? One way to look at this is through the number of variables and constraints required to represent these polyhedral sets. We can first observe that to formulate \( \Xi_{\text{bud}} \) we require \(|W||T|\) variables, \(|W||T|^2\) inequality constraints, and \(|T|\) equality constraints. By removing equality constraints, we can also formulate this polyhedral set using \(|W||T - T|\) variables, and \(|W||T|^2\) inequality constraints. Now, in order to efficiently represent the budget uncertainty set \( \Xi_{\text{bud}} \) we can use an extended formulation of this set to handle the absolute values in its formulation. In fact, \( \Xi_{\text{bud}} \) is the projection of a set with \(|W||T|^2\) variables and \(|3|W||T + T|\) inequality constraints (ALVARO: Shouln’t this be \(|4|W||T + T|\) Constraints?). As a summary, \( \Xi_{\text{bud}} \) requires a total of \(|5|W||T + T|\) variables and inequality constraints, while \( \Xi_{\text{bud}} \) only requires a total of \(|3|W||T - T|\) variables and inequality constraints.

Finally, an important special case arises when the budget uncertainty set corresponds to a “box” set. This case is obtained when \( \Delta_t = |W| \) for all \( t \). In this case, the set of minimal elements of the budget uncertainty set contains a single element and the overall max-min problem consequently collapses to a single minimization problem. The next Theorem summarizes this result.

**Theorem 5.** If \( \Delta_t = |W| \) for all \( t \) then \( \Xi_{\text{bud}} = \left\{ \xi - \xi \right\} \), the robust UC with dispatchable wind (1) becomes
\[
\min_{x \in X} \left\{ b^T x + \max_{\xi \in \Xi_{\text{bud}}(p, w) \in \Omega(x, \xi)} \left( c^T p + d^T w \right) \right\}
\]
\[
= \min_{x \in X, (p, w) \in \Omega(x, \xi)} b^T x + c^T p + d^T w. \]
The robust UC formulation (1) including budget of uncertainty (7) is given by \( \xi = \hat{\xi} - \hat{\xi} \). The rest follows directly from Lemma 4.

D. Consequences in the Unified Stochastic-Robust UC

The unified stochastic-robust UC is defined as

\[
\min_{x \in X} \left\{ b^\top x + (1 - \alpha) \max_{\xi \in \Xi} \min_{(p,w) \in \Omega(x,\xi)} \left( c^\top p + d^\top w \right) \right. \\
+ \alpha \sum_{s=1}^{S} \pi_s \min_{(p_s,w_s) \in \Omega(x,\xi)} \left( c^\top p_s + d^\top w_s \right) \right\},
\]

(10)

where there are two characterizations of uncertainty for available wind power \( \xi \); the uncertainty set \( \Xi \), as in problem (1), and \( S \) scenarios \( \xi_s \), with respective probabilities \( \pi_s \). Now, for the stochastic part, \( p_s \) and \( w_s \) are the dispatch decisions of thermal and wind units for scenario \( s \), respectively. The objective is to minimize the sum of commitment costs \( b^\top x \), and a weighted combination of worst-case dispatch cost (with weight \( 1 - \alpha \)) and expected cost (with weight \( \alpha \)), where parameter \( \alpha \in [0,1] \).

This unified stochastic-robust UC approach was first proposed in [3]. In this paper, we explore the consequences of the special structure of wind dispatchability in the problem, to show that under this setting a very powerful and efficient stochastic-robust UC approach is obtained with just a few scenarios, exploiting the structure of the max-min problem representing worst-case dispatch costs.

Finally, since the worst-case wind dispatch scenario is composed by the set of minimal elements, we can then relate the dispatch of the stochastic and worst-case scenarios as

\[
w_s \geq w, \quad \forall s \in S
\]

(11)

which guarantees that all the wind dispatch scenarios for the stochastic part are greater than or equal to the worst-case wind dispatch of the robust part, thus ensuring that the stochastic solution is indeed protected by the robust solution. Therefore, any uncertain wind realization above \( w \) is protected since, in the worst-case, it can be curtailed to \( w \). This relation between the stochastic and robust parts through (11) significantly improve the robustness of the pure unified robust-stochastic solution, as shown in Section III-C2.

### III. Numerical Results

To validate the proposed formulations with dispatchable wind, we compare the performance of the following models:

**RO:** The robust UC formulation proposed in this paper, see (9) in Theorem 5, where its level of conservatism is adjusted by shrinking the uncertainty set. For this, we introduce the parameter \( \pi \) to control the level of conservatism: \( [\hat{\xi} - \hat{\xi}] \pi, \hat{\xi} + [\hat{\xi}] \pi \). Since for RO, the worst-case scenario lies on the lower bound of the uncertainty set, then by changing \( \pi \) from 0 to 1 the worst-case scenario changes from \( \hat{\xi} \) to \( [\hat{\xi} - \hat{\xi}] \).

**ROB:** The robust UC formulation (1) including budget of uncertainty (7). Similarly to RO, the level of conservatism is controlled by \( \pi \in [0,1] \), thus expressing the budget of uncertainty as \( \Delta_t = \pi \cdot |W| \). The two-level optimization problem is solved using a column-and-constraint generation algorithm and the alternating direction method [2], [6], further details and advantages compared with other methods are discussed in [5], [6].

**SO:** The stochastic UC formulation.

**SR:** The proposed unified stochastic-robust UC formulation (10) and (11), where RO is used for the robust part.

**SRB:** The unified stochastic-robust UC formulation (10) and (11), where ROB is used for the robust part.

After detailing the experimental setup, this section studies the impact of different scenarios on the stochastic and unified formulations. For SR and SRB, we then stable the weight \( \alpha \) for the stochastic part in the objective function. Also we analyse the effect of changing the penalty for wind curtailment. Finally, we compare the performance of the robust and unified formulations when changing their level of conservatism.

#### A. Experimental Setup

As a case study, we use the IEEE 118-bus test system, which was adapted to consider startup and shutdown power trajectories [11]. This system has 186 transmission lines, 54 thermal generator units, 91 loads, and three buses with wind production. The penalty costs for demand-balance and transmission-limits violations are set to 10000 $/MWh and 5000 $/MWh [12], respectively. In addition, wind curtailment is penalized as 300 $/MWh (about ten times the average wind bid in some US markets [13], [14]) to simulate cases where wind curtailment is not desired.

All experiments are solved using CPLEX 12.6.3 with default parameters [15] and are run on an Intel-Xeon 3.7-GHz personal computer with 16 GB of RAM memory. All instances are solved until they reach an optimality tolerance of \( 5 \cdot 10^{-4} \).

To compare the performance of the different UC models, we make a clear distinction between the scheduling and (out-of-sample) evaluation stages. The scheduling stage solves the different UC models and obtains their commitment policy...
using a small representative number of wind scenarios (up to 50) for the stochastic formulations, and uncertainty sets for the robust formulations. The evaluation stage, for each fixed commitment policy, solves a network-constrained economic dispatch problem repetitively for a set of 1000 out-of-sample wind scenarios (see Fig. 1), thus obtaining an accurate estimate of the expected performance of each UC policy.

Wind production uncertainty is characterized with an uncertainty set for the robust formulations and with scenarios for the stochastic formulations. We assume that wind production follows a multivariate normal distribution, and it is truncated for nonnegative values. For the robust formulations RO, ROB, SR and SRB, the uncertainty set is defined by a nominal value $\hat{\xi}$ and a deviation $\tilde{\xi}$ corresponding to 98.8% confidence level (2.5 standard deviations). For the stochastic formulations SO, SR and SRB, the scenarios are generated following a predicted wind production.

We use Latin Hypercube Sampling (LHS) to generate scenarios for the uncertain wind power production. The main advantage of LHS is that it optimally distributes the samples aiming to explore the whole area in the experimental region, avoiding the creation of scenarios that are too similar (clusters) [16]. That is, for a given number of scenarios N, LHS generates them reducing their correlation and maximizing the minimum distance between them [17]. Fig. 1 shows some set of the scenarios generated for one of the three buses containing wind production.

B. Robust, Stochastic, and Unified

Table I shows an overview of the problem size for RO, and for SO and SR for 1, 5 and 30 scenarios. The sizes of ROB and SRB are not listed in Table I since they are not constant, they start with an initial size similar to RO and SR, respectively, but they increase through the iterations of the column-and-constraint generation algorithm [4]. Note that for a given number of scenarios, SR is slightly larger than SO because SR adds an extra scenario representing the worst-case scenario. Despite the number of modelled scenarios, all the UCs have the same number of binary variables, because they all obtain the same quantity of binary (first-stage) decisions.

In this section, we set the level of conservatism ($\pi$) of RO, ROB, SR and SRB at 0.6, 0.5, 0.6 and 0.4, respectively. As further discussed Section III-C, the formulations performed the best at these levels of conservatism.

1) Stochastic (SO) vs. Robust (RO and ROB) formulations: In Table II, we assess the performance of the different UC policies on five aspects. For the scheduling stage: 1) the fixed production costs (FxdCost [k$]), which includes non-load, startup and shutdown costs, and 2) the time required to solve the UC problem (CPU Time [s]). For the evaluation stage we record 3) the average of the total production costs including the wind curtailment penalization (AvgTC [k$]); 4) the maximum total cost of the 1000 out-of-sample scenarios, representing the worst-case scenario (WorstTC); 5) the total accumulated number of violations in both demand-balance and transmission-limits constraints (# Viol); and 7) the average percentage of wind that was curtailed (% WCurt).

We can clearly observe the differences widely discussed in the literature, e.g., [2], [3]. First, as expected, the higher the number of scenarios, the better the SO performance (lower AvgTC and Viol). Second, on the one hand, the stochastic formulation SO using 30 scenarios guarantees robustness (Viol=0), and a SO with a higher number of scenarios presents a small improvement at the expense of higher computational cost: from 30 to 50 scenarios, SO improves AvgTC in less than 0.01% and takes more than 2x longer to solve. On the other hand, the robust formulations RO and ROB guarantee robustness by only optimizing for the worst-case scenario, but they scheduled too few reserves (lower FxdCost) hence not ensuring that higher wind production levels could be dispatched (WCurt was around 10x larger than SO). The robust formulations ignore the possibility of optimistic (high wind) scenarios hence not taking advantage of them. However, compared with SO with 30 scenarios and ROB, the formulation RO proposed in this paper could be solved more than an order of magnitude faster (above 71x and 13x, respectively). Furthermore, since RO only considers one scenario, like a pure deterministic formulation (SO with one scenario, see Table I), it solves in similar time as SO with one scenario, but RO reduced the violations to zero, lowered the AvgTC and WorstTC by 20.5% and 83.4%, respectively, although increased WCurt by more than twice.

### Table I

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### Table II

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<td>38.0</td>
</tr>
<tr>
<td>RO</td>
<td>63.20</td>
<td>184.8</td>
</tr>
<tr>
<td>RO</td>
<td>65.89</td>
<td>277.0</td>
</tr>
<tr>
<td>RO</td>
<td>66.33</td>
<td>266.8</td>
</tr>
<tr>
<td>RO</td>
<td>65.04</td>
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</tr>
<tr>
<td>RO</td>
<td>64.52</td>
<td>980.6</td>
</tr>
<tr>
<td>RO</td>
<td>65.46</td>
<td>1971.2</td>
</tr>
</tbody>
</table>

### Notes

1. Stochastic (SO) vs. Robust (RO and ROB) formulations: In Table II, we assess the performance of the different UC policies on five aspects. For the scheduling stage: 1) the fixed production costs (FxdCost [k$]), which includes non-load, startup and shutdown costs, and 2) the time required to solve the UC problem (CPU Time [s]). For the evaluation stage we record 3) the average of the total production costs including the wind curtailment penalization (AvgTC [k$]); 4) the maximum total cost of the 1000 out-of-sample scenarios, representing the worst-case scenario (WorstTC); 5) the total accumulated number of violations in both demand-balance and transmission-limits constraints (# Viol); and 7) the average percentage of wind that was curtailed (% WCurt).

2. B. Robust, Stochastic, and Unified: Table I shows an overview of the problem size for RO, and for SO and SR for 1, 5 and 30 scenarios. The sizes of ROB and SRB are not listed in Table I since they are not constant, they start with an initial size similar to RO and SR, respectively, but they increase through the iterations of the column-and-constraint generation algorithm [4]. Note that for a given number of scenarios, SR is slightly larger than SO because SR adds an extra scenario representing the worst-case scenario. Despite the number of modelled scenarios, all the UCs have the same number of binary variables, because they all obtain the same quantity of binary (first-stage) decisions.

3. In this section, we set the level of conservatism ($\pi$) of RO, ROB, SR and SRB at 0.6, 0.5, 0.6 and 0.4, respectively. As further discussed Section III-C, the formulations performed the best at these levels of conservatism.
2) Stochastic (SO) vs. Unified robust-stochastic (SRB) formulations: Notice in Table I how the robust part of SR drastically improves the performance of the stochastic formulations SO despite the quantity of scenarios used. Even when very few scenarios are considered, SR presents a significantly higher performance than SO: for the case of one and five scenarios, SR presented no violations, instead of 698 and 191, a cost reduction of more than 24% and 4%, and a worst-case (WorstTC) reduction of more than 84% and 71%, respectively.

The most important part, however, is that, compared with SO, adding the worst-case scenario into SR comes at almost no extra costs in terms of run time (it was even lower in some cases), where, in average, SR solved all the cases less than 1% faster than SO. Compared with SO with 25 scenarios, which had 50 violations, SR with 5 scenarios presented even lower costs (AvgTC) and a further reduction of volatility (WorstTC), still completely avoiding violations. Moreover, it solved an order of magnitude faster (19.3x) than a comparable solution of SO (with 30 scenarios).

In general, the performance of a stochastic formulation with few scenarios is dramatically improved by adding the worst-case scenario to the formulation. This is because the stochastic part lowers the expected costs and the robust part avoids violations, as also concluded in [3]. Furthermore, the computational burden of SR is not significantly affected, compared to SO.

3) Unified robust-stochastic formulations, SR vs SRB: Notice that, for five and higher number of scenarios, SRB presented similar performance compared to SRB in the out-of-sample evaluation. SRB presented violations when only one (the nominal) scenario was considered. Bear in mind that the level of conservatism of SRB was set to $\pi = 0.4$, which was too low to provide robustness for the one-scenario case; however, for a $\pi = 0.5$, SRB with one scenario could achieve 0 violations, an AvgTC of 769.99 k$, a WorstTC of 890.41 k$, and a WCurt of 0.34%. Although SR and SRB perform similarly in the out-of-sample evaluation, SR solved more than 6.8x faster because it can be solved directly as single-level MIP problem, instead of requiring ad-hoc algorithms to solve the two-level MIP problem including a bilinear inner problem.

**a) Quantity of scenarios needed by SR and SRB:** Unlike the stochastic SO formulation, which needs a large quantity of scenarios to guarantee robustness, the stochastic-robust formulations SR and SRB are robust despite the quantity of scenarios used. Therefore, few scenarios can be used to obtain a good performance in terms of costs (AvgTC) and wind curtailment (WindCurt). Although SR and SRB using one scenario presents already a better performance than SO using 25 scenarios, henceforth, we use 5 scenarios for the stochastic part of SR and SRB since it further decreases AvgTC, WorstTC and WindCurt, and they still solved the UC problems in less than one and four minutes, respectively. Moreover, considering more than 5 scenarios adds a very little performance improvement at a very high computational cost, e.g., when using 10 scenarios instead of 5, SR and SRB took more than 3x and 2.7x longer to solve, respectively.

**b) Different Objective Weights for SR and SRB:** Here, we aim to establish the optimal balance between the costs of the worst-case component and the other scenarios in the objective function of the SR and SRB formulations. Fig. 2 presents the results of SR and SRB using 5 scenarios for objective weight $\alpha$ ranging from 0 (all weight to worst-case scenario) to 1 (all weight to stochastic scenarios). In addition, similar results where found when 30scenarios where used for the stochastic part.

From the results we observe that as $\alpha$ increases, the average total costs and average curtailment decrease. This is because the problem becomes less conservative when the robust part of SR has a smaller weight. The same behaviour was previously observed in [3]. Similarly to the total average costs, the worst-case cost also decreased as $\alpha$ increases, achieving its lowest value when $\alpha = 1$. The lowest wind curtailment for both SR and SRB was obtained when $\alpha = 0.9$. We can also observe that results are not very sensitive to $\alpha$: in the range [0.2, 1] we see a difference in total costs lower than 1.5%.

The stochastic parts of SR and SRB help to accommodate high values of wind (lower wind curtailment) and to lower the AvgTCW, but when $\alpha = 0$, this effect disappears completely. As a consequence, the SR and SRB solutions tend to become the same as the RO and ROB solutions, respectively (see Table II), but the problem takes much longer to be solved (5x and 8x, respectively) because the SR and SRB formulations are significantly larger (see Table I). In average, all the SR cases were solved in 54.8 seconds and more than an order the magnitude faster (10.4x) than SRB.

It is important to highlight that none of the SR and SRB cases presented any violations, even when the robust part is ignored in the objective function ($\alpha = 1$). This is because the worst-case scenario is still in the set of constraints hence guaranteeing robustness.

In short, as the weight of the worst-case cost component $(1 - \alpha)$ decreases, the average total costs and wind curtailment decrease, which is not surprising as the problem then becomes less conservative. However, when ignoring the worst-case ($\alpha = 1$), the curtailment is a bit higher, so based on these results, we set $\alpha = 0.9$ for the remaining experiments, noting that the results are rather robust against the selected value.

![Fig. 2. Different levels of $\alpha$ for SR and SRB using 5 scenarios. Upper graph: Average total costs over the 1000 out-of-sample scenarios. Lower graph: Average wind curtailment of the 1000 scenarios.](image-url)
4) **Wind Penalization:** The approach we put forward in this paper is enabled by wind curtailment. In some power systems, wind curtailment may be undesirable; however, violations of the demand balance and of transmission-capacity limits are even worse. As explained earlier all these types of violations are penalized in the UC formulation (demand balance with 10000 $/MWh and transmission limits with 5000 $/MWh). The objective of this experiment is to study the effect of different penalties on wind curtailment, which is equivalent to different negative values of wind bids [14].

Fig. 3 shows the performance of the following formulations for different penalties on wind curtailment: the robust formulations RO and ROB, the stochastic-robust formulations RS and RSB with 5 scenarios, and the stochastic formulations SO. In general for a given UC formulation, as the wind curtailment penalization increases, wind curtailment decreases. This results from the need to schedule more resources (reserves) to better accommodate different wind realizations. This, in turn, also increases the average total costs.

When wind curtailment is not penalized (zero penalization) the UC approaches accommodate the least quantity of wind, hence they also carry the least quantity of reserves. Consequently, SO with 30 scenarios reported violations for the case in which the penalization of wind curtailment was set to zero. Therefore, the least quantity of reserves was insufficient to avoid violations in the out-of-sample evaluation stage. Surprisingly, even though RO and ROB were expected to be over-conservative despite the value of wind curtailment penalty, when this penalty is equal to zero, RO and ROB obtained curtailment and average costs similar to the stochastic and stochastic-robust formulations.

For non-zero wind-curtailment penalties, RO and ROB present the highest curtailment. This is a result of their conservative policy, which avoids infeasible solutions but cannot guarantee that a high wind production will be dispatched. This also leads RO and ROB to have the higher average total costs compared to SR, SBR and SO.

In general, higher penalties lead to lower wind curtailment and higher production costs. The robust formulations RO and ROB present very similar performance, but RO solved the UC problems 17x faster. The stochastic and stochastic-robust and formulations also presented similar performance with each other, where SO presented slightly lower costs for penalties different than zero; however, SR solved the problems more than 7x faster than SBR, and more than an order of magnitude faster (20x) than SO.

**C. Level of Conservatism: Budget and Box of Uncertainty**

This section compares the performance of the robust and stochastic-robust formulations when changing their level of conservatism.

1) **Robust Formulations, RO vs ROB:** As detailed at the beginning of this section, the level of conservatism of the robust formulations RO and ROB can be controlled through the parameter $\pi \in [0, 1]$. Fig. 4 shows the average total costs and the worst-case scenario of the 1000 out-of-sample scenarios for RO and ROB when changing the level of conservatism from 0 to 1. Bear in mind that both formulations are exactly equivalent when the budget of uncertainty is 0 (nominal scenario) or 1 (complete box). RO presented violations (served energy & transmission limits) for levels of conservatism (box-size) values lower and equal to 0.5, while ROB presented violations for levels of conservatism (budget of uncertainty) values below 0.4.

For low values of $\pi$, below 0.4, the robustness of ROB dominates presenting fewer violations, hence lower average costs and also lower worst-case scenario (see lower graph in Fig. 4). This is the main advantage of ROB which is protecting against a worst-case scenario even, when $\pi$ is low, considering the full limits of the box of uncertainty, instead of reducing the box completely as RO does.

On the other hand, for levels of conservatism above 0.4, both formulations present similar average and worst-case costs, but RO presents slightly lower average costs because it is less conservative. It is important to highlight that the ideal value of conservatism should be low to avoid expensive operation.
costs due to an over-conservative policy but high enough so avoiding violations completely. That is why we set $\pi = 0.6$ and $\pi = 0.5$ for RO and ROB, respectively, which are the values where the average costs are the lowest while avoiding any possible violation.

2) Unified Stochastic-Robust Formulations: Apart from the previous two stochastic-robust formulations SR and SRB, the following two formulations are also implemented:

SRI: The same as SR but disregarding constraint (11), hence making the set of constraints of the stochastic part completely independent from those of the robust part.

SRBI: The same as SRB but disregarding constraint (11).

Fig. 5 compares the four unified robust-stochastic formulations. When compared with RO and ROB, the highest average cost of the four stochastic-robust formulations (782.29 k$ for SRI and SRBI at $\pi = 0.3$, see upper part of Fig. 5) is lower than the lowest one of RO and ROB (792.12 k$ for RO at $\pi = 0.1$, see upper part of Fig. 4). This is mainly because the stochastic part of the unified formulations helps to minimize expected cost by also accommodating different scenarios of wind (and not just minimal elements of the uncertainty set), which at the end result in lower wind curtailment reducing operating costs, as shown in Fig. 6. That is, pure robust planning leads to considerably more costly operation, even when including budget of uncertainty, with stochastic-robust. Also the robustness of the solution is considerably improved when adding the stochastic part to the robust part: notice that for any given level of conservatism, the highest worst cost scenario of the unified formulations (lower part Fig. 5) were always lower than those of the pure robust formulations (lower part Fig. 4).

Similarly to the pure robust formulations, all unified formulations SR, SRB, SRI and SRBI present similar average costs for levels of conservatism above 0.5; however, between 0.4 and 0.5, the budget of uncertainty made SR and SRB more robust. On the other hand, for values below 0.4, SR and SRB fewer violations and lower worst-case cost (lower part Fig. 5) than SR and SRB.

When adding (11) to the stochastic-robust formulations, the robustness improves considerably for low values of $\pi$, and for for higher values of $\pi$ (above 0.5) SR and SRB are very similar to SRI and SRBI, respectively, because (11) becomes not active. However, for values between 0.4 and 0.5, (11) starts to dominate forcing that the worst-case dispatch wind scenario remains below all the stochastic scenarios even though this worst-case scenario could be very similar to the mean wind value. For values below 0.4, the lower elements of the stochastic scenarios dominate increasing the robustness of the solution despite the low values of $\pi$. That is, the unified formulation protects itself using the lower elements of its own stochastic part.

We set the levels of conservatism to $\pi = 0.6$ and $\pi = 0.4$ for SR and SRB, respectively. These are the values where the average costs are the lowest while avoiding any possible violations.

3) Computational Performance: Notice that although the two formulations RO and ROB solve a robust UC, RO is a single level MIP formulation compared with the two-level MIP of ROB, which also include a bilinear term, hence RO solved more than 6x faster than ROB in average. Moreover, the slowest case of RO ($\pi = 0.5$) solved in 39.8 seconds, was faster than the fastest case of ROB ($\pi = 0.3$), which took 43.3 seconds. These computational comparisons are made for $\pi$ different than 0, since RO and RS are computationally equivalent to ROB and RSB, respectively, when $\pi = 0$. Similarly, although SR and SRB present similar performance in average costs, wind curtailment and robustness, SR solved 3.9x faster where its slowest case ($\pi = 0.5$), solved in 85.9 seconds, was faster than the fastest case of SRB ($\pi = 0.2$), which took 181.2 seconds.

As mentioned above, the robustness of the stochastic-robust formulations is improved by linking the stochastic and robust parts using (11). However, this extra constraint also brings an extra computational burden, resulting in 60%, in average, extra computational burden for SR and SRB.

It is interesting to note that SR outperformed ROB in all aspects, average costs, robustness and wind curtailment, despite the value chosen for $\pi$, while simultaneously SR solved the problems always faster (1.6x in average).

IV. Conclusions and Future Work

This paper presented a single-level mixed-integer linear programming formulation (MIP) for fully adaptive robust unit commitment (UC) problem with dispatchable wind. We
showed that this is possible by allowing wind curtailment and considering a box of uncertainty set for wind. We also showed that the worst-case wind (available capacity) scenario can be found before solving the robust UC. The resulting complete bi-level and bilinear robust UC formulation thus is equivalent to a single-level MIP problem, which solves considerably faster than traditional robust formulations. Moreover, the level of conservatism can be controlled by shrinking the box of uncertainty, providing similar robustness as a formulation considering a budget of uncertainty, as shown in the numerical experiments. Furthermore, we also showed that our formulation reduce the complexity of the robust UC even when considering budget of uncertainty. The efficient formulation is used as part of an unified stochastic-robust UC formulation [3], thus reducing the over-conservatism of pure robust UC because an expected value is now optimized on a set of scenarios, and it does not require the large quantity of scenarios to guarantee feasibility as pure stochastic UC does. The results of this paper allow us to link the wind dispatch constraints between the stochastic and the robust part further increasing the robustness of the unified formulation. More importantly, the computational burden of the unified approach remains low, since the proposed robust UC solution just adds a single extra scenario to the stochastic UC. Numerical experiments showed that under hundreds of out-of-sample wind scenarios a unified UC with 5 scenarios present a similar performance as a pure stochastic formulation using 30 scenarios, but solving above an order of magnitude faster. Moreover, the proposed unified UC outperformed a traditional pure robust UC, including budget of uncertainty, in all aspects, average costs, robustness and wind curtailment, despite the level of conservatism chosen, while simultaneously the unified UC solved the problems significantly faster. The formulation has been evaluated only for wind curtailment, but it would be useful for any uncertain source that can be curtailed, in particular generation from solar panels. As shown experiments, the robustness of a single-scenario deterministic formulation is greatly improved by adding the worst-case scenario, hence another straightforward application of the results in this paper would be to incorporate the worst-case solution to any deterministic UC formulation based on reserves, thereby greatly improving its robustness without significantly affecting its computational burden.

REFERENCES