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RESEARCH ARTICLE

Fault detection and identification for a class of continuous piecewise affine systems with unknown subsystems and partitions

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Summary

This paper establishes a novel online fault detection and identification strategy for a class of continuous piecewise affine (PWA) systems, namely, bimodal and trimodal PWA systems. The main contributions with respect to the state-of-the-art are the recursive nature of the proposed scheme and the consideration of parametric uncertainties in both partitions and in subsystems parameters. In order to handle this situation, we recast the continuous PWA into its max-form representation and we exploit the recursive Newton-Gauss algorithm on a suitable cost function to derive the adaptive laws to estimate online the unknown subsystem parameters, the partitions, and the loss in control authority for the PWA model. The effectiveness of the proposed methodology is verified via simulations applied to the benchmark example of a wheeled mobile robot.

KEYWORDS

fault detection and identification, online parameter estimation, piecewise affine unknown systems, unknown partitions

1 | INTRODUCTION

With the increased demand of reliability for control systems, much attention has been devoted by the control community in fault detection techniques for complex systems.¹⁻⁴ Piecewise affine (PWA) systems constitute a special class of complex (in particular, hybrid) systems that has been extensively studied in the literature in many application domains: production control systems,⁵ robotics,⁶ and flight control systems,⁷ among others. A classical problem in the aforementioned application domains is the detection and identification of faults, which might appear in the form of plant structural changes (usually associated to variations in the state matrix) or actuator faults (usually associated to changes in the input matrix). In the classical (nonhybrid) setting, the fault detection and identification (FDI) problem can be reformulated in terms of an estimation problem, ie, it is assumed that faults in the system are reflected in a change of the parameters of the system model.⁸ The situation with PWA systems is, however, more complex than classical estimation because an extra uncertainty might occur, ie, the partitions describing the switching from one mode to another might be uncertain or even change with time. Therefore, FDI of a PWA system involves the estimation of both the parameters of the submodels and the regions (hyperplanes) defining the partition of the state space. In other words, despite the more complex setting, also the FDI for PWA systems can be, in principle, reformulated as a parametric estimation problem.

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With reference to partitioning, two alternative assumptions can be distinguished: the partition is assumed known and fixed a priori or the partition is unknown along with the unknown submodels. For the first case, estimation of the submodels can be carried out using standard linear identification techniques; therefore, no particular challenge appears. For the second case, both the subsystems and the partitions corresponding to each subsystem must be estimated. This issue implies a classification problem where each data point must be associated to the most suitable mode. Then, the regions are shaped to clusters of data where the strict relation among data classification, parameter estimation, and region estimation makes the FDI problem hard to solve.⁹ Despite the challenging task, there is a number of approaches in the literature of PWA systems dealing with this problem: Ferrari-Trecate et al.¹⁰ propose a statistical clustering approach to classify the data points and estimate the submodel parameters in order to reconstruct the polyhedral partition of the regressor domain. Further results dealing with the estimation problem include the Bayesian statistical-based approach,¹¹ the bounded-error procedure,¹² and the mixed-integer programming procedure.¹³ A survey on further recent results for the estimation of PWA systems can be found in the work of Garulli et al.¹⁴ It has to be noted though that the vast majority of results for the estimation of PWA systems focuses on the development of estimation algorithms that work offline, ie, from batches of data.

On the other hand, literature has provided also alternative sets of tools (non necessarily based on parameter estimation) for FDI in complex systems: a brief overview is given here. Model-based tools focusing on the detection and identification of the partial loss of control authority in PWA systems, frequently used to model actuator faults, are studied in the works of Zhou et al.¹⁵ and Ding.¹⁶ Most recently, a robust three-stage unscented Kalman filter is introduced in the works of Xiao et al.,^{17,18} for the simultaneous state and fault estimation of nonlinear systems with unknown inputs. An observer-based fault estimation approach for discrete PWA systems is presented in the work of Xu et al.,¹⁹ whereas Tabatabaeipour and Bak²⁰ provide sufficient conditions in terms of linear matrix inequalities for the input-to-state stability of the estimator. A message passing algorithm for automatically propagating the effects of uncertainties in interconnected bilinear systems and derive probabilistic fault thresholds is proposed in the work of Ferrari et al.²¹ In the work of Bashi et al.,²² a clustering approach based on the maximized expectation algorithm is used, and it is proven to identify effectively sudden or preexisting faults into a hybrid, mixed discrete mode continuous-time state setting. An online learning algorithm using a Lyapunov-based approach is carried out in the work of Trunov and Polycarpou²³ to prove robust fault detection for the case of multiple-input-multiple-output nonlinear systems. Estimation-based and observer-based FDI of PWA systems with parametric uncertainties and known partitions is studied in the works of Baldi et al.²⁴ and Satyavada and Baldi,²⁵ respectively. A map-based approach using parameter-estimation techniques is presented in the work of Schwaiger and Krebs,²⁶ where the unknown parameters are estimated online and they are used to detect faults in the model. A dual estimation scheme is developed in the work of Baldi et al.²⁷ to detect parametric changes with partial state information. A comprehensive review presenting the state-of-the-art FDI methods in the literature and their applications are given in the works of Hwang et al.²⁸ and Samy et al.²⁹

Closely related to FDI, special attention has been devoted to fault-tolerant controller (FTC) synthesis for complex systems, which aim to cope with the identification of partial loss of the control action and compensate for the later in the closed-loop hybrid or PWA systems. FTC architectures can be divided into two main categories: passive FTC methods, which provide controller synthesis proven to guarantee stable performance both when the system works in nominal operation and under faulty conditions, and active FTC methods, which are characterized by the reconfiguration of the controller when faulty conditions are detected.³⁰ In the work of Nayeapanah et al.,³¹ a FTC is proposed to guarantee stabilization and satisfactory system performance in case of partial loss of control authority in the control loop. A reconfigurable control approach for continuous PWA systems susceptible to actuator and sensor faults is given in the work of Richter et al.³²: by solving a set of linear matrix inequalities, this approach is proven robust to closed-loop stability and guarantees reference tracking. Overviews of the diverse FTC schemes and their applications are given by the works of Witczak³³ and Zhang and Jiang.³⁴

None of the aforementioned FTC and FDI approaches can deal with PWA systems with parametric uncertainties in both partitions and in subsystems parameters. Therefore, to the best of the authors' knowledge, there is currently no online FDI technique developed for continuous PWA systems with joint unknown subsystems and partitions. The main contribution of this work is tackling, in a parameter estimation framework, the FDI problem for a class of continuous-time PWA systems, namely, bimodal and trimodal continuous PWA systems, where the subsystems and the partition are jointly unknown. Without loss of generality, the unknown system partition is assumed to be generated by the so-called "centers" as defined in the work of Bako et al.³⁵ By exploiting this particular description, a novel parametric model is derived via the max-form of the PWA system. A cost function depending on the estimation error is derived, which is used to develop a recursive Gauss-Newton algorithm to obtain online the adaptive laws for all the parameters (ie, the subsystem parameters

and the centers). It has to be noted that, differently from the literature on estimation in PWA systems, the developed algorithm is completely online.

Online FDI algorithms produce unknown system estimates at each time instant, by processing and evaluating the current signals measurements. Because of this, they are commonly referred to as recursive FDI algorithms, to be distinguished from the *offline* or nonrecursive ones. For the latter case, also found in the literature as the batch FDI estimation algorithms, all signals' measurements are collected offline over large time interval horizons. In both online or offline methods, the unknown parameters are calculated by using optimization techniques on some appropriately chosen cost function. However, while parameter values estimated using online fault detection and parameter estimation architectures can vary with time as new data arrive, the parameters estimated using offline techniques do not (unless new batches of data are collected). Compared to the offline scheme, online recursive algorithms measure the system's signals continuously so as to update and correct the parameter estimates. Because they can update for fault occurrence incidents in the system and compensate for their resultant detrimental effect while the system is in operation, online estimation algorithms are conceptually superior for fault detection problems when compared to the offline ones.^{36,37}

The rest of this paper is organized as follows. Section 2 presents preliminaries for the PWA system representation and Sections 3 and 4 present the online FDI problem and the main result of this work, for bimodal and trimodal PWA systems, respectively. The effectiveness of the online identification methodology is illustrated via simulations in Section 5. Finally, Section 6 concludes this paper summarizing the main findings and giving some recommendations for future work.

The notations used in this paper are standard:

\mathbb{R} : the set of real numbers;

\mathbb{N} : the set of nonnegative integers;

Given a vector $x = [x_1 \ x_2 \ \cdots \ x_m]^T \in \mathbb{R}^m$, the superscript T denotes its transpose and $\text{diag}(x) = \begin{bmatrix} x_1 & 0 & \cdots & 0 \\ 0 & x_2 & \cdots & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & \cdots & x_m \end{bmatrix}$;

$r_i(X)$ denotes the i th row of matrix X .

2 | PRELIMINARIES IN PWA SYSTEMS

We consider the bimodal PWA system of the form

$$\dot{x} = \begin{cases} A_1x + B_1\Lambda_1u + e_1, & \text{if } (x, u) \in \mathcal{X}_1 \\ A_2x + B_2\Lambda_2u + e_2, & \text{if } (x, u) \in \mathcal{X}_2, \end{cases} \quad (1)$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^m$ is the input, $B_i \in \mathbb{R}^{n \times m}$ are known matrices, $A_i \in \mathbb{R}^{n \times n}$ and $e_i \in \mathbb{R}^n$, are unknown matrices and affine terms, respectively, $\Lambda_i \in \mathbb{R}^{m \times m}$ are unknown diagonal matrices, for $i \in \{1, 2\}$. The term $B_i\Lambda_i$ models partial loss of control authority, and $\{\mathcal{X}_1, \mathcal{X}_2\}$ are polyhedral partitions of the state-input space. The regions \mathcal{X}_1 and \mathcal{X}_2 are polyhedral partitions into \mathbb{R}^{n+m} (the state-input space), generated by the centers as defined in the work of Bako et al.³⁵ In fact, for general PWA systems (non necessarily bimodal), given $N \in \mathbb{N}, N \geq 2$ vectors $c_1, c_2, \dots, c_N \in \mathbb{R}^{n+m}$ representing the centers, for each point $z = [x, u]^T \in \mathbb{R}^{n+m}$ in the state-input space, the polyhedral regions are defined as

$$\begin{aligned} \mathcal{X}_j &= \{z \in \mathbb{R}^{n+m} \mid \|z - c_j\|_2 \leq \|z - c_k\|_2, k \neq j\} \\ &= \{z \in \mathbb{R}^{n+m} \mid \mathcal{A}_j z \leq q_j\}, \end{aligned} \quad (2)$$

where

$$\begin{aligned} \mathcal{A}_j &= 2[c_1 - c_j \ c_2 - c_j \ \cdots \ c_N - c_j]^T, \\ q_j &= [\beta_{1,j} \ \beta_{2,j} \ \cdots \ \beta_{N,j}], \end{aligned}$$

with $\beta_{k,j} = c_k^T c_k - c_j^T c_j$ for $j = 1, 2, \dots, N$. For bimodal PWA systems with partitions \mathcal{X}_1 and \mathcal{X}_2 , we have only two centers, ie, c_1 and c_2 from (2). The regions \mathcal{X}_1 and \mathcal{X}_2 are given by the following relations:

$$\mathcal{X}_1 = \left\{ (x, u) \mid 2(c_2 - c_1)^T \begin{bmatrix} x \\ u \end{bmatrix} - (c_2^T c_2 - c_1^T c_1) \leq 0 \right\} \quad (3a)$$

$$\mathcal{X}_2 = \left\{ (x, u) \mid 2(c_2 - c_1)^T \begin{bmatrix} x \\ u \end{bmatrix} - (c_2^T c_2 - c_1^T c_1) \geq 0 \right\}. \quad (3b)$$

System (1) is an extension in a PWA sense of classical uncertain systems used in adaptive and fault-tolerant control of multivariable linear systems.^{38,39} FDI in classical uncertain systems can be performed by using parameter estimation techniques, eg, by assuming that faults in the system are reflected in a change of the (nonfaulty) parameters in the system model.⁴⁰ A similar idea applies (albeit the more challenging task) to the PWA extension (1): the FDI problem then involves detecting any change in the system parameters of (1), as formulated in the following.

Problem 1. Derive a recursive (online) FDI algorithm with the capability of estimating the unknown system parameters, the unknown loss of control authority, and the unknown partitions of the PWA system (1). In addition, embed in the FDI algorithm a finite-memory (or forgetting) mechanism so as to be able to detect (slowly) changes in the system parameters.

2.1 | Max-form representation of bimodal PWA systems

It is assumed that system (1) is continuous in the state space. By referring to the work of Thuan and Camlibel,⁴¹ continuity of the system is equivalent to the existence and uniqueness of an $h \in \mathbb{R}^n$ such that

$$\begin{bmatrix} A_1 & B_1 \Lambda_1 \end{bmatrix} - \begin{bmatrix} A_2 & B_2 \Lambda_2 \end{bmatrix} = 2h(c_2 - c_1)^T \quad (4a)$$

$$e_1 - e_2 = -h(c_2^T c_2 - c_1^T c_1). \quad (4b)$$

In view of (4), system (1) can be written into its max-form representation as follows:

$$\dot{x} = \begin{bmatrix} A_2 & B_2 \Lambda_2 \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix} + e_2 - h \max \left\{ 2(c_1 - c_2)^T \begin{bmatrix} x \\ u \end{bmatrix} - (c_1^T c_1 - c_2^T c_2), 0 \right\}. \quad (5)$$

One can see that there are infinitely many pairs of centers (c_1, c_2) that can generate the polyhedral regions \mathcal{X}_1 and \mathcal{X}_2 in (3). However, if we fix one center to an arbitrary value, the other center is uniquely determined. Therefore, without loss of generality, we fix the center c_2 to be equal to a given value \tilde{c} and we use the notations c, A, B, e , and Λ in place of c_1, A_2, B_2, e_2 , and Λ_2 , respectively. Then, (5) becomes

$$\dot{x} = Ax + B \text{diag}(u)\lambda + e - h \max \left\{ 2(c - \tilde{c})^T \begin{bmatrix} x \\ u \end{bmatrix} - (c^T c - \tilde{c}^T \tilde{c}), 0 \right\}, \quad (6)$$

where $\lambda \in \mathbb{R}^m$ in (6) is defined in vector form as $\lambda = [\lambda_1 \ \lambda_2 \ \dots \ \lambda_m]^T$, such that $\Lambda = \text{diag}(\lambda)$.

Remark 1. Note that the clear benefit of (6) with respect to (1) is its economy with respect to parameters. In fact, in (1), we need to estimate $2(n^2 + m + n)$ parameters for the subsystems and $(n + m + 1)$ parameters for the partitions: on the other hand, in (6), we have $n^2 + m + n$ parameters for the subsystem $(A, B\Lambda, e)$ and $2n + m$ parameters for h and c . This is because (6) exploits explicitly the continuity of the PWA system.

3 | ONLINE IDENTIFICATION OF BIMODAL PWA SYSTEMS

By following a FDI approach based on parameter estimation as in the work of Simani et al,⁴⁰ Problem 1 for the PWA system (6) can be recast to the minimization of the following cost function:

$$J(t, \hat{\theta}) = \frac{1}{2} \int_0^t e^{-\xi(t-s)} \left\| x(s) - \hat{x}(s, \hat{\theta}) \right\|^2 ds,$$

which can be componentwisely written as

$$J(t, \hat{\theta}) = \frac{1}{2} \int_0^t e^{-\xi(t-s)} \sum_{i=1}^n (\hat{x}_i(s, \hat{\theta}) - x_i(s))^2 ds, \quad (7)$$

where $\xi > 0$ corresponds to the forgetting factor that is a design parameter, θ denotes the unknown parameter that contains all the healthy (nonfaulty) or faulty values of the parameters, which appear in the form of plant structural changes (associated to variations in the state matrix A and the affine vector e), actuator faults (associated to changes in the input

vector λ), or mode partition faults (associated to changes in the vector h and the center c). In addition, after collecting the true parameters in

$$\theta = \begin{bmatrix} \theta_1 \\ \vdots \\ \theta_n \\ \lambda \\ c \end{bmatrix}, \quad \text{with } \theta_i = \begin{bmatrix} r_i(A)^T \\ e_i \\ h_i \end{bmatrix} \quad \text{for } i = 1, 2, \dots, n, \quad (8)$$

where e_i and h_i in (8) are the scalar components of the vectors e and h , we have that $\hat{\theta}$ are the estimated values of θ computed by the minimization of (7). The state $\hat{x}(s, \hat{\theta})$ is the observed state for system (6), which is computed through the following Luenberger-like observer:

$$\begin{aligned} \dot{\hat{x}}(s, \hat{\theta}) &= A_m \hat{x}(s, \hat{\theta}) + (\hat{A} - A_m)x(s) + B \text{diag}(u(s))\hat{\lambda} + \hat{e} \\ &\quad - \hat{h} \max\{\Psi(\hat{c}, x(s), u(s)), 0\}, \end{aligned} \quad (9)$$

where $\Psi(\hat{c}, x(s), u(s)) = 2(\hat{c} - \bar{c})^T \begin{bmatrix} x(s) \\ u(s) \end{bmatrix} - (\hat{c}^T \hat{c} - \bar{c}^T \bar{c})$ and A_m is a Hurwitz matrix. The Luenberger-like observer (9) is an extension in PWA sense of the parallel-series estimator used for classical linear systems.³⁷ The solution of (9) can be calculated explicitly as follows:

$$\hat{x}(s, \hat{\theta}) = e^{A_m s} x_0 + \int_0^s e^{A_m(s-\tau)} \left\{ \begin{bmatrix} [x^T \mathbf{1} - \max\{\Psi, 0\}] \cdots & 0 \\ 0 & \ddots & 0 \\ 0 & \cdots & [x^T \mathbf{1} - \max\{\Psi, 0\}] \end{bmatrix} \begin{bmatrix} r_1(\hat{A} - A_m) \\ \hat{e}_1 \\ \hat{h}_1 \\ \vdots \\ r_n(\hat{A} - A_m) \\ \hat{e}_n \\ \hat{h}_n \end{bmatrix} + B \text{diag}(u)\hat{\lambda} \right\} d\tau. \quad (10)$$

The unknown parameter θ is estimated with the recursive Gauss-Newton algorithm. Then, $\hat{\theta}$ is updated online via the following adaptive law:

$$\dot{\hat{\theta}}(t) = -\Gamma U(t)^{-1} \Phi(t) \begin{bmatrix} \frac{\partial J(t, \hat{\theta})}{\partial \hat{x}_1} \\ \vdots \\ \frac{\partial J(t, \hat{\theta})}{\partial \hat{x}_n} \end{bmatrix} \Big|_{\hat{\theta}(0)=\hat{\theta}_0}, \quad (11)$$

where $\Gamma > 0$ is the adaptation gain decided by the designer and

$$\dot{U}(t) = -\xi U(t) + \Phi(t)\Phi(t)^T, \quad U(0) = 0 \quad (12)$$

with

$$\Phi(t) = \begin{bmatrix} \frac{\partial \hat{x}_1(t, \hat{\theta})}{\partial \hat{\theta}_1} & \frac{\partial \hat{x}_2(t, \hat{\theta})}{\partial \hat{\theta}_1} & \cdots & \frac{\partial \hat{x}_n(t, \hat{\theta})}{\partial \hat{\theta}_1} \\ \frac{\partial \hat{x}_1(t, \hat{\theta})}{\partial \hat{\theta}_2} & \frac{\partial \hat{x}_2(t, \hat{\theta})}{\partial \hat{\theta}_2} & \cdots & \frac{\partial \hat{x}_n(t, \hat{\theta})}{\partial \hat{\theta}_2} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial \hat{x}_1(t, \hat{\theta})}{\partial \hat{\lambda}} & \frac{\partial \hat{x}_2(t, \hat{\theta})}{\partial \hat{\lambda}} & \cdots & \frac{\partial \hat{x}_n(t, \hat{\theta})}{\partial \hat{\lambda}} \\ \frac{\partial \hat{x}_1(t, \hat{\theta})}{\partial \hat{c}} & \frac{\partial \hat{x}_2(t, \hat{\theta})}{\partial \hat{c}} & \cdots & \frac{\partial \hat{x}_n(t, \hat{\theta})}{\partial \hat{c}} \end{bmatrix}. \quad (13)$$

In order to calculate recursively all the terms in (13), one can see that $\hat{x}(t, \hat{\theta})$ can be written in the following form:

$$\hat{x}(t, \hat{\theta}) = g_0(t) + g_1(t)\hat{\theta} + g_2\hat{\lambda} \quad (14)$$

with

$$\begin{aligned} g_0(t) &= e^{A_m t} x_0 - A_m \int_0^t e^{A_m(t-\tau)} x(\tau) d\tau, \\ g_1(t) &= \int_0^t e^{A_m(t-\tau)} \begin{bmatrix} [x(\tau)^T \mathbf{1} - \max\{\Psi, 0\}] \cdots & 0 \\ 0 & \ddots & 0 \\ 0 & \cdots & [x(\tau)^T \mathbf{1} - \max\{\Psi, 0\}] \end{bmatrix} d\tau, \\ g_2(t) &= \int_0^t e^{A_m(t-\tau)} B \text{diag}(u(\tau)) d\tau, \end{aligned}$$

where Ψ is intended as $\Psi(\hat{c}, x(\tau), u(\tau))$. By using (10), the following relations are true:

$$\frac{\partial \hat{x}(t, \hat{\theta})}{\partial \hat{\theta}} = g_1(t) \quad (15a)$$

$$\hat{x}(t, \hat{\theta}) - x(t) = g_0(t) - x(t) + g_1(t)\hat{\theta} + g_2(t)\hat{\lambda} \quad (15b)$$

$$\frac{\partial \hat{x}(t, \hat{\theta})}{\partial \hat{\lambda}} = g_2(t) \quad (15c)$$

and

$$\frac{\partial \hat{x}(t, \hat{\theta})}{\partial \hat{c}} = - \int_0^t e^{A_m(t-\tau)} \hat{h} \begin{bmatrix} w_1(\tau) \\ \vdots \\ w_{n+m}(\tau) \end{bmatrix}^T d\tau, \quad (15d)$$

where

$$w_j(\tau) = \begin{cases} 2x_j(\tau) - 2\hat{c}_j(\tau), & \Psi(\hat{c}, \tau) = \max\{\Psi(\hat{c}, \tau), 0\} \\ 0, & \text{otherwise,} \end{cases} \quad (16)$$

for $j = 1, 2, \dots, n$, where

$$x_j(\tau) = \begin{cases} x_j(\tau), & j = 1, 2, \dots, n \\ u_{j-n}(\tau), & j = n+1, \dots, n+m. \end{cases}$$

From (7) and (15b), it can be proven

$$\frac{d}{dt} \left(\frac{\partial J(t, \hat{\theta})}{\partial \hat{x}} \right) = -\xi \frac{\partial J(t, \hat{\theta})}{\partial \hat{x}} + g_0(t) - x(t) + g_1(t)\hat{\theta}(t) + g_2(t)\hat{\lambda}(t), \quad (17)$$

and because of (13), (15a), (15c), (15d), relation (13) is equivalently represented by

$$\Phi(t) = \begin{bmatrix} g_1^T(t) \\ g_2^T(t) \\ \frac{\partial \hat{x}(t)}{\partial \hat{c}}^T \end{bmatrix}. \quad (18)$$

To update g_0, g_1, g_2 , and $\frac{\partial \hat{x}(t)}{\partial \hat{c}}$, we use the fact that

$$\dot{g}_0 = A_m g_0 - A_m x, \quad g_0(0) = x(0) \quad (19a)$$

$$\dot{g}_1 = A_m g_1 + \begin{bmatrix} [x^T 1 - \max\{\Psi, 0\}] \cdots & 0 \\ 0 & \ddots & 0 \\ 0 & \cdots & [x^T 1 - \max\{\Psi, 0\}] \end{bmatrix}, \quad (19b)$$

$$g_1(0) = 0$$

$$\dot{g}_2 = A_m g_2 + B \text{diag}(u), \quad g_2(0) = 0 \quad (19c)$$

$$\frac{d}{dt} \left(\frac{\partial \hat{x}}{\partial \hat{c}} \right) = A_m \frac{\partial \hat{x}}{\partial \hat{c}} - \hat{h} \begin{bmatrix} w_1 \\ \vdots \\ w_{n+m} \end{bmatrix}^T, \quad \frac{\partial \hat{x}}{\partial \hat{c}}(0) = 0 \quad (19d)$$

with w_1, w_2, \dots, w_{n+m} defined in (16). The recursive design is complete and the local optimality of the resulting FDI method for PWA systems is remarked hereafter.

Remark 2. Because (1) is nonlinear with respect to the estimated parameters, the cost function (17) is nonconvex with respect to $\hat{\theta}$, even after the max-form representation (6). As a consequence, a global optimum minimizing the cost function (17) cannot be guaranteed for every initial condition (even in the presence of persistency of excitation). In other words, only convergence to local optima can be guaranteed in general: therefore, the Gauss-Newton algorithm will exhibit best performance when the initial estimate $\hat{\theta}_0$ lies in a small neighborhood of θ . To the best of the authors' knowledge, there is no estimation method for PWA systems with joint unknown subsystems and partitions that can guarantee global optimality.

Remark 3. Note that, in case the partitions $\{\mathcal{X}_1, \mathcal{X}_2\}$ are known, the parameter c is given, and (6) results in a linear-in-the-parameter model for which standard converge results apply,⁴² after a slight revision of the proposed method in order to get rid of $\frac{\partial \hat{x}(t)}{\partial \hat{c}}$.

4 | ONLINE IDENTIFICATION OF TRIMODAL PWA SYSTEMS

The proposed framework can be extended to trimodal continuous PWA systems with minor modifications. Similarly to the bimodal PWA system case studied in Section 2, the trimodal PWA system reads as

$$\dot{x} = \begin{cases} A_1x + B_1\Lambda_1u + e_1, & \text{if } (x, u) \in \mathcal{X}_1 \\ A_2x + B_2\Lambda_2u + e_2, & \text{if } (x, u) \in \mathcal{X}_2 \\ A_3x + B_3\Lambda_3u + e_3, & \text{if } (x, u) \in \mathcal{X}_3, \end{cases} \quad (20)$$

where

$$\begin{aligned} \mathcal{X}_1 &= \left\{ (x, u) \mid 2(c_2 - c_1)^T \begin{bmatrix} x \\ u \end{bmatrix} - (c_2^T c_2 - c_1^T c_1) \leq 0, 2(c_3 - c_1)^T \begin{bmatrix} x \\ u \end{bmatrix} - (c_3^T c_3 - c_1^T c_1) \leq 0 \right\}, \\ \mathcal{X}_2 &= \left\{ (x, u) \mid 2(c_2 - c_1)^T \begin{bmatrix} x \\ u \end{bmatrix} - (c_2^T c_2 - c_1^T c_1) \geq 0, 2(c_3 - c_2)^T \begin{bmatrix} x \\ u \end{bmatrix} - (c_3^T c_3 - c_2^T c_2) \leq 0 \right\}, \\ \mathcal{X}_3 &= \left\{ (x, u) \mid 2(c_3 - c_2)^T \begin{bmatrix} x \\ u \end{bmatrix} - (c_3^T c_3 - c_2^T c_2) \geq 0, 2(c_3 - c_1)^T \begin{bmatrix} x \\ u \end{bmatrix} - (c_3^T c_3 - c_1^T c_1) \geq 0 \right\}. \end{aligned}$$

4.1 | Max-form representation of trimodal PWA systems

In order to write the max-form presentation of the PWA system in (20), one has to distinguish between two cases.

Case 1. The centers $c_1, c_2,$ and c_3 lie on a line. Without loss of generality, it is assumed that the center c_2 lies on the segment $[c_1, c_3]$. Similarly to the bimodal PWA system case, the continuity of the PWA system (20) is equivalent to the existence and uniqueness of $h_1, h_2 \in \mathbb{R}^n$ such that (20) can be equivalently written as

$$\begin{aligned} \dot{x} &= [A_2 \ B_2\Lambda_2] \begin{bmatrix} x \\ u \end{bmatrix} + e_2 \\ &\quad - h_1 \max \left\{ 2(c_2 - c_1)^T \begin{bmatrix} x \\ u \end{bmatrix} - (c_2^T c_2 - c_1^T c_1), 0 \right\} \\ &\quad - h_3 \max \left\{ 2(c_1 - c_3)^T \begin{bmatrix} x \\ u \end{bmatrix} - (c_1^T c_1 - c_3^T c_3), 0 \right\}. \end{aligned} \quad (21)$$

Case 2. The centers $c_1, c_2,$ and c_3 do not lie on a line. The continuity of (20) is equivalent to the existence and uniqueness of $h_1, h_2, h_3 \in \mathbb{R}^n$ such that

$$\begin{aligned} [A_1 \ B_1\Lambda_1] - [A_2 \ B_2\Lambda_2] &= 2h_1(c_2 - c_1)^T, \\ e_1 - e_2 &= -h_1(c_2^T c_2 - c_1^T c_1), \end{aligned} \quad (22a)$$

$$\begin{aligned} [A_2 \ B_2\Lambda_2] - [A_3 \ B_3\Lambda_3] &= 2h_2(c_3 - c_2)^T, \\ e_2 - e_3 &= -h_2(c_3^T c_3 - c_2^T c_2), \end{aligned} \quad (22b)$$

$$\begin{aligned} [A_3 \ B_3\Lambda_3] - [A_1 \ B_1\Lambda_1] &= 2h_3(c_3 - c_1)^T, \\ e_3 - e_1 &= -h_3(c_3^T c_3 - c_1^T c_1). \end{aligned} \quad (22c)$$

Lemma 1. For the vectors $h_1, h_2,$ and h_3 in (22), it is true that

$$h_1 = h_2 = -h_3. \quad (23)$$

Proof. Relation (22) gives

$$(h_3 + h_2)c_3^T + (h_1 - h_2)c_2^T - (h_1 + h_3)c_1^T = 0. \quad (24)$$

If $c_1, c_2,$ and c_3 are linearly independent, it follows from (24) that $h_1 = h_2 = -h_3$. For the case that $c_1, c_2,$ and c_3 are linearly dependent, one center can be written as a linear combination of the two other centers. Without loss of generality, let $c_3 = \alpha c_1 + \beta c_2$, with $\alpha, \beta \in \mathbb{R}$ such that $\alpha + \beta \neq 1$. It follows that

$$\begin{aligned} (h_3 + h_2)c_3^T &= \alpha(h_3 + h_2)c_1^T + \beta(h_3 + h_2)c_2^T, \\ (h_3 + h_2)c_3^T &= (h_1 + h_3)c_1^T + (h_2 - h_1)c_2^T, \end{aligned} \quad (25)$$

implying $h_2 + h_3 = (\alpha + \beta)(h_2 + h_3)$, and hence, $h_2 = -h_3$. Substituting this result in (24), it follows that $h_1 = h_2$ and the lemma is proved. \square

In view of (22) and Lemma 1, if we define $h_1 = h_2 = -h_3 = h$, then the PWA system (20) is given in its max-form presentation as

$$\dot{x} = \begin{bmatrix} A_3 & B_3 \Lambda_3 \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix} + e_3 - h \max \left\{ 2(c_2 - c_3)^T \begin{bmatrix} x \\ u \end{bmatrix} - (c_2^T c_2 - c_3^T c_3), 2(c_1 - c_3)^T \begin{bmatrix} x \\ u \end{bmatrix} - (c_1^T c_1 - c_3^T c_3), 0 \right\}. \quad (26)$$

Remark 4. As demonstrated from the above discussion, the max-form presentation of the trimodal PWA system in (20) can have two different forms, (21) or (26), depending on whether the centers lie on a line or not. Once the appropriate max-form is determined, the adaptive update laws are developed in similar fashion as in the bimodal PWA system case.

5 | SIMULATION RESULTS

5.1 | Bimodal PWA system

In this section, we evaluate the effectiveness of the online FDI technique on the wheeled mobile robot (WMR) shown in Figure 1 and presented in the work of Nayeapanah et al.⁴³

The WMR is assumed to be rigid and it is driven by a torque T to control the heading angle ψ . The forward velocity of the robot u_0 is in the direction of the X -body axis and it is assumed to be constant, by designing appropriately a cruise controller. The heading angle of the WMR ψ is measured with respect to the positive X -axis in the inertial frame. The kinematic equations for the WMR are

$$\begin{aligned} \dot{y} &= u_0 \sin(\psi) \\ \dot{\psi} &= R, \end{aligned} \quad (27)$$

and the dynamic equation of the WMR is

$$\dot{R} = 0.75 \frac{1}{I} T, \quad (28)$$

where T is the input to the system, corresponding to the torque generated by the direct current motors, 0.75 is the unknown actuator effectiveness, and $I = 1 \text{ kg}\cdot\text{m}^2$ (which is known) corresponds to the moment of inertia of the WMR with respect to the center of its mass. Inspired by this example, we consider as the actual system the bimodal PWA system in the form (1), with

$$\begin{aligned} A_1 &= \begin{bmatrix} 0 & \frac{2}{\pi} u_0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & -\frac{2}{\pi} u_0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \\ B_1 &= B_2 = \begin{bmatrix} 0 & 0 & \frac{1}{I} \end{bmatrix}^T, \quad \Lambda_1 = \Lambda_2 = 0.75, \\ e_1 &= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad e_2 = \begin{bmatrix} 2u_0 \\ 0 \\ 0 \end{bmatrix}, \end{aligned}$$

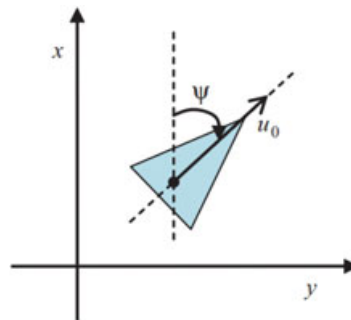


FIGURE 1 Schematic representation of the wheeled mobile robot⁴³ [Colour figure can be viewed at wileyonlinelibrary.com]

with $u_0 = 1$, which is unknown. The aforementioned matrices arise from approximating, in the range $[-\pi/2, 3\pi/2]$, the sinusoid with two straight lines (one straight line passes through the origin with slope $2/\pi$, while the other one passes through the point $(\pi, 0)$ with slope $-2/\pi$). As a consequence, the switching surface between the two subsystems is given by

$$\begin{bmatrix} 0 & \frac{2}{\pi} & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ u \end{bmatrix} - 1 \leq 0, \quad (\geq 0), \quad (29)$$

where $[x_1 \ x_2 \ x_3 \ u] = [y \ \psi \ R \ T]$. The surface can be equivalently expressed by the two centers c_1 and c_2 , defined as follows:

$$c_1 = \left[0.25 \ \frac{\pi}{2} - 0.25 \ 0.25 \ 0.25 \right]^T,$$

$$c_2 = \left[0.25 \ \frac{\pi}{2} + 0.25 \ 0.25 \ 0.25 \right]^T.$$

Note that the definition of c_1 and c_2 is not unique: however, by fixing c_2 , the other center c_1 would be uniquely determined. We acknowledge that, in this particular example, the partitions might be known: however, to be consistent with our setting and illustrate the proposed method, we assume that the partitions are unknown.

In view of the structure of the matrices, only five parameters are unknown and need to be determined: the nonzero term in the first row of A_2 , the nonzero term in e_2 (representing uncertainties or changes in the cruise speed), the scalar term Λ_2 (representing uncertainties or changes in the actuator effectiveness), the unique nonzero term in h , and the second entry of c_1 (representing uncertainties in the partition). Therefore, by defining θ properly, it is possible to use a priori knowledge of the matrix structure and derive a Gauss-Newton method that estimates only the relevant five parameters (details are not shown for compactness). The design parameters have been taken as

$$A_m = \begin{bmatrix} 0 & -0.637 & 0 \\ 0 & 0 & 1 \\ 0.003 & -0.054 & -0.114 \end{bmatrix}, \quad \xi = 0.5, \quad \Gamma = \text{diag}(0.01, 0.03, 0.85, 0.03, 0.01),$$

where the eigenvalues of A_m are stable (one real eigenvalue and one complex conjugate pair). The initial state is taken as $x_0 = [1 \ \pi/2 \ 0]^T$. In order to provide enough persistency of excitation, the input is a series of steering and countersteering sinusoids at frequency 0.2, 0.8, and 1.6 rad/s.

In order to check the consistency of the approach, we have selected many initial estimates $\hat{\theta}(0)$ randomly (zero mean Gaussian noise with covariance 0.1) in a neighborhood of θ . For all initial conditions, the convergence was consistent, and Figures 2 and 3 show one simulation. In addition, Figures 4 and 5 show the capability to track some (slow) variation in time of the parameters: these variations have been simulated by slightly increasing u_0 and decreasing the actuator effectiveness.

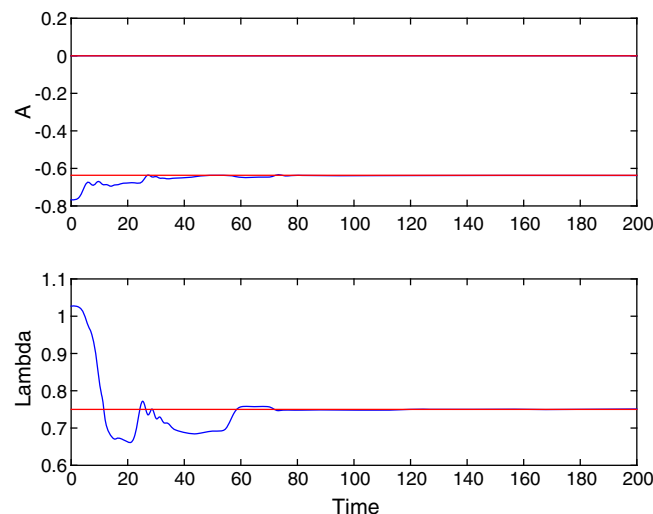


FIGURE 2 Online identification of A_2 and Λ_2 when c_2 is known (the true parameter values are shown in red color lines) [Colour figure can be viewed at wileyonlinelibrary.com]

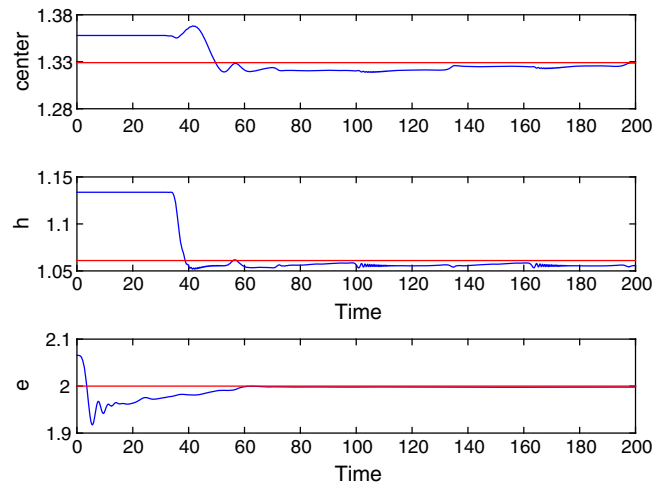


FIGURE 3 Online identification of e_2 , h , and c_1 when c_2 is known (the true parameter values are shown in red color lines) [Colour figure can be viewed at wileyonlinelibrary.com]

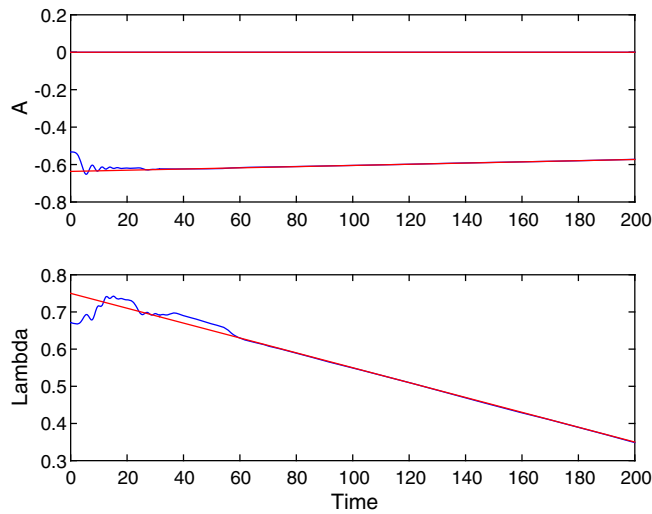


FIGURE 4 Online identification of A_2 and Λ_2 when c_2 is known for slow variations (the true parameter values are shown in red color lines) [Colour figure can be viewed at wileyonlinelibrary.com]

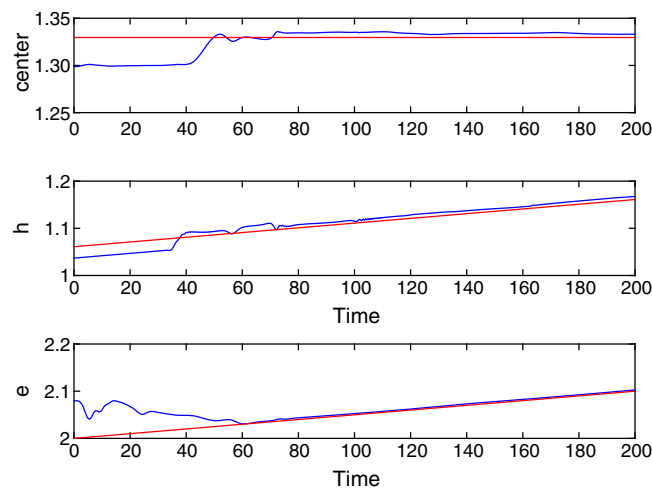


FIGURE 5 Online identification of e_2 , h , and c_1 when c_2 is known for slow variations (the true parameter values are shown in red color lines) [Colour figure can be viewed at wileyonlinelibrary.com]

TABLE 1 Performance depending on the initial estimate

Var $[\theta - \hat{\theta}(0)]$	Avg $\left[\frac{\ \theta - \hat{\theta}_{st}\ }{\ \theta\ } \right]$
0.03	0.2%
0.1	0.4%
0.3	0.8%
1.0	4.2%
3.0	18.8%

Remark 5. In order to highlight nonlinearity of the problem and the possibility of getting trapped into local minima, Table 1 shows the distance between the true and the estimated parameters (at steady state) $\|\theta - \hat{\theta}_{st}\| / \|\theta\|$, as a function of the variance of $\theta - \hat{\theta}(0)$. The Table highlights that, when the initial condition is very far from the true parameter, the steady-state distance also increases: this happens because the Gauss-Newton algorithm may not converge to the actual parameters.

5.2 | Trimodal PWA system

In order to show the effectiveness of the proposed approach also in a trimodal setting, we take the example from the work of Kersting and Buss.⁴⁴ This example has all the centers on a line, and notice that e_1 and e_3 have been modified with respect to the aforementioned work⁴⁴ so as to make the PWA system continuous. In particular, we have

$$\begin{aligned} A_1 &= \begin{bmatrix} 0 & 1 \\ -1.5 & -1 \end{bmatrix}, & B_1 &= \begin{bmatrix} 0 \\ 1.5 \end{bmatrix}, & e_1 &= \begin{bmatrix} 0 \\ 1.4 \end{bmatrix}, \\ A_2 &= \begin{bmatrix} 0 & 1 \\ -2 & -1 \end{bmatrix}, & B_2 &= \begin{bmatrix} 0 \\ 1.5 \end{bmatrix}, & e_2 &= \begin{bmatrix} 0 \\ 0.4 \end{bmatrix}, \\ A_3 &= \begin{bmatrix} 0 & 1 \\ -2.5 & -1 \end{bmatrix}, & B_3 &= \begin{bmatrix} 0 \\ 1.5 \end{bmatrix}, & e_3 &= \begin{bmatrix} 0 \\ 1.4 \end{bmatrix}, \end{aligned}$$

and $\Lambda_1 = \Lambda_2 = \Lambda_3 = 0.75$. The switching surface is defined in terms of the three centers

$$\begin{aligned} c_1 &= [-4 \ 0 \ 0]^T, \\ c_2 &= [0 \ 0 \ 0]^T, \\ c_3 &= [4 \ 0 \ 0]^T. \end{aligned}$$

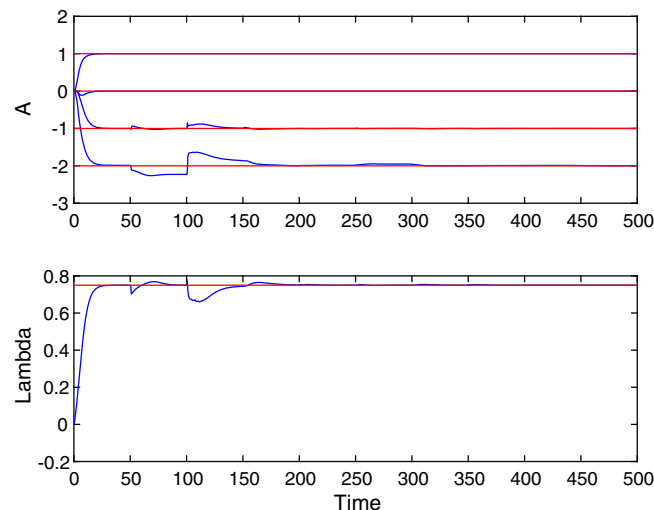


FIGURE 6 Online identification of A_2 and Λ_2 when c_2 is known (the true parameter values are shown in red color lines) [Colour figure can be viewed at wileyonlinelibrary.com]

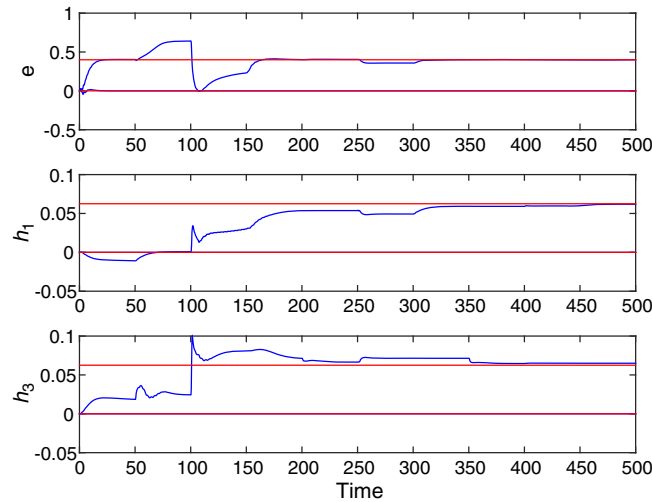


FIGURE 7 Online identification of e_2 , h_1 , and h_3 when c_2 is known (the true parameter values are shown in red color lines) [Colour figure can be viewed at wileyonlinelibrary.com]

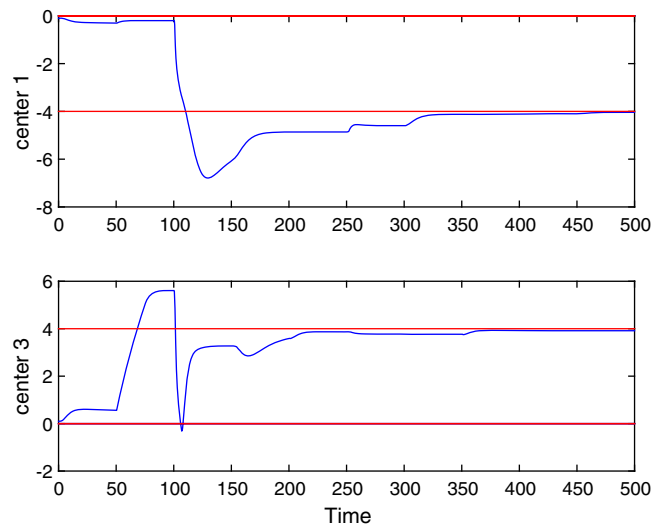


FIGURE 8 Online identification of c_1 and c_3 when c_2 is known (the true parameter values are shown in red color lines) [Colour figure can be viewed at wileyonlinelibrary.com]

By exploiting a similar form as in (21), we formulate the FDI problem as the one of estimating the parameters of A_2 , Λ_2 , e_2 , the vectors h_1 and h_3 , and the centers c_1 and c_3 (we assume that the center c_2 is known). We have used $x_0 = [0.5 \ -0.5]^T$, a multisinusoid input (with 3 sinusoids), and the design parameters

$$A_m = \begin{bmatrix} -10 & 0 \\ 0 & -10 \end{bmatrix}, \xi = 0.05, \Gamma = \text{diag}(1, 1, 1, 1, 1, 1, 1, 0.05, 0.05, 40, 40),$$

where the zero components of h_1 , h_3 , c_1 , and c_3 are not estimated. The results from the proposed online FDI algorithm are given in Figure 6 (for A_2 and Λ_2), Figure 7 (for e_2 , h_1 , and h_3), and Figure 8 (for c_1 and c_3). It is observed that all estimates converge to the correct values after some transient.

6 | CONCLUSION

This paper has established a novel online FDI strategy for a class of continuous PWA systems, namely, bimodal and trimodal PWA systems. The approach is estimation based, ie, it is assumed that faults in the system are reflected in a change of the parameters of the system model. The main contributions with respect to the state-of-the-art are the recursive

nature of the proposed scheme and the consideration of parametric uncertainties in both partitions and in subsystems parameters. In order to handle this situation, we recast the continuous PWA into its max-form representation and we exploited the recursive Newton-Gauss algorithm on a suitable cost function to derive the adaptive laws to estimate online the unknown subsystem parameters, the partitions, and the loss in control authority for the PWA model. The effectiveness of the proposed methodology was verified via simulations applied to the benchmark example of a WMR. Future work could include the extension beyond trimodal systems: a possible idea to deal with this situation is to have multiple bimodal or trimodal estimators and a switching logic, according to architectures as in the work of Baldi et al.⁴⁵

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