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A Test Case Prioritization Genetic Algorithm guided by the Hypervolume Indicator

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Abstract—Regression testing is performed during maintenance activities to assess whether the unchanged parts of a software behave as intended. To reduce its cost, test case prioritization techniques can be used to schedule the execution of the available test cases to increase their ability to reveal regression faults earlier. Optimal test ordering can be determined using various techniques, such as greedy algorithms and meta-heuristics, and optimizing multiple fitness functions, such as the average percentage of statement and branch coverage. These fitness functions condense the cumulative coverage scores achieved when incrementally running test cases in a given ordering using Area Under Curve (AUC) metrics. In this paper, we notice that AUC metrics represent a bi-dimensional (simplified) version of the hypervolume metric, which is widely used in many-objective optimization. Thus, we propose a Hypervolume-based Genetic Algorithm, namely HGA, to solve the Test Case Prioritization problem when using multiple test coverage criteria. An empirical study conducted with respect to five state-of-the-art techniques shows that (i) HGA is more cost-effective, (ii) HGA improves the efficiency of Test Case Prioritization, (iii) HGA has a stronger selective pressure when dealing with more than three criteria.

Index Terms—Test Case Prioritization, Genetic Algorithm, Hypervolume.

1 INTRODUCTION

The goal of regression testing is to verify that software changes do not affect the behavior of unchanged parts [2]. Many approaches have been proposed in literature to reduce the effort of regression testing [2], [3], which remains a particular expensive post-maintenance activity [4]. One of these approaches is test case prioritization (TCP) [5], [6], whose goal is to execute the available test cases in a specific order that increases the likelihood of revealing regression faults earlier [7]. Since fault detection capability is unknown before test execution, most of the proposed techniques for TCP use coverage criteria [2] as surrogates with the idea that test cases with higher code coverage will have a higher probability to reveal faults. Once a coverage criterion is chosen, search algorithms can be applied to find the order maximizing the selected criterion.

Greedy Algorithms have been widely investigated in literature for test case prioritization, such as simple greedy algorithms [2], additional greedy algorithms [5], 2-optimal greedy algorithms [3], or hybrid greedy algorithms [8]. Other than greedy algorithms, meta-heuristics have been applied as alternative search algorithms to test case prioritization. To allow the application of meta-heuristics, proper fitness functions have been developed [3], such as the Average Percentage Block Coverage (APBC) or the Average Percentage Statement Coverage (APSC). Each fitness function measures the Area Under Curve (AUC) represented by the cumulative coverage and cost scores achieved when incrementally executing the test cases according to a specific prioritization (or order). As such, multiple points in the cost-coverage space are condensed into a single scalar value that can be used as a fitness function for meta-heuristics, such as single-objective genetic algorithms. Later work on search-based TCP also employed multi-objective genetic algorithms considering different AUC-based metrics as different objectives to optimize [9], [10], [11], [12].

We observed that the AUC metric used in the related literature for TCP represents a simplified version of the well-known hypervolume [13], which is a metric used in many-objective optimization. Indeed, the problem of condensing multiple points in the objective space (i.e., a Pareto front) has been already investigated in many-objective optimization using the more general concept of hypervolume under manifold [13], which is a generalization of the AUC-based metrics used in previous TCP studies but for the higher dimensional objective space. We argue that the hypervolume can be used to condense not only a single cumulative code coverage criteria (as done by previous AUC metrics used in TCP literature) but also multiple testing criteria, such as the test case execution cost or further coverage criteria (e.g., branch, and past-fault coverage), in only one scalar value.

In our previous work [1], we introduced a Hypervolume-based Genetic Algorithm (HGA) to solve the TCP problem with multiple testing criteria. We conducted a preliminary study on six open-source programs and we compared HGA with the Additional Greedy algorithm [5], [14] when optimizing up to three testing criteria. Our preliminary results
showed that HGA is not only much faster than the greedy algorithm but that the generated test orderings reveal more regression faults than the alternative algorithm for large software programs. However, despite these encouraging results, further studies are needed to answer the following questions: (i) How does HGA perform compared to other state-of-the-art techniques for the TCP problem? (ii) To what extent does HGA scale when dealing with more than three testing criteria? (iii) To what extent does HGA scale when dealing with large software systems containing real faults?

To answer the aforementioned open questions, in this paper we provide an extensive evaluation of Hypervolume-based and state-of-the-art approaches for TCP when dealing with up to five testing criteria (four objectives). In particular, we carry out a first case study to assess the cost-effectiveness and the efficiency of the various approaches. We compare HGA with respect to three state-of-the-art techniques: a cost cognizant Additional Greedy algorithm [5], [14], a single objective Genetic Algorithm based on an AUC metric (GA) [3], and Non-dominated Sorting Genetic Algorithm II (NSGA-II), a multi-objective search-based algorithm [9], [10], [11], [12].

A well-known limitation in many-objective optimization is that traditional multi-objective evolutionary algorithms (e.g., NSGA-II) do not scale when handling more than three criteria. This happens because the number of non-dominated solutions increases exponentially with the number of objectives [15], [16], [17] (selection resistance). Therefore, we perform a second case study to assess the selective pressure capabilities of HGA when dealing with more than three criteria, comparing it with two many-objective search algorithms, namely Generalized Differential Evolution 3 (GDE3) [18] and Multi-objective Evolutionary Algorithm Based on Decomposition (MOEA/D-DE) [19]. Finally, we conduct a third case study with the aim of evaluating the performance of HGA when dealing with large software systems containing real faults. The studies are designed to answer the following research questions:

- **RQ1**: What is the cost-effectiveness and efficiency of HGA, compared to state-of-the-art test case prioritization techniques?
- **RQ2**: How does HGA perform with respect to many-objective test case prioritization techniques?
- **RQ3**: How does HGA perform on a large software system with real faults?

Our results suggest that the solution (test ordering) produced by HGA is more cost-effective than the solution generated by Additional Greedy, GA, and NSGA-II. In terms of efficiency, HGA is much faster than GA and NSGA-II. Moreover, with respect to Additional Greedy, its efficiency does not decrease as the size of the software program and of the test suite increase. When comparing HGA with many-objective search algorithms (e.g., GDE3 and MOEA/D-DE), we observe that it is not only more or equally effective, but it is also up to 3 times more efficient. Finally, when dealing with large software systems such as MySQL, we observe similar results to those achieved in the first case study.

The contributions of this paper compared to the conference paper [1] can be summarized as follows:

1) We extend the empirical evaluation by conducting two new case studies.
2) We partially replicate a previous study [12] on a large real-world software system, namely MySQL.
3) We compare our algorithm with five state-of-the-art algorithms for the Test Case Prioritization problem, namely Additional Greedy [5], [14], a Genetic Algorithm based on an AUC metric [3], Non-dominated Sorting Genetic Algorithm II [20], Generalized Differential Evolution 3 [18], and Multi-objective Evolutionary Algorithm Based on Decomposition [19].
4) We provide a comprehensive replication package [21] including all the raw data and working data sets of our studies.

In addition, we provide more details of the HGA algorithm, expand the discussion of related work, and provide a more qualitative discussion of the results. The remainder of the paper is organized as follows. Section 2 discusses the related literature, while Section 3 presents the proposed algorithm. Sections 4, 5, and 6 describe our empirical studies including the research questions and the results that we obtained. Section 7 discusses the threats that could affect the validity of the results achieved. Finally, Section 8 concludes the paper.

## 2 Background and Related Work

The Test Case Prioritization (TCP) problem consists of generating a test case ordering \( \tau' \in PT \) that maximizes fault detection rate \( f \) [7].

**Definition 1.** — Given a test suite \( T \), the set of all permutations \( PT \) of test cases in \( T \), and a function \( f: PT \to \mathbb{R} \).

\[
\text{Problem: find } \tau' \in PT \text{ such that } \left( \forall \tau'' \in PT \right)(\tau'' \neq \tau')\left(f(\tau') \geq f(\tau'')\right)
\]

However, the fault detection capability case is not known to the tester before test execution. Therefore, researchers have proposed to use surrogate metrics, which are in some way correlated with the fault detection rate [2], to determine test case execution order. They can be divided into two main categories [22]: white-box metrics and black-box metrics.

Code coverage is the most widely used metric among white-box ones, e.g., branch coverage [5], statement coverage [6], block coverage [23], and function or method coverage [24]. Other prioritization criteria were also used instead of structural coverage, such as interactions [25], [26], requirement coverage [27], statement and branch diversity [28], [29], and additional spanning statement and branches [30]. Other than white-box metrics also black-box metrics have been proposed. For example, Bryce et al. proposed the \( t \)-wise approach that considers the maximum interactions between \( t \) model inputs [31], [32] [33]. Other approaches considered the input diversity calculated using NCD [34], the Jaccard distance [35], [36], and the Levenshtein distance [37], [38] between inputs. Finally, Henard et al. considered also the number of killed model mutants [36], [39]. Henard et al. [22] compared various white-box and black-box criteria for TCP, showing that there is a “little difference between black-box and white-box performance”.

In addition, we provide more details of the HGA algorithm, expand the discussion of related work, and provide a more qualitative discussion of the results. The remainder of the paper is organized as follows. Section 2 discusses the related literature, while Section 3 presents the proposed algorithm. Sections 4, 5, and 6 describe our empirical studies including the research questions and the results that we obtained. Section 7 discusses the threats that could affect the validity of the results achieved. Finally, Section 8 concludes the paper.
In all the aforementioned works, once a prioritization criterion is chosen, a greedy algorithm is used to order the test cases according to the chosen criterion. Two main greedy strategies can be applied [8] [40]: the total strategy selects test cases according to the number of code elements they cover, whereas the additional strategy iteratively selects the test case that yields the maximal coverage of code elements not covered yet by previously selected test cases. Recently, Hao et al. [8] and Zhang et al. [40] proposed a hybrid approach that combines total and additional coverage criteria showing that their combination can be more effective than the individual components. Greedy algorithms have also been used to combine multiple testing criteria such as code coverage and cost. For example, Elbaum et al. [41] and Malishevsky et al. [42] considered code coverage and execution cost, where the additional greedy algorithm was customized to condense the two objectives in only one function (coverage per unit cost) to maximize. Three-objective greedy algorithms have also been used to combine statement coverage, historical fault coverage, and execution cost [2], [43].

2.1 Search-Based Test Case Prioritization

Other than greedy algorithms, meta-heuristics have been investigated as alternative search algorithms to test case prioritization. Li et al. [3] compared additional greedy algorithm, hill climbing, and genetic algorithms for code coverage-based TCP. To enable the application of meta-heuristics they developed proper fitness functions: APBC (Average Percentage Block Coverage), APDC (Average Percentage Decision Coverage), or APSC (Average Percentage Statement Coverage). For a generic coverage criterion (e.g., branch coverage), the corresponding fitness function is defined as follows:

Definition 2. — Let \( E = \{e_1, \ldots, e_m\} \) be a set of target elements to cover; let \( \tau = \langle t_1, t_2, \ldots, t_n\rangle \) be a given test case ordering; let \( TE_i \) be the position of the first test in \( \tau \) that covers the element \( e_i \in E \); the Average Percentage of Element Coverage, i.e., the AUC metric, is defined as:

\[
APEC = 1 - \frac{\sum_{i=1}^{m} \sum_{j=TE_i}^{m} C_j - \frac{1}{2} C_{TE_i}}{\sum_{i=1}^{n} c_i \times n} \tag{1}
\]

In the definition above, the target elements in \( E \) can be branches (Equation 2 would correspond to APDC), statements (APSC), basic blocks (APBC), etc. Equation 2 condenses the cumulative coverage scores (e.g., branch coverage) achieved when considering the test cases in the given order \( \tau \) using the Area Under Curve (AUC) metric. This area is delimited by the cumulative points whose \( y \)-coordinates are the cumulative coverage scores (e.g., statement coverage) achieved when varying the number of executed test cases (\( x \)-coordinates) according to a specified ordering [3].

Equation 1 relies on the assumption that all test cases have equal cost. However, such an assumption is unrealistic in practice and, as consequence, test orderings optimizing Equation 1 may become sub-optimal when measuring the test execution cost. In principle, the cost of each test could be measured as its actual execution time. As argued by previous studies [14], [43], such a measurement is not reliable because it depends on several external factors such as different hardware, application software, operating system, etc. Therefore, researchers used different metrics as proxy for the actual execution time, such as counting the number of executed statements [14], the number of executed basic blocks in the control flow graph [43], or estimating the monetary cost of each test case [44], or re-using the test execution measurements from past regression testing activities [42].

Given a measurement of the test execution cost, the “cost-cognizant” variant of Equation 1 has been defined in the literature [12] as follows:

Definition 3. — Let \( E = \{e_1, \ldots, e_m\} \) be a set of target elements to cover; let \( \tau = \langle t_1, t_2, \ldots, t_n\rangle \) be a given test case ordering; let \( C = \{c_1, \ldots, c_m\} \) be the cost of tests in \( \tau \); let \( TE_i \) be the position of the first test in \( \tau \) that covers the element \( e_i \in E \); the “Cost-cognizant” Average Percentage of Element Coverage is defined as:

\[
APEC_c = \frac{\sum_{i=1}^{m} \sum_{j=TE_i}^{m} C_j - \frac{1}{2} C_{TE_i}}{\sum_{i=1}^{n} c_i \times m} \tag{2}
\]

When assuming that all tests have the same cost (i.e., \( \forall c_i \in C, c_i = 1 \)), Equation 2 becomes equivalent to Equation 1 [45]. This “cost-cognizant” variant measures the AUC delimited by the cumulative points whose \( y \)-coordinates are the cumulative coverage scores (e.g., statement coverage) while their \( x \)-coordinates are the cumulative test execution costs for a specified test ordering \( \tau \).

Since these metrics allow to condense multiple cumulative points in only one scalar value, single-objective genetic algorithms can be applied to find an ordering maximizing the AUC. According to the empirical results achieved by Li et al. [3], in most of the cases, the difference between the effectiveness of permutation-based genetic algorithms and additional greedy approaches is not significant.

2.2 Multi-objective Test Case Prioritization

Later works highlighted that given the multi-objective nature of the TCP problem, permutation-based genetic algorithms should consider more than one testing criterion. For example, Li et al. [9] proposed a two-objective permutation-based genetic algorithm to optimize APSC and execution cost required to reach the maximum statement coverage (cumulative cost). They use a multi-objective genetic algorithm, namely NSGA-II, to find a set of Pareto optimal test case orderings representing optimal compromises between the two corresponding AUC-based criteria.

Based on the concept of Pareto optimality [46], in this formulation of the problem, a test cases permutation \( \tau_A \) is better than another permutation \( \tau_B \), (and vice versa), if and only if \( \tau_A \) outperforms \( \tau_B \) in at least one objective and it is not worse in all other objectives. Formally:

Definition 4. — Given two permutations of test cases, \( \tau_A \) and \( \tau_B \), and a set of \( n \) functions (objectives) \( f : PT \rightarrow \mathbb{R} \), \( \tau_A \) dominates \( \tau_B \) \((\tau_A \prec \tau_B)\) if an only if:

\[
f_i(\tau_A) \geq f_i(\tau_B), \forall i \in 1, 2, \ldots, n
\]

\[
\text{and}
\]

\[
\exists i \in 1, 2, \ldots, n : f_i(\tau_A) > f_i(\tau_B)
\]

(3)
Definition 5. — Given the concept of Pareto dominance and a set of feasible solutions $\Omega$, a solution $\tau^*$ is Pareto optimal if a solution able to dominate it does not exist, namely:

$$\not\exists \tau_A \in \Omega : \tau_A \prec \tau^*$$

Definition 6. — A Pareto Front is a set composed of Pareto optimal solutions.

$$P^* = \{\tau^* \in \Omega\}$$

It is worth considering that multi-objective approaches for test case prioritization return a Pareto front of permutations, that is a set of Pareto optimal test orderings.

Islam et al. [10] and Marchetto et al. [11] used NSGA-II to find Pareto optimal test case orderings representing trade-offs between three different AUC-based criteria: (i) cumulative code coverage, (ii) cumulative requirement coverage, and (iii) cumulative execution cost. Similarly, Epitropakis et al. [12] compared greedy algorithms, MOEAs (NSGA-II e TAEA), and hybrid algorithms. As already done by Islam et al. [10] and Marchetto et al. [11], they considered different AUC-based fault surrogates: statement coverage (APSC), $\Delta$-coverage (APDC), and past fault coverage (APPFD). They showed that three-objective MOEAs and hybrid algorithms are able to produce more effective solutions with respect to those produced by additional greedy algorithms based on a single AUC metric.

In this paper, we notice that these approaches [9], [10], [11], [12] to test case prioritization have important drawbacks. First of all, these measures are computed considering the Area Under Curve obtained plotting the value of the metric with respect to the test cases position in a Cartesian plan [11] and then computing a numerical approximation of the Area Under Curve, using the Trapezoidal rule [47]. These values are projections of a manifold of cumulative points (e.g., a projection of a volume into two areas). Therefore, despite the AUC metrics being strictly dependent on each other, the different AUC metrics are calculated independently (we will show an example in Section 3). Moving to this multi-objective paradigm where AUC metrics are treated as independent objectives has an additional overhead compared to a single-objective search. In multi-objective search, the computational complexity of computing the dominance relation for all pairs of candidate test permutations is $O(n^2 \times m)$, where $n$ is the number of test permutations and $m$ is the number of AUC metrics. Instead, in single-objective search, the cost of sorting the individuals to select the best ones is $O(n \times \log n)$ for stochastic selection or $O(n)$ with tournament selection. In general, the selection in single-objective search is less expensive than the selection in a multi-objective paradigm.

Moreover, the tester has to inspect the Pareto front in order to find the most suitable solution with respect to the testing criteria but no guidelines are provided for selecting the ordering (Pareto optimal solution) to use. The Pareto efficient solutions generated by a multi-objective search are trade-offs in the space of the AUC metrics and not in the space of the original testing criteria, which are the actual aspects that decision-makers (a.k.a. testers) look at for regression testing purpose. Furthermore, each solution in the Pareto front represents a permutation of tests and selecting a different permutation requires re-evaluating all the test cases in that permutation.

Another important limitation of these classical multi-objective approaches is that they lose their effectiveness as the problem dimensionality increases, as demonstrated by previous work in numerical optimization [48]. Therefore, other non-classical many-objective solvers must be investigated when dealing with multiple (many) testing criteria. Finally, in [3], [9], [10], [12] there is a lack of strong empirical evidence of the effectiveness of MOEAs with respect to simple heuristics, such as greedy algorithms, in terms of cost-effectiveness.

In this paper, we notice that the most natural way to deal with the multi-objective TCP problem is represented by the hypervolume-based solvers since the AUC metrics used in the related literature for TCP represent a specific simplified version of the hypervolume metric [13]. Indeed, in many-objective optimization, the hypervolume metric is widely used to condense points from a higher dimensional objective space in only one scalar value. For these reasons, in this paper, we propose to use a hypervolume metric to solve the multi-objective TCP problem. Moreover, because of the monotonicity properties of the coverage criteria, the computation of the hypervolume for TCP requires polynomial time versus the exponential time required for traditional many-objective problems.

### 2.2.1 Hypervolume-based many-objective optimization

Multi-objective meta-heuristics have been successfully applied in the literature to solve a number of software engineering problems, such as software refactoring [49], test data generation [50], defect prediction [51], [52], and regression testing [53], [43]. These problems have often been solved with algorithms like the Nondominated Sorting Genetic Algorithm II (NSGA-II) [20] or the improved Strength Pareto Evolutionary Algorithm (SPEA2) [54], which are very effective for problems with two or three objectives. However, handling more than four objectives is particularly challenging as the number of non-dominated solutions may exponentially increase with the number of objectives to optimize. In this scenario, the classical non-dominated sorting algorithms or other classical environmental selection mechanisms are not able to promote some solutions over the others within a given population (selection resistance [15], [16]) because all solutions are incomparable (i.e., they do not dominate each other).

To address this problem, researchers in the evolutionary computation community developed a new class of meta-heuristics, often referred to as many-objective algorithms, for problems with more than three search objectives. According to a recent survey by Li et al. [15], strategies to address the selective resistance include diversity-based, reference set based, and indicator-based algorithms. For example, the Generalized Differential Evolution 3 (GDE3) [18] relies on a diversity-based mechanism to improve the selection pressure. GDE3 extends differential evolution (DE) for constrained multi-objective and many-objective optimization where the population for the next generation is generated by combining the non-dominated sorting with a pruning algorithm for the non-dominated set. The non-dominated set is pruned according to the solution diversity,
which is measured with a crowding estimation metric based on
the nearest neighbors. Solutions having the smallest dis-
tance to their neighbors are the most crowded ones in the
non-dominated set and can be pruned out. Kukkonen and
Deb [55] showed that GDE3 with crowding estimation is
effective and efficient in producing well-diversified trade-
offs for problems with more than two objectives.
MOEA/D is decomposition-based evolutionary algorithm
[19] which decomposed a multi- or a many-objective prob-
lem into multiple single-objective problems obtained via
sum-scalarization. Specifically, it specifies beforehand a set
of predefined search directions uniformly distributed over
the entire Pareto-optimal front. These directions are ob-
tained by normalizing the search objectives and combin-
ing them using a weighted sum approach, where different
weights are used to specify the different search directions
to consider. Then, the MOEA/D promotes solutions that are
closer to these directions, which therefore correspond to
well-distributed reference points the search aims to reach.
The idea of using predefined reference points has been
proved to be so effective that it has been reused and ex-
tended in more recent many-objective algorithms, such as
NSGA-III [56], the surface-based evolutionary algorithm
(SEA) [57], and other decomposition-based evolutionary
algorithms [58].

The closest many-objective algorithms to HGA are the
indicator-based evolutionary algorithms. The first algorithm
proposed in the literature and falling in this category is
IBEA [59], an evolutionary algorithm that selects solutions
based on a binary hypervolume indicator that compares
the portion of hypervolume they dominate. Emmerich et al.
[60] proposed SMS-EMOA, which is a steady-state evolu-
tionary algorithm that combines non-dominated sorting
with an hypervolume-based selection. It first uses the non-
dominated sorting to determine the set of non-dominated
solutions in each generation. Then, solutions with the least
hypervolume contribution are discarded if the number of
non-dominated solutions is larger than the fixed population
size. Recently, Jiang et al. [61] proposed a more-efficient
algorithm for the exact computation of the hypervolume.
However, no analysis or proof is given about its worst-case
computational complexity.

While the aforementioned hypervolume-based evolu-
tionary algorithms help to generate better solutions than
classical multi-objective algorithms (e.g., NSGA-II, SPEA2),
they are particularly expensive due to the algorithms used
to compute the hypervolume, whose complexity is expo-
nential in the number of objectives [62], [63]. Indeed, previ-
ous studies showed that there is no polynomial algorithm
available for the exact computation of the hypervolume
dominated by a generic set of non-dominated solutions
[62]. To cope with the computation cost of the exact hy-
pervolume computation, researchers have proposed various
approximating strategy. Bader and Zitzler [63] used Monte
Carlo simulation to approximate the exact hypervolume
values. Ishibuchi et al. [64] used a scalarizing function-based
method to approximate the hypervolume metric.

Our approach follows the line of research defined by
the aforementioned hypervolume-based evolutionary algo-
rithms. However, we introduce a novel polynomial-time
algorithm for the exact computation of the hypervolume but
that is applicable when the functions used in the hypervol-
ume computation are monotonic, such as in case of the TCP
problem. Therefore, defining a polynomial-time algorithm
for the exact computation of the hypervolume indicator
for any set of solutions (i.e., for any problem) still remain
an open challenge. Since our algorithm provides an exact
computation of the hypervolume for TCP, there is no need
for the usage of approximation strategies in our context.

3 Hypervolume Genetic Algorithm for
Test Case Prioritization

This section describes the proposed hypervolume metric for
the multi-objective test case prioritization problem. It also
highlights connections and differences with the AUC-based
metrics used in previous work on search-based test case
prioritization [3], [9], [10], [11], [12].

3.1 Hypervolume indicator

In many-objective optimization, there is a growing trend to
solve many-objective problems using quality scalar indicators
to condense multiple objectives into a single objective [13].
Therefore, instead of optimizing the objective functions
directly, indicator-based algorithms are aimed at finding
a set of solutions that maximize the underlying quality
indicator [13]. One of the most popular indicators is the
hypervolume, which measures the quality of a set of solutions
as the total size of the objective space that is dominated by
one (or more) of such solutions (combinatorial union [13]).
For two-objective problems, the hypervolume corresponds
to the area under the curve, i.e., the portion of the area that
is dominated by a given set of candidate solutions, while for
three-objective problems it is represented by the volume.

Hypervolume in two-objective TCP. To illustrate intu-
atively the proposed hypervolume metric, let us consider for
simplicity only two testing criteria: (i) maximizing the state-
ment coverage and (ii) minimizing the execution cost of a
test suite. When considering the test cases in a specific order,
the cumulative coverage and the cumulative execution cost
reached by each test case draw a set of points within the
objective space.

For example, let us consider the test suite $T = \{t_1, t_2, \ldots, t_n\}$ with the following statement coverage
$Cov = \{cov_S(t_1), cov_S(t_2), \ldots, cov_S(t_n)\}$ and execution
cost $Cost = \{cost(t_1), cost(t_2), \ldots, cost(t_n)\}$. As
depicted in Figure 1-(a), if we consider the ordering $\tau =
(t_1, t_2, \ldots, t_n)$ we can measure the cumulative scores as
follows: the first test case $t_1$ covers a specific set of
code statements $cov_S(p_1) = cov_S(t_1)$ with cost equal to
$cost(p_1) = cost(t_1)$ (first cumulative point $p_1$); the sec-
ond test case in the ordering $t_2$ reaches a new cumulative
statement coverage $cov_S(p_2) = cov_S(p_1) \cup cov_S(t_2)$
with $cost(p_2) = cost(p_1) + cost(t_2)$ (second cumulative
point $p_2$). In general, $cov_S(p_i) = cov_S(p_{i-1}) \cup cov_S(t_i)$ and
$cost(p_i) = cost(p_{i-1}) + cost(t_i)$. Thus, each test case priori-
ritzation corresponds to a set of points in the two-objective
space denoted by the two testing criteria, i.e., statement
coverage and execution cost in our example (see Figure 1-
(a)). These points are weakly monotonically increasing since
cumulative cost increases, while cumulative coverage is stable.
or increases when adding a new test case from the ordering, i.e., $\text{cov}_S(p_i) \leq \text{cov}_S(p_{i+1})$ and $\text{cost}(p_i) \leq \text{cost}(p_{i+1})$. Note that in Figure 1-(a) $|\text{cov}_S(p_i)|$ denotes the cardinality of the set $\text{cov}_S(p_i)$.

Given this set of points, we can measure how quickly the given ordering $\tau$ optimizes the two objectives by measuring the proportion of the area dominated by the corresponding cumulative points $P(\tau)$, denoted by the gray area in Figure 1-(a). The dominated area is represented by all points in the objective space that are worse than the cumulative points according to the concept of dominance in Definition 4. Notice that by definition [65], the area dominated by a given point $A = (x_a, y_a)$ within the bi-dimensional objective space $F = \{\text{cost}, |\text{cov}|\}$ (i.e., cumulative cost and cumulative coverage) is the rectangle (area) delimited by all points in $F$ such that $\text{cost} \geq x_a$ and $|\text{cov}| \leq y_a$. For example, the area dominated by a cumulative point $p_i$ in Figure 1-(a) is the rectangle (area) delimited by $\text{cost} \geq \text{cost}(p_i)$ and $|\text{cov}| \leq |\text{cov}_S(p_i)|$. Given a set of non-dominated points $P(\tau)$ within the bi-dimensional objective space $F = \{\text{cost}, |\text{cov}|\}$, the overall dominated area is given by the union of the area (rectangle) dominated by each single point $p_i \in P(\tau)$ [65].

Two different orderings correspond to two different sets of cumulative points and then two different dominated areas. Therefore, we can compare the corresponding fraction of dominated areas to decide whether one candidate test case ordering is better or not than another one (fitness function): larger dominated areas imply faster statement coverage rate. In this two-objective space, the dominated area can easily be computed as the sum of the rectangles of width $|\text{cost}(p_{i+1}) - \text{cost}(p_i)|$ and height $|\text{cov}_S(p_i)|$ as reported in Figure 1-(a).

**Hypervolume in three-objective TCP.** Similarly, if we consider a third testing criterion (such as past faults coverage $|\text{cov}_{PF}(p_i)|$) each candidate prioritization corresponds to a set of points in a three-dimensional space and, in this case, the dominated proportion of the objective space is represented by a volume instead of an area, as depicted in Figure 1-(b). Since even in this three-objective space the cumulative points are always weakly monotonically increasing, the dominated volume can be computed as the sum of the parallelepipeds of width $|\text{cost}(p_{i+1}) - \text{cost}(p_i)|$, height $|\text{cov}_S(p_i)|$, and depth $|\text{cov}_{PF}(p_i)|$.

**Hypervolume in N-objective TCP.** For more than three testing criteria the objective space dominated by a set of cumulative points is called a hypervolume and represents a generalization of the area for a higher dimensional space. Without loss of generality, let $T = \{t_1, t_2, t_3, \ldots, t_n\}$ be a test suite of size $n$ and $F = \{\text{cost}, \text{Cov}_1, \ldots, \text{Cov}_m\}$ a set of testing criteria used to prioritize the test cases in $T$, where $\text{cost}$ denotes the execution cost of each test case while $\text{Cov}_1, \ldots, \text{Cov}_m$ are the remaining $m$ testing criteria to maximize. Given a permutation $\tau$ of test cases in $T$ we can compute the corresponding set of cumulative points $P(\tau) = \{p_1, \ldots, p_n\}$ obtained by cumulating the scores $\text{cost}, \text{Cov}_1, \ldots, \text{Cov}_m$ achieved by each test case in $\tau$.

**Definition 7.** — The hypervolume dominated by a permutation $P(\tau)$ of test cases can be computed as follows:

$$I_H(\tau) = \sum_{i=1}^{n-1} \left[|\text{cost}(p_{i+1}) - \text{cost}(p_i)| \times |\text{Cov}_1(p_i)| \times \cdots \times |\text{Cov}_m(p_i)|\right]$$

(6)

where $|\text{cost}(p_{i+1}) - \text{cost}(p_i)| \times |\text{Cov}_1(p_i)| \times \cdots \times |\text{Cov}_m(p_i)|$ measures the hypervolume dominated by a generic cumulative point $p_i$ but non-dominated by the next point $p_{i+1}$ in the ordering $\tau$. Since in test case prioritization the maximum values of all the testing criteria are known (e.g., the maximum execution cost or the maximum statement coverage are already known), we can express the hypervolume as a fraction of the whole objective space as follows:

**Definition 8.** — The fraction of the hypervolume dominated by a permutation $P(\tau)$ of test cases is:

$$I_{HF}(\tau) = \frac{\sum_{i=1}^{n-1} \left[|\text{cost}(p_{i+1}) - \text{cost}(p_i)| \times |\text{Cov}_1(p_i)| \times \cdots \times |\text{Cov}_m(p_i)|\right]}{\sum_{i=1}^n |\text{cost}(p_i)| \times |\text{Cov}_{min}^1| \times \cdots \times |\text{Cov}_{min}^m|}$$

(7)
where \( \text{cost}(p_n) \) is the execution cost of the whole test suite \( T \) and \( |Cov_i^{\text{max}}| \) denotes the maximum values for the \( i \)-th coverage criterion. Such a metric ranges in the interval \([0; 1]\). It is equal to +1 in the ideal case where the test case ordering allows to reach the maximum test criteria scores independently from the execution cost value \( \text{cost}(p_i) \). A higher \( I_{HP}(\tau) \) mirrors a higher ability of the prioritization \( \tau \) in maximizing the testing criteria with lower cost.

### 3.1.1 Hypervolume complexity

As pointed out by Auger et al. [13], the computation of the hypervolume indicator is usually not a trivial task and it is strongly impacted by the choice of the reference points and the distribution of solutions on the Pareto front. Despite this, it is worth noting that in the case of Test Case Prioritization a candidate test case ordering corresponds to a set of monotonically increasing cumulative scores. For this reason, we can use Equation 7 to compute the dominated hypervolume instead of the more expensive algorithm proposed by Auger et al. [13]. Indeed, the \( I_{HP}(\tau) \) metric sums up the slices of dominated hypervolume delimited by two subsequent cumulative points. Thus, let \( m \) be the number of the testing criteria and let \( n \) be the number of cumulative points (corresponding to the size of the test suite), \( I_{HP}(\tau) \) requires to sum the \( n \) hypervolume slices, each one computed as the multiplication of \( m \) test criteria scores. Thus, the overall computational time is \( O(n \times m) \). Conversely, in traditional many-objective optimization the points delimiting the non-dominated hypervolume are non-monotonically increasing and thus, the computation of the hypervolume metric requires a more complex algorithm which is exponential with respect to the number of objectives \( m \) [13], or testing criteria for TCP.

#### 3.1.2 Efficient hypervolume computation

To speed up the computation of the hypervolume metric, we use Algorithm 1. Given a permutation of test cases \( \tau \), the corresponding execution cost array \( \text{cost} \), and a set of testing criteria to maximize \( Cov_1, ..., Cov_m \), the algorithm initializes the cumulative coverage scores (line 3 of Algorithm 1). Such scores are then incrementally updated for each test case in the given order \( \tau \) (main loop in lines 4-11). In particular, for each test \( t \) in \( \tau \), the algorithm computes the cumulative cost (line 5) and cumulative coverage scores (lines 6-7), one cumulative coverage score for each testing criterion \( Cov_i \in F \). Then, the cumulative scores are used to compute the actual \( I_{HP}(\tau) \) (lines 8-9). If the maximum coverage is reached earlier for all \( Cov_i \in F \) (i.e., before iterating over all \( t \in \tau \)), the loop is terminated (lines 10-11). The remaining portion of the \( I_{HP}(\tau) \) metric is added in lines 12-13 of Algorithm 1: it corresponds to the hypervolume of size \( (\text{cost}^{\text{max}} - \text{cumCost}) \times |Cov_1^{\text{max}}| \times \cdots \times |Cov_m^{\text{max}}| \). Finally, \( I_{HP}(\tau) \) is normalized in lines 14-16. The core idea of Algorithm 1 is to reduce the number of iterations needed to compute \( I_{HP}(\tau) \) given the fact that the remaining portion of the hypervolume is known a priori when the maximum cumulative coverage is reached for all testing criteria in \( F \).

To better understand how Algorithm 1 works, let us consider the example of the test suite shown in Table 1. The test suite contains five test cases, whose execution time and coverage information are also shown in the table. Table 2 shows how the hypervolume is computed in each step of Algorithm 1 for the prioritization \( \tau = (t_5, t_3, t_4, t_2, t_1) \). First, the hypervolume and the cumulative scores are initialized as specified in line 3 of Algorithm 1. In the first iteration of the algorithm, the cumulative scores are updated based on \( t_5 \), which is the first test case in the permutation. \( t_5 \) covers four branches, five statements, and its cost is 14s. Therefore, the hypervolume score is updated according to Equation 7 as \( I_{HP}(\tau) = (61s - 14s) \times 4 \) (branches) \times 5 (statements) = 940. In the second iteration, the coverage scores are updated by considering the second test in the permutation \( \tau \), i.e., \( t_3 \). Such a test covers two additional branches and six additional statements compared to \( t_5 \). Therefore, the new hypervolume is \( I_{HP}(\tau) = 940 \) (previous value) + \( (105s - 61s) \times 6 \) (branches) \times 11 (statements) = 3,844. The third test case in the permutation is \( t_4 \), which covers two additional branches and two additional statements with an additional cost of 44s. Thus, in the third iteration of Algorithm 1, the new hypervolume value is computed as \( I_{HP}(\tau) = 3,844 \) (previous value) + \( (124s - 105s) \times 8 \) (branches) \times 13 (statements) = 5,820. The first three test cases already allow to reach 100% of branch and 100% of statement coverage; thus, the main loop in lines 4-11 of Algorithm 1 is terminated without iterating over the remaining two test cases \( t_1 \) and \( t_2 \). In the second last row of Table 2, the hypervolume is updated according to lines 12-13 of Algorithm 1. Specifically, \( I_{HP}(\tau) = 5,820 \) (previous value) + \( (165s - 124s) \times 8 \) (branches) \times 13 (statements) = 10,084. Finally, the hypervolume is normalized by diving \( I_{HP}(\tau) \) by the hypervolume of the hyper-rectangle whose sides are equal to the overall cost and coverage achievable by running all tests in \( \tau \). Specifically, the final hypervolume score for the permutation \( \tau \) is \( I_{HP}(\tau) = 10,084 / (165s \times 8 \times 13) \approx 0.5876 \).

### 3.2 Hypervolume-based Genetic Algorithm

In this paper, we consider the \( I_{HP}(\tau) \) metric as a suitable fitness function to guide search algorithms in finding the
In a single one. Each solution is a permutation of integers in
fitness function, it is possible to combine multiple objectives
single-objective problems, using the (Hypervolume-based Genetic Algorithm), depicted in Al-
individual from the current generation with a given probability,
function, (ii) a crossover operator, that recombines two indi-
offsprings) are created by applying
new individuals (\(\tau\) in multi-objective test case prioritization.
for every test \(t\), we specify which
branches and statement are covered by \(t\) as well as its execution cost (time in s).

\[
\begin{array}{cccccccccc}
\text{Tests} & b_1 & b_2 & b_3 & b_4 & b_5 & b_6 & b_7 & b_8 & \text{Cost} \\
\hline
\text{t}_1 & X & X & & & & & & & 41 \\
\text{t}_2 & X & X & X & & & & & & 19 \\
\text{t}_3 & X & & X & X & & & & & 47 \\
\text{t}_4 & X & & X & X & X & & & & 44 \\
\text{t}_5 & X & X & X & X & X & X & & & 14 \\
\end{array}
\]

TABLE 1
An example of test suite \(T = \{t_1, t_2, t_3, t_4, t_5\}\) for a small program with eight branches, 13 statements. For every test \(t\), we specify which branches and statement are covered by \(t\) as well as its execution cost (time in s).

<table>
<thead>
<tr>
<th>Tests</th>
<th>Branches</th>
<th>Statements</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>t₁</td>
<td>X X</td>
<td>X</td>
<td>41</td>
</tr>
<tr>
<td>t₂</td>
<td>X X X</td>
<td>X X X X X</td>
<td>19</td>
</tr>
<tr>
<td>t₃</td>
<td>X X X X X</td>
<td>X X X X X</td>
<td>47</td>
</tr>
<tr>
<td>t₄</td>
<td>X X X X</td>
<td>X X X X</td>
<td>44</td>
</tr>
<tr>
<td>t₅</td>
<td>X X X X X</td>
<td>X X X X X</td>
<td>14</td>
</tr>
</tbody>
</table>

TABLE 2
Walkthrough of Algorithm 1 for the test suite in Table 1 and the prioritization \(\tau = (t_5, t_3, t_4, t_2, t_1)\).

<table>
<thead>
<tr>
<th>Steps</th>
<th>Selected Tests</th>
<th>Cumulative scores</th>
<th>(I_{HP})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initialization</td>
<td>-</td>
<td>0 0 0 0</td>
<td>0 0</td>
</tr>
<tr>
<td>Iteration 1</td>
<td>(t_5)</td>
<td>14 4 5</td>
<td>940</td>
</tr>
<tr>
<td>Iteration 2</td>
<td>(t_3)</td>
<td>61 6 11</td>
<td>3,844</td>
</tr>
<tr>
<td>Iteration 3</td>
<td>(t_4)</td>
<td>105 8 13</td>
<td>5,820</td>
</tr>
<tr>
<td>Adding the last part of the volume</td>
<td>(t_1, t_2)</td>
<td>165 8 13</td>
<td>10,084</td>
</tr>
<tr>
<td>Normalization</td>
<td>-</td>
<td>- - -</td>
<td>0.5876</td>
</tr>
</tbody>
</table>

Algorithm 2: Hypervolume Genetic Algorithm

Input: Solution representation: permutation of test cases
Fitness function: \(I_{HP}(\tau)\)
Result: the best permutation of test cases according to \(I_{HP}(\tau)\)

\[\begin{align*}
\text{begin} & \quad \text{initialize population with random candidate solutions} \\
& \quad \text{while max \# of generations has not been reached do} \\
& \quad \quad \text{select best individuals based on } I_{HP}(\tau) \text{ using binary tournament selection} \\
& \quad \quad \text{recombine pairs of individuals using PMX-Crossover} \\
& \quad \quad \text{mutate individuals using SWAP-Mutation} \\
& \quad \text{evaluate each candidate solution} \\
\text{end} \]

optimal ordering \(\tau\) in multi-objective test case prioritization.
In particular, we applied the Genetic Algorithm (GA) [66],
a stochastic search technique based on the mechanism of
natural selection and natural genetics. We selected this
algorithm because it has been used to solve a wide range of
optimization problems that are not solvable in polynomial
time. Moreover, with respect to other search algorithms, it is
highly parallelizable [67].

GA starts with a random population of solutions. Each
individual (i.e., chromosome) represents a solution of the
optimization problem. The population evolves through sub-
sequent generations where individuals are evaluated based
on a fitness function to be optimized. At each generation,
new individuals (i.e., offsprings) are created by applying
three operators: (i) a selection operator, based on the fitness
function, (ii) a crossover operator, that recombines two indi-
viduals from the current generation with a given probability,
and (iii) a mutation operator, which modifies the individuals
with a given probability.

We propose a new genetic algorithm named HGA
(Hypervolume-based Genetic Algorithm), depicted in Al-
gorithm 2. Despite, GAs are commonly used for solving
single-objective problems, using the hypervolume indicator as
fitness function, it is possible to combine multiple objectives
in a single one. Each solution is a permutation of integers in
which each element represents a test case to be executed
and the population is represented by a set of different
test case permutations. The selection operator is the binary
tournament selection (line 5), which randomly picks two
individuals for the tournament and selects the one with the
better fitness function. The crossover operator is the PMX-
Crossover (line 6), which swaps the permutation elements
at a given random crossover point. The mutation operator
is the SWAP-Mutation (line 7) that randomly swaps two
chosen permutation elements within each offspring. More
details on the parameter settings are reported in Section
4.1.4. The fitness function that drives the GA evolution is
the hypervolume indicator described in Section 3.1. HGA
can be briefly summarized as (i) generating test cases orderings,
(ii) evaluating the permutations using the \(I_{HP}(\tau)\) metric,
and (iii) using this value to drive the GA evolution.

3.3 The Relationship between Hypervolume and AUC-
based Metrics

The \(I_{HP}(\tau)\) metric proposed in this paper can be viewed as
a generalization of the AUC-based metrics (e.g., APSC) used
in prior work on search-based test case prioritization. For
example, the APSC metric measures the average cumulative
fraction of statements coverage as the Area Under Curve
delimited by the test case ordering with respect to the
cumulative statement coverage scores [3]. In light of the
proposed hypervolume metric, APSC can be viewed as a
simplified version of \(I_{HP}(\tau)\) where all test cases have exe-
cution cost equal to one and only the statement coverage is
considered as a testing criterion. A similar consideration can
be made for all the other cumulative fitness functions used
in previous work on search-based test case prioritization [3],
[9], [10].

Finally, as explained in Section 2, despite the AUC
metrics being strictly dependent on each other, they are
calculated independently in test case prioritization based
on multi-objective Genetic Algorithms. Indeed, these values
are projections of a manifold of cumulative points (e.g., a
projection of a volume into two areas). For example, let
us consider again the example of the test suite in Table
1. Figure 2-(a) depicts the cumulative coverage and cost
scores for the prioritization \(\tau = (t_5, t_3, t_4, t_2, t_1)\). Applying
AUC-based metrics to assess the fitness of \(\tau\) require us to
calculate two metrics, i.e., APBCc and APSCc. These metrics
correspond to the grey areas in Figure 2, which correspond
to the projections of the hypervolume on the geometric
plane Statement-Cost and Branch-Cost. An important dif-
fERENCE between the AUC-based metrics (e.g., APSC) and
\(I_{HP}(\tau)\) lies in how they measure the area dominated by a
given test case permutation/ordering \(P(\tau)\). The AUC-based
that the three original testing criteria correspond to only one objective paradigm with the theoretical motivation that having multiple Pareto optimal solutions helps to accommodate for different views from decision-makers that may profit of the variants to refine their views during the solution evaluation process. In the following, we show, through an example, that choosing trade-offs in the space of the AUC-based metrics is different from choosing trade-offs among the original testing criteria being condensed in these metrics. To this aim, let us consider again the example of test suite of the two permutations \(\tau_B\) and \(\tau_S\) correspond to the two corners of the Pareto front as shown in Figure 3-(a). The third Pareto optimal solution is the permutation \(\tau_H = \{T_5, T_2, T_3, T_1\}\), which corresponds to the solutions with the largest hypervolume score.

Let us assume that the decision-maker wants to give higher priority to statement coverage over branch coverage and therefore he/she chooses the solution \(\tau_S\) with the largest statement coverage rate. In theory, we may conclude that \(\tau_S\) is indeed the best test prioritization for statement coverage. To refute this hypothesis, let us now look at the solution \(\tau_S\) projected in the space of the original coverage criteria (for simplicity we consider only cost and statement coverage) rather than in the objective space (i.e., the space of the AUC-metrics). Figure 3-(b) depicts the statement coverage over execution cost achieved by \(\tau_S\) compared to the solution with \(\tau_H\). As we can observe, \(\tau_S\) achieves the maximum statement coverage earlier than \(\tau_H\) (105s for the former compared to 121s for the latter). However, \(\tau_H\) achieves better statement coverage than \(\tau_S\) during the first 60s of test execution time. Similarly, the best permutation is \(\tau_H\) if our goal is to reach higher statement coverage in 100s of test execution time.

Remark 2: when not enough resources are available to run the entire test suite, choosing a solution among the trade-offs produced with AUC-based metrics can lead to suboptimal results.

Furthermore, we notice that any test case permutation is by itself a set of trade-offs in the space of the original testing criteria. Indeed, \(\tau_S\) corresponds to six points/trade-offs between execution cost and statement coverage as already shown in Figure 3. Similarly, \(\tau_H\) corresponds to six trade-offs in the space of the testing criteria. Decision-makers can, in theory, choose not only which permutation to select but also whether stopping the execution of the test suite earlier (if he/she has not enough resources and time to run the entire suite). In our example, analyzing the trade-offs in the space of execution cost and statement coverage depicted in Figure 3 provides better insights about the pros and the cons of the two permutations \(\tau_S\) and \(\tau_H\) when varying the amount of resources (time) we want to spend on regression testing.

Remark 3: each test permutation is by definition a set of trade-offs in the space of the testing criteria.
4 Evaluating the Hypervolume Genetic Algorithm with Less Than Three Criteria

We conduct a first empirical study to assess the performances of HGA. In particular, we investigate the following high-level research question:

RQ1: What is the cost-effectiveness and efficiency of HGA, compared to state-of-the-art test case prioritization techniques?

To better clarify it, we detailed it in two research questions:

- RQ1.1: What is the cost-effectiveness of HGA, compared to state-of-the-art test case prioritization techniques? This research question aims at evaluating to what extent the test case ordering obtained by HGA is able to detect faults (effectiveness) earlier (lower execution cost) in comparison with three state-of-the-art techniques: a cost cognizant additional greedy algorithm [5], [41], a single objective genetic algorithm based on an AUC metric (GA) [3], and a multi-objective search based algorithm namely NSGA-II [20] used in prior test case prioritization [14], [12]. This reflects the developers' needs to discover regression faults with minimum cost.

- RQ1.2: What is the efficiency of HGA, compared to state-of-the-art test case prioritization techniques? With this second research question, we are interested in comparing the running time (efficiency) required by HGA to find an optimal test ordering, in comparison with the three experimented test case prioritization techniques.

4.1 Study Design

This subsection describes the design of the study.

4.1.1 Context of the Study

The context consists of five GNU utilities —namely Bash, Flex, Grep, GZip, Sed—from the Software-artifact Infrastructure Repository (SIR) [68]. The characteristics of these five programs are reported in Table 3, including their size (in terms of lines of code), test suite size, and type of faults.

In total, the selected programs have a size ranging between 5,680 and 59,846 LOC, while the number of test cases varies between 214 and 1,061. We selected these programs since they have been used in previous work on regression testing [3], [12], [14], [69], [70], [71]. Moreover, they have different size, number of tests, and context applications. As faults, we consider the seeded faults that are available in SIR. Please consider that, when seeding the faults, the authors of the repository assumed that the programmer that made the changes inserted the faults. Thus, the seeded faults can be located only within the changes between versions (calculated with the assistance of a diff tool). More specifically, SIR provides a list of seeded faults with the corresponding test-fault coverage information. In our study, we considered the non-trivial faults, i.e., faults that can be exposed by a very few test cases, as suggested in the SIR guidelines [68]. For the sake of this analysis, we always selected the largest hard matrices (i.e., matrices of faults that are killable by few tests) in case of multiple fault matrices available in the SIR repository.

4.1.2 Testing Criteria

To answer our research questions, we considered different testing criteria widely used in previous test case prioritization work [3], [12], [72]:

- **Statement coverage criterion.** We measured statement coverage achieved by each test case using gcov, a profiling tool that is part of the GNU C compiler (gcc).

- **Execution cost criterion.** To compute the execution cost, we could just measure the test case execution time. However, this measure depends on several external factors such as different hardware, application software, operating system, etc. In this paper, we addressed this issue by counting the number of executed instructions in the production code, instead of measuring the actual execution time. To this aim, we used gcov to measure the execution frequency of each source code instruction for the programs from the GNU. Notice that approximating the execution cost as the number of

1. https://sir.unl.edu/content/c-fault-seeding.php
that have been widely used in prior studies (mine which coverage criteria have the higher likelihood namely (i) Additional Greedy

We compared HGA formulations of the TCP problem:

We considered the two different testing criteria described above, we examined two different formulations of the TCP problem:

- **Two-criteria (Single-objective).** The goal is to find an optimal ordering of test cases which (i) minimizes the execution cost and (ii) maximizes the statement coverage.

- **Three-criteria (Two-objective).** For this formulation, we considered the past faults coverage as a third criterion to be maximized.

### 4.1.3 Evaluated Algorithms

We compared HGA with three state-of-the-art algorithms, namely (i) Additional Greedy [5], [14], [41], (ii) GA [3], and (iii) NSGA-II [14], [20]. In particular, we compared HGA with Additional Greedy and GA in the single objective formulation (two criteria) and with Additional Greedy and NSGA-II in the two-objective formulation (three criteria).

**Additional Greedy.** This algorithm instantiated for the TCP problem [5], [14] considers coverage and cost at the same time by maximizing the coverage per unit of time of the selected test cases (cost cognizant additional greedy). Similarly, for what concerns the three-criteria formulation of the problem, we used the algorithm proposed by Yoo and Harman [7], which conflates code coverage, execution cost and past coverage in one objective function to minimize.

**Additional Greedy is an iterative deterministic search algorithm that starts with an empty order of test cases \( \tau_0 = \emptyset \); then, it selects the test case \( t_{\text{max}} \) having the highest value of code coverage per time unit (greedy step), i.e., \( \tau_1 = (t_{\text{max}}) \). In each of the subsequent iterations, it selects the test case yielding the highest (additional) increment of code coverage in the last previous iteration of the algorithm. The loop ends when the highest coverage per unit time is reached, i.e., when adding any un-selected test does not lead to an increment in coverage. To complete the test order, the un-prioritized test cases that do not contribute to the additional coverage could be ordered using any strategy (e.g., using a random order). In this way, we recursively re-applied the Additional Greedy algorithm to the un-prioritized tests until all are ordered, as done in previous work [3].

When multiple coverage criteria are used (as for the three-criteria formulation), the additional coverage per unit time of each test \( t \) is computed using the following equation:

\[
g(t) = \frac{1}{m} \times \frac{1}{\text{cost}(t)} \times \sum_{i=1}^{i=m} f_i(t)\]

where \( F = \{f_1, \ldots, f_m\} \) is the set of coverage criteria to consider and \( \text{cost}(t) \) denotes the execution cost of the test \( t \).

**Genetic Algorithm.** Genetic Algorithms (GAs) represent a class of search techniques based on the natural selection processes defined by Darwin’s theory of biological evolution. A typical GA procedure starts with an initial population \( P \) of individuals. Selected pairs of individuals are combined and mutated to generate new individuals that will be part of the population of the next generation. A GA is an approximated algorithm that does not guarantee to converge. For this reason, the search continues for a number of generations until a stop condition is reached. Individuals of the population are represented by their chromosome (e.g., the sequences of their variables/parameters). We select a GA because it is one of the best single-objective algorithms for the test case prioritization problem [3].

**NSGA-II.** The Non-dominated Sorting Genetic Algorithm II [12] is a computationally fast and elitist multi-objective evolutionary algorithm based on a non-dominated sorting approach. As any population-based evolutionary algorithms, NSGA-II starts with a set of solutions (test case orderings in our case) randomly generated within the solution space. At each generation, offsprings are generated by combining pairs of fittest individuals through three genetic operators: selection, crossover and mutation. To form the population for the next generation, parents and offsprings are ordered using the non-dominated sorting algorithm, which assigns to each candidate solution a fitness score that combines the non-domination relation (see Equation 3) and the crowding distance. The individuals are sorted and the fittest ones are selected to form the new population. The process is repeated until a maximum number of

<table>
<thead>
<tr>
<th>Program</th>
<th>Description</th>
<th>Version</th>
<th>LOC</th>
<th># Tests</th>
<th># Faults</th>
<th>Language</th>
<th>Fault Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bash</td>
<td>Shell Language Interpreter</td>
<td>V2</td>
<td>59,846</td>
<td>1,061</td>
<td>5</td>
<td>C</td>
<td>Seeded</td>
</tr>
<tr>
<td>Flex</td>
<td>Fast Lexical Analyzer</td>
<td>V2</td>
<td>10,459</td>
<td>567</td>
<td>15</td>
<td>C</td>
<td>Seeded</td>
</tr>
<tr>
<td>Grep</td>
<td>Regular Expression Utility</td>
<td>V2</td>
<td>10,068</td>
<td>809</td>
<td>10</td>
<td>C</td>
<td>Seeded</td>
</tr>
<tr>
<td>GZip</td>
<td>Compression Tool</td>
<td>V2</td>
<td>5,680</td>
<td>214</td>
<td>11</td>
<td>C</td>
<td>Seeded</td>
</tr>
<tr>
<td>Sed</td>
<td>Non Interactive Text Editor</td>
<td>V2</td>
<td>14,427</td>
<td>360</td>
<td>5</td>
<td>C</td>
<td>Seeded</td>
</tr>
</tbody>
</table>

**TABLE 3** Programs used in the study.
performances. In particular, we use the following (default) search-based software engineering does not assure better is a long and expensive process that in the context of reasonable choice, even considering that parameter tuning studies [76], [77] demonstrated that default values are a conscious studies on TCP [12], [14]. This is because previous the performance of all the algorithms reducing the size of theposed by Epitropakis usage compaction algorithm data using the lossless.

4.1.4 Implementation Details and Parameter Setting

All the algorithms have been implemented using JMetal [75], a Java-based framework for multi-objective optimization with meta-heuristics. To reduce the execution time needed for the experiments, we pre-processed the coverage data using the lossless coverage compaction algorithm proposed by Epitropakis et al. [12]. This technique improves the performance of all the algorithms reducing the size of the coverage matrices by a factor between 7 and 488 [12].

We used the default parameters values used in previous studies on TCP [12], [14]. This is because previous studies [76], [77] demonstrated that default values are a reasonable choice, even considering that parameter tuning is a long and expensive process that in the context of search-based software engineering does not assure better performances. In particular, we use the following (default) parameter values:

- **Population size**: 250 individuals.
- **Selection**: binary tournament selection. It randomly picks two individuals for the tournament and selects the fittest one. The winner of each tournament is the solution with the best \( I_{HP(t)} \) (Equation 7) in HGA or the permutation with the best AUC-based metric for GA. For NSGA-II, the winner of the tournament is the test case with the best non-dominance rank, or with the highest crowding distance at the same level of non-dominance rank.
- **Crossover**: PMX-Crossover with crossover probability of \( p_c = 0.90 \). This operator swaps elements at a given random crossover point.
- **Mutation**: SWAP-Mutation that randomly swaps two chosen permutation elements within each offspring with a mutation probability of \( p_m = 1/n \), where \( n \) is the number of test cases.
- **Stopping criterion**: 100 generations, corresponding to 25,000 fitness evaluations.

To account for the inherently random nature of search-based algorithms [78], we performed 30 independent runs for each program and for each search algorithm in our study.

4.1.5 Evaluation Metrics

To address RQ1.1 we used the cost-cognizant Average Percentage of Faults Detected metric (APFD\(_c\)) proposed by Elbaum et al. [41]. This metric measures the ability of a test permutation to reveal faults earlier [41]. The larger the APFD\(_c\), the lower the average cost needed to detect the same number of faults. Since we performed 30 independent runs, we report the mean and the standard deviation of the APFD\(_c\) scores achieved for each program and for each formulation. It is worth noting that for NSGA-II we report the mean and the standard deviation of all the solutions in the Pareto set. The cost-cognizant Average Percentage of Faults Detected per unit cost can be computed as follows:

\[
APFD_c = \frac{\sum_{i=1}^{m} \left( \left( \sum_{j=1}^{c_i} c_j - \frac{1}{2} c \right) \right)}{\sum_{i=1}^{m} c_i \times m}
\]

where \( T = \{t_1, t_2, \ldots, t_n\} \) is the test suite to be optimized, with cost \( C = \{c_1, c_2, \ldots, c_n\} \), TS is the first test case in an ordering \( T' \) of \( T \) that is able to cover the statement \( i \).

4.2 Results of the empirical study

This subsection discusses the results of our first study, thus, answering the research questions.

4.2.1 Results for Two-criteria (Single objective) formulation

Table 4 reports the APFD\(_c\) values and the running time obtained by HGA and the state-of-the-art algorithms on the five programs from the Software-artifact Infrastructure Repository (SIR) [68].

Results for RQ1.1. From the comparison between HGA and Additional Greedy, we observe that the former achieves statistically higher APFD\(_c\) scores than the latter...
in four out of five programs (i.e., $\hat{A}_{12} > 0.5$ and $p$-value < 0.05). Moreover, the $\hat{A}_{12}$ statistics reveal that in all these cases the effect size is large. The improvements range between a minimum of +0.60% and a maximum of +41.10% achieved for Sed and GZip, respectively. Instead, Additional Greedy produced a significantly higher APFD$_c$ score for Bash although the difference is quite small: -2.10% on average. Bash is a particular program since all algorithms almost achieve an optimal APFD$_c$, which is very close to one. This is due to the fact that the mutation test requires between 0.01 1.00 Large $\hat{A}_{12}$ and 1.00 Large $\hat{A}_{12}$, respectively. In other words, the larger the test suite, the more time Additional Greedy needs in terms of execution time.

The comparison between GA and HGA, we can notice that for all the programs we can reject the null hypothesis for the Wilcoxon test. However, the Vargha-Delaney ($\hat{A}_{12}$) statistics, in all programs, HGA outperforms (is more efficient than) GA with large effect size. Indeed, GA requires between 2.00 (e.g., GZip) to 3.20 times (e.g., Grep) the execution times required for HGA. On average HGA is 1.80 times faster than GA. As we already noticed in the comparison with Additional Greedy, the number of test cases strongly influences the performance of GA. Indeed, as the number of test cases increases the ratio between the time required by GA and HGA increases. These observations are also confirmed by the permutation test: the differences (improvements/worsening) between the execution time of the two algorithms and the number of the test cases to prioritize ($p$-value=4.14 × 10^{-4}). In other words, the larger the test suite, the more time Additional Greedy needs in terms of execution time.

4.2.2 Results for Three-criteria (Two-objective) formulation

Table 5 reports the APFD$_c$ values and the running time obtained by HGA, Additional Greedy, and NSGA-II.

Results for RQ$_{1.1}$. We observe that HGA outperforms Additional Greedy in four out of five programs with a large effect size. HGA improves APFD$_c$ values up to +48.40% with respect to Additional Greedy, while in the opposite case the difference is low (e.g., -2.00% on Bash). Looking at the results obtained when comparing NSGA-II with HGA, we notice that in four cases out of five we can
reject the null hypothesis. In three of those cases, HGA outperforms NSGA-II: in two cases with a small effect size and in another one with a medium effect size.

Only for Bash, Additional Greedy produces a better test permutation with respect to HGA with a medium effect size. Furthermore, there is also only one program (i.e., Sed), in which NSGA-II is better than HGA with large effect size. Both these two programs are characterized by very large APFD$_c$ scores for all search algorithms (i.e., APFD$_c > 0.90$). To shed light on these close-to-being-optimal results, we manually investigated the permutations generated by the algorithms. While the regression faults are non-trivial for both the two programs —they are detectable by 1.5% of tests on Bash and 16% of tests in case of Sed—all fault-revealing test case have very high statement coverage and are, therefore, always selected very early. The differences in APFD$_c$ scores between the three algorithms are due to very few test cases that differ in the corresponding test permutations. Given the fact that all algorithms achieve very high APFD$_c$ scores, these differences are negligible in practice although statistical significant.

Notice that for the comparison above we considered all Pareto-optimal solutions produced by NSGA-II (between nine and 50 solutions). However, different solutions in the Pareto fronts may provide different APFD$_c$ scores. Figure 4 compares the APFD$_c$ scores by HGA (single points) with the boxplots of NSGA-II (i.e., the distributions of APFD$_c$ scores of the entire Pareto front). The purpose of this comparison is two-fold: (i) we want to measure whether the majority of the solutions by NSGA-II are better than the single solution by HGA; and (ii) we want to measure the variability of the APFD$_c$ values by NSGA-II.

As we can observe, in four systems, the single solution provided by HGA is better or equal to the median solution of NSGA-II. For GZip, we observe a huge variation in the APFD$_c$ distribution yielded by NSGA-II: it ranges between 0.40 and 1.00, with a median value of 0.400. For this project, choosing a proper solution from the Pareto front is very critical since not all its solutions have better APFD$_c$ scores than HGA. In particular, only 26% of the Pareto front is more cost-effective than the single solution achieved by HGA. Once again, no guideline exists that helps the testers choosing the most cost-effective solutions in the Pareto front as the APFD$_c$ scores can be computed only by executing all test permutations.

To better understand how the three algorithms optimize the selected testing criteria, Figure 5 plots—for Grep and Sed—the Pareto front produced by NSGA-II, and the single solutions generated by Additional Greedy and HGA with respect to the objectives optimized by NSGA-II (APSC$_c$ and APPFD$_c$). The complete set of plots for all the programs in our study is available in our online appendix [21]. This comparison allows understanding whether the solutions generated by one algorithm dominate (i.e., are better than) the solutions produced by an alternative algorithm in the space of the AUC-metrics. It is possible to notice that in all the cases the solutions generated by NSGA-II and HGA always dominate the solutions produced by Additional Greedy.

Furthermore, we observe that the single solution yielded by HGA is never dominated by the Pareto front generated by NSGA-II. Vice versa, for Grep, the single solution by HGA dominates the majority of the Pareto fronts produced by NSGA-II. These results are very unexpected considering that NSGA-II explicitly optimizes APSC$_c$ and APPFD$_c$ as two contrasting objectives. Instead, HGA optimizes the hypervolume indicator, which generalizes and combines

![Fig. 4. APFD$_c$ scores achieved by Additional Greedy (□), NSGA-II (boxplots), and HGA (*) on the three-criteria (two-objective) formulation of the TCP problem.](image-url)
**Summary for RQ1.** HGA outperforms Additional Greedy in most of the cases in terms of cost-effectiveness but it is less efficient. On the two-criteria formulation, as expected from the theory, HGA and GA are equivalent in terms of fault detection capability. However, the former is more efficient than the latter thanks to our algorithm for the fast computation of the hypervolume metric. On the three-criteria formulation, HGA is often more effective and always more efficient than NSGA-II.

**5 Evaluating the Hypervolume Genetic Algorithm with up to Five Criteria**

This section discusses the second empirical study we carried out to assess the performances of HGA compared to state-of-the-art many-objective algorithms when handling up to five testing criteria. Thus, we formulated the additional following high-level research questions:

- **RQ2:** How does HGA perform with respect to many-objective test case prioritization techniques?

To better clarify it, we detailed it in two research questions:

- **RQ2.1:** What is the cost-effectiveness of HGA, compared to many-objective test case prioritization techniques? This research question aims at evaluating the selective pressure of HGA, that is “the degree to which the better individuals are favored during the computation” [82]. In particular, similarly to RQ1.1, it analyses to what extent the test case ordering obtained by HGA is able to detect faults (effectiveness) earlier (lower execution cost) in comparison to two state-of-the-art many-objective algorithms, namely GDE3 and MOEA/D-DE.

- **RQ2.2:** What is the efficiency of HGA, compared to many-objective test case prioritization techniques? Similarly to RQ1.2, with this research question, we are interested, in comparing the running time (efficiency) required by HGA to find an optimal test ordering compared to the alternative many-objective algorithms in cases that require a strong selective pressure.
5.1 Study Design

5.1.1 Context of the Study and Testing Criteria

The context of the study is the same as that of the first empirical study in Section 4. For the testing criteria, we added two additional criteria with respect to the first study, namely branch and function coverage criteria. In particular, we considered the following criteria:

- **Branch and function coverage criterion.** Also, in this case, we measured statement coverage achieved by each test case using the gcov tool part of the GNU C compiler (gcc).

With these additional testing criteria, we examined two different many-objective formulations of the TCP problem:

- **Four-criteria (Three-objective).** The goal is to find an optimal ordering of test cases which (i) minimizes the execution cost, (ii) maximizes the statement coverage, (iii) maximizes the past faults coverage, and (iv) maximizes the branch coverage. We applied this formulation for the ten programs from SIR [68].

- **Five-criteria (Four-objective).** For this formulation, we considered the function coverage as a fifth criterion to be maximized. We applied this formulation on the same programs already used for the four-objective formulation.

5.1.2 Evaluated Algorithms and Parameter Setting

We compared the results of HGA with the those achieved by two algorithms, namely (i) GDE3 [18] and (ii) MOEA/D-DE [19]. The former implements a diversity-based mechanism to address the problem of selective resistance, while the latter is a reference-point based mechanism to guarantee well distributed Pareto fronts for many-objective problems. These two algorithms inspired many other many-objective meta-heuristics and are representative for the two classes of algorithms discussed in section 2.2.1. As already highlighted in the first study in Section 4, it is worth noting that for GDE3 and MOEA/D-DE the objective functions to optimize are AUC-based metrics.

For GDE3 and MOEA/D-DE, we used their implementation available in JMetal [75] and preprocessed the coverage data using the lossless coverage compaction algorithm proposed by Epitropakis et al. [12]. For both algorithms, we used their default parameters values [12], [14]:

- **Population size:** 250 individuals as for HGA.
- **Selection:** For GDE3 and MOEA/D-DE, the fittest individuals are selected using the differential evolution selection operator.
- **Crossover:** we used the PMX-Crossover with crossover probability \( p_c = 0.90 \) and MOEA/D-DE need also to set another parameter, namely \( CR \). This parameter indicates how single sub-problems are separable (i.e., the lower the value, the more the problems are separable). We applied the default values (e.g., 0.50 for GDE3 and 1.00 MOEA/D-DE).
- **Mutation:** as mutation operator, we used the DE/1/2-DE and MOEA/D-DE need an additional parameter \( F \). This scaling factor controls the speed and robustness of the search (i.e., with a lower value the algorithm converges faster, but it has a higher risk of stacking in a local optimum). Also, in this case, we applied the default value (i.e., 0.50).
- **Stopping criterion:** the evolutionary algorithms end when reaching 100 generations, corresponding to 25,000 fitness evaluations.

We used default parameters considering that previous studies [76], [77] demonstrated that they are a reasonable choice, even considering that parameter tuning is a long and expensive process that in the context of search-based software engineering does not assure better performance.

To account for the inherently random nature of search-based algorithms [78], we performed 30 independents runs for each program and for each search algorithm in our study.

5.1.3 Evaluation Metrics

We used the same evaluation metrics used in Section 4. In particular, for RQ2.1 we relied on the same evaluation metrics used for RQ1.1, while for RQ2.2 we relied on the same evaluation metrics used for RQ1.2.

5.2 Results of the empirical study

This section discusses the results of our second study, thus, answering the research questions.

5.2.1 Results for Four-criteria (Three-objective) formulation

Table 6 reports the APFD values and the running time obtained by HGA and the state-of-the-art algorithms for the five programs from the Software-artifact Infrastructure Repository (SIR) [68].

**Results for RQ2.1.** We observe that HGA achieves equal or better APFD values with respect to GDE3 and MOEA/D-DE. In particular, the Wilcoxon t-test revealed that in three out of five programs the differences between HGA and GDE3 are statistically significant (in two cases with large effect size and in one case with large effect size). Notice that GDE3 returns many solutions (test permutations), whose number ranges between 20 (e.g., GZip) and 38 (e.g., Flex). Figure 6 shows that the APFD values achieved by GDE3 may vary across the solutions in the Pareto fronts. In all the programs, the solution by HGA is more or equally cost-effective than the median score yielded by all the solutions by GDE3. For example, on Bash the APFD scores by GDE3 vary between 0.65 and 0.98, with a median value of 0.875. On this project, the solution by HGA outperforms 70% of the Pareto optimal solutions obtained with GDE3.

When comparing HGA with MOEA/D-DE we notice that in three out of five cases the null hypothesis cannot be rejected according to the Wilcoxon t-test. For the remaining programs, HGA is better than MOEA/D-DE (in one case with large effect size and in the other one with small effect size). Moreover, not all Pareto efficient solutions yielded by MOEA/D-DE achieve the same APFD values as shown in Figure 6. One exemplary case is observable for GZip: for this project, the APFD scores of the Pareto optimal solutions by MOEA/D-DE range between 0.40 and 1.00, with a mean value of 0.602. Instead, the single solution by HGA is better than 75% of Pareto optimal solutions generated by MOEA/D-DE.
Figure 7 plots —for Flex and Sed— the Pareto fronts produced by GDE3 and MOEA-D/DE as well as the single solutions generated by HGA with respect to the AUC-based metrics. As we can observe, the solutions by GDE3 and MOEA-D/DE do not dominate the solution generated by HGA. Vice versa, the solution generated by HGA dominates a large portion of the solutions by these algorithms. Using the hypervolume indicator as fitness function, HGA is able to optimize the AUC-based metrics even if the baselines use such metrics as objectives to optimize. This further poses the question of whether it is worth using multi-objective or many-objective algorithms for the test case prioritization problem, given the difficulty to discriminate the best solution among those produced by these algorithms. The complete set of plots of all the programs is available in our online appendix. [21].

Results for RQ2.2. In all the programs, the HGA is statistically faster (i.e., $A_{12} < 0.5$ and p-value < 0.05) than the GDE3 with large effect size. The improvements range between a minimum of 1.65 times and a maximum of 3.00 times, achieved on Bash and Sed, respectively. On average, HGA is 1.81 times faster than GDE3. Similarly to the results achieved for NSGA-II, the number of test cases strongly influences the performance of GDE3. Indeed, the ratio between the execution time needed by GDE3 and the execution time required by HGA increases as the number of test cases grows. This insight is further confirmed by the permutation test: the differences (improvements/worsening) between the execution time of HGA and GDE3 significantly interacts with the test suite size (p-value=7.9 × 10^{-5}).

These results are confirmed when comparing HGA with MOEA-D/DE. Indeed, for all the programs we can reject the null hypothesis for the Wilcoxon t-test with large effect size. MOEA-D/DE requires between 1.30 (e.g., Bash) to 3 times (e.g., Sed) the execution times required for HGA. On average HGA is 1.49 times faster than MOEA-D/DE. To assess the interaction between the number of test cases and the performance gap between HGA and MOEA-D/DE, we performed the permutation test achieving a p-value equal to 2.20 × 10^{-16}.

By comparing the running time of HGA across the different programs, we can notice that HGA took more time to converge on Bash (51s) and Grep (19s) with respect to the other programs in our study. The computation cost of HGA is polynomial to (i) the number of criteria, (ii) to the population size, (iii) the cost of computing the hypervolume metric. While the number of criteria and the population size is the same for all programs, the cost of computing the hypervolume metric varies. Indeed, the cost of computing such a metric depends on two factors: (1) the test suite size (i.e., the total number of test cases) and (2) the percentage of test cases in the test suite required to reach the maximum coverage. Compared to the other programs, Bash and Grep have the largest test suites in our study. Furthermore, reaching the maximum coverage scores requires to run almost all test cases in their suites. In such a scenario, the algorithm for computing the hypervolume metric (Algorithm 1) performs a large number of iterations (equal to the number of test cases needed to reach the maximum coverage score).

### 5.2.2 Results for Five-criteria (Four-objective) formulation

Table 7 reports the APFD values and the running time obtained by HGA and the state-of-the-art algorithms for the five programs from the Software-artifact Infrastructure Repository (SIR) [68].

Results for RQ2.1. The results are very similar to those achieved in the four-criteria formulation. Indeed, when
comparing HGA and GDE3, we notice that on 4 out of 5 programs the former algorithm achieves statistically better scores than the latter (three times with large effect size and one time with medium effect size). Analyzing the results for MOEA-D/DE, we observe that in all cases, we cannot reject the null hypothesis. Hence, the results achieved by HGA and MOEA-D/DE are comparable in terms of APFDc.

It is important to highlight that Table 6 reports the performance of these algorithms on the five-criteria (four-objective) formulation of TCP. The table does not describe the distributions of the scores. To this aim, Figure 8 compares these distributions. We can notice that the solution by HGA is more cost-effective than the solutions produced with GDE3 and MOEA-D/DE. Moreover, the performance of these algorithms has a large variation. For example, on GZip, the scores for GDE3 vary between 0.20 and 1.00 while for MOEA-D/DE they vary between 0.40 and 1.00. Choosing a proper solution in these contexts is very hard. Indeed, for GZip only four out of 144 solutions by MOEA-D/DE are more cost-effective than the solution achieved by HGA. Moreover, there is no guideline that helps in choosing the most cost-effective solutions in the Pareto front as the fault detection capability can be computed only after a posteriori.

Results for RQ2.2. The comparison between HGA and GDE3 shows that, in all the programs, the former algorithm is statistically faster (i.e., $\hat{A}_{12} < 0.5$ and p-value < 0.05) than the latter with large effect size. The improvements range between a minimum of 1.49 times and a maximum of 3.50 times, achieved on Bash and on Sed respectively. On average, HGA is 1.75 times faster than GDE3.

When comparing HGA and MOEA-D/DE, we notice that for all the programs we can reject the null hypothesis.
We evaluated three formulations of the TCP problem: et al. from issue tracker of the software system with “closed” times (e.g., Grep) to 3 times (e.g., Sed) the execution times required for HGA. On average HGA is 1.53 times faster than MOEA-D/DE.

**Summary for RQ2.** HGA often outperforms GDE3 and MOEA-D/DE in terms of cost-effectiveness. The single solution provided by HGA is not dominated in the objectives space by those generated by the many-objective algorithms. Finally, HGA is more efficient than GDE3 (up to 1.65 and 1.75 times for four and five criteria formulations) and MOEA-D/DE (up to 3 and 3.50 times depending on the formulation).

### 6 Evaluating the hypervolume genetic algorithm on a large software system

We conduct a third empirical study to assess the performances of HGA, partially replicating a previous study [12]. In particular, we investigate the following research questions:

- **RQ3**: How does HGA perform on a large software system with real faults?

To better clarify it, we detailed it in two research questions:

- **RQ3.1**: What is the cost-effectiveness of HGA on a large software system with real faults? This research question aims at evaluating to what extent the test case ordering obtained by HGA is able to detect faults (effectiveness) earlier (lower execution cost) in comparison with two state-of-the-art techniques: a cost cognizant additional greedy algorithm [5], [41], a single objective genetic algorithm based on an AUC metric (GA) [3], and a multi-objective search based algorithm namely NSGA-II [20] used in prior test case prioritization [14], [12] on a large software system, namely MySQL, containing real faults.

- **RQ3.2**: What is the efficiency of HGA on a large software system with real faults? With this second research question, we are interested in comparing the running time (efficiency) required by HGA to find an optimal test ordering, in comparison with the three experimented test case prioritization techniques on a large software system, namely MySQL, containing real faults.

#### 6.1 Study Design

The context consists of MySQL, a large real-world system that has been previously studied by Epitropakis et al. [12]. MySQL is developed in Java. It comprises 1,283,433 LOC and has 2,005 test cases. We used the same real faults from the original study [12], where the authors collected 20 real faults from issue tracker of the software system with “closed” status and available fix patches.

We considered the same testing criteria used by Epitropakis et al. [12]. In particular, we considered statement coverage, Δ-coverage, past faults coverage, and execution cost. We evaluated three formulations of the TCP problem:

- **Two-criteria (Single-objective)** that (i) minimizes the execution cost and (ii) maximizes the statement coverage.
- **Three-criteria (Two-objective)** that considers the past faults coverage as a third criterion to be maximized.
- **Four-criteria (Three-objective)** that considers Δ-coverage as a fourth criterion to be maximized.

In particular, we used the statement coverage matrix and the execution cost array provided by Epitropakis et al. [12] and built using the software profiling tool Valgrind. The Δ-coverage criterion represents the difference of statement coverage between two consecutive versions of a program. The conjecture behind the use of this information is that changed lines of code are more likely to introduce faults in the system. It was calculated by applying the diff program between two consecutive coverage matrices. It is worth to notice that Epitropakis et al. [12] considered only the Four-criteria formulation and that we added the Two- and Three-criteria formulations for sake of completeness.

We compared the results of HGA with those achieved by Additional Greedy [5], [14], [41], (ii) GA [3], and (iii) NSGA-II [14], [20]. More details on these algorithms are provided in Section 4.

As done for our previous studies, we implemented these algorithms using JMetal [75]. Moreover, as in the original study [12], we pre-processed the coverage data using the lossless coverage compaction algorithm proposed by Epitropakis et al. [12].

We set up the parameters of the algorithms using the same values as the original study [12]. In particular, with respect to our two previous studies, we ran the algorithm for 200 generations (i.e., 50,000 fitness evaluations). We performed 30 independents runs for each program and for each search algorithm. Finally, we used the same evaluation metrics used in Section 4 and 5.

#### 6.2 Results of the empirical study

This section discusses the results of our third study, thus, answering the research question.

**Results for RQ3.1.** Table 8 reports the APFDc values and the running time obtained by HGA and the state-of-the-art algorithms on MySQL. In all the formulations, HGA achieves a higher AFDPc, with respect to Additional Greedy. This difference is statistically significant (p-value < 0.05) with a large effect size. Looking at the two-criteria formulation, we can notice that there is no statistically significant difference between HGA and GA. This result is expected since, as explained in Section 2 and empirically evaluated in Section 4, the hypervolume and the AUC-based metric are equivalent. More interesting are the comparisons on the three- and four-criteria formulations between HGA and NSGA-II. Even in these cases there are no statistically significant differences in terms of cost-effectiveness demonstrating that even on a large software system, despite using only one fitness function (i.e., the hypervolume indicator), HGA is competitive with NSGA-II.

Since NSGA-II produces multiple solutions, Figure 9 compares the APFDc values of NSGA-II with those of HGA and Additional Greedy for the three- and four-criteria formulations. As we can observe, the single solution produced by Additional Greedy is worse than all solutions generated by NSGA-II as well as the one produced by HGA. In both the formulations, the distribution of APFDc scores
by NSGA-II presents a large variation with a median value that is very close to the value obtained by HGA.

Figure 10 plots, the Pareto front produced by NSGA-II and the solutions generated by Additional Greedy and HGA on the four-criteria formulation with respect to APSC_c, APDC_c and APPFD_c, (i.e., the objectives optimized by NSGA-II). The solution by HGA is able to dominate the whole Pareto front of NSGA-II and the solution of Additional Greedy. Even on this large systems, we can notice that the better results produced by NSGA-II in terms of cost-effectiveness (Figure 9) are not always related to the AUC metrics that are optimized.

Results for RQ_3.2. Table 8 reports the results in terms of efficiency. More specifically, for all the formulations, the comparison between HGA and Additional Greedy shows that Additional Greedy algorithm is statistically faster (i.e., $\hat{A}_{12} > 0.5$ and $p-value < 0.05$) with large effect size than HGA, despite being less cost-effective as already shown. Even when comparing HGA with GA and NSGA-II, we can reject the null hypothesis of the Wilcoxon t-test. In particular, on the four-criteria formulation, NSGA-II requires 87% more execution time than HGA.

Summary for RQ_3. NSGA-II is more cost-effective than Additional Greedy. The solution provided by HGA is not dominated by those generated by NSGA-II. Additional Greedy is statistically more efficient than NSGA-II and HGA, while HGA is faster than GA and NSGA-II.

7 Threats to Validity

This section discusses the threats to the validity of our empirical evaluation, classifying them into construct, internal, external, and conclusion validity.

Construct Validity. In this study, they are mainly related to the choice of the metrics used to evaluate the characteristics of the different test case prioritization algorithms. To evaluate the optimality of the experimented algorithms (e.g., HGA, Additional Greedy, GA, NSGA-II, GDE3, and MOEA-D/DE) we used the APFD_c [41], a well-known metric used in previous work on multi-objective test case prioritization [24, 83]. Another construct validity threat involves the correctness of the measures used as test criteria: statement coverage, fault coverage and execution cost. To mitigate such a threat, the code coverage information was collected using two open-source profiler/compiler tools (GNU gcc and gcov). The execution cost has been measured by counting the number of source code blocks expected to be executed by the test cases [2, 43], while the original fault coverage information has been extracted from the SIR repository [68].

Internal Validity. To address the random nature of the GAs themselves [78], we run HGA, GA, NSGA-II, GDE3, and MOEA-D/DE 30 times for each subject program (as done in previous work [3, 14, 69]), and considered the median APFD_c scores. The tuning of the EA’s parameters is another factor that can affect the internal validity of this work. In
this study, we use the same genetic operators and the same parameters used in previous work on test case prioritization [3, 10]. It is worth remarking that previous studies [76, 77] demonstrated that default values are a reasonable choice, even considering that parameter tuning is a long and expensive process that in the context of search-based software engineering does not assure better performances.

External Validity. We consider six open source and proprietary programs, that were used in previous work on regression testing [3, 12, 69, 70, 71, 84]. In details, we firstly compared HGA on two different formulations of the test case prioritization problem, with respect to three state-of-the-art algorithms for test case prioritization (e.g., Additional Greedy, GA, and NSGA-II). Secondly, we look at two new formulations of the problem considering more criteria and comparing with two many-objective meta-heuristic algorithms (i.e., GDE3 and MOEA/D-DE). Finally, we partially replicated the study by Epitropakis et al. [12] on a large software system (e.g., MySQL), comparing HGA with Additional Greedy and NSGA-II.

Conclusion Validity. We interpret our findings using appropriate statistical tests. In particular, to test the significance of the differences we used (i) Welch’s t-test [79] and (ii) Wilcoxon t-test [79], while to estimate the magnitude and the effect size of the observed differences we used the Vargha-Delaney statistic [80]. Conclusions are based only on statistically significant results.

8 Conclusion and Future Work

This paper proposed a hypervolume-based genetic algorithm (HGA) to improve multi-criteria test case prioritization. Specifically, we use the concept of hypervolume [13], which is widely investigated in many-objective optimization, to generalize the traditional Area Under Curve (AUC) metrics used in previous work on test case prioritization [3, 9, 10, 11, 12]. Indeed, the hypervolume metric condenses multiple testing criteria through the proportion of the objective space, while AUC based metrics can manage only one cumulative code coverage criterion per time [3].

We performed three empirical studies with three main goals. First of all, we aimed at evaluating the cost-effectiveness and efficiency of HGA, compared to three state-of-the-art algorithms for the Test Case Prioritization problem, namely Additional Greedy [5], GA [3], and NSGA-II [14, 20]. Secondly, we intended to analyze the degree to which they handle the selective pressure as the number of objectives grows. Thus we compared HGA with two many-objective evolutionary algorithms, i.e., GDE3 [18] and MOEA/D-DE [19]. Finally, we aimed at analyzing the performance of HGA in terms of cost-effectiveness and efficiency when dealing with large software systems with respect to two state-of-the-art algorithms such as Additional Greedy [5] and NSGA-II [14, 20].

Our results show that HGA is more or equally cost-effective than the state-of-the-art approaches in most cases. The single solution provided by the algorithm is able to dominate most of the solutions provided by NSGA-II in terms of cost-effectiveness. Moreover, the performance of HGA does not decrease when larger programs and more objectives are considered. Looking at the execution time we note that the efficiency of Additional Greedy is strictly related to the number of test cases, while HGA is faster than GA and NSGA-II in all the considered programs and formulations. Moreover, we show that, in terms of cost-effectiveness, HGA is equivalent or better than GDE3 and MOEA/D-DE, while being much more efficient in terms of execution time.

As future work, we plan to incorporate diversity measures proposed in previous studies on multi-objective test case selection [43, 69] to improve the performance of HGA for software systems with highly redundant test suites, where greedy algorithms are particularly competitive. We plan to apply the proposed HGA also for other test case optimization problems, such as Test Suite Minimization and Test Case Selection. Finally, starting from the considerations made in the empirical studies, we plan to perform a new empirical study to investigate which testing criteria are more able to discover new faults.

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References


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