Healthy speed control of belt conveyors on conveying bulk materials

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\textbf{ABSTRACT}

Belt conveyors play an important role in the dry bulk material handling process. Speed control is a promising method of reducing the power consumption of belt conveyors. However, inappropriate transient operations might cause risks like material spillage away from the belt conveyor. The unexpected risks limit the applicability of speed control. Current studies on speed control mainly focus on designing energy models of belt conveyors or building control algorithms of variable speed drives, while rare researchers take into account the risks in transient operations and the dynamic performance of belt conveyors under speed control. The paper proposes an Estimation-Calculation-Optimization (ECO) method to determine the minimum speed adjustment time to ensure healthy transient operations. The ECO method is composed of three steps and takes both risks in transient operations and the conveyor dynamics into account. In the Estimation step, an estimator is built to approximate the permitted maximum acceleration by treating the belt as a rigid body. Taking the belt’s visco-elastic property into account, the Calculation step computes the conveyor dynamics by using a finite-element-method. With respect to the risks in transient operations, the Optimization step improves the conveyor’s dynamic behaviors and optimizes the speed adjustment time. A case of a long belt conveyor system is studied and the ECO method is applied. The secant method is also used to improve the optimization efficiency. According to the experimental results, the ECO method is successfully used to determine the minimum speed adjustment time to ensure healthy transient operations, including both the accelerating and the decelerating operations. With the suggested adjustment time, unexpected risks are avoided and the belt conveyor shows an appropriate dynamic behavior. Accordingly, the ECO method ensures healthy transient operations and improves the applicability of speed control with the consideration of the potential risks and the conveyor dynamics.

\section{1. Introduction}

A belt conveyor system is a typical continuous transport system conveying dry bulk solid materials without any interruptions. For more than a century, belt conveyors have been an important part of material handling for both in-plant and overland transportation \cite{1,2}. After the Second World War, rubber technologies began a period of rapid development and these changes promoted the improvement of conveyor systems. In the previous decades, belt conveyors have become longer and faster, with higher capacity and less environmental impact \cite{3}. Moreover, the belt conveyor systems have proven themselves one of the most cost-effective ways of handling bulk solid materials.

Due to their inherent advantages, such as high capacity and low labor requirements, belt conveyors play a significant role in bulk solids handling and conveying, especially in some areas where the transport infrastructure is underdeveloped or non-existent \cite{4}. According to \cite{5}, there are more than 2.5 million belt conveyors annually operating in the world. Considering the extensive use of belt conveyors, their operations involve a huge amount of electricity. Hiltermann \cite{6} gives an example, showing that belt conveyors are responsible for 50% to 70% of the total electricity consumption in a dry bulk terminal. However, most electricity today is still generated by burning fossil fuels like coal and oil. As Goto et al. \cite{7} further suggest, the coal-fired power plants currently provide 41% of the global electricity, while coal makes up over 45% of the world’s carbon dioxide emissions from fuels \cite{8}. Therefore, by taking into account the relevant economic and social challenges, there is a strong request to improve the energy efficiency of belt conveyors and to reduce the carbon footprint.
Speed control has been proved a promising approach of improving the energy efficiency of belt conveyors. The method of adjusting the conveyor speed to match the actual material flow as to reduce the energy consumption is the so-called speed control [6]. Generally, belt conveyors are running at a constant normal speed. They always are part of a bulk material handling chain and the actual feeding rate is determined by the upper-stream handling process. Then due to the variation of bulk material flow discharged onto the belt conveyors, they are only partially filled in most cases. In such cases, if the actual material flow or the peak of the upcoming material flow can be predicted, the conveyor speed can be adjusted to match their variations. Then, according to a calculation model derived from the standard DIN 22101 [9], the belt conveyor’s energy consumption is expected to be reduced. Besides the promising energy savings, extra benefits can be achieved by speed control, such as less maintenance [10].

The research into speed control can be dated back to the end of the last century. Over the past few years, several important results have been achieved. Based on the standard DIN 22101, Hiltemann et al. [11] proposed a method of calculating the energy savings achieved by speed control. Zhang and Xia [12] put forward an alternative calculation model which combined energy calculations of DIN 22101 and of ISO 5048 [13] and prososed a model-predictive-control method to optimize the operating efficiency of belt conveyors. By considering the dynamics of belt conveyors, the authors of this paper proposed a fuzzy control method to adjust the conveyor speed in a discrete manner [14]. Another fuzzy logic controller was built by Ristic et al. [15] for the purpose of applying speed control to belt conveyors.

However, previous research did not cover some issues, such as potential risks in transient operations. Transient operations are the operations of adjusting the conveyor speed to match the actual material flow. In transient operations, a large ramp rate of conveyor speed might result in very high tension on the belt [14]. The unexpected high tension is the major reason of belt breaking at the splicing area. Besides the risk of belt over-tension, there exist some other risks in transient operations, including the risk of belt slipping around the drive pulley, the risk of material spilling away from the belt, the risk of motor over-heating, and the risk of pushing a motor into the regenerative operation. However, few studies discussed the conveyor’s dynamic performance in transient operations of speed control. Although some researchers and engineers have studied conveyor dynamics for decades [16–19], they mainly focus on the realization of soft start-ups or soft stops. Compared to the normal start-up or stop, the transient operations for speed control should be given more attention, since the belt conveyors often have a high filling ratio in these operations. Moreover, the conveyor dynamics in transient operation are of complexity, especially in the case where a long-distance and high-capacity belt conveyor is frequently sped-up or slowed-down to match a variable material flow.

This paper is one further step of our previous work [20,21]. The purpose of the paper is to seek a method to keep speed control healthy with the consideration of the conveyor dynamics, specially in transient operations. To realize a healthy speed control, an Estimation-Calculation-Optimization (ECO for short) method is proposed to decide the demanded minimum speed adjustment time. The ECO method is composed of three steps and takes both risks in transient operations and the dynamic performance of a belt conveyor into account. In the Estimation step, an estimator is built to compute the minimum acceleration time. Since the estimator does not consider the belt’s visco-elastic property, the Calculation step needs to be carried out to analyze the conveyor dynamics when the conveyor speed is adjusted with the estimated acceleration time. If risks like belt slippage around the drive pulley are observed in the Calculation step, further simulations should be carried out in the Optimization step to improve the transient operation and to find the minimum acceleration time. Beyond our previous work [20,21], this paper discusses both the accelerating and the decelerating operations. In addition, it also considers different speed adjustment ranges. Moreover, an iteration method is used to reduce the simulation times in the Optimization step to improve the optimization efficiency.

The layout of this paper is as follows. Section 2 defines the conception of the transient operation and analyzes potential risks in transient operations. Section 3 details the ECO method whose components are detailed step-by-step. A long belt conveyor system is studied in Section 4 where the ECO method is used to achieve the minimum speed adjustment time of transient operations. The last section concludes the results and findings of this study.

2. Potential risks in transient operations

The operations of belt conveyors can be distinguished into two groups: the stationary operation and the transient operation [22]. The stationary operations include three cases: the case where the belt is totally stopped, the case where the belt is running at nominal speed with the steady state, and the case where the belt is running at a fixed non-nominal speed caused by speed control. If we do not take into account other issues, such as the efficiency of the driving system, the non-nominal speed can be any value between zero and the nominal speed. The operation of a belt conveyor is often at a steady-state. When the material feeding rate has a considerable change, the belt conveyor will speed up or slow down to match the actual material flow. In this paper, the operations between adjacent stationary operations are defined as the transient operations. As shown in Fig. 1, the transient operations include both the accelerating and decelerating processes.

In transient operations, the motor speed or torque is controlled so that the conveyor speed can follow a planned speed profile. Keeping transient operations healthy is an important prerequisite of the belt conveyor speed control, since the dynamic performance during transient operations has a significant impact on the service life of conveyor components. In the case of an improper transient operation, a large acceleration might result in a very high tension in the belt. However, as German Institute for Standardization [9] suggests, the over-tensioning is the major reason of belt breaking at the splicing area. Besides the risk of belt over-tension, several other risks also should be accounted in transient operations, including:

- The risk of belt slipping around the drive or brake pulley,
- The risk of material spilling away from the belt, and
- The risk of motor over-heating.

Additionally, the deceleration operations should specially consider the risk of pushing a motor into the regenerative operation.

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![Fig. 1. Stationary operations and transient operations.](image-url)
2.1. Belt over-tension at the splicing area

The belt strength is mainly determined by the strength of its carcass. For a certain conveyor belt, the belt tension rating is a given and thus constant. However, as Nordell [23] stated, “the chain is only as strong as its weakest link. In conveyor belts... the weak link is the splice”. The German Institute for Standardization [9] suggests the maximum safe working tension is far less than the belt rating due to issues such as splice fatigue and degradation. In [9], the ratio between the working tension and the actual breaking tension is referred to as the service factor or safety factor (SF). According to the German Institute for Standardization [9], the ratio between the maximum safe working tension and the breaking tension is referred to as the minimum safety factor:

\[ S_{A,\min} = \frac{T_{\max, A}}{kN B} \]  
(1)

\[ S_{B,\min} = \frac{T_{\max, B}}{kN B} \]  
(2)

where

- \( S_{A,\min} \): minimum safety factor in transient operations
- \( S_{B,\min} \): minimum safety factor in steady operating conditions
- \( T_{\max, A} \): maximum safe working tension in transient operations in kN
- \( T_{\max, B} \): maximum safe working tension in the steady operating condition in kN
- \( kN \): belt tension rating, minimum breaking strength in kN/m
- \( B \): belt width in m

The German Institute for Standardization [9] further suggests that the belt conveyor should be designed with a belt rating larger than 8.0 times that of maximum belt tension in stationary operations under normal operating conditions. Moreover, in transient operations under normal operating conditions, the maximum belt tension should be no larger than 1/5.4 times that of the belt rating. Otherwise, it increases the risk of belt over-tension. The belt over-tension may cause the breakage of the belt splice and can cause the reduction of the service life of pulleys. Therefore, the belt tension must be limited to a safe level in transient operations.

2.2. Belt slippage around the drive pulley

The power between drive pulley and a conveyor belt is transmitted by their friction connection. Euler [24] and Entelwein [25] suggest that the maximum available friction \( F_{f,\max} \) between the drive pulley and the belt can be approximated by

\[ F_{f,\max} = T_2 (\mu \cos \alpha - 1) \]  
(3)

where \( T_2 \) is the belt tension after the drive pulley, \( \alpha \) is the belt’s wrapping angle around the drive pulley, and \( \mu \) is the coefficient of friction between the belt and the drive pulley. However, as they further suggest, belt slippage occurs whenever the driving force \( F_d \) attempts to exceed the maximum available frictions \( F_{f,\max} \). Then the drive pulley fails to drive the belt as planned, since the available driving forces are less than the demanded. If the belt slippage continues to the extent that it slows down the conveyor, then the spillage of the bulk solid material or the blockage of the belt’s feeder chute may occur. Additionally, the relative movement wears out the bottom cover of the belt and the surface cover of the drive pulley. More seriously, a fire case is mentioned by Nel and Shrott [26] which was caused by a long-period’s continuous belt slippage. Therefore the transient operations must strictly control the driving forces so as to prevent the risk of belt slippage.

2.3. Material spillage away from the belt

Even though a belt conveyor is carefully designed, material spillage from the carrying side of the belt may occur at the loading point and elsewhere along the belt conveyor. There are many factors that can lead to the spillage, such as the material off-center loading at the transfer point and the belt mistracking along the conveying route. This paper mainly focuses on the unnecessary spillage caused by the conveyor’s inappropriate dynamic behavior in transient operations. For example, in case of large speed fluctuations, the instantaneous conveying capacity can be less than the actual loading rate, and so that the belt at the loading area may be overloaded. The overload can lead to the escape of bulk materials from the belt.

In addition, the excessive belt tension also can result in the material spillage. In transient operations, the belt tension is varying along the conveying route and fluctuating over time. On the one hand, if the improper transient operation results in an excessively low belt tension, the belt may drop significantly between neighboring idler stations. According to Conveyor Equipment Manufacturers Association [27], the bulk material may spill over the edges of the belt when the conveyor belt sags more than 3 \% of the idler spacing, see Fig. 2a. On the other hand, in the case of dipping belt conveyors, the large belt tension can lead to the lift of the belt from the idler stations, see Fig. 2b. The lift also can cause the material spillage away from the belt.

If the bulk solid material escapes from the carrying belt, it may land on the return side of the belt. When it moves to the tail pulley, the escaped material can cause the overstretching of the belt’s carcass and the cover damage of the pulley. Consequently, the maintenance costs increase. Moreover, if the bulk material conveyed is sticky, the escaped material may adhere to the surface of idle rollers. This can cause not only the wear of conveyor components, but also the loss of belt alignment. The misalignment of the belt increases the spillage in return. Besides the maintenance cost, the spillage also increases the cleaning cost if it falls on the floor. More severely, the spilled material can pose threats to personal safety if it falls from a height. Therefore, by taking all these negative influences into account, the
unnecessary material spillage from the belt must be prevented in transient operations.

2.4. Motor overheating

As a motor operates, it converts electrical energy to mechanical energy. In this conversion, part of the energy is lost due to motor losses. The motor temperature rises due to the heat generated from the motor losses during operation. If the winding’s temperature is above the rated temperature, the motor is overheated. Overheating occurs due to a number of factors, one of which can be the unhealthy transient operation with rapid acceleration. Normally, motors can provide larger shaft torque than the nominal for a short period of time without overheating. However, if the overload continues for a long time or the load exceeds the permitted load greatly, it increases the risk of overheating. Especially in the cases where a motor is operated at a low frequency and the cooling fan is mounted on the rotor shaft, the reduced cooling efficiency increases the risk of motor overheating.

Overheating is a serious problem for a motor, and can cause a number of performance problems. As Wiedenbrug [28] suggests, the winding’s insulation life is cut in half for every 10 °C above the rated temperature. For example, in the case of a motor with a service life of 20 years and with the rated temperature 40 °C, the life of the motor is cut to about 1 year if it runs at 80 °C. Moreover, as Mirza [29] suggests, more than 55% of the insulating failures are caused by overheating. Although modern insulating materials are more durable, the overheating still considerably shortens the service life of a motor. Therefore, the transient operations should avoid overloading the driving machine.

2.5. Pushing a motor into the regenerative operation

Pushing a driving motor into the regenerative operation may occur in an improper deceleration operation. In such operation, negative driving forces or braking forces sometimes are required in order to keep the conveyor following a planned speed profile. The negative driving forces can be provided by a brake system or by pushing the driving motor into the regenerative operation. However, in the case of a conveyor system without neither the braking system nor the function of regenerating, the driving system cannot provide a negative torque. In such cases, the driving torque can be suddenly lost, and then the belt fails to run as planned. Therefore, in the cases where the driving system cannot produce generative driving torque, the risk of pushing the motor into the regenerative operation should be paid more attention, especially in deceleration operations.

3. ECO method

In the design of belt conveyors, existing standards, such as DIN 22101 [9], CEMA [27] and ISO5048 [13], are usually used to calculate the required driving forces on the driving pulleys and to approximate the minimum startup time. It is important to note that in the calculation of these standards, the conveyor belt is viewed as a rigid body and the belt elastic-visco properties are not taken into account. In the case of short-distance and low-capacity belt conveyor systems, this may lead to acceptable dynamic behaviors of the belt. However, in the case of long-distance and high-capacity belt conveyor systems, the ignorance of belt dynamics can cause operational problems like the premature collapse of the belt and the damage of drive systems. Therefore, a method is demanded to ensure healthy transient operations to protect a belt conveyor from potential risks, especially in the cases where the conveyor speed is frequently adjusted. Based on our previous work [20,21], this paper proposes an Estimation-Calculation-Optimization (ECO) method to determine the minimum speed adjustment time, by taking both the potential risks and the conveyor dynamics into account.

The ECO method is based on simulations and consists of three steps: Estimation, Calculation and Optimization. In the Estimation step, an estimator is built to estimate the minimum adjustment time. Taking the potential risks listed in Section 2 into account, the estimator considers the belt as a rigid element and computes the permitted maximum acceleration in transient operations. Then by accounting for the acceleration profiles, the demanded minimum speed adjustment time can be roughly estimated. Section 3.1 describes the estimation in detail. The purpose of the Calculation step is to observe the belt’s dynamic performance in transient operations. This step takes the effect of belt hysteresis into account and carries out a computer simulation. The simulation is on the basis of a finite element method which is detailed in Section 3.2. In the case of a long-distance and high-capacity belt conveyor, the conveyor dynamics in a transient operation is complex with regard to the belt’s viscoelasticity. Therefore, the transient operation observed in the Calculation step may result in, for instance, the risk of belt over-tension. In such cases, optimizations are required to improve the dynamic performance and in order to achieve the demanded minimum speed adjustment time. Section 3.3 presents the Optimization step.

3.1. Estimation

Taking the potential risks in transient operations into account, an estimator is built in this step to estimate the minimum adjustment time. In order to describe the estimator clearly, a case as shown in Fig. 3 is exemplified. The belt conveyor is driven by a head pulley. In order to provide a pre-tension, a single sheave gravity take-up device is mounted at the return side after the head pulley. In the following, both the permitted maximum acceleration and deceleration are computed.

3.1.1. The maximum acceleration

As regards the risk of belt breaking at the splice caused by over-tension, the belt tension along the conveying route must be maintained in a safe level. In cases as shown in Fig. 3, the maximum belt tension generally occurs right before the drive pulley. The tension $T_1$ before the drive pulley in a steady state can be approximated by

$$T_1 = T_2 + F_d$$  \hspace{1cm} (4)

where $F_d$ is the driving force and $T_2$ is the belt tension after the drive pulley. As shown in Fig. 3, if the horizontal distance between the drive pulley and the gravity take-up device with mass $m_f$ is small, the belt tension $T_2$ after the drive pulley can be approximated by

$$T_2 = \frac{1}{2} m_f g$$ \hspace{1cm} (5)

![Fig. 3. A belt conveyor with a single sheaved gravity take-up.](image)
where $g$ is the gravity acceleration. According to Eq. (1), the permitted belt tension $T_{1,\text{max}}$ before the drive pulley can be estimated by

$$T_{1,\text{max}} = \frac{k_0 B}{S_{\text{Amin}}}$$

Then the combination of Eqs. (4) to (6) gives the maximum permitted driving forces

$$F_{d,\text{max,tension}} = \frac{k_0 B}{S_{\text{Amin}}} - \frac{1}{2} m T g$$

in respect of the risk of belt over-tension.

Belt slippage is another major risk in acceleration operations. As stated by Euler [24] and Entelwein [25], belt slippage occurs whenever the driving force exerted on the drive pulley exceeds the maximum available frictions. Then according to Eq. (3), the maximum demanded driving forces are limited to

$$F_{d,\text{max,slip}} = F_f,\text{max} = T_2 (e^{a_{\text{max}}} - 1)$$

and the maximum driving force $F_{d,\text{max,heat}}$ onto the drive pulley is

$$F_{d,\text{max,heat}} = i_{gf} \frac{\tau_{\text{motor,nom}}}{R_d} = \frac{i_{gf} \tau_{\text{motor,nom}}}{R_d}$$

in which $i_{gf}$ is the gearbox reduction ratio and $R_d$ is the drive pulley’s radius.

Then taking these three risks in acceleration operations into account, the permitted maximum driving forces $F_{d,\text{max}}$ in transient operations are

$$F_{d,\text{max}} = \min (F_{d,\text{max,tension}}, F_{d,\text{max,slip}}, F_{d,\text{max,heat}})$$

According to Newton’s Second Law, the acceleration is the net result of any and all forces acting on belt conveyors. Then in acceleration operations, the permitted acceleration is

$$a_{d,\text{max}} = \frac{F_{d,\text{max}} - F_f}{m}$$

where $F_f$ is the total motional resistances, and $m$ is the total lump mass of the belt, rollers and the bulk material on the belt. According to the German Institute for Standardization [9], the motional resistances along the conveying route can be estimated by

$$F_f = C g L (m_{\text{roll}} + (2 m_{\text{belt}} + m_{\text{bulk}}) \cos \hat{o}) + m_{\text{bulk}} g H$$

and the total mass is

$$m = L (m_{\text{roll}} + 2 m_{\text{belt}} + m_{\text{bulk}})$$

in which

- $C$ the coefficient of secondary resistances
- $f$ the artificial coefficient of friction resistances
- $L$ the conveying length of conveyor
- $m_{\text{roll}}$ the mass of rollers in unit length
- $m_{\text{belt}}$ the linear density of belt
- $m_{\text{bulk}}$ the linear density of bulk material
- $\hat{o}$ the inclination angle of a belt conveyor
- $H$ the height difference between the loading and unloading areas of a belt conveyor

Finally, the maximum permitted acceleration can be estimated by substituting Eqs. (13) and (14) into Eq. (12).

### 3.1.2. The maximum deceleration

In a soft deceleration operation, the driving force exerted on drive pulleys decreases gradually and the conveyor speed is reduced smoothly. Differing from the acceleration operation, the deceleration operation mainly considers the risk of pushing a motor into the regenerative operation. In the case of belt conveyors which cannot provide the negative driving forces, the maximum deceleration can be estimated by:

$$a_{d,\text{max}} = \frac{-F_f}{m}$$

### 3.1.3. The minimum speed adjustment time

The speed adjustment time is the time required in acceleration or deceleration operations, and its value is dependent on the planned acceleration profile. Conventionally, the linear acceleration was widely used in start-up operations. However, due to the sudden change of the acceleration at the beginning and the ending of transient operations, the linear acceleration operation often results in a large mechanical jerk. In order to reduce the mechanical jerk and to enable a soft-start operation, Harrison [31] recommended a sinusoidal acceleration profile as shown in Fig. 4. The figure includes the curves of an acceleration operation and of a deceleration operation. Mathematically, the acceleration and the speed can be expressed:

$$a(t) = \pi \frac{\Delta v}{t_0} \sin \frac{\pi t}{t_0}$$

$$v(t) = v_0 + \frac{\Delta v}{2} \left(1 - \cos \frac{\pi t}{t_0} \right)$$

where $\Delta v$ is the speed change between the initial and final speed.

![Fig. 4. Acceleration profiles and speed curves in the transient operation.](image-url)
where $\Delta v$ is the speed adjustment range, $t_0$ is the speed adjustment time, $t$ is the instantaneous time ($0 \leq t \leq t_0$), and $v_0$ is the initial speed before the transient operation. According to Eq. (16), the maximum acceleration $a_{\text{max}}$ occurs at $t = t_0/2$ and

$$a_{\text{max}} = a \left( \frac{t_0}{2} \right) = \frac{\pi \Delta v}{2 t_0}$$

(18)

Then in transient operations with sinusoidal acceleration profiles, the required minimum speed adjustment time is

$$t_{\text{ac, min}} = \frac{\pi \Delta v}{2 a_{\text{ac, max}}}$$

(19)

$$t_{\text{de, min}} = \frac{\pi \Delta v}{2 a_{\text{de, max}}}$$

(20)

where the subscripts $\text{ac}$ and $\text{de}$ represent the operations of the acceleration and deceleration, respectively.

### 3.2. Calculation

In the Estimation step, an estimator is built to approximate the permitted acceleration and the demanded adjustment time. The estimator views the belt as a rigid element, and it assumes that all masses are accelerated or decelerated at the same time with the same rate. However, the neglect of belt elasticity in high-capacity and/or long-distance conveyors may lead to operational problems. Moreover, the estimator fails to take into account the risk of material spillage caused by the low speed or by the low belt tension. Therefore, the dynamic analyses play an important role to detect whether the potential risks occur in transient operations.

Our previous work [3] gives a detailed historic overview of modeling belt conveyor dynamics. Finite element models of belt conveyor systems have been developed to calculate the conveyor dynamic behavior, especially during staring and stopping. Although these models only determine the longitudinal response of the belt by mainly using truss like elements, they have been successful in predicting the elastic response of the belt during starting and stopping. Lodewijks and Kruse [32] give an important case study in which both the field measurements and the finite-element-model-based simulations were carried out. The experimental results show that the deviation between the measured results and the calculated results could be restricted within 5%, which falls within the accuracy of the field measurements. Therefore, the finite element model, taking into account the belt viscosity-elasticity, is capable of analyzing the conveyor dynamics in transient operations.

The finite element model used in [32] is derived from our previous work [33] which has been widely accepted and used by other researchers and engineers. In the finite element approach, the distributed mass of the belt, the idle rollers and the bulk material are divided into finite number of elements. In Fig. 5, these elements are presented by nodes and numbered in sequence. As the figure shows, the conveyor is evenly divided into N-1 segments with N nodes. The sum of the belt mass, idler mass, bulk material mass (on the belt carrying side) is treated as a singular lumped mass on the each node. On the carrying side, the lump-mass of the node equals the sum of equivalent belt mass, equivalent idler mass and the equivalent bulk material mass. On the return side, the lump-mass of the node equals the sum of belt mass and the idler mass. An important notation is that the lump mass of (i + 1)th node includes the mass of the tail pulley. In addition, due to the belt’s viscosity-elasticity, the adjacent nodes are coupled by a spring-damping connector, briefly presented by a spring element in Fig. 5.

As Lodewijkis [33] suggests, it is reasonable to suppose that the belt is laid in a horizontal direction and moves towards one direction. Fig. 6 illustrates the one dimensional model of a single drive belt conveyor system. In this model, the driving force $F_d$ is placed on the 1sr node and the pre-tension is placed on both the 1sr and Nth nodes. The displacement of all nodes can be expressed as the relative displacement of Nth node. Then according to Newton’s Second Law, the motion of a belt conveyor can be described as

$$M \ddot{x} + C \dot{x} + Kx = F$$

where

- $M$ mass matrix, size $N \times N$
- $C$ matrix of damping factors, size $N \times N$
- $K$ matrix of spring factors, size $N \times N$
- $\dot{x}$ vector of nodal displacement, size $N \times 1$
- $x$ vector of nodal velocity, size $N \times 1$
- $\ddot{x}$ vector of nodal acceleration, size $N \times 1$
- $F$ vector of force, size $N \times 1$

### 3.3. Optimization

With respect to the belt elasticity-viscosity, the dynamic performance of the belt conveyor is complex. Due to the fact that the estimator views the belt as a rigid object, the conveyor’s transient operation in the Calculation step might result in, for instance, the risk of belt slippage. If so, further studies must be carried out to improve the conveyor dynamic performance in transient operations. Table 1 summarizes some solutions, including replacing a new belt with higher tension rating, optimizing the mass of the take-up device, applying a softer deceleration profile and increasing the speed adjustment time. With respect to the fact that changing the construction or components of an existing conveyor is not practical to some extent, the general method of improvement is to extend the speed adjustment time. Then the third step, Optimization, is carried out to find the minimum speed adjustment time.

The optimization is realized by using finite-element-model-based simulations. The optimizing procedure can be viewed as a process of root finding. Taking the risk of belt slippage for instance. The maximum available friction resistances between the belt and the drive pulley are $F_{d, \text{bd}}$. It is assumed that during the transient operations, the driving force can exceed the maximum available friction $F_{d, \text{bd}}$. It is further assumed that $F_d(t)$ represents the maximum driving force over different acceleration time $t$. For instance $F_d(30)$ represents the maximum driving force during a specific transient operation within the acceleration time 30 s. Then with respect to the risk of belt slippage, the minimum acceleration time can be approached by finding the root of function

$$f(t) = F_d(t) - F_{d, \text{bd}} = 0$$

(22)

Although the computer nowadays is very powerful and can quickly complete the calculations, iteration methods are suggested so as to
find the solutions more quickly. These methods mainly include the bisection method, the false position method, the Newton-Raphson method and the secant method. Because the research of iteration methods is beyond the scope of this paper, the iteration methods will not be further discussed here. More details of the iteration methods can be found in [34–36].

4. Case study: a long-distance and high-capacity belt conveyor

In order to show how the ECO method is used to determine the minimum speed adjustment time, this section studies a horizontal long-distance and high-capacity belt conveyor. The belt conveyor is designed by Lodewijks [33]. The designed capacity is 2500 MTPH for conveying coal with a density of 850 kg/m$^3$, and the conveying distance is 1000 m. A fabric belt, with Young’s modulus of 370 MPa and tension rating of 500 kN/m, is selected. The linear density of the belt is 14.28 kg/m. The carrying part of the belt is supported by three-roller stations with an average mass-per-unit-length of 14.87 kg/m. The return part is supported by one-roller stations with an average mass-per-unit-length of 7.72 kg/m. The conveyor is driven by a 250 kW motor whose nominal torque is 1592 Nm. More parameters of the belt conveyor system are illustrated in Table 2. As examples of the case study, one acceleration operation from 2 m/s to 4 m/s and one deceleration operation from 4 m/s to 2 m/s are studied in sub-sections 4.1 and 4.2, respectively.

4.1. Acceleration operation from 2 m/s to 4 m/s

4.1.1. Step 1: Estimation

On the basis of data in Table 2, Eq. (7) suggests that the permitted driving forces are

$$F_{d,\text{max,tension}} = \frac{k Nh_{\text{roll}}}{\Delta v_{\text{min}}} - \frac{1}{2} M g = 86.3 \text{kN}$$

(23)

with respect to the risk of belt over-tension. Taking the risk of belt slippage around the drive pulley into account, Eq. (8) yields the permitted driving forces

$$F_{d,\text{max,slip}} = \frac{1}{2} m_t g (e^{\Delta v} - 1) = 49.6 \text{kN}$$

(24)

In addition, with respect to the risk of motor over-heating, the maximum driving forces are approximated by Eq (10):

$$F_{d,\text{max,heat}} = \frac{i_r T_{\text{motor,nom}}}{R_d} = 54.9 \text{kN}$$

(25)

Then by taking these three risks into account, the permitted maximum driving forces are

$$F_{d,\text{max}} = \min \left( F_{d,\text{max,tension}}, F_{d,\text{max,slip}}, F_{d,\text{max,heat}} \right) = 49.6 \text{kN}$$

(26)

The result in Eq. (26) suggests that in an acceleration operation of this specific belt conveyor system, the highest risk is the belt slippage around the drive pulley. Moreover, it further suggests that due to the phenomenon of belt slippage, both the risk of belt over-tension and the risk of motor over-heating are prevented. Then taking the motional resistances along the conveying route into account, Eq. (12) suggests the maximum acceleration value

$$a_{\text{ac, max}} = \frac{F_{d,\text{max}} - C_{\text{fgL}} (m_{\text{roll}}' + 2m_{\text{belt}}' + m_{\text{bulk}}')}{L (m_{\text{roll}}' + 2m_{\text{belt}}' + m_{\text{bulk}}')} = 0.076 \text{m/s}^2$$

(27)

Accordingly, it requires at least

$$t_{\text{ac, min}} = \frac{\pi}{2} \frac{\Delta v}{a_{\text{ac, max}}} = 41 \text{s}$$

(28)

to complete the acceleration operation from 2 m/s to 4 m/s with a sinusoidal acceleration profile.

4.1.2. Step 2: Calculation

In this step, simulations are carried out to examine the conveyor dynamic behavior in the acceleration operation from 2 m/s to 4 m/s within 41 s. Simulations are based on the following suppositions:

- The conveyor belt is fully loaded over the simulations. The whole belt is visually divided into 21 pieces. As Fig. 5 shows, the nodes are numbered from the discharged area of the carrying side. The length of nodes 1 and 21 is 50 m, and the other nodes’ length is 100 m.
- Whenever the driving force attempts to exceed the available friction resistances, the belt is slipping around the drive pulley, and the forces exerted on the drive pulley equal the maximum available frictions between the belt and the drive pulley.

In addition, some other assumptions are given that the maximum permitted belt sag ratio is 3%, and that the permitted speed deviation is 15%. Therefore, the belt tension along the carrying side should be no less than

$$T_{\text{min}} = \frac{g (m_{\text{belt}}' + m_{\text{bulk}}') l_c}{8h_{\text{rel}}} = 9.05 \text{kN}$$

(29)

where $l_c$ is the idler spacing of the carrying side, and $h_{\text{rel}}$ is the maximum permitted belt sag ratio. In addition, the belt speed at the tail pulley should be no less than 3.4 m/s after the acceleration operation.

Fig. 7 illustrates the conveyor dynamics in the acceleration operation. The acceleration starts from the time point 1 s. The diagram of Fig. 7a illustrates the driving force exerted on the drive pulley. It shows that at the beginning of the accelerating operation, the driving force increases gradually. Around the time point 16 s, the driving force reaches the limit 49.6 kN. Due to the fact that the drive pulley cannot provide more friction resistances to couple the exceeding driving forces, the belt is then slipping around the drive pulley and the driving forces remain at 49.6 kN. The belt slippage continues for 16 s, and after that the driving force decreases. Due to the belt...
elasticity, the driving force fluctuates around a certain level after the acceleration operation. Meanwhile, due to the belt viscosity, the fluctuation amplitude is decreasing over time.

Fig. 7b presents the belt tension at each nodal point. The diagram shows that along the carrying side, the minimum belt tension is over 20 kN during and after the acceleration operation. That means in this acceleration, the risk of material spillage caused by the excessive low belt tension is avoided. However, the diagram further shows when the belt is slipping around the drive pulley, an uncontrolled belt tension wave travels from the head pulley to the tail. After that, the belt tension oscillates with dampened amplitudes.

Fig. 7c compares the reference speed and the dynamic speed of the belt around the drive pulley over time. As the figure shows, the belt speed at the drive pulley gently increases and follows the reference speed when the acceleration starts. However, due to the phenomenon of belt slippage, the belt speed at the drive pulley suddenly shocks at the time point 16 s. Due to the fact that the drive pulley cannot provide sufficient driving forces during the period of belt slippages, the dynamic speed is lower than the reference speed until the disappearance of belt slippage. After that, the belt speed at the drive pulley successfully catches up the reference speed. At the time point 42 s, it finally reaches the desired speed 4 m/s, and follows a period of fluctuations.

Fig. 7d illustrates the belt speed at each node over time. During the accelerating operation, the belt speed at each node increases successively. Similar to the belt speed around the drive pulley, the speed curves of other nodes are not thus smooth due to the uncontrolled acceleration waves caused by the belt slippage. The belt speed also fluctuates with a dampened amplitude for a certain period in response to the belt’s viscosity-elasticity. The amplitude at the carrying side is less than that at the return side. However, it clear shows that after accelerating, the minimum speed of belt at the tail pulley is larger than 3.4 m/s. Therefore, material spillage caused by an excessive speed deviation does not occur in this acceleration operation.

4.1.3. Step 3: Optimization

When an acceleration time of 41 s is applied to the acceleration operation from 2 m/s to 4 m/s, the driving force attempts to exceed the friction limit. It causes the belt slipping around the drive pulley.

![Fig. 7. Belt conveyor dynamic performance in the acceleration operation with ΔV = 2 m/s and t_ac = 41 s. (a) Driving forces exerted on the drive pulley. (b) Belt tension at each nodal point. (c) Belt speed at the drive pulley. (d) Belt speed at each nodal point along the conveying route.](image-url)
Besides, it excites an uncontrolled belt tension wave and an uncontrolled acceleration wave. Therefore, an optimization is required to find the minimum acceleration time and to improve the conveyor’s dynamic behaviors. By taking the belt slippage into account, the optimization process is defined as a problem of finding the root of function \( f(t) \):

\[
 f(t) = F_{d_{\text{max}}}(t) - 49.6 = 0 \tag{30}
\]

where an important assumption is given that the driving force can be larger than the maximum available friction resistances between the belt and the drive pulley.

In order to reduce the computational time, the secant method is applied. Compared to the bisection method and the false position method, the secant method generally can find the root faster. In addition, the secant method is more applicable than the Newton-Raphson, due to the fact that the derivative of the maximum driving force over time is hardly expressed in a mathematical way. In the implementation of the secant method, the two initial time points are \( t_0 = 41 \) and \( t_1 = 71 \). The stopping criterion is the function value \( f(t) \) less than 0.1. The calculation results are shown in Table 3. As the data shows the secant method costs only three iterations for finding the minimum acceleration time. The data suggests the optimum acceleration time approaches to 51 s.

Fig. 8 presents the examination results of the conveyor dynamics during the execution of the 51 seconds’ acceleration profile. The diagram of Fig. 8a clearly shows during the acceleration, the maximum driving force is 49.5 kN. It suggests that the risk of belt slippage is successfully avoided. The diagrams of Fig. 8b to d also show that in the acceleration, both the tension force and the speed vary smoothly over time. From the presented results it can be concluded that it requires 51 s to speed up the studied belt conveyor from 2 m/s to 4 m/s with regard to both the potential risks and the conveyor dynamics in the acceleration operation.

4.2. Deceleration operation from 4 m/s to 2 m/s

4.2.1. Step 1: Estimation

Pushing a motor into the regenerative operation is the major risk during a deceleration operation. With respect to this risk, the driving torque of the drive pulley should be non-negative. Accordingly, the maximum deceleration is

\[
 a_{d_{\text{e, max}}} = -C_{Fg} = -0.19 m/s^2 \tag{31}
\]

Then based on the formula (20), the estimated deceleration time is

\[
 t_{d_{\text{e, min}}} = \frac{\pi}{2} \frac{\Delta V}{a_{d_{\text{e, max}}}} = 16.5 s \approx 17 s \tag{32}
\]

With respect to the risk of material spillage caused by the excessive low belt speed, the belt speed at the tail pulley should be more than 1.7 m/s after the deceleration operation.

4.2.2. Step 2: Calculation

Fig. 9 illustrates the examination results of the deceleration operation from 4 m/s to 2 m/s within 17 s. Fig. 9a presents the driving forces exerted on the drive pulley. The diagram shows at the start of the decelerating procedure, the driving force decreases gradually over time. Meanwhile, both the belt tension and the belt speed change smoothly, see Fig. 9b and d respectively. However, the diagram in Fig. 9a shows at the time point 11 s, the driving force reduces to the bottom 0 kN, and then the driving forces are lost. This phenomenon continues for about 8 s. As Fig. 9b to c shows, the belt tension and speed along the conveyor are uncontrollable during this period. The diagram in Fig. 9c also clearly shows that the belt speed at the drive pulley fails to follow the reference speed. Therefore, the deceleration operation from 4 m/s to 2 m/s within 17 s is unacceptable.

Besides the sudden loss of the driving force, the deceleration also results in the belt slippage around the drive pulley. As the diagram in Fig. 9a shows, at the time point 26 s, the driving force reaches its limit and then the belt slippage occurs. The slippage causes an uncontrollable acceleration wave as seen in Fig. 9c and d. The diagram in Fig. 9c further shows that the slippage results in an unexpected speed deviation which lasts for about 10 s.

The diagram in Fig. 9d also shows a large speed deviation during and after the deceleration. The largest deviation between the drive and tail pulleys can be over 1 m/s. The figure further shows at the time point 24 s, the tail pulley speed drops to 0.9 m/s, less than half of the desired speed. This may result in material spillage away from the belt at the loading area.

4.2.3. Step 3: Optimization

From the examination results of Fig. 9, several risks like pushing the motor into the generative operation are observed in the deceleration activity from 4 m/s to 2 m/s within 17 s. These unexpected risks must be prevented in a healthy deceleration operation. By accounting for the risks showed in Fig. 9, the problem of finding the demanded minimum deceleration time can be defined as:

\[
 t^* = \max(t^*_1, t^*_2, t^*_3) \tag{33}
\]

in which

- \( t^*_1 \): the root of function \( f(t) = F_{d_{\text{d, max}}}(t) - 49.6 = 0 \) with respect to the risk of belt slippage
- \( t^*_2 \): the root of function \( g(t) = F_{d_{\text{d, min}}}(t) = 0 \) with respect to the risk of pushing the motor into the generative operation
- \( t^*_3 \): the root of function \( h(t) = \min(t) - 1.7 = 0 \) with respect to the risk of material spillage caused by the excessively low belt speed.

The secant method is also used for finding the root of functions \( f(t), g(t) \) and \( h(t) \). Then by comparing these roots it yields the demanded minimum deceleration time for the deceleration operation from 4 m/s to 2 m/s with an acceptable dynamic performance. It is important to note that in order to implement the secant method, the values of the driving forces are allowed to be more than 49.5 kN or less than 0 kN.

We first try to find the root of function \( f(t) \). The initial points are set to 17 and 47, and the stop criteria is \(| \epsilon | < 0.1 \). The root is approached after four times of iterations and Table 4 presents the root finding result. The data of Table 4 shows that the minimum deceleration time is 30.37 s with respect to the risk of belt slippage. The data in Table 4 also includes the function values of \( g(t) \) and \( h(t) \) at point \( t^*_3 \). It shows when a deceleration time of 30.37 s is used, the minimum driving force is far larger than 0 kN during the deceleration operation. Therefore, the activity of finding root of \( g(t) \) can be canceled. However, it also shows there still exists the risk of material spillage when \( t^*_3 = 30.37 \). Therefore, the root finding of function

<table>
<thead>
<tr>
<th>( t )</th>
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</thead>
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<tr>
<td>1</td>
<td>41</td>
</tr>
<tr>
<td>2</td>
<td>71</td>
</tr>
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<td>3</td>
<td>55.01</td>
</tr>
</tbody>
</table>

Table 3 Root finding function of \( f(t) = F_{d_{\text{max}}}(t) - 49.6 = 0 \) with the secant method. The stop criteria is \( | \epsilon | < 0.1 \).
$h(t)$ should be carried out. The data in the second row further suggests that the root of function $h(t)$ is larger than 35.2. Then in the root finding of function $h(t)$, the initial points are set to 35.2 and 47. After three times of iterations, the root 35.44 is successfully found by using the secant method. Table 5 details the root finding of $h(t)$. Finally, the required minimum deceleration time is obtained which equals 35.44 s. If we continue round the root to the nearest integer

which is no less than the root, the minimum deceleration time of this deceleration operation is around 36 s.

Fig. 10 presents the dynamic performance of the belt conveyor during the deceleration operation from 4 m/s to 2 m/s within 36 s. The figure shows during and after the execution of the 36 seconds’ deceleration operation, the value of the driving forces always stays positive but below the friction limit. Meanwhile, both the belt tension

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**Fig. 8.** Belt conveyor dynamics in the acceleration operation from 2 m/s to 4 m/s within 51 s. (a) Driving forces exerted on the drive pulley. (b) Belt tension at each nodal point. (c) Belt speed at the drive pulley. (d) Belt speed at each nodal point along the conveying route.

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**Fig. 9.** Belt conveyor dynamic behaviors in the deceleration from 4 m/s to 2 m/s within 17 s. (a) Driving forces exerted on the drive pulley. (b) Belt tension at each nodal point. (c) Belt speed at the drive pulley. (d) Belt speed at each nodal point along the conveying route.
and the belt speed along the conveying route are varying smoothly. Apparently, this deceleration operation is acceptable.

In the case of belt conveyors with a variable material feeding rate, the belt speed is expected to be accurately adjusted to match the varying mass flow. However, as Pang and Lodewijks [14] suggest, the speed adjustment should be in a discrete way to prevent a continuous high stress. In terms of the studied belt conveyor, it is assumed that the selected speeds are 2 m/s, 2.5 m/s, 3 m/s, 3.5 m/s, 4 m/s, 4.5 m/s, 5 m/s and 5.2 m/s. By using the ECO method, more researches are carried out to find the minimum speed adjustment time for different transient operations. The results of the optimized adjustment time, including accelerating and decelerating operations, are illustrated in Fig. 11. These results can be directly used in the future application of speed control in terms of the studied belt conveyor.

5. Conclusions and recommendations

This paper proposed the ECO method to ensure healthy transient operations of a long-distance and high-capacity belt conveyors on handling bulk solid materials. The ECO method was used to calculate the demanded minimum speed adjustment time for different transient operations, both in acceleration and deceleration operations. Based on the computational simulation results, the following conclusions can be drawn:

- The conveyor dynamic performance is complex, especially in the case of a long-distance and high-capacity belt conveyor system. The finite element method can be used to calculate the conveyor dynamics in transient operations under speed control.
- The ECO method can be used to determine the demanded minimum adjustment time, taking both the risks in transient operation and the conveyor dynamics under speed control into account.
- The secant method improves the efficiency of finding the minimum speed adjustment time.
- The ECO method ensures healthy transient operations and improves the applicability of speed control.

Besides, some important recommendations for the future research are formulated as follows:

- Optimization of belt conveyor design. According to the simulation results, it is clear to see that in terms of the studied belt conveyor, the phenomenon of belt slipping around the drive pulley may happen more often than other potential risks. Therefore, in the future design of belt conveyor systems, the mass of the gravity take-up device should be optimized so as to (1) reduce the risk of belt slippage to make the belt conveyor system healthier; (2) decrease the friction coefficient of the belt conveyor systems to gain a higher energy efficiency.
- Frequency of transient operations and value of the transient safety factor. If the loading keeps constant in long term operations or it varies moderately in between long term operations, the conveyor speed is fixed in a certain period of time, in spite of small and/or temporary variations in the material flow. In

Table 4
Root finding function of \( f(t) = \frac{v_{\text{min}}(t) - 49.6}{0.5} = 0 \) with the secant method. The stop criteria is \( |c| < 0.1 \). The function values \( g(f(t)) \) and \( h(t) \) are also given.

<table>
<thead>
<tr>
<th>( t_1 )</th>
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<th>( t_3 )</th>
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</table>

Table 5
Root finding function of \( h(t) = \frac{v_{\text{min}}(t) - 1.7}{0.5} = 0 \) with the secant method. The stop criteria is \( |c| < 0.01 \). The function values \( f(t) \) and \( g(t) \) are also given.

<table>
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<th>( t_3 )</th>
<th>( f(t) )</th>
<th>( g(t) )</th>
</tr>
</thead>
<tbody>
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<td>47</td>
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</tr>
<tr>
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<td>-6.7318</td>
</tr>
</tbody>
</table>

Fig. 10. Belt conveyor dynamics in the deceleration operation from 4 m/s to 2 m/s within 36 s. (a) Driving forces exerted on the drive pulley. (b) Belt tension at each nodal point. (c) Belt speed at the drive pulley. (d) Belt speed at each nodal point along the conveying route.
References

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33. G. Lodewijks, Dynamics of belt systems, TU Delft, Delft University of Technology, Delft, the Netherlands, 1996.(Doctoral thesis).

such cases, the frequency of transient operations is very low. However, in the cases with moderately varying loading degree in between short term operations, if the material flow variation over time is taken into account, the number of transient operations will increase significantly and the speed adjustment will be more frequent. In these cases, a higher transient safety factor is suggested so as to remain the conveyor healthy. So in the future research, the relationship between the frequency of transient operations and the value of the transient safety factor should be studied. In addition, the energy savings should be evaluated over different number of stress cycles caused by transient operations.

Fig. 11. Minimum speed adjustment time for different speed adjustment ranges. (a) Minimum acceleration time. (b) Minimum deceleration time.