A Phase-Domain Readout Circuit for a CMOS-Compatible Hot-Wire CO₂ Sensor

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Abstract—This paper presents a readout circuit for a carbon dioxide (CO₂) sensor that measures the CO₂-dependent thermal time constant of a hot-wire transducer. The readout circuit periodically heats up the transducer and uses a phase-domain ΔΣ modulator to digitize the phase shift of the resulting temperature transients. A single resistive transducer is used both as a heater and as a temperature sensor, thus greatly simplifying its fabrication. To extract the transducer’s resistance, and hence its temperature, in the presence of large heating currents, a pair of transducers is configured as a differentially driven bridge. The transducers and the readout circuit have been implemented in a standard 0.16-μm CMOS technology, with an active area of 0.3 and 3.14 mm², respectively. The sensor consumes 6.8 mW from a 1.8-V supply, of which 6.3 mW is dissipated in the transducers. A resolution of 94-ppm CO₂ is achieved in a 1.8-s measurement time, which corresponds to an energy consumption of 12 mJ per measurement, >10× less than prior CO₂ sensors in CMOS technology.

Index Terms—Carbon dioxide (CO₂) sensor, CMOS compatible, delta-sigma modulator, phase-domain readout, resistive sensor, thermal conductivity (TC).

I. INTRODUCTION

CARBON dioxide (CO₂) measurement is an important function in home and building automation [1]–[4]. CO₂ concentration is an indicator of indoor air quality. It can be used to estimate the occupancy of a building and correlates with the degree of comfort experienced by the occupants [1], [2]. Adverse effects on human productivity have been reported for average CO₂ concentrations as low as 1000 ppm [2], [3]. Regulations limit indoor CO₂ concentration to 5000 ppm [2]. Applications such as electrochemical methods or solid electrolytes, have also been proposed in recent years [12], [13]. They are generally less expensive than optical sensors, but their accuracy and long-term stability still need further investigation.

Thermal-conductivity (TC)-based sensors, due to their CMOS compatibility, are an attractive alternative [14]–[18]. They exploit the fact that the TC of CO₂ is lower than that of the other constituents of air, so that CO₂ concentration can be indirectly measured via the heat loss of a suspended heated wire to ambient. Although TC-based CO₂ sensors have an inherently poor selectivity, this is not a problem in indoor-air monitoring, since exhalation of CO₂ is then the main cause of changes in air composition [19]. Furthermore, cross-sensitivities to temperature, humidity, and pressure can be compensated for by integrating additional sensors with the CO₂ sensor. However, compared to TC-based sensors for gases like helium or hydrogen [20], [21], the sensitivity of TC-based CO₂ sensors is typically rather low due to the relatively small difference in the TCs of CO₂ and air. In fact, a 1-ppm change in CO₂ concentration will only result in a 0.37-ppm change in the TC of air. In practice, the actual sensitivity may be even lower, due to heat loss to the substrate.

To measure TC accurately, a steady-state approach is usually employed, in which the power dissipation of a transducer is maintained at a well-defined level, and the resulting temperature rise is measured [15]–[17]. To relax the requirements on the stability of the transducer’s power dissipation, Cai et al. [14] propose a ratiometric approach, in which the temperature and power dissipation of CO₂-sensing and reference transducers are measured, and then used to calculate a CO₂-dependent TC ratio. This approach successfully avoids the need for a stable power reference. However, the required...
CO₂-insensitive reference transducers require capping, which complicates the packaging procedure. Moreover, the implementation in [14] uses switched-capacitor (SC) circuits to sample the voltage drop across the transducer resistance, causing the resulting resolution to be limited by $kT/C$ noise. As a result, a high degree of oversampling is needed to reach a resolution of 200 ppm, translating into a relatively long conversion time of 30 s, and a correspondingly high energy consumption of about 340 mJ per measurement.

As an alternative to a steady-state measurement, TC can also be derived from a transient measurement of the transducer’s thermal time constant $\tau_{\text{th}}$, i.e., the product of its thermal capacitance ($C_{\text{th}}$) and its thermal resistance to ambient ($R_{\text{th}}$), since this also depends on the TC of the surrounding air [15]–[17]. This approach has the important advantage that the measurement of $\tau_{\text{th}}$ relies on a time reference, and hence the absolute temperature and power levels do not need to be accurately stabilized or measured. In prior work, $\tau_{\text{th}}$ was determined by periodically generating heat pulses in a resistive transducer, and then measuring the phase shift of the temperature transients with a separate temperature sensor: either a similar transducer [16] or a thermopile [24], [25]. In these designs, the heating and sensing elements are separate devices, and hence the thermal signal to be sensed is naturally separated from the electrical driving signal, which simplifies the process of signal conditioning. However, this inevitably results in a more complex fabrication process, in contrast with the simple CMOS-compatible single-wire transducers used in [14].

In contrast with earlier TC sensors based on transient measurements [16], [24], we present a readout circuit that allows the heating and temperature-sensing functions to be combined in a single resistive transducer [26]. This greatly simplifies fabrication, since only a single extra etch step is required to realize a tungsten hot-wire transducer in the via layer of a standard CMOS process [14]. The readout circuit is based on a continuous-time phase-domain delta–sigma modulator (PD$\Delta$ΣM), thus circumventing the $kT/C$ noise limitations associated with SC readout circuits. To reduce the required dynamic range of the modulator, the large transients associated with the periodic heating pulses, and the offset associated with the baseline resistance of the resistive transducer are cancelled by employing two transducers in a novel bridge-type architecture. Experimental results show that the proposed CO₂ sensor achieves a resolution of 94 ppm in a conversion time of 1.8 s while dissipating only 12 mJ per measurement, or 2$\times$ more resolution than the state of the art [14], at an energy consumption that is $>10\times$ lower.

This paper is organized as follows. In Section II, the operation of steady-state TC sensing is first presented, followed by a discussion of transient TC sensing, as used in this paper. In addition, the principles of PD$\Delta$ΣM are briefly reviewed. Section III is devoted to the circuit implementation of the readout circuit, including the design of the PD$\Delta$ΣM and the techniques used to reduce its dynamic range requirements. Experimental results and discussions are presented in Section IV, and the conclusion is provided in Section V.

**II. OPERATING PRINCIPLE**

### A. Amplitude-Domain Thermal-Conductivity Sensing

Fig. 1(a) shows a cross section of the CMOS-compatible transducer reported in this paper, which is the same as that described in [14]. It consists of a suspended tungsten wire made in the via layer of the metal stack of a 0.16-μm CMOS process, which is released by means of a single etching step, using an approach similar to that described in [27]. As shown in Fig. 1(b), the temperature rise of the wire relative to ambient temperature ($\Delta T$) caused by electrical power dissipation ($P$) is directly proportional to the equivalent thermal resistance $R_{\text{th}}$ to ambient: the parallel combination of $R_{\text{th}}$ with its thermal resistance to air ($R_{\text{th,air}}$) and that to the substrate ($R_{\text{th,sub}}$),

$$\Delta T = P \cdot R_{\text{th}} = P \cdot (R_{\text{th,air}} \parallel R_{\text{th,sub}}) .$$

(1)

Since different gases have different TCs, $R_{\text{th,air}}$ is a function of the air composition. The TC of CO₂ is slightly lower than the average TC of air, and thus a higher CO₂ concentration leads to a slightly higher temperature rise $\Delta T$. Hence, $\Delta T$ can be used as a proxy for CO₂ concentration. To minimize the transducer’s heat loss to the substrate, it is typically suspended above the substrate [14], [28], [29].

Resistance $R$ of a hot-wire transducer can be approximated by a linear function of temperature $T$

$$R = R_0 \cdot (1 + \alpha \cdot (T - T_0))$$

(2)

where $\alpha$ is the temperature coefficient of the electrical resistance of the transducer, and $R_0$ is its nominal value at
temperature $T_0$. For our tungsten transducers, $R_0$ is 110 $\Omega$, and $\alpha$ is 0.0017 $/K$ (determined by experimental characterization of test devices). Thus, the temperature of the transducer can be measured via changes in its resistance [30], [31].

It is obvious that the accuracy of steady-state TC sensing depends on the accuracy with which the power dissipation $P$ and the temperature rise $\Delta T$ (i.e., the temperature of the hotwire resistor relative to the ambient temperature) can be measured. Achieving less than 200-ppm error in the CO$_2$ concentration, corresponds to less than 80-ppm errors in TC, and therefore require similar levels of accuracy for both the measurement of $P$ and $\Delta T$ [14]. Since this is well beyond the state of the art, it is very difficult to make integrated CMOS-compatible ambient CO$_2$ sensors based on steady-state TC sensing [14].

**B. Time-Domain Thermal-Conductivity Sensing**

Rather than measuring the steady-state temperature rise of a heated wire, an alternative method is to characterize its thermal time constant $\tau_{th}$, which is the product of its thermal resistance to ambient ($R_{th}$) and its thermal capacitance ($C_{th}$) [15]–[17]. When the wire is driven with a current $I_d$ pulsed at a frequency $f_{drive}$, and is thus periodically heated, its temperature transients are delayed relative to the driving pulses. The delay is determined by the thermal time constant $\tau_{th}$, which in turn depends on the TC of the surrounding air (Fig. 2).

Such a TC sensor can be modeled as a first-order low-pass filter. Using a fixed driving frequency will then result in phase-delayed temperature transients relative to the driving pulses, from which $\tau_{th}$ can be derived. The optimal driving frequency equals the filter’s pole frequency, i.e., $1/2\pi \tau_{th}$, at which the sensitivity of the phase shift to the changes of $\tau_{th}$ is maximal. For our devices, $\tau_{th} \approx 17 \mu s$, leading to an optimal $f_{drive}$ around 9–10 kHz.

Earlier TC sensors based on transient measurements employed separate resistive heaters and temperature sensors, either thermistors [16] or thermopiles [24], which were mounted together on a thermally isolated membrane. This separates the temperature transients from the electrical transients, and thus simplifies the readout circuitry, at the cost of fabrication complexity and hence cost.

In earlier work, sine waves [16] and square waves [24] have both been used to drive the heater. In terms of circuit implementation, a square wave is much easier to generate than a sine wave. The advantage of a sine wave is that it has no harmonics, and thus the phase shift of the temperature signal can be determined by filtering and zero-crossing detection [16]. In contrast, a square wave consists of a series of harmonics. The phase shift of the fundamental can then be detected by synchronous detection [24]. By employing a synchronous detector as the summing node of a delta–sigma modulator, a PDA$\Delta$S$\Sigma$M can be realized with which the phase shift of the fundamental can be digitized with high resolution [32], [33].

Since resistive transducers can be used both as a heater and a temperature sensor, the heating and sensing functions can, in principle, be combined in a single resistor, provided an appropriate readout scheme is devised. This will be discussed in Section III-A.

**C. Phase-Domain Delta–Sigma Modulator**

When driven at $f_{drive}$, the phase shift $\phi_{sig}$ of the transducer’s temperature transients can be found by synchronous detection, i.e., by multiplying the transients by a reference signal at the same frequency and with a phase $\phi_{ref}$, as illustrated in Fig. 3(a). Assuming sine waves for simplicity, the result is a dc component proportional to the cosine of the phase difference ($\phi_{sig} - \phi_{ref}$), and a component at 2$f_{drive}$ that can be removed by a low-pass filter

\[
A \cdot \sin(2\pi f_{drive} t + \phi_{sig}) \cdot \sin(2\pi f_{drive} t + \phi_{ref}) = 0.5 \cdot A \cdot [\cos(\phi_{sig} - \phi_{ref}) - \cos(4\pi f_{drive} t + \phi_{sig} + \phi_{ref})].
\]

As shown in Fig. 3(b), a PDA$\Delta$S$\Sigma$M can be realized by embedding the synchronous detector in a delta–sigma ($\Delta$S) loop [32], [33]. The loop’s integrator serves as a low-pass filter and feedback is applied in the phase domain, by toggling $\phi_{ref}$ between two phase references $\phi_0$ and $\phi_1$ depending on the bitstream output $bs$. The feedback loop, on average, nulls the input of the integrator and thus ensures that the average...
phase reference tracks the phase of the input signal, which can, therefore, be derived from the average value of the bitstream. From (3) it can be seen that in order to allow the ΔΣ modulator to track $\phi_{\text{sig}}$, the reference phase $\phi_{\text{ref}}$ should toggle between two values such that the polarity of the $\cos(\phi_{\text{sig}} - \phi_{\text{ref}})$ term also toggles. This implies that, with respect to the input signal, the two reference phases ($\phi_0$ and $\phi_1$) should be slightly less and slightly more than 90° (i.e., $\phi_0 = 90^\circ + \Delta \phi/2$ and $\phi_1 = 90^\circ - \Delta \phi/2$, where $\Delta \phi$ is the full scale of the PDΔΣM). Given the relatively low frequency, such reference phases can be readily derived from a higher frequency master clock in practice.

The resolution with which the phase shift can be determined depends on the oversampling ratio (OSR), i.e., the number of clock cycles $N$ that the ΔΣ modulator is operated per measurement, and equals $(\phi_1 - \phi_0)/N$ for a first-order ΔΣ modulator (given that the cosine non-linearity can be neglected over the relatively small range $\phi_1 - \phi_0$) [33]. Simulation shows that the phase shift induced by a 1-ppm change in CO$_2$ concentration is roughly 7 μ°. The required OSR to arrive at the desired CO$_2$ resolution can thus be estimated from this. For example, for a full scale $\Delta \phi = \phi_0 - \phi_1 = 4^\circ$, the required OSR for a quantization step equivalent to 100-ppm CO$_2$ is about 6000. Although this number could be reduced by using a second-order modulator, this would have little benefit in our design, because an even higher OSR is needed to reduce the thermal noise to the 100-ppm level, as will be shown in Section III-D. In designs in which the thermal-noise target can be reached at a lower OSR, increasing the order of the modulator could reduce the conversion time and thus result in less energy consumption per measurement.

**III. Circuit Implementation**

**A. Front-End Dynamic Range Reduction Technique**

While the voltage across the transducer in Fig. 2 contains temperature information, its sensitivity to temperature is proportional to the current level. As a result, the sensitivity will be much higher when the transducer is biased at the drive-current level to heat it up, than when it is biased at a lower current level. This complicates the extraction of the transducer’s temperature transients from the voltage across it. To mitigate this, a small sense current $I_s$, switched at a much faster rate $f_{\text{sense}}$, produces a modulated voltage proportional to $R(t)$ with a sensitivity to temperature that is reasonably independent of the drive current [Fig. 4(a)]. To ease the detection of this voltage in the presence of the large voltage transients at $f_{\text{drive}}$ (about 300-mV peak-to-peak), a pair of transducers are heated simultaneously by pulsed currents $I_d$ (= 2 mA), and read out differentially via out-of-phase sense currents $I_s$ (= 0.5 mA), switched at $f_{\text{sense}} = 15 \times f_{\text{drive}}$ [Fig. 4(b)]. Thus, the signal at $f_{\text{drive}}$ is converted into a common-mode signal and can be rejected, while the differential signal is demodulated using a chopper switch, resulting in an output voltage $V_s$ that contains the temperature transients at $f_{\text{drive}}$. Each transducer is also biased by an additional constant sense current $I_s$ (= 0.5 mA) to provide a voltage signal to be sensed when $I_d$ is switched OFF. The sense current flowing through the transducers leads to a small baseline power dissipation that adds to the period heating due to the switching of $I_d$. For a given total average power dissipation, increasing the ratio $I_d/I_s$ increases the amplitude of the temperature transients, and thus increases the signal to be sensed. At the same time, as will be detailed in Section III-D, a smaller value of $I_s$ increases the noise level. The chosen 4:1 ratio between $I_d$ and $I_s$ is a tradeoff between these two effects and optimizes the SNR.

The ratio of 15 between $f_{\text{sense}}$ and $f_{\text{drive}}$ separates the drive and sense signals by more than a decade in the frequency spectrum, thus minimizing the thermal transients due to sense signal, and facilitating the filtering of the upconverted drive signal by the PDΔΣM. An odd ratio is chosen to prevent errors due to the downconversion of harmonics of the drive signal. This is because any mismatch between the drive signals will cause a fraction of the common-mode drive signal to be converted into a differential-mode signal. If $f_{\text{sense}}$ is an even multiple of $f_{\text{drive}}$, the odd harmonics of this differential-mode signal will be downconverted to $f_{\text{drive}}$ by the chopper.
Current trimming DACs

In practice, however, the mismatch between the transducers and the poly-resistors leads to ac ripple, which reduces the modulator’s effective resolution affecting the decimated results. If \( I_{\text{s}} \) is chosen to be an odd multiple of \( I_{\text{drive}} \), any downconverted harmonics will end up at dc, and will be rejected by the PDAΣM.

Even with this arrangement, a large dynamic range is still required, since the temperature-induced resistance increase \( (\Delta R \approx 3 \, \Omega) \) is small compared to the baseline resistance \( (R_0 = 110 \, \Omega) \), while the changes in \( R_0 \) due to changes in CO2 concentration are even smaller (about 1.5 \( \mu \Omega \) per ppm CO2). To cancel the voltage steps associated with \( R_0 \), two poly-resistors \( R_{p1,2} (=R_0) \) are connected in series with the transducers, and the sense currents are routed such that the additional voltage drop \( I_s \cdot R_p \) cancels out \( I_d \cdot R_0 \) [Fig. 4(c)].

The remaining differential signal \( V_s \) is ideally equal to \( I_s \cdot \Delta R \), and reflects the transient temperature change, which is about 1.5 mV, 200\( \times \) smaller than the initial 300-mV transients.

Note that all switches in Fig. 4(c) are either in series with current sources or in voltage-sensing paths in which no significant current flows. As a result, the finite on-resistance of the switches, to first order, does not lead to measurement errors. The resistance of the interconnect between the transducers and the poly-resistors should be kept small compared to the nominal resistance of the transducers.

B. Current Trimming DACs

In practice, however, the mismatch between the transducers and the poly-resistors leads to ac ripple, which reduces the modulator’s effective resolution and increases the requirements on its dynamic range. To minimize the ripple, three current digital-to-analog converters (DACs) are used to trim the drive and sense currents. As shown in Fig. 5, one 6-bit drive-current DAC \( (I_{DAC1}, \text{LSB} = 0.1\%I_{\text{REF}} = 0.025\%I_d) \) is used to trim the two drive current sources and thus compensate for the mismatch between the two transducers \( R_{t1} \) and \( R_{t2} \). Two 6-bit sense-current DACs \( (I_{DACs1} \text{ and } I_{DACs2}, \text{LSB} = 0.4\%I_s) \) can be connected to two of the three sense current sources through a 2–3 multiplexer, to compensate for the mismatch between the two poly-resistors, and between the poly-resistors and the nominal resistance of the transducers.

If there would be no mismatch, the following would hold:

\[
I_d1 R_{t1} = I_d2 R_{t2} \tag{4}
\]

\[
(I_s1 + I_s2) R_{t1} = I_s3 (R_{t2} + R_{p2}) \tag{5}
\]

\[
I_s1 (R_{t1} + R_{p1}) = (I_s2 + I_s3) R_{t2} \tag{6}
\]

Hence, the target of the current trimming is to configure the trimming DACs such that these conditions are reached.

The procedure used to find the proper trimming settings is shown in Fig. 6. First, only the two drive current sources are used to bias transducers \( R_{t1} \) and \( R_{t2} \). The PDAΣM is disconnected and the voltage across \( V_s \) across the transducers is measured using an (off-chip) multimeter. Due to mismatch, \( V_s \) can be non-zero. The drive-current DAC is then used to compensate for this mismatch such that \( V_s \approx 0 \) and hence (4) holds. Second, only the sense current sources are switched on. When the switches are configured as shown, \( V_s \) is the voltage difference between the left and right branches, which ideally should be 0 as demonstrated by (5). Due to the variation of poly-resistor \( R_{p2} \), this voltage can be above or below 0. Here, it is assumed that \( V_s > 0 \) as an example to demonstrate the trimming procedure. To reduce \( V_s \) to 0, \( I_{DACs1} \) is connected as shown to add current to \( I_s3 \) and thus null \( V_s \), so that (5) holds. Third, the switches for the sense current sources are changed. Again, due to mismatch, \( V_s \neq 0 \). Here, it is again assumed that \( V_s > 0 \). To reduce \( V_s \) to 0, \( I_{DACs2} \) is connected as shown to add current into \( I_s2 \) and thus \( V_s \), so that (6) holds.

In summary, current trimming is used to modify the sense currents \( I_{s1}, I_{s2}, \) and \( I_{s3} \) so as to compensate for the mismatch.
of the resistors and drive the steady-state voltage difference between the left and right branches to 0. The example shown here is for $V_s > 0$ in both steps 2 and 3. For other cases, the proper settings of the DACs can be found in a similar way. By analyzing all possible cases, it is found that two multiplexed DACs for the three sense currents are sufficient, though three DACs may simplify the trimming procedure. To save die area, this design uses one trimming DAC for drive current and two trimming DACs for sense current.

The resolution of the trimming DACs is determined by the level of the residual ripple that the modulator can allow. An LSB of 0.4% of $I_s$ current DAC can correct errors of resistance mismatch down to 0.4%, and the corresponding voltage ripple amplitude is about 0.2 mV, which is sufficiently small compared with the signal amplitude after baseline cancellation (1.5 mV). It should be noted that although this voltage ripple can increase the required dynamic range of the modulator and reduce the effective resolution, the relationship between the amplitude of the ripples and the CO$_2$ resolution is not straightforward.

The current trimming is done at room temperature, i.e., around the midpoint of the indoor temperature range (10 °C–40 °C). If the resistors only have mismatch in their baseline resistance but have the same temperature coefficient, the mismatch will be corrected by the current trimming, independent of further temperature variations. Mismatch in the temperature dependence of the resistance will lead to a (small) residual ripple that is larger for temperatures further away from the trimming temperature. Since the sensor operates over a very limited temperature range, we expect this residual ripple to be sufficiently small to avoid degrading the resolution.

C. Phase-Domain Delta–Sigma Modulator

The phase shift of the temperature-related differential signal $V_s(\approx I_s \cdot \Delta R)$ is digitized by a low-noise PDΔΣM similar to that described in [25]. As shown in Fig. 7, before demodulation by $f_{\text{sense}}$, a low-noise transconductor $g_{\text{m}}$ converts the differential voltage $V_s$ into a current. This current passes through a chopper switch, which serves to dual purpose of demodulation by $f_{\text{sense}}$ (like the chopper switch in Fig. 4), and multiplication with the phase-shifted versions of $f_{\text{drive}}$ as a function of the bitstream [as shown in Fig. 3(b)]. This combination is realized by multiplying the phase-shifted versions of $f_{\text{drive}}$ with $f_{\text{sense}}$ by means of XOR gates. The resulting demodulated current is proportional to the phase difference between $V_s(t)$ and the selected phase reference. This difference is integrated on capacitors $C_{\text{int}}$ of an active integrator and quantized using a clocked comparator, to form a ΔΣ loop which nulls the input of the integrator and thus ensures that the average phase reference tracks the phase of $V_s(t)$, which can, therefore, be derived from the average value of the bitstream.

To ensure that the noise from the transconductor is lower than that from the transducer and its bias circuit, $g_{\text{m}}$ of the transconductor should be at least 400 $\mu$S. The transconductance of the $g_{\text{m}}$ stage is about 560 $\mu$S. Fig. 8 shows the schematic of the $g_{\text{m}}$ stage. It employs a gain-boosted folded-cascode structure for high output impedance to minimize the leakage of the integrator [25]. The chopper demodulator is embedded into the transconductor. The input pair of the transconductor is sized such that its $1/f$ noise corner frequency is below $f_{\text{sense}}$, ensuring that $1/f$ noise of the transconductor does not affect the measurement. The sampling frequency is chosen the same as $f_{\text{drive}}$. Both $f_{\text{drive}}$ and $f_{\text{sense}}$, including the feedback signals at $f_{\text{drive}}$ with reference phases $\phi_0$ and $\phi_1$, are derived from a single off-chip master clock. Capacitor $C_{\text{int}}$ in the integrator is 50 pF.

D. Noise Analysis

The noise of the phase-domain ΔΣ analog-to-digital converter (ADC) can be analyzed using a charge-balancing analysis similar to that described in [34]. During a complete ΔΣ conversion, the total charge accumulated in the integrator is approximately 0. This includes all signal charge accumulated in the ΔΣ cycles when $bs = 0$ and when $bs = 1$, as well as the noise charge

$$Q_{\text{acc}} = Q_{\text{sig}} + q_{\text{n,acc}} = N(1 - \mu)Q_0 + \mu Q_1 + q_{\text{n,acc}} = 0$$

(7)
in which \( Q_{\text{sig}} \) is the total signal charge and \( q_{n,\text{acc}} \) is the total noise charge. \( N \) is the total number of \( \Delta \Sigma \) cycles during one conversion, \( \mu \) is the average value of the bitstream, \( Q_0 \) is the accumulated signal charge in one \( \Delta \Sigma \) cycle when \( bs = 0 \), and \( Q_1 \) is the accumulated signal charge in one \( \Delta \Sigma \) cycle when \( bs = 1 \).

Rearranging (7) results in the expression for \( \mu \)

\[
\mu = \frac{Q_0}{Q_0 - Q_1} + \frac{q_{n,\text{acc}}}{N(Q_0 - Q_1)} \tag{8}
\]

in which the first part is the average value of the bitstream in the noise-free case, and the second part represents the noise contribution. The total accumulated noise charge is related to the noise charge, \( q_n \), and the contribution. The total accumulated noise charge is related to the noise charge, \( q_n \), and the

\[
\sigma_\mu = \frac{1}{Q_0 - Q_1} \sqrt{\frac{q_{n,\text{acc}}^2}{N}} \tag{9}
\]

Referring to Fig. 3, the signal charge difference \( Q_0 - Q_1 \) can be approximated as

\[
Q_0 - Q_1 = \int_{t_{\text{clk}}} \cos(\phi_{\text{sig}} - \phi_0) - \cos(\phi_{\text{sig}} - \phi_1) \\, dt
\]

\[
\approx \int_{t_{\text{clk}}} (\phi_0 - \phi_1) \tag{10}
\]

in which \( I_{\text{int}} \) is the output current of the transconductor \( g_m \), \( t_{\text{clk}} \) is the inverse of the sampling frequency, and the approximation is justified as both \( (\phi_{\text{sig}} - \phi_0) \) and \( (\phi_{\text{sig}} - \phi_1) \) are close to \( \pi/2 \).

Noise charge accumulated in every cycle includes noise from the current source, the transducers, the transconductor \( g_m \), and the chopper switches

\[
q_n^2 = q_{n,\text{cs}}^2 + q_{n,Rt}^2 + q_{n,ch}^2 + q_{n,gm}^2 = 4kT R_n \cdot g_m^2 \cdot t_{\text{clk}}^2 \cdot B \tag{11}
\]

in which \( q_{n,\text{cs}} \) is the noise charge due to the current source, \( q_{n,Rt} \) is the noise charge due to the transducers (including the poly-resistors), \( q_{n,ch} \) is the noise charge generated by the chopper switches, \( q_{n,gm} \) is the noise from the transconductor \( g_m \), \( k \) is Boltzmann’s constant, \( T \) is the absolute temperature, \( B \) is the equivalent noise bandwidth (= 1/2\( t_{\text{clk}} \)), and \( R_n \) is the equivalent noise resistance. The latter is given by

\[
R_n = \gamma_1 g_{m,\text{cs}} R_t^2 + R_t + R_{\text{on,ch}} + \frac{\gamma_2}{g_m} \tag{12}
\]

in which \( \gamma_1 \) and \( g_{m,\text{cs}} \), are the excess noise factor and the transconductance of the current source, respectively, \( R_t \) is the resistance of the transducers, \( R_{\text{on,ch}} \) is the on-resistance of the chopper switches, and \( \gamma_2 \) is the excess noise factor of the transconductor \( g_m \). Substituting (10) and (11) into (9) results in an expression for the standard deviation of the output-referred noise

\[
\sigma_\mu = \frac{g_m}{I_{\text{int}}(\phi_0 - \phi_1)} \sqrt{\frac{2kT R_n}{N \cdot t_{\text{clk}}}} = \frac{16}{I_s \cdot \Delta R \cdot (\phi_0 - \phi_1)} \sqrt{\frac{2kT R_n}{N \cdot t_{\text{clk}}}} \tag{13}
\]

in which \( I_{\text{int}} = (1/16)I_s \cdot \Delta R \cdot g_m \). Note that this derivation approximates the drive and sense currents, as well as the feedback signals as sinusoidal signals.

This expression shows that the standard deviation of \( \mu \) and hence the resolution can be improved by decreasing the equivalent noise resistance, by increasing the total conversion time, or by increasing the signal amplitude \( (I_s \cdot \Delta R) \). Note that the latter also requires an increase in the size of the integration capacitors, as the sampling frequency of the PD\( \Delta \Sigma \)M should not be higher than the drive frequency \( f_{\text{drive}} \). System-level simulations show that a 50-pF integration capacitor can handle a signal amplitude of about 1.5 mV. The resistance of the transducer is 110 \( \Omega \) in this paper. We have chosen the drive current (2.5 mA) such that the temperature-induced resistance \( \Delta R \) is limited to 3 \( \Omega \), and the sensing current \( I_s \) is 0.5 mA. The resulting calculated resolution, expressed in terms of an equivalent CO2 concentration, is shown in Fig. 11 as a function of the number of \( \Delta \Sigma \) cycles \( N \) (i.e., the OSR).

IV. EXPERIMENTAL RESULTS AND DISCUSSION

Both the transducers and the readout circuit have been implemented in the same 0.16-\( \mu \)m CMOS technology (Fig. 9), with an active area of 0.3 and 3.14 mm\(^2\), respectively. For flexibility, they have been realized on separate chips and connected on the PCB, and hence they can readily be co-integrated. When doing so, the impact of on-chip thermal gradients should be taken into account. Given that the hot-wire transducers lose the majority of their heat to the ambient air rather than to the substrate, and that the substrate is a good thermal conductor, we expect these gradients to be manageable. The readout circuit consumes 6.8 mW from a 1.8-V supply, a 6.3 mW of which is dissipated in the transducers.

All control signals, including \( f_{\text{drive}}, f_{\text{sense}} \), the phase-shifted feedback reference signals, and control signals for the current DACs, are generated from a 10-MHz master clock using a field-programmable gate array (FPGA), and can be similarly generated on-chip in a future implementation. Jitter requirements for the clocks can be derived from the thermal time constant and required resolution. The transducer in this paper has a thermal time constant of about 17 \( \mu \)s.
A 100-ppm change in CO₂ corresponds to about 40-ppm change in TC, and so about 40-ppm change in thermal time constant (\(\tau_{th} = R_{th}C_{th}\), and \(C_{th}\) is assumed to be constant), which is about 680 ps. To leave some margin, 100-ps jitter would be sufficient. The clock edges of \(f_{drive}\) and \(f_{sense}\) should be aligned to avoid possible mixing errors.

Fig. 10 shows the measured bitstream spectrum of the PD\(\Delta\Sigma\)M, demonstrating a first-order noise shaping similar to that of a conventional amplitude-domain first-order \(\Delta\Sigma\) modulator. Note that the large amplitude in the first bin of the fast Fourier transform (FFT) is due to the non-zero dc value of the bitstream (which represents the phase shift to be digitized). The slightly sloped low-frequency noise could be due to \(1/f\) noise of the current sources in the bias circuit.

Fig. 11 shows the measured resolution (standard deviation of 20 consecutive measurements) and energy per measurement as a function of OSR.

Fig. 12. Measured phase shift as a function of the drive frequency.

Fig. 13. Relative changes in the decimated results as well as in power consumption as a function of drive current.

\(f_{samp} = f_{drive} = 9.26\ kHz\), and an energy consumption of 12 mJ per measurement. This is a significant improvement compared with the amplitude-domain readout based on an SC integrator described in [14]. While the energy efficiency of the readout circuit in [14] could be further improved with some tradeoffs (i.e., by using larger sampling capacitors or a higher sampling rate), the continuous-time modulator used in this paper should still be better in terms of energy efficiency, as it avoids the inherent noise folding associated with SC circuitry.

The thermal delay, or equivalently the measured phase shift, caused by the thermal resistance and thermal capacitance, should present a first-order behavior as a function of the driving frequency, like a first-order electrical low-pass filter. This is confirmed by measurements, as shown in Fig. 12. The measured phase shift as a function of the drive frequency shows a good agreement with the ideal first-order behavior associated with the hot-wire’s thermal time constant (measured using a larger full scale \(\Delta\phi = \phi_0 - \phi_1 = 12^\circ\) for clarity).

To demonstrate the sensitivity of the sensor’s output to variations in power consumption, Fig. 13 shows the decimated results and the power consumption as a function of the drive current \(I_d\). With \(\pm 10\%\) change in current, the power consumption changes by \(\pm 20\%\), while the decimated results change by 2.5%, equivalent to a variation of about 1.5% in CO₂ concentration (given the sensitivity of 1.6 ppm per ppm CO₂). For 200-ppm accuracy in CO₂ measurement, this
implies that variations in the power dissipation should be less than ±2600 ppm. Compared to the steady-state TC sensing, for which the required stability of the power dissipation would be <80 ppm when measuring in amplitude domain, the time-domain readout reduces the sensitivity to power level by 30–50×. The residual dependence could come from two possible sources. One is the temperature-dependent sensitivity [14]. Due to the change in power dissipation, the temperature of the hot-wire changes. It has been found in the previous work that the sensitivity of the TC of air to CO2 concentration is not constant but temperature dependent. This could lead to power (temperature)-dependent measurement results. A second possible source of the residual current dependence is residual mismatch after trimming. This mismatch will appear as a dc input to the $PD\Delta SM$, which causes ripple at the output of the integrator. Since the $PD\Delta SM$ has a finite ability to reject this ripple, and the amplitude of the ripple is proportional to the current level, the output of the $PD\Delta SM$ will depend on the current level.

To measure the CO2 response, the sensor was placed in a sealed box along with an NDIR reference CO2 sensor [5]. Like other TC-based sensors [14], [16], the sensor is cross-sensitive to ambient variations, such as temperature, humidity, and pressure sensors were placed in the sealed box to facilitate cross-sensitivity compensation. The results after compensation are shown in Fig. 14, demonstrating a good agreement between the readings of our sensor and the CO2 concentration measured by the reference CO2 sensor K30.

Table I summarizes the performance of the chip and compares it with the prior art. The proposed TC-based CO2 sensor achieves a resolution of 94 ppm while dissipating only 12 mJ per measurement, which represents a significant improvement in energy efficiency compared to the state of the art for TC-based CO2 sensors. Compared with the NDIR-based counterpart, the proposed sensor has advantages in cost (×10) and volume (×100), due to its CMOS-compatibility, and also consumes less energy. Implementation of on-chip cross-sensitivity compensation is future work, and remains essential for the realization of a practical TC-based CO2 sensor. Several reported low-power, small-form-factor silicon sensors could be tailored for this application [35]–[37].

V. C O N C L U S I O N

In this paper, we have presented a CMOS-compatible CO2 sensor that senses the CO2-dependent variations in the ambient air. Rather than measuring the steady-state temperature rise of a hot-wire transducer, we detect its thermal time constant $\tau_{th}$, thus obviating the need for heating-power stabilization and accurate temperature sensing. The thermal time constant is the product of the wire’s thermal capacitance and its thermal resistance to ambient, which in turn depends on the TC of the surrounding air. It is sensed by periodically heating up the wire and digitizing the phase shift in the resulting temperature transients by means of a low-noise $PD\Delta SM$. The temperature transients are sensed through the resistance changes of the heater resistor, greatly simplifying the fabrication process compared to prior designs that employ separate resistive or thermopile-based temperature sensors closely integrated with the heater. In order to reduce energy consumption, the required dynamic range of the readout circuit is substantially reduced by cancelling the baseline resistance and by removing the impact of the large electrical driving signals. The sensor achieves a CO2 resolution of 94 ppm at only 12-mJ energy consumption per measurement,
the best reported resolution in TC-based CO₂ sensors and the lowest energy consumption compared to the prior art. This makes this design a promising candidate for CO₂ sensing in cost- and energy-constrained applications.

ACKNOWLEDGMENT
The authors would like to thank L. Pakula and Z.-Y. Chang for their technical support.

REFERENCES

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