TU Delft

Delft University of Technology

Untangling decision tree and real options analyses

a public infrastructure case study dealing with political decisions, structural integrity and price uncertainty

van den Boomen, M.; Spaan, M. T.J.; Schoenmaker, R.; Wolfert, A. R.M.

DOI
10.1080/01446193.2018.1486510

Publication date
2019

Document Version
Final published version

Published in
Construction Management and Economics

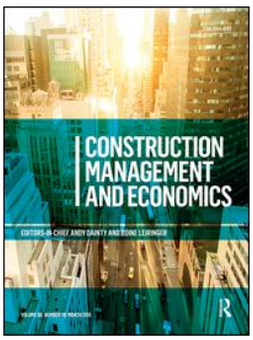
Citation (APA)
van den Boomen, M., Spaan, M. T. J., Schoenmaker, R., & Wolfert, A. R. M. (2019). Untangling decision tree and real options analyses: a public infrastructure case study dealing with political decisions, structural integrity and price uncertainty. *Construction Management and Economics*, *37*(1), 24-43. https://doi.org/10.1080/01446193.2018.1486510

Important note
To cite this publication, please use the final published version (if applicable).
Please check the document version above.

Copyright
Other than for strictly personal use, it is not permitted to download, forward or distribute the text or part of it, without the consent of the author(s) and/or copyright holder(s), unless the work is under an open content license such as Creative Commons.

Takedown policy
Please contact us and provide details if you believe this document breaches copyrights.
We will remove access to the work immediately and investigate your claim.

This work is downloaded from Delft University of Technology.
For technical reasons the number of authors shown on this cover page is limited to a maximum of 10.



Routledge
Taylor & Francis Group

Construction Management and Economics

ISSN: 0144-6193 (Print) 1466-433X (Online) Journal homepage: http://www.tandfonline.com/loi/rcme20

Untangling decision tree and real options analyses: a public infrastructure case study dealing with political decisions, structural integrity and price uncertainty

M. van den Boomen, M. T. J. Spaan, R. Schoenmaker & A. R. M. Wolfert

To cite this article: M. van den Boomen, M. T. J. Spaan, R. Schoenmaker & A. R. M. Wolfert (2018): Untangling decision tree and real options analyses: a public infrastructure case study dealing with political decisions, structural integrity and price uncertainty, Construction Management and Economics, DOI: 10.1080/01446193.2018.1486510

To link to this article: https://doi.org/10.1080/01446193.2018.1486510

© 2018 The Author(s). Published by Informa UK Limited, trading as Taylor & Francis Group.

Published online: 04 Oct 2018.

Submit your article to this journal

Article views: 146

View Crossmark data

Full Terms & Conditions of access and use can be found at
http://www.tandfonline.com/action/journalInformation?journalCode=rcme20

CONSTRUCTION MANAGEMENT AND ECONOMICS
https://doi.org/10.1080/01446193.2018.1486510

Routledge
Taylor & Francis Group

OPEN ACCESS Check for updates

Untangling decision tree and real options analyses: a public infrastructure case study dealing with political decisions, structural integrity and price uncertainty

M. van den Boomen, M. T. J. Spaan, R. Schoenmaker and A. R. M. Wolfert

Delft University of Technology, Delft, the Netherlands

ABSTRACT

Managerial flexibility in infrastructure investment and replacement decisions adds value. Real options analysis (ROA) captures this value under uncertain market prices. The concept of ROA is that future unfavourable payoffs can be deferred as soon as more information about market prices becomes available. The popularity of ROA is seen in a growing number of case studies on real assets. Despite its increasing popularity, ROA has not gained a foothold in public infrastructure decision making. One of the difficulties in the application of ROA is the required estimation of market variables. To avoid this, a simplified but not correct version of ROA is easily applied, referred to as a Decision Tree Approach (DTA) to ROA. Another difficulty is that infrastructure assets are subject to other types of uncertainties, defined here as asset uncertainties. This study investigates the value of managerial flexibility in a public infrastructure replacement decision. The uncertainty drivers are the strength of a bridge, political decisions regarding traffic flow and the price development of construction costs. Three valuation approaches are compared: DTA, ROA and the DT approach to ROA. Although it is complex, ROA certainly adds value in public infrastructure decision making when market price uncertainty is prevalent. However, in the absence of reasonable estimates of market variables, the DT approach to ROA is the best alternative. In the absence of market price uncertainties, ROA should be avoided DTA is to be preferred.

ARTICLE HISTORY
Received 1 February 2018
Accepted 2 June 2018

KEYWORDS
Real options; decision analysis; infrastructure planning; replacement; uncertainty

1. Introduction

The real options analysis (ROA) of the last decade is advocated as a promising technique in valuing the flexibility of managerial decisions in infrastructure life cycle decisions. The theory of real options originates from financial options valuation. It values financial options that give a holder the right to defer unfavourable payoffs. Groundwork for financial option pricing was laid by Black and Scholes (1973) and Merton (1973). Their efforts brought about the famous Black–Scholes–Merton formula, which provides a closed-form solution to value European call options.

A call option provides the right but not the obligation to buy a share of common stock at a predetermined price before or at a predetermined date. An owner will only exercise the option if the option payoff is positive. The market wants to be compensated for bearing this risk, which is reflected in a risk-adjusted discount rate, or equivalently, a risk-adjusted stream of cash flows discounted at a risk-free discount rate. In response to the closed form formula for a special case of option pricing, Cox *et al.* (1979) developed a discrete binominal approach, that uses basic mathematics and allows for more flexibility in exercising calls and valuing options. The approach of Cox *et al.* (1979) is widely adopted as a standard for option pricing.

It is easy to draw a parallel between financial options and ROA. Unfavourable future payoffs on tangible real assets or projects can be deferred by managerial decisions. Real options are for example expansion, replacement, switching or abandonment of real assets. The theory behind real options is well-documented by authors like Amram and Kulatilaka (1999); Copeland and Antikarov (2003); Mun (2006); Guthrie (2009); Peters (2016); and Brealey *et al.* (2017) and applied in a growing number of case studies in the literature. For example, a practical spreadsheet approach to value real options of investment strategies for a garage parking case study is presented by

CONTACT M. van den Boomen m.vandenboomen@tudelft.nl Faculty of Civil Engineering and Geosciences, Delft University of Technology, Building 23, Room 3.40, P.O. Box 5048, 2628 CN Delft, the Netherlands

© 2018 The Author(s). Published by Informa UK Limited, trading as Taylor & Francis Group.
This is an Open Access article distributed under the terms of the Creative Commons Attribution-NonCommercial-NoDerivatives License (http://creativecommons.org/Licenses/by-nc-nd/4.0/), which permits non-commercial re-use, distribution, and reproduction in any medium, provided the original work is properly cited, and is not altered, transformed, or built upon in any way.

de Neufville *et al.* (2006). In this study, the uncertainty driving investment decisions is the future demand.

Cheah and Liu (2006) applied ROA to value the concessions of a government in a private sector Design, Build, Finance, Maintain and Operate (DBFMO) project. The authors demonstrated their approach using a case study on a causeway. The main uncertainty driver in this case study was traffic volume and growth. A toll road example is provided by Ford *et al.* (2002) to encourage wider use of ROA in construction projects. Chow and Regan (2011) integrated ROA in a network design optimization challenge with uncertain demand. Richardson *et al.* (2013) used ROA to determine optimized replacement cycles of heavy mobile equipment under volatile operational expenditures and long lead times of orders. Kim *et al.* (2017) applied ROA in order to value the potential damage reduction under optimized adaptation strategies and volatile future climate scenarios. Electricity demand and public acceptance is driving the uncertainty in a recent case study for nuclear power plant, investigated by Cardin *et al.* (2017).

The commonality in most case studies on ROA is a sole source of an external uncertainty such as weather conditions, operational expenses or demand, which may influence future costs and benefits.

A second observation in the case studies is a lack of consistency in the use of discount rates. Some authors use the weighted average cost of capital of an organization (WACC) with or without a risk premium. Other authors use a discount rate without explanation. Some authors use the concept of risk-neutral probabilities in combination with a risk-free discount rate on bonds, obtained from the ROA theory. These discrepancies demonstrate the major difficulties in applying ROA theory to real assets.

A third observation is the confusion regarding approaches to value flexibility: ROA and DTA. In this context, Neely and De Neufville (2001) introduce the term "hybrid real option valuation". The authors clearly separate non-diversifiable risks from diversifiable risks. There are no mitigating measures to avoid non-diversifiable risk. In contrast, diversifiable risk can be mitigated. ROA theory applies to non-diversifiable market price uncertainties, which need a different type of valuation approach than diversifiable asset uncertainties. Some ROA authors like Copeland and Antikarov (2003) even state that DTA is wrong. However, DTA is not wrong; the valuation method of options in a DTA should be aligned to the type of uncertainty involved. Schwartz and Trigeorgis (2001) sharply describe ROA as "a special and economically corrected version of DTA which recognizes market opportunities to trade and borrow". To avoid further confusion, De Neufville and Scholtes (2011) and Cardin *et al.* (2013) introduce the phrase "flexibility in engineering design" to designate that ROA is not the only approach to value real options. De Neufville and Scholtes (2011) state that for applying ROA, two conditions must be met: a replicating market portfolio with shares and bonds should exist and its compositions should be tradable. In other words, the market should contain a tradable portfolio of shares and bonds that exactly mimics the cash flows of the real asset or engineering project being considered. De Neufville and Scholtes (2011) conclude that these conditions seldom apply to real-life engineering assets or projects.

A fourth observation is that ROA case studies on *public* infrastructure assets, especially maintenance and replacement decisions, are hard to find. Woodward *et al.* (2014) incorporate real options to establish adaptive flood management strategies under an uncertain sea level rise. The study emphasizes that flexibility has value, which should be incorporated into the design and life cycle strategies of infrastructure assets. Herder *et al.* (2011) identify several barriers as to why ROA does not seem to gain a foothold in public infrastructure investment decisions. One reason mentioned by the authors is the increased difficulty for public sector organizations to find underlying comparable market information necessary for the correct valuation of real options. The most prominent barrier identified by the authors, is the political, institutional and organizational context in which public investment decisions are made. Decisions for large public infrastructure investments are seldom driven by economic reasons only. Infrastructure investments are often driven by societal and political interests. Investment decision may be influenced by anticipation to time consuming legal environmental impact assessments. Interesting is also the authors' observation that ROA may have problems of reputation, as a consequence of the financial crises in 2008/2009. Finally, the authors suggest that public organizations may face problems of lock-in. Endemic routines in combination with the absence of a ROA-toolkit could be a barrier for the application of ROA.

To summarize the literature: there is confusion and academic debate on the correct valuation of managerial flexibility in engineering practice. In addition, there is a lag in the application of valuing flexibility in public infrastructure decision making. The purpose of this study is to investigate the valuation of flexibility in public infrastructure replacement decisions and to

disentangle the debate on how to correctly value flexibility. In Section 2, a model is developed for the valuation of options in a case study for a common infrastructure replacement problem. A clear distinction is made between asset uncertainties and market price uncertainties. This case study reveals some of the prominent difficulties in valuing flexibility and especially in the application of ROA in public sector investment and replacement decisions. Three approaches are compared: valuing flexibility of options in the absence of price uncertainty (DTA), valuing flexibility of options subject to price uncertainty (ROA), and the application of the popular but not fully correct DT approach to ROA. Conclusions are presented in Section 4.

2. Model development for a bridge replacement

The case study is an existing old bridge in the city centre of the capital city of the Netherlands. The bridge was built before 1900. Around 1925, the bridge was expanded and reinforced. Around 1980, major renovation work took place. The case study is of interest because it is exemplary for many similar ageing bridges in an urban environment. Second, the case study contains multiple uncertainties of a different nature which need different treatment in a DTA and ROA. Third, the identified dominant uncertainties are difficult to quantify but influence the preservation of capital and the capital investment planning.

The purpose of this case study is to develop an infrastructure replacement optimization model that is capable of inclusion of different types of uncertainty and contains the flexibility to respond to these uncertainties. The approach or method development is generic; however, the case study and its underlying assumptions are specific.

Recently, mandatory structural safety investigations and calculations were carried out for this bridge according to national standards for structural safety assessments of existing structures (NEN 8700:2011 nl). This national standard builds on and is an extension to the well-known European reference design codes for new structures (NEN-EN 1990:2002 en). The assessment comprises structural integrity calculations based on strength-load combinations for an extended design life of 15 years and assesses for compliance to a safety limit state at disapproval level. In-depth field investigations were carried out to assess the load and bearing capacity of the structure and soil. Motorized traffic prognoses for the corridor of this bridge are stable (10,752 motorized vehicles per day in 2020) and show a slight decline towards 2030 with 10,393 motorized vehicles per day (Amsterdam 2018). The main concern is the strength of the piles. In Amsterdam (and other cities), bridges built around 1900 are founded on timber piles which are subject to bacteriological deterioration, a process well described by Klaassen (2008) and Klaassen and van Overeem (2012). This bacteriological decay of submerged timber piles is not only a Dutch problem (Nilsson and Björdal 2008, Zelada-Tumialan *et al.* 2014). As part of the structural safety assessment, under-water samples were taken from accessible wooden piles and analysed, resulting in predictions for bacteriological decay and residual strength. Also, site exploration was carried out to evaluate the load-bearing capacity of the soil.

The results of the field investigations were used to carry out the structural safety assessment calculations for a reference period of 15 years. The theoretical calculations demonstrate current compliance to the requirements of the standard (limit state for disapproval level). However, the main uncertainty is bacteriological decay of the piles and the development of the load-bearing capacity of the piles. Therefore, deformation of the bridge will be measured yearly to guarantee safe usage. In the case that a threshold for deformation is exceeded, the bridge will be closed for traffic and immediate replacement will be initiated.

We define a generic probability function $b(t)$ to model the time-variant probability of a premature unplanned replacement (corrective replacement) when a certain threshold is exceeded. For the case study, $b(t)$ is the probability that the measured deformation exceeds the permitted threshold. As a practical assumption for $b(t)$, first a current failure probability is estimated based on actual failures. Three ageing bridges out of an initial population of 160 similar bridges under investigation, were taken out of service in the past 2 years because thresholds were exceeded, which is an approximated 2%. As a conservative future estimate, a yearly additional percentage of 0.5% is added for the remaining reference period of 15 years. This approach can be seen as a managerial strategy of an asset manager to incorporate a certain level of additional risk costs as a consequence of a probable premature replacement. Naturally, such estimates and decisions are taken by a team of (field) experts.

Bridge failure probability modelling under specific circumstances is clearly more complex and a research field on its own. Sańchez-Silva and Klutke (2016) provide a comprehensive overview of fundamental and state of the art probabilistic degradation models for

infrastructure ranging from regression analysis to modelling of degradation caused by shocks. In addition, a time-variant capacity-load approach could be considered (Leira 2013). However, these mathematical models need data to validate their statistical properties in order to establish a time-variant probability for the remaining life time of a structure. Data to perform such modelling is currently absent for the case study and will only become available in the future.

That certainly does not impede the method development in the current research which aims to develop strategies and budget forecasts under uncertain conditions. Dealing with uncertainty is discussed in Section 3. One of the uncertainty reduction approaches is data gathering and reflective learning. As soon as better information becomes available, the model is easily updated.

The consequence of exceeding the deformation threshold is an immediate replacement with investment costs that are a factor of 1.5 higher than a preventive replacement. This factor is based on experience within the municipality.

A second uncertainty is about urban planning. At present, cars are allowed in the city centre but banning cars from the city centre is currently a hot political issue in the city council. Its success depends on the composition of the city council and elections take place every 4 years. Other cities may face other types of political decisions. Political decisions are uncertain and difficult to predict. The probability of this decision in one of these years is designated with $p(t)$. For the case study, its value is based on the expected probability that one of the parties will grow. One could argue that these types of events are highly uncertain. However, these events are part of real-life uncertainties that a decision maker faces and addressed in discussions about infrastructure replacement planning. A decision to ban cars from the city centre would offer the opportunity to build a smaller bridge, which would significantly reduce the future life cycle costs (investment and maintenance costs). Operational expenses of the current and new bridges are estimated at 10% of their new investment costs based on past data. Periodic major overhauls (intermediate large maintenance works such as asphalt replacement and conservation work) are included in this figure. The last uncertainty is the development of construction costs. Exogenous market forces may influence future construction costs.

The extreme scenarios are to replace the bridge immediately with a large bridge or wait for another 15 years and build a large or small bridge depending on political decisions to ban cars from the city centre. There are two clear incentives for waiting: the benefits of postponing large investments and waiting for more information that allows for the building of a smaller bridge. There is one incentive for not-waiting: the potential risk costs, more specifically an unexpected corrective replacement. The question of interest is: what is the best strategy for this municipality and what is the value of waiting for information?

A model is developed in two stages for this case study. The first stage is described as the DT approach, which omits market price uncertainty. Hereafter the model is extended with market price uncertainty for which the ROA approach is incorporated. Finally, the incorrect application of the ROA theory (the DT approach to ROA) is applied to evaluate the deviations. The input data for the case study are presented in Table 1. Planned and unplanned replacements are designated with preventive and corrective replacements respectively. This is common terminology in maintenance engineering and allows for using the indices p and c for preventive (planned) and corrective (unplanned) replacements. The index u is reserved for an up-move in the ROA-modelling in Section 2.2.

2.1. Valuing flexibility in the absence of price uncertainty

Looking at this question from a decision tree perspective first requires identifying the two events that influence decisions. The first event is that cars are allowed in the city centre. This situation is designated as state large (L). The second event is that cars are not allowed in the city centre. This event is designated as state small (S). The current situation is state L. A decision maker cannot influence the events, but can wait and base future decisions on the outcome of political decisions made in the Years 4, 8 and 12. In these years, a transition from state L to S is possible with a probability of $p(t)$. A transition from state L to S does not automatically imply that a decision maker will build a smaller bridge immediately. A decision maker will maximize the value of potential decisions and chooses the best option from the range of options available. A political decision to ban cars from the city centre, is considered to be irreversible for the coming decades.

The decision nodes (not yet the decisions) and possible transitions from state L to state S are shown in Figure 1. For example: in Year 4, there is a probability of $p(t)$ for entering state S and a probability of $1 - p(t)$ for remaining in state L. Once in state S there

Table 1. Data with descriptions, symbols and values (monetary amounts in million €).

Description	Symbol	Value
Investment in a preventive replacement with a large bridge	I_L^P	5
Investment in a corrective replacement with a large bridge	I_L^C	$1.5 \cdot I_L^P = 7.5$
Investment in a preventive replacement with a small bridge	I_S^P	$0.6 \cdot I_L^P = 3$
Investment in a corrective replacement with a small bridge	I_S^C	$1.5 \cdot 0.6 \cdot I_L^P = 4.5$
Yearly operational expenses of large bridges	E_L	$0.1 \cdot I_L^P = 7.5$
Yearly operational expenses of small bridges	E_S	$0.1 \cdot I_S^P = 0.3$
Present value of a perpetuity of investments and operational expenses of a preventive and corrective replacement with a large bridge	L^P and L^C	See Table 2
Present value of a perpetuity of investments and operational expenses of a preventive and corrective replacement with a small bridge	S^P and S^C	See Table 2
Probability each year for a corrective replacement	$b(t)$	$2\%+0.5\% \cdot t \ \forall \ t$
Probability for the decision to ban cars from the city centre	$p(t)$	30% *if* $t = 4,\ 8,\ 12,$ *otherwise* 0
Maximum allowable life time extension of old bridge	T	15 *years*
Time/period to evaluate in years	t	$0 \leq t \leq T$
Technical life cycle of a new bridge	n	100 *years*
Risk-adjusted discount rate of the municipality	r_a	3.5%

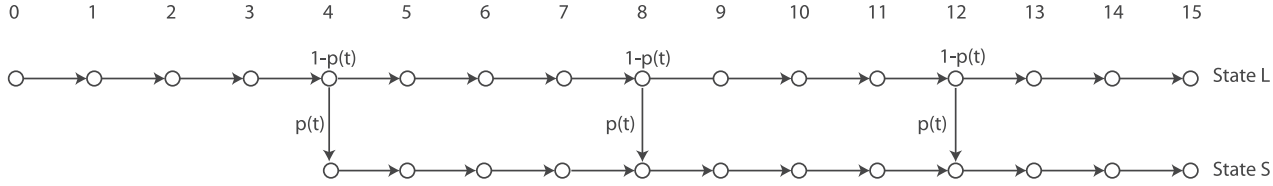


Figure 1. Decision nodes for the case study with recombined branches.

is no possibility of transferring to state *L*. Figure 1 is a recombined version of an extended tree. Recombining enhances the efficiency of following recursive calculations and makes the interpretation of results easier. The process of recombining decision trees is well described by Guthrie (2009).

Having identified state *S* and *L* and the possible decision nodes, the decision tree is built. Figure 2 shows the options in a decision node of state L and S. Figure 3 shows the complete decision tree. The options in any decision node in state *S* and *L* are defined as:

$$A = \{wait, replace\} \tag{1}$$

In decision nodes of a large state *L*, a decision maker is faced with two options each year, except for the last year. These are illustrated in Figure 2. A decision node is represented by a non-filled dot. The first option is to replace the old bridge with a new one. This is a preventive replacement and requires a large rebuild including all future life cycle costs L^P. The second option is to wait. Waiting comprises the probability of an unplanned (corrective) large investment $b(t)L^C$ and the benefits of postponing the investment with a probability of $1 - b(t)$, represented by *W*. The chance node is designated with a square in Figure 2. In the last year, the only option left is to replace the old bridge with a large one: L^P. After a replacement (L^P or L^C) in any of the nodes the decisions are terminated by a perpetuity of future life cycle costs (investments and exploitation expenditures). Termination nodes are represented by a black filled dot in Figure 2 and Figure 3. In every node of the decision tree, there is a possibility that the decision tree ends, as a consequence of a planned replacement or unplanned replacement.

In decision nodes of a small state *S*, cars are not allowed in the city centre. This creates an opportunity for a less expensive replacement with a small bridge. Again, a decision maker has two options as depicted in Figure 2: preventively replace the old bridge with a small one, including all future life cycle costs S^P or

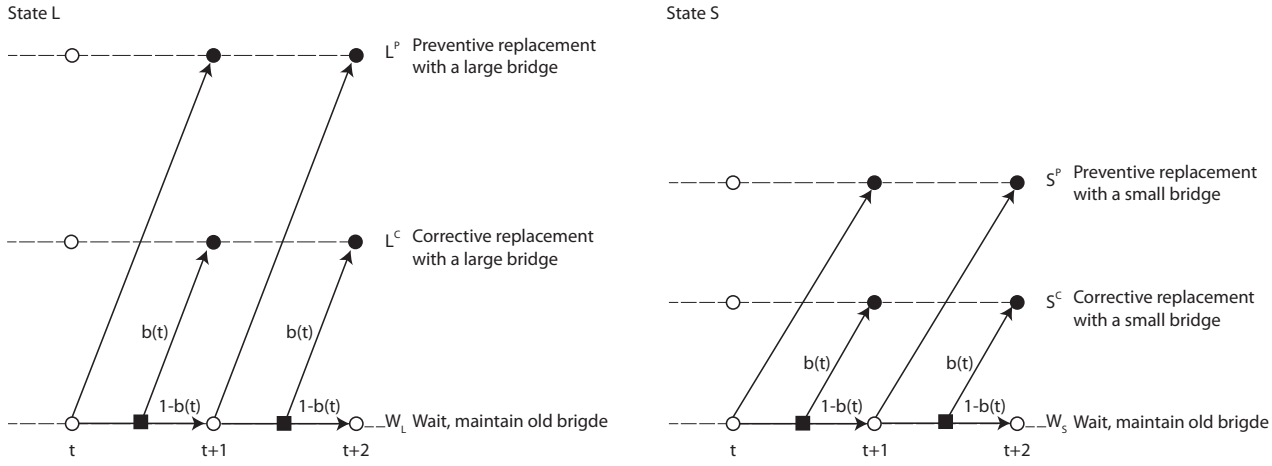


Figure 2. Decisions available in a large and small state.

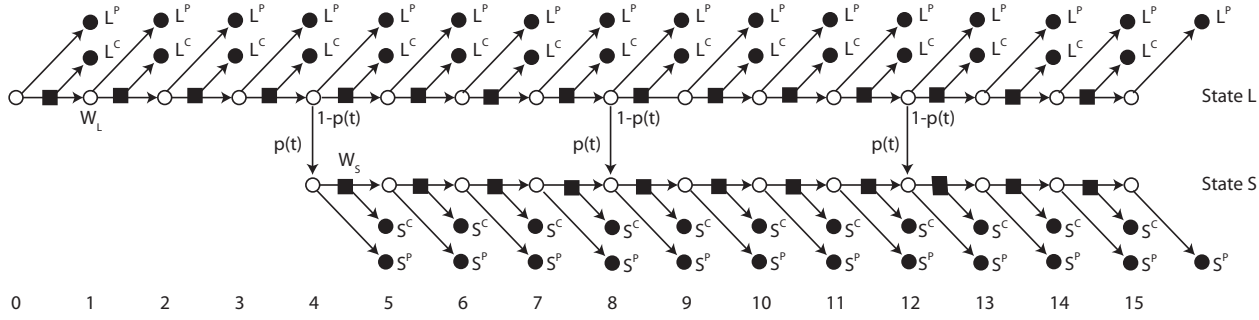


Figure 3. Full decision tree for the case study.

Table 2. Calculation of boundary conditions: discounted perpetual future life cycle costs without price uncertainty.

	Symbol for perpetuity of replacements	Life cycle, n (years)	Investment (x million €)	Yearly operational expenses (x million €)	Present value of perpetuity of replacements $V_I + V_E$ (x million €)
Perpetual preventive replacement of a large bridge	L^P	100	$I_L^P = 5$	$E_L = 10\% \times I_L^P = 0.5$	$5.17 + 14.29 = 19.45$
Perpetual corrective replacement of a large bridge	L^C	100	$I_L^C = 1.5 \cdot I_L^P = 7.5$	$E_L = 10\% \times I_L^P = 0.5$	$7.75 + 14.29 = 21.95$
Perpetual preventive replacement of a small bridge	S^P	100	$I_S^P = 0.6 \cdot I_L^P = 3$	$E_S = 10\% \times I_S^P = 0.3$	$3.10 + 8.57 = 11.67$
Perpetual corrective replacement of a small bridge	S^C	100	$I_S^C = 1.5 \cdot 0.6 \cdot I_L^P = 4.5$	$E_S = 10\% \times I_S^P = 0.3$	$4.65 + 8.57 = 13.17$

wait. Waiting encompasses a probability $b(t)$ for a corrective replacement with a small bridge S^C, and the benefits of postponing the investment W with a probability of $1 - b(t)$. If the old bridge remains in place, the only option left at the expiration date of postponement is a preventive replacement with a small bridge S^P.

The decision tree for the case study combines Figures 1 and 2 and is presented in Figure 3. The discounted value of perpetual life cycle costs of replacements L^P, L^C, S^P and S^C are determined after definition of the model and presented in Table 2.

The decision tree is solved by using backward recursion for discounting of costs. Knowing the boundary constraints at the year in which the possibility of postponement of a replacement expires (year T), allows for working back year by year until the present value at $t = 0$ is found. A clear and simplified example of this principle of backward recursion in a decision tree that contains options and probabilities is provided by Brealey *et al.* (2017). The boundary constraints at T for the two states are:

$$V_L(T) = L^P \tag{2}$$

$$V_S(T) = S^P \tag{3}$$

where, $V_L(T)$ and $V_S(T)$ are the discounted values in year T of the cash flows from Year 15 to infinity for a large and small state, respectively. The values are calculated in Table 2. If the existing bridge still functions at the end of Year 15, the only decision left is to replace the bridge with costs L^P or S^P, depending on the state.

The present value of a waiting option in a small state $Q_S(t, wait)$ in a decision node at time t is given by the recursive relationship:

$$Q_S(t, wait) = b(t)S^C + (1-b(t)) \cdot \left(E_L + \frac{V_S(t+1)}{1+r_a}\right) \quad (4)$$

The waiting option comprises three cost components: (i) the more expensive corrective replacement, including all future life cycle costs, with a probability $b(t)$ resulting in costs: $b(t)S^C$, (ii) the yearly regular maintenance costs of the existing bridge with a probability $(1-b(t))$ resulting in costs $(1-b(t))E_L$ and (iii), the discounted value of all future cash flows with a probability of $(1-b(t))$ resulting in costs: $((1-b(t)) \cdot V_S(t+1)/(1+r_a)$. The risk costs $b(t)S^C$ and operational expenses $(1-b(t))E_L$ are considered to be incurred at the time of the decision to wait, as a principle of prudence. It is also justified to incur these costs in the middle or end of the year, and discount them appropriately.

The present value of a preventive replacement in a small state in a decision node at time t is:

$$Q_S(t, replace) = S^P \quad (5)$$

The objective in each decision node in a small state becomes:

$$V_S(t) = \min_{a \in A} Q_S(t, a) \quad \forall 4 \leq t \leq 15 \quad (6)$$

$V_S(t)$ represents the discounted life cycle costs in a small state, in Year t under optimal decisions. This equation minimizes the discounted costs of a preventive investment in a small bridge (S^P), including all future life cycle costs and the discounted costs of waiting. As an example, the cash flows of recursive Equation (6), to be evaluated at $t = T-1$, are graphically depicted in Figure 4.

Similarly, the recursive relationships in the case of a large state are:

$$Q_L(t, wait) = b(t)L^C + (1-b(t)) \cdot \left(E_L + \frac{V_L(t+1)}{1+r_a}\right) \quad (7)$$

$$Q_L(t, replace) = L^P \quad (8)$$

The objective function for a large state must incorporate the possibility of transition from a large state to a small state and becomes:

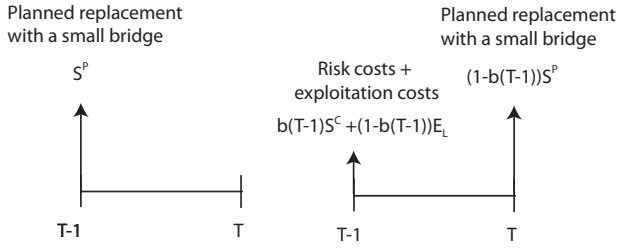


Figure 4. Cash flows of Equation (6) to be evaluated at year T-1: replace or wait.

$$V_L(t) = (1-p(t)) \min_{a \in A} Q_L(t, a) \quad p(t) \min_{a \in A} Q_S(t, a) \quad \forall 0 \leq t \leq 15 \quad (9)$$

For example, at the end of Year 12, the decision maker will know the outcome of the political decision to ban cars from the city centre in that year. From today's point of view, the decision maker will evaluate the cash flows in a small and large state with probabilities of respectively $p(t)$ and $1-p(t)$. For solving the recursive relationships, the boundary conditions L^P, L^C, S^P and S^C first need to be determined.

2.1.1. Boundary conditions without price increases

The value in a termination node is estimated by the discounted value of all future replacements and exploitation expenditures. Therefore, the boundary conditions L^P, L^C, S^P and S^C are calculated based on a combination of traditional discounted cash flow formula for the perpetuity of replacements and the perpetuity of operational expenses. The generalized present value of a perpetuity of identical investment costs is calculated as:

$$V_I = I \cdot \left(1 + \left(\frac{1}{1+r_a}\right)^n + \left(\frac{1}{1+r_a}\right)^{2n} + \left(\frac{1}{1+r_a}\right)^{3n} + \ldots\right) = I \cdot \frac{1}{1-\left(\frac{1}{1+r_a}\right)^n} \quad (10)$$

where, n is the interval of the replacement cycles and r_a is the risk-adjusted discount rate required by the organization.

The generalized present value of the perpetuity of identical yearly operational expenses follows from the well-known capitalized equivalent worth relationship (Park 2011):

$$V_E = E \cdot \frac{1}{r_a} \quad (11)$$

In which E are the yearly operational expenses. When yearly operational expenses are not constant as

Table 3. Results of the present value calculations (× million €) for the case study without price uncertainty.

Year	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Option value																
$V_L(t)$	15.9	15.9	15.7	15.6	15.3	16.8	16.6	16.4	16.1	17.8	17.6	17.4	17.1	19.5	19.5	19.5
$V_S(t)$					11.7	11.7	11.7	11.7	11.7	11.7	11.7	11.7	11.7	11.7	11.7	11.7
Strategy																
Large	W/L^C	W/L^C	W/L^C	W/L^C	W/L^C	W/L^C	W/L^C	W/L^C	W/L^C	W/L^C	W/L^C	W/L^C	L^P	L^P	L^P	L^P
Small					S^P	S^P	S^P	S^P	S^P	S^P	S^P	S^P	S^P	S^P	S^P	S^P

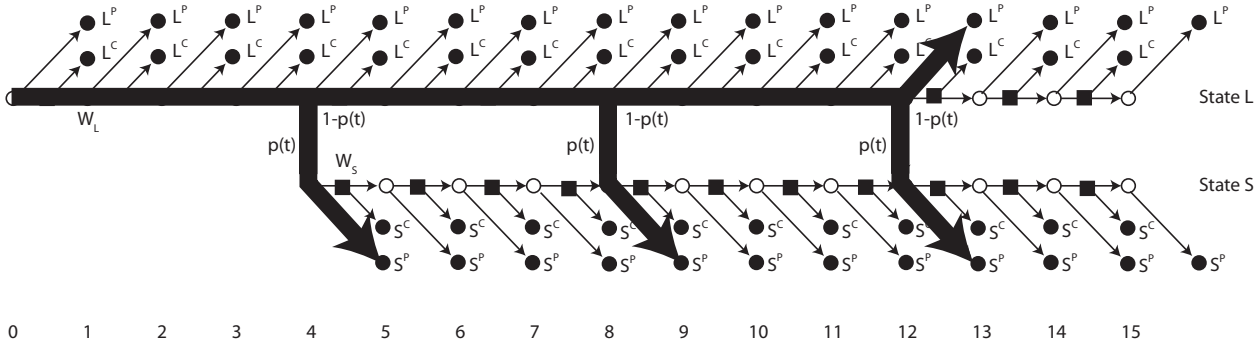


Figure 5. Optimized paths for case study subject to structural integrity and political decisions.

a consequence of major overhauls or ageing, the life cycle costs should first be translated into (constant) equivalent annual costs (EAC) over the life cycle of the asset by means of the discounted cash flow annuity factor.

Combining V_I and V_E delivers the total present value for a perpetuity of preventive replacements at any time because no price increases are involved yet (this will be done in Section 2.2). A small correction is required for calculating the present value of perpetual replacements, which start with a more expensive corrective replacement. In this case, under the assumption that subsequent future replacements will be planned, the difference between a preventive and corrective investment needs to be added to Equation (10). The calculations for the present values of the perpetuities L^P, L^C, S^P and S^C are presented in Table 2.

2.1.2. Results of DTA

With the values for the boundary conditions L^P, L^C, S^P and S^C, the recursive relationships (Equations (6) and (9)) are solved. The results for the case study are presented in Table 3. The optimized strategy is depicted in the bottom part of Table 3 and in Figure 5. The strategy follows directly from the minimum options chosen in the recursive relationships at decision nodes. The strategy in the bottom part of Table 3 is read from left to right.

In the first 4 years, cars will be allowed in the city centre. The strategy here is to wait and to accept the potential risk of a corrective large replacement (W/L^C). At the end of Year 4, the politics may decide to ban cars from the city centre. In that case, the strategy is to replace the old bridge immediately with a preventive small replacement (S^P) at which the options end. The explanation here is that the risk costs of a corrective small replacement, the operational expenses of the old bridge minus the benefits of postponement from Year 4 to 5 including future options, exceed the costs of an immediate small replacement. In contrast, if the political decisions taken at the end of Year 4 decide not to ban cars from the city centre, the best strategy is to wait. Here, the benefits of delaying the preventive replacement of a large bridge outweigh the costs of such a replacement.

The best strategies for all possible scenarios are derived from the bottom part of Table 3. Assume that cars are banned from the city centre at Year 8 or Year 12, respectively, then the best strategy for the case study is a small preventive replacement at the end of Year 8 or 12 and to accept the probability of an earlier corrective large replacement. If cars are not banned from the city centre in Year 8, then the best strategy is to wait and accept the probability of an earlier corrective large replacement. If cars are not banned from the city centre in Year 12, then the best strategy is a preventive replacement for a large bridge. This is because the increasing risk costs do not allow a further postponement of the replacement.

It is obvious that the outcome of the calculations and the strategy depends on the ratio between the

cost components and the risk function. The presented model allows for easy adaptation of input variables.

Although, backward recursion is hardly ever applied in present value calculations, the advantage of this approach is twofold. First, only a few calculations are required to calculate the expected present value of all possible scenarios. Second, the backward recursion provides the decision maker with a strategy.

2.2. Valuing flexibility subject to market price uncertainty: ROA approach

Technical and political uncertainties are considered in the previous section. In this section, market price uncertainty is incorporated into the model. Market price uncertainty is treated differently than asset uncertainty because it is a non-diversifiable risk (Cox *et al.* 1979, Neely and de Neufville 2001). For the correct valuation of options under market price uncertainty, a risk-adjustment of the discount rate or cash flows is required. There are two approaches to obtain this information from the market. The first is known as the replicating portfolio approach and the second, the risk-neutral probability approach and both deliver the same results.

The replicating portfolio approach directly obtains a risk-adjusted discount rate r_m from the market and uses actual probabilities for up and down moves of market prices. The equivalent risk-neutral probability approach obtains a risk-free discount rate r_f from the market and corrects the up and down moves of market prices with so-called risk-neutral probabilities.

Risk-neutral probabilities have no physical meaning. It is a theoretical concept that allows for discounting with a risk-free discount rate instead of a risk-adjusted discount rate. The advantage of using risk-neutral probabilities is that the risk-free discount rate can directly be observed in the market when options are to be priced over multiple periods. In contrast, the market risk-adjusted discount rate r_m will change in each period when asymmetric option payoffs are introduced. Options are asymmetric when their present values are not a common multiple of a traded security that mimics the option payoffs. Therefore, the risk-neutral probability approach often has computationally advantages over the replicating portfolio approach (Copeland and Antikarov 2003). The risk-neutral probabilities for the case study are defined after the formulation of the mathematical model.

For correct valuation, two important assumptions underlie the ROA theory: the payoffs of a project are spanned by traded securities in the market (called a twin security or spanning asset) and arbitrage profits do not exist. The latter means that the market is efficient and financial assets are always correctly priced. There are no possibilities for investors to achieve quick wins by exploiting price differences between similar financial assets (Cox *et al.* 1979, Guthrie 2009, De Neufville and Scholtes 2011). The validity of these assumptions for many public infrastructure projects is questioned by authors including De Neufville and Scholtes (2011) and Herder *et al.* (2011). Other authors like Copeland and Antikarov (2003) and Mun (2006) argue that the market will always contain replicating traded securities, even if they are not easy to find.

For the case study, construction prices per unit (X) are identified as the market state variable or spanning asset. All cost components in the case study are a common multiple of the initial construction costs (Tables 1 and 2). This leads to the special situation of symmetrical option payoffs for which a constant risk-adjusted discount rate r_m applies. The case study, however, will use and demonstrate the more generic risk-neutral probability approach. Results have been verified with the replicating portfolio approach.

The Central Bureau for Statistics (CBS) in the Netherlands publishes historical quarterly data on construction costs for bridges and tunnels (CBS, 2017). The data are calculated based on a compiled bundle that contains labour, materials and equipment. The case study assumes that this bundle suffices as the spanning asset and that somewhere in the market a tradable replicating portfolio with this spanning asset and risk-free bonds can be found.

A common convention used to estimate market prices development is the assumption of a geometric Brownian Motion (GBM). A GBM assumes that the natural logarithm of the price X follows a random walk with an annualized drift μ and volatility σ:

$$X_{t+1} = X_t \cdot \exp\left(\mu \Delta t_m + \sigma \sqrt{\Delta t_m} \cdot \varepsilon\right) \tag{12}$$

Δt_m is the proportional time step used in the calculation for the future price development and is 1 for yearly time steps (Δt_m would be 0.25 if quarterly time steps were used). $\varepsilon \sim N(0,1)$ is a shock, which is normally distributed with a mean of zero and a standard deviation of 1.

The drift μ and volatility σ are obtained from the mean and standard deviation of historical log price differences (equal to the log of the returns). Analysing the quarterly data of construction prices in the Netherlands from 2000 until 2017 leads to an annualized drift of 0.015 and volatility of 0.027. Based on these data, some possible scenarios for the price

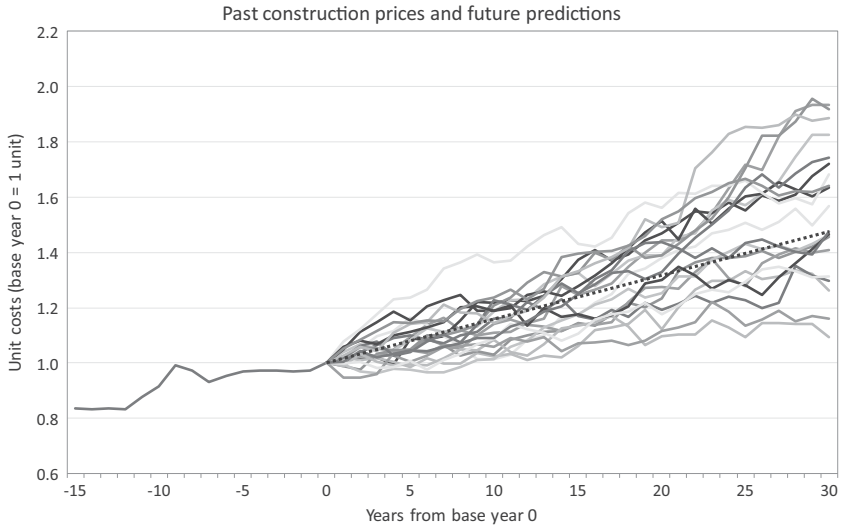


Figure 6. Possible scenarios for the price development of construction costs assuming a GBM with an annualized drift of 0.015 and volatility 0.027. The dotted line expresses the drift and expected value.

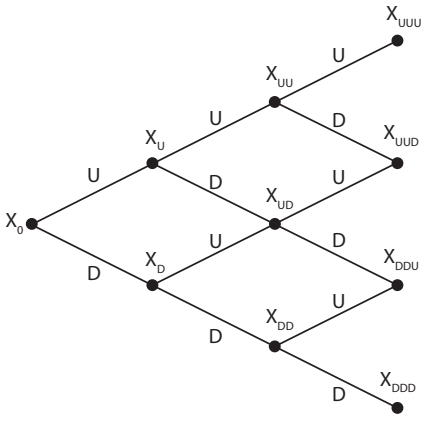


Figure 7. Binominal lattice with up moves *U* and down moves *D*.

development of construction costs per unit are depicted in Figure 6.

A GBM is a continuous stochastic process, which can be converted into a discreet simulation process in the form of a recombining binominal lattice with up moves *U* and down moves *D* (Figure 7). The size of an up move *U* is calculated as:

$$U = \exp\left(\sigma\sqrt{\Delta t_m}\right) \tag{13}$$

For a recombining lattice, the size of a down move *D* must satisfy:

$$D = \frac{1}{U} \tag{14}$$

Using Equations (13) and (14), the discrete simulation of the market state variable $X(i,t)$ of 1 unit of construction costs is presented in Appendix Table A1. The index i represents the number of down moves D and t represents the time. For example, and with reference to Figure 7: $X_{UU} = X(0,2)$ and $X_{DDU} = X(2,\ 3)$.

Equation (15) provides a direct relationship to calculate $X(i,t)$. This relation will be used after development of the model to derive equations for the new boundary constraints.

$$X(i,t) = X(0,0) \cdot \exp\left((t-2i)\sigma\sqrt{\Delta t_m}\right) \tag{15}$$

The next step is the inclusion of market price uncertainty in the model of the case study. Market price uncertainty affects all cost components which now becomes a function of the number of down moves i and time t.

The adjusted boundary constraints at expiration year T are:

$$V_S(i,T) = S^P(i,T) \tag{16}$$

$$V_L(i,T) = L^P(i,T) \tag{17}$$

The discounted value of a waiting option at node (i,t) in a small state is calculated as under the new circumstances:

$$Q_S(i,t,wait) = b(t)S^C(i,t) + (1-b(t)) \cdot \left(E_L(i,t) + \frac{\eta_U V_S(i,t+1) + (1-\eta_U)V_S(i+1,t+1)}{1+r_f}\right) \tag{18}$$

In this equation, η_U and r_f represent a risk-neutral probability for an up movement and a risk-free discount rate respectively, which will be further explored after defining the model. The symbol i represents the number of down moves.

The discounted value of a preventive replacement at node (i,t) in a small state becomes:

$$Q_S(i,t,replace) = S^P(i,t) \tag{19}$$

The objective in each decision node in a small state now reads:

$$V_S(i,t) = \min_{a\in A} Q_S(i,t,a) \quad \forall 4 \le t \le 15 \tag{20}$$

Similar, the discounted value of a waiting option at node (i,t) in a large state becomes:

$$Q_L(i,t,wait) = b(t)L^C(i,t) + (1-b(t)) \cdot \left(E_L(i,t) + \frac{\eta_U V_L(i,t+1) + (1-\eta_U)V_L(i+1,t+1)}{1+r_f}\right) \tag{21}$$

The discounted value of a preventive replacement at node (i,t) in a large state becomes:

$$Q_L(i,t,replace) = L^P(i,t) \tag{22}$$

The objective function of a decision node in a large state needs to incorporate the probability of transferring from a large state to a small state. Recall that $p(t)$ is only > 0 for nodes 4, 8 and 12. The generic objective function for decision nodes in state L becomes:

$$V_L(i,t) = p(t)\min_{a\in A} Q_S(i,t,a) + (1-p(t))\min_{a\in A} Q_L(i,t,a) \tag{23}$$

The inclusion of market uncertainty quickly complicates the model. First, it significantly increases the number of calculations as a consequence of considered up and down moves. Second, the estimated fluctuations of one unit of investment costs still needs to be converted to the present values of the future life cycle costs (perpetuities) of preventive and corrective replacements at all nodes (i,t). The boundary conditions calculated in Table 2, need to be adjusted to incorporate price increases. Third, motivated estimations are required for the upward risk-neutral probability η_U and the risk-free discount rate r_f.

We begin with the estimations for the risk-neutral probabilities and market risk-free discount rate. The risk-neutral probability approach requires an estimation of a risk-free interest rate. The risk-free interest rate is a secure bond that serves as a standard for pricing other risky assets. Since 2009, the financial crisis has caused risk-free interest rates to decline rapidly. The current short term risk-free interest rate is close to zero in the Euro-zone. This poses a problem for the application of ROA and other economic instruments to value derivatives (Hull and White 2013, ECB 2014, 2017, Frankema 2016). This is an ongoing debate between econometrists and beyond the scope of this study, which intends to apply ROA theory in engineering practice. In accordance with the current policy of the ECB, the long-term risk-free interest rate for the case study is estimated from the Euro area yield curve that contains the long-term structure of the interest rates of AAA-rated Euro area central government bonds. Based on the instantaneous forward Euro area yield curve, an average risk-free interest rate of 0.8% is estimated for the case study.

The next step is to estimate the risk-neutral probabilities. In the absence of dividends payment or a market risk premium and systematic market risks for holding the spanning assets, the risk-neutral probability of an up move is derived by Cox *et al.* (1979) as:

$$\eta_U = \frac{(1+r_f)-D}{U-D} \tag{24}$$

In the case study, market risk premiums and systematic market risks are present and cannot be ignored. The Dutch National Task Force for Discount Rates advises that for these types of investments (for costs and benefits) use a 3% market risk premium and a β-coefficient of 1 [Werkgroep Discontovoet (National Taskforce for the Societal Discount Rate), 2015]. A β-coefficient accounts for the systematic market risk. A β of 1 implies that the net benefits (benefits minus costs) of the project move along with the economy. Although benefits are not included in the case description, benefits are present. The case study just assumes that the different alternatives have equal societal benefits whether it will be a large or small bridge. It is hard to differentiate between social benefits in the case study. If cars are banned from the city centre, there will probably be benefits to tourism, the local economy (more restaurants, cafés) and improvements in air quality. If cars are not banned from the city centre, the benefits are found in better accessibility and probably shorter travel times for motorized traffic.

Guthrie (2009) provides an approach to address the market's attitude by incorporating a market risk premium and β-coefficient in the dividend free risk-neutral probability Equation (24):

$$\eta_U = \frac{\mathrm{K}-\mathrm{D}}{U-D} \tag{25}$$

with:

$$K = \left(\frac{\phi_U X_U + \phi_D X_D}{X}\right) - (\text{market risk premium}) \cdot \beta \tag{26}$$

The first term between brackets on the right side of the equation is the expected return on the state variable using the actual probabilities of up and down moves: φ_U and φ_D.

To determine K, first the actual probability of an up move is calculated based on the observed data. Cox *et al.* (1979) recommend to estimate φ_U as:

$$\phi_U = \frac{1}{2} + \frac{1}{2}\frac{\mu\sqrt{\Delta t_m}}{\sigma} \tag{27}$$

The mean μ and volatility σ were already obtained from analysing the historical data of the construction prices. Knowing that $\varphi_D = 1 - \varphi_U$, allows for calculating the risk-adjusted growth factor K and risk-neutral probabilities η_U, and $\eta_D = 1 - \eta_U$.

To complete the analysis, the one period risk-adjusted discount rate r_m for the state variable can be derived from equivalence relationship between the risk-neutral probability approach and the replicating portfolio approach. The equivalence between a market risk-adjusted discount rate r_m, the actual probabilities of up and down moves and the risk-free interest rate and risk-neutral probabilities in ROA valuation is in its simplest form, for a one-time period, represented by (Cox *et al.* 1979, Copeland and Antikarov 2003, Guthrie 2009):

$$V_0 = \frac{\eta_U \cdot V_U + \eta_D \cdot V_D}{1 + r_f} = \frac{\phi_U \cdot V_U + \phi_D \cdot V_D}{1 + r_m} \tag{28}$$

where, V_0 = a generic option value, V_U = option payoff after one period in an up-state and V_D = option payoff after one period in a down-state. The risk-adjusted discount rate r_m for one period follows from Equation (28):

$$r_m = \left(\frac{\phi_U V_U + \phi_D V_D}{V_0}\right) - 1 \tag{29}$$

To find r_m for the first timestep of the case study (and the following because of the special situation of symmetrical option payoffs), V_U is substituted with $X(0,1)$, V_D is substituted with $X(1,\ 1)$ and V_0 is defined by Equation (28). The intermediate calculations for the market variables are presented in Table 4. At this point, it is interesting to notice that not compensating for the market risk premium and systematic market risk would result in a risk-neutral probability for an up move of $\eta_U = 0.643$ (Equation (24)) and a risk-adjusted discount rate r_m of 1.6% (Equation (29)). The latter is far below even the risk-adjusted discount rate of the municipality of 3.5% and would not provide a realistic result. This observation demonstrates the difficulty in correctly estimating market variables in an infrastructure case study.

2.2.1. Boundary conditions under price increases

As a result of the price uncertainty of the development of construction costs, the boundary conditions for the case study change. The case study needs values for a perpetual stream of life cycle costs indexed by (i,t) for the four types of replacements. Four additional tables like Appendix Table A1 need to be constructed for $L^P(i,t)$, $L^C(i,t)$, $S^P(i,t)$ and $S^C(i,t)$.

Under the assumption that the yearly operational expenses $E(i,t)$ after replacement remain a fraction of the initial construction costs and are subject to the same uncertainty as the state variable $X(i,t)$, the expected discounted value at (i,t) of a continuation of operational expenses with growth rate g and a

Table 4. Intermediate calculations of market variables.

Symbol	Value	Description	Source
X(0,0)	1	State variable: one unit of construction costs	–
σ	0.027	Annualized volatility of observed historical price data of construction costs	Data analysis
μ	0.015	Annualized mean of observed historical price data of construction costs	Data analysis
U	1.027	One up move of the state variable	Equation (13)
D	0.974	One down move of the state variable	Equation (14)
φ_U	0.789	Actual probability of an up move	Equation (27)
φ_D	0.211	Actual probability of a down move	1 – Equation (27)
mrp	3%	Market risk premium	Data
β	1	Coefficient for systematic market risk	Data
K	0.986	Risk-adjusted growth factor	Equation (26)
η_U	0.228	Risk-neutral probability of an up move	Equation (25)
η_D	0.772	Risk-neutral probability of a down move	1 – Equation (25)
r_f	0.8%	Risk-free interest rate	Data
r_m	0.039	Risk-adjusted discount rate	Equation (29)

risk-adjusted discount rate r_m, is in generalized form given by:

$$E[V_E(i,t)] = E(i,t) \cdot \frac{1+g}{r_m - g} \tag{30}$$

This formula is derived from a standard discounted cash flow gradient annuity factor in which we allow n to approach infinity (Park 2011, Sullivan *et al.* 2012). The actual expected annual growth rate g is 0.0159 and follows from:

$$(1+g) = \frac{\phi_U X_U + \phi_D X_D}{X} \tag{31}$$

As a consequence of the assumptions mentioned above, its equivalent relationship in the risk-neutral world reads as (Guthrie 2009):

$$E[V_E(i,t)] = E(i,t) \cdot \frac{K}{R_f - K} \tag{32}$$

where, $R_f = 1 + r_f$.

In generalized form, the discounted value of a perpetuity of repeating risky replacement costs $I(i,t)$ with interval n at (i,t) is derived as:

$$E[V_I(i,t)] = I(i,t) \cdot \left(1 + \left(\frac{1+g}{1+r_m}\right)^n + \left(\frac{1+g}{1+r_m}\right)^{2n} + \left(\frac{1+g}{1+r_m}\right)^{3n} + \dots\right) = I(i,t)\frac{1}{1 - \left(\frac{1+g}{1+r_m}\right)^n} \tag{33}$$

Due to the above-mentioned assumptions (all cost elements are proportional to the state variable $X(i,t)$), its risk-neutral equivalent expression is (Guthrie 2009):

$$E[V_I(i,t)] = I(i,t)\frac{1}{1 - \left(\frac{K}{R_f}\right)^n} \tag{34}$$

Again, a small correction for the one-time occurrence of a more expensive corrected replacement needs to be made. To be accurate, the difference in investment costs between a preventive and corrective replacement needs to be added to Equation (33) or (34) to calculate the perpetuity of a corrective replacement, followed by future preventive replacements.

The discounted expected values capture all probable future cash flows for each i and t. This has been verified by an alternative calculation in which the actual or risk-neutral probabilities throughout the binominal lattice are used to calculate the expected values of the cash flow at Year t and discounting these values to the present with r_m or r_f depending on the probabilities used (an equivalent but more time-consuming calculation).

Table 5. Input data for case-specific boundary conditions subject to price uncertainty. The proportional values k are derived from Table 1.

$k_1 = 5$
$k_2 = 1.5$
$k_3 = 0.6$
$k_4 = 0.1$
$I_L^P(i,t) = k_1 \cdot X(i,t)$
$I_L^C(i,t) = k_2 \cdot I_L^P(i,t) = k_1 \cdot k_2 \cdot X(i,t)$
$I_S^P(i,t) = k_3 \cdot I_L^P(i,t) = k_1 \cdot k_3 \cdot X(i,t)$
$I_S^C(i,t) = k_2 \cdot I_S^P(i,t) = k_1 \cdot k_2 \cdot k_3 \cdot X(i,t)$
$E_L(i,t) = k_4 \cdot I_L^P = k_1 \cdot k_4 \cdot X(i,t)$
$E_S(i,t) = k_4 \cdot I_S^P = k_1 \cdot k_3 \cdot k_4 \cdot X(i,t)$

Combining the relationships (Equations (15, 32 and 34)), correcting them for proportional fractions of cost components (Table 1) to the state variable $X(i,n)$ as expressed in Table 5 and rewriting the relationships, results in direct equations for the case-specific boundary conditions underprice uncertainty.

$$L^P(i,t) = k_1 \cdot X(0,0)\exp[(t-2i)\sigma] \cdot \left[\frac{1}{1 - \left(\frac{K}{R_f}\right)^n} + k_4 \cdot \frac{K}{R_f - K}\right] \tag{35}$$

$$L^C(i,t) = k_1 \cdot X(0,0)\exp[(t-2i)\sigma] \cdot \left[(k_2 - 1) + \frac{1}{1 - \left(\frac{K}{R_f}\right)^n} + k_4 \cdot \frac{K}{R_f - K}\right] \tag{36}$$

$$S^P(i,t) = k_1 \cdot k_3 \cdot X(0,0)\exp[(t-2i)\sigma] \cdot \left[\frac{1}{1 - \left(\frac{K}{R_f}\right)^n} + k_4 \cdot \frac{K}{R_f - K}\right] \tag{37}$$

$$S^C(i,t) = k_1 \cdot k_3 \cdot X(0,0)\exp[(t-2i)\sigma] \cdot \left[(k_2 - 1) + \frac{1}{1 - \left(\frac{K}{R_f}\right)^n} + k_4 \cdot \frac{K}{R_f - K}\right] \tag{38}$$

2.2.2. Results of ROA

At this point, the case study has all the information it needs to solve the risk-neutral recursive relationships (Equations (20) and (23)). The results of the final recursive calculations are shown in Appendix Tables A2, A3, A4 and A5. The optimized strategy underprice uncertainty is shown in Appendix Tables A4 and A5 and

Table 6. Comparison of different valuation methods for the case study.

Case study bridge replacement	DTA	ROA	DT approach to ROA (wrong application of ROA theory)
Uncertainties	• Strength of bridge • Political decisions	• Strength of bridge • Political decisions • Prices (drift and volatility)	• Strength of bridge • Political decisions • Prices (drift and volatility)
Assumptions for prices	• Constant • No drift • No volatility	• GBM • Drift and volatility obtained from historical data	• GBM • Drift and volatility obtained from historical data
Assumptions for discount rate	• Static: minimum acceptable rate of return provided by organization	• Dynamic: influenced by market forces (trading) • Use of equivalent risk neutral probability approach and risk-free discount rate	• Static: minimum acceptable rate of return provided by organization
Option value $V_L(0,0)$	15.9	22.9	26.6
Replacement Strategy	Identical	Identical	Identical

Option values expressed in discounted costs (×million €).

equals the strategy without price uncertainty (Figure 5). A decision maker should wait for political decisions on banning cars from the city centre and incur the risk costs under the current assumptions until Year 12. Only when entering a small state (a decision to ban cars from the city centre) the current bridge should immediately be replaced by a small bridge.

The volatility of construction prices does not influence the strategy in the case study. This is deducted from Appendix Table A4. All the strategies in one column are identical. There is no incentive to act purely on the volatility derived from analysis of historical construction prices.

2.3. The DT approach to ROA

The difficulty in applying ROA in engineering practice lies in the establishment of reasonable assumptions for market behaviour as demonstrated in Section 2.2. A spanning asset or twin security needs to be found and analysed. Assumptions that predict future prices are required for the process (Table 4). Systematic market risks and risk premiums need to be estimated. A prediction of the future risk-free interest rate should be obtained from the financial market. Unfortunately, risk-free interest rates fluctuate and are only constant for an agreed term. Presently, the short-term risk-free interest is close to zero in the Netherlands. Solutions to value real options under zero or negative risk-free interest rates are not readily available and require in-depth economic expertise.

It is understandable that in engineering practice, ROA is adapted to become what is often called a DT approach to ROA. Price development is modelled according to standard ROA practices, for example a GBM expressed in a recombining binominal lattice. However, the difference is that instead of calculating with risk-neutral probabilities $(\eta_U,\ \eta_D)$ and discounting the adapted cash flows with a risk-free discount rate r_f, actual probabilities $(\varphi_U,\ \varphi_D)$ are used and discounted with the minimum accepted rate of return of the organization ($r_a = 3.5\%$ in the case study). This also affects the perpetuities of the boundary constraints which are now calculated by using the annual growth rate g and the organization's discount rate r_a. Performing these calculations for the case study results in an option value $V_{L,\ DT\ approach\ to\ ROA}\ (0,0)$ of 26.6 instead of 22.9. The optimized strategy does not alter in the current case study. This DT approach to ROA is incorrect in its definition of ROA because it allows for the possibility of arbitrage on the financial market.

2.4. Comparison

The purpose of this study is methodology development and to demonstrate how and when to apply different approaches: DTA, ROA and the DT approach to ROA. Table 6 summarizes the three approaches used for the case study. The approaches that value options without (DTA, Section 2.1) and with market price uncertainty (ROA, Section 2.2) show a difference in option value. This difference is a consequence of two different approaches and their underlying assumptions. The basic rule for applying ROA instead of DT is whether or not market prices are involved.

Comparing the ROA (Section 2.2) with the DT approach to ROA (Section 2.3) also shows a difference in option values but here the explanation is clear: the DT approach to ROA is an incorrect application of the ROA theory. However, all these applications still result in the same strategy for the case study. This is a consequence of construction prices with low drift and volatility, and a market discount rate close to the discount rate of the organization.

3. Discussion

Section 2 is focused on model development for an infrastructure replacement decision under different types of uncertainty, including the managerial flexibility to respond to these uncertainties. Although a specific case study is used to demonstrate the model development, the approach is generic and in principle applicable to a wide range of design, build and operate decisions. The approach identifies the uncertainties and managerial options, which are combined in a decision tree. The decision tree is solved with backward recursion. Each decision node is evaluated for the best option out of a range of options for all prevailing states of uncertainty at that particular time.

The underlying mathematics for solving a decision tree (the backward recursion) are generic but the modelling of a decision tree and the inclusion of different types of uncertainty in a ROA/DTA is not. Probably the major barrier in the practical application of the method is the identification and quantification of uncertainties. Perminova *et al.* (2008) conducted a comprehensive research on defining uncertainty in projects. The authors observe the absence of a common understanding on the definition of uncertainty in and between different disciplines such as project management and economics. Uncertainty is not self-explanatory. Uncertainty is often used to designate the probability of events, but also as the probable outcome of these events to which others refer to as risk. A third definition of uncertainty is the unknown unknown: events that cannot be anticipated on because they are totally unknown. The current research follows a commonly applied convention in the discipline of project management for the definition of uncertainty and defines uncertainty as a (time-variant) probability of an event. Perminova *et al.* (2008) conclude that reflective learning and information sharing are methods to manage and reduce uncertainty and stress the importance of future research to develop tools that assist managers in decision making under uncertainty. The current study developed one of these tools by integrating different types of uncertainty in a DTA/ROA. Neely and De Neufville (2001) referred conceptually to such an approach as "hybrid" real options.

Section 2 already emphasizes the importance of separating market price uncertainty from other types of uncertainty as they require different treatment in discounting approaches. The case study in Section 2 also demonstrates the difficulty in estimating expected values of the uncertainty variables. Expected values can be obtained by wide range of approaches such as expert judgement, data-analysis, testing, using reference data of similar assets or projects and mathematical prediction modelling. Hereafter, uncertainty bounds for the expected value of variables need to be defined. Again, various approaches are available to model uncertainty bounds such as using random walks (geometric or arithmetic Brownian Motions), shock models, working with (time-variant) probability distributions or with non-probabilistic uncertainty bounds as does the info-gap decision theory and the sensitivity analysis approach. When uncertainty variables influence each other, more sophisticated techniques like Markov chains, Baysian networks, and artificial learning come into view.

Uncertainty modelling is complex (Perminova *et al.* 2008, Scope *et al.* 2016, Ilg *et al.* 2017). And even uncertainty models are subject to uncertainty. Would that be a reason for practice to refrain from the application of DTA and ROA? To answer this question, we first argue that uncertainty is inherent to every analysis conducted, including conventional LCC analysis. An extensive research on uncertainty in LCC modelling was conducted by Scope *et al.* (2016). The authors identify numerous approaches for dealing with uncertainty and classify these approaches in deterministic, probabilistic, possibilistic and practical methods. The above mentioned techniques identified by the current study are easily classified within these categories. Scope *et al.* (2016) also observe the absence of a holistic model in dealing with uncertainty in LCC analyses and conclude that choosing the right approach does not follow generic decision rules. Although uncertainty approaches can be grouped, their selection and application remain case-specific. Therefore, the authors stress the importance of developing case studies and learning by example.

The current research is a case-specific application of uncertainty modelling. A DTA/ROA incorporates all possible scenario's in a condensed decision tree and the backward recursion provides for choosing the best option in any decision node. By navigating through the tables with results, the best strategy in each decision node and uncertainty state is provided (Appendix A).

That still leaves the issue of selecting and quantifying uncertainties which may refrain practitioners from the application of DTA/ROA. Modelling price uncertainty is not an insuperable obstacle, because the ROA theory offers well-defined approaches and, historic price indices of construction costs and materials are often available. The estimation of boundary constraints, especially the perpetuities of replacements and life cycle costs *under price uncertainty*, are not yet

available in the engineering economy discipline and only partly available in the ROA discipline. This approach has been developed in the current research.

A difficult part in the application of ROA is estimating long-term market variables, required for the calculation of the risk neutral probabilities. A pragmatic solution is to omit this process and discount with fixed discount rates. In Section 1, De Neufville and Scholtes (2011) provide arguments that support this pragmatic solution. Second, the discrepancy between short-term market behaviour and long-term infrastructure life cycles, also calls into question the long-term validity of these risk neutral probabilities.

The second category of uncertainty is the infrastructure asset or project-related uncertainty. The current research demonstrates how these types of uncertainty can be incorporated in a DTA/ROA. A pragmatic approach based on failure data and expert judgement is used to provide reasonable estimates. Although reliability modelling of infrastructure is complex, often reasonable and pragmatic estimates for the current type of calculations suffice.

Taking the case study as an example, the strategy for the first 4 years is to wait and see what happens in Year 4. At present deformation, monitoring is initiated. In 4 years' time, results of measurements will become available and the model can be adjusted with better predictions for the probability of exceeding a deformation threshold in the future. Deformation monitoring is also initiated on other bridges in this city, which will provide the data required for establishing uncertainty bounds. This process of managing and reducing uncertainty is an example of reflective learning as referred to by Perminova *et al.* (2008) and an example of a practical method for dealing with uncertainty as referred to by Scope *et al.* (2016).

4. Conclusions

This study investigates the application of DTA and ROA in a common public infrastructure challenge, that of replacing a bridge in an urban environment. The concept of DTA and ROA is an incentive to wait for more information that allows decision makers to optimize future decisions. This managerial flexibility has value, which should be incorporated into traditional investment or replacement analyses. Both DTA and ROA can capture the value of flexibility.

The theory of ROA originates from valuing financial options and is strongly tied to the behaviour of financial markets. Therefore, applying ROA requires a careful estimation of market variables such as the choice of a spanning asset whose price can be observed in the market, market risk premiums, systematic market risks and risk-free interest rates. The estimation of market variables is subject to an inherent uncertainty regarding long-term market behaviour.

In the last decade, an academic debate on real options has revealed some interesting perspectives. A growing number of case studies demonstrate the application of ROA on real assets and advocate a wider application. Other literature warns against using ROA formula in the absence of price uncertainty. Two mistakes are easily made: ROA is applied to value flexibility in the absence of market price uncertainty and a DT approach to ROA is applied to value flexibility subject to market price uncertainty. It is correct to apply ROA to value flexibility subject to market price uncertainty and apply DTA to value flexibility in the absence of market price uncertainty.

At the same time, ROA has not gained foothold in public infrastructure investment decisions. The dominant reasons are its complexity, its difficulty in estimating market variables and the political context of public decision making. Investment or replacement decisions in public infrastructure are seldom driven by economic reasons alone. The current research, however, demonstrates with a case study that ROA can be applied to public sector investment decisions when market prices are observable. Second, even after high-level political investment decisions are made, there is no reason to ignore the value of flexibility and to address the question of timing.

The complexity of ROA is easily reduced by an incorrect application of ROA (referred to as the DT approach to ROA) that partially omits the process of estimating market variables. Here experts on ROA claim that this will lead to an incorrect valuation of flexibility under uncertain market prices. However, in the case study used in this research, the differences in these monetary values resulting from the application of different methods do not result in different optimized strategies. Although the monetary values of flexibility differ, the optimized replacement strategy does not alter because the discount rate of the organization is close to the discount rate obtained from the market. Second, the case study demonstrates that having capital-intensive options (replace or wait and accept risk costs) quickly dominates the impact of the volatility of market prices.

This leads us to the primary conclusion of this research. In the absence of market price uncertainty, ROA should be avoided and DTA used instead. In the presence of market price uncertainty, ROA is the first

choice to value the flexibility of engineering options. However, when market variables like market prices, systematic market risks, risk premiums and risk-free interest rates, cannot reasonably be estimated, the DT approach to ROA is the best approximation for ROA. If the discount rate of the organization is close to the discount rate that would be obtained from the market, and capital-intensive options are involved, then it is very unlikely that the DT approach to ROA will result in a different strategy. These conditions often apply to public infrastructure assets.

Disclosure statement

No potential conflict of interest was reported by the authors.

References

Amram, M. and Kulatilaka, N., 1999. Real option. Managing strategic investment in an uncertain world. *Financial management association survey and synthesis series*. Boston: Harvard Business School Press.

Amsterdam, G., 2018. *Verkeersprognose Amsterdam (Traffic Prognosis Amsterdam)* [online]. Available from: http://www.verkeersprognoses.amsterdam.nl/ [Accessed 20 April 2018].

Black, F. and Scholes, M., 1973. The pricing of options and corporate liabilities. *The journal of political economy*, 81 (3), 637–654.

Brealey, R. E., Myers, S. C., and Allen, F., 2017. *Principles of corporate finance*. 12th ed. New York: McGraw-Hill Education.

Cardin, M.-A., *et al.*, 2013. Empirical evaluation of procedures to generate flexibility in engineering systems and improve lifecycle performance. *Research in engineering design*, 24 (3), 277–295.

Cardin, M.-A., Zhang, S., and Nuttall, W.J., 2017. Strategic real option and flexibility analysis for nuclear power plants considering uncertainty in electricity demand and public acceptance. *Energy economics*, 64 (1), 226–237.

Central Bureau for Statistics (CBS), 2017. *Web page: price development for bridges and tunnels* [online]. Netherlands, Central Bureau for Statistics. Available from: http://statline.cbs.nl/Statweb/publication/?DM=SLNL&PA=81139NED&D1=0&D2=4&D3=a&VW=T [Accessed 15 January 2018].

Cheah, C.Y.J. and Liu, J., 2006. Valuing governmental support in infrastructure projects as real options using Monte Carlo simulation. *Construction management and economics*, 24 (5), 545–554.

Chow, J.Y.J. and Regan, A.C., 2011. Real option pricing of network design investments. *Transportation science*, 45 (1), 50–63.

Copeland, T. and Antikarov, V., 2003. *Real options. A practitioner's guide*. UK: The Penguin Group.

Cox, J.C., Ross, S.A., and Rubinstein, M., 1979. Option pricing: A simplified approach. *Journal of Financial Economics*, 7 (3), 229.

De Neufville, R. and Scholtes, S., 2011. *Flexibility in engineering design*. Massachusetts: Massachusetts Institute of Technology.

de Neufville, R., Scholtes, S., and Wang, T., 2006. Real options by spreadsheet: Parking garage case example. *Journal of infrastructure systems*, 12 (2), 107–111.

European Central Bank (ECB), 2014. Euro area risk-free interest rates: Measurement issues, recent developments and relevance to monetary policy. *ECB monthly bulletin*, July, 1–15.

European Central Bank (ECB), 2017. *Webpage: Euro area yield curves* [online]. Available from https://www.ecb.europa.eu/stats/financial_markets_and_interest_rates/euro_area_yield_curves/html/index.en.html [Accessed 15 January 2018].

Ford, D.N., Lander, D.M., and Voyer, J.J., 2002. A real options approach to valuing strategic flexibility in uncertain construction projects. *Construction management and economics*, 20 (4), 343–351.

Frankema, L. H., 2016. *Pricing and heging options in a negative interest rate environment*. Thesis (Master's), Delft University of Technology, Delft.

Guthrie, G., 2009. *Real options in theory and practice*. Oxford: Oxford University Press.

Herder, P.M., *et al.*, 2011. Buying real options - Valuing uncertainty in infrastructure planning. *Futures*, 43 (9), 961.

Hull, J. and White, A., 2013. Libor vs. OIS: The derivatives discounting dilemma. *Journal of investment management*, 11 (3), 14–27.

Ilg, P., *et al.*, 2017. Uncertainty in life cycle costing for long-range infrastructure. Part I: Leveling the playing field to address uncertainties. *The international journal of life cycle assessment*, 22 (2), 277–292.

Kim, K., Ha, S., and Kim, H., 2017. Using real options for urban infrastructure adaptation under climate change. *Journal of cleaner production*, 143, 40–50.

Klaassen, R.K.W.M., 2008. Bacterial decay in wooden foundation piles—Patterns and causes: A study of historical pile foundations in the Netherlands. *International biodeterioration & biodegradation*, 61 (1), 45–60.

Klaassen, R.K.W.M., and van Overeem, B.S., 2012. Factors that influence the speed of bacterial wood degradation. *Journal of cultural heritage*, 13 (Suppl. 3), S129–S134.

Leira, B.J., 2013. *Optimal stochastic control schemes within a structural reliability framework*. Cham, Heidelberg: Springer International Publishing.

Merton, R.C., 1973. Theory of rational option pricing. *The bell journal of economics and management science*, 4 (1), 141–183.

Mun, J., 2006. *Real options analysis. Tools and techniques for valuing strategic investment and decisions*. New Jersey: Wiley Finance Series - Wiley & Sons.

Neely, J.E. and De Neufville, R., 2001. Hybrid real options valuation of risky product development projects. *International journal of technology, policy and management*, 1 (1), 29–46.

NEN-EN 1990:2002 en. *Eurocode: Basis of structural design*. Delft: Netherlands Normalisatie Instituut.

NEN 8700:2011 nl. Beoordeling van de constructieve veiligheid van een bestaand bouwwerk bij verbouw en afkeuren - Grondslagen [Assessment of existing structures in case of reconstruction and disapproval - Basic rules]. Delft: Netherlands Normalisatie Instituut.

Nilsson, T., and Björdal, C., 2008. Culturing wood-degrading erosion bacteria. *International biodeterioration & biodegradation*, 61 (1), 3–10.

Park, C. S., 2011. *Contemporary engineering economics*. 5th ed. New Jerssey: Pearson.

Perminova, O., Gustafsson, M., and Wikstrom, K., 2008. Defining uncertainty in projects - A new perspective. *International journal of project management*, 26 (1), 73–79.

Peters, L., 2016. *Real options illustrated*. Switzerland: Springer International Publishing.

Richardson, S., Kefford, A., and Hodkiewicz, M., 2013. Optimised asset replacement strategy in the presence of lead time uncertainty. *International journal of production economics*, 141 (2), 659.

Sańchez-Silva, M. and Klutke, G.-A., 2016. *Reliability and lifecycle analysis of deteriorating systems*. Switzerland: Springer International Publishing.

Schwartz, E. and Trigeorgis, L., 2001. *Real options and investment under uncertainty: Classical readings and recent contributions*. Massachusetts: Massachusetts Institute of Technology.

Scope, C., *et al.*, 2016. Uncertainty in life cycle costing for long-range infrastructure. Part II: Guidance and suitability of applied methods to address uncertainty. *The international journal of life cycle assessment*, 21 (8), 1170–1184.

Sullivan, W.G., Wicks, E.M., and Koeling, C.P., 2012. *Engineering economy*. Boston: Pearson, Prentice Hall.

Werkgroep Discontovoet, 2015. *National taskforce for the societal discount rate* [online]. Rapport werkgroep discontovoet. Available from http://www.tweedekamer.nl/kamerstukken/detail?id=2015D43793 [Accessed 15 January 2018].

Woodward, M., Kapelan, Z., and Gouldby, B., 2014. Adaptive flood risk management under climate change uncertainty using real options and optimization. *Risk analysis: an official publication of the society for risk analysis*, 34 (1), 75–92.

Zelada-Tumialan, G., *et al.*, 2014. Untreated submerged timber pile foundations: Part 2 – Estimating remaining service life. *Structural forensics*, January, 10–13.

Appendix A

Table A1. Tabular representation of the binominal lattice for the state variable $X(i,t)$: yearly price increases of 1 unit of construction costs with a move up $U = 1.027$ and move down $D = 0.974$. The number of down moves is represented by i.

$X(i,t)$	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	1.00	1.03	1.05	1.08	1.11	1.14	1.17	1.21	1.24	1.27	1.31	1.34	1.38	1.42	1.45	1.49
1		0.97	1.00	1.03	1.05	1.08	1.11	1.14	1.17	1.21	1.24	1.27	1.31	1.34	1.38	1.42
2			0.95	0.97	1.00	1.03	1.05	1.08	1.11	1.14	1.17	1.21	1.24	1.27	1.31	1.34
3				0.92	0.95	0.97	1.00	1.03	1.05	1.08	1.11	1.14	1.17	1.21	1.24	1.27
4					0.90	0.92	0.95	0.97	1.00	1.03	1.05	1.08	1.11	1.14	1.17	1.21
5						0.87	0.90	0.92	0.95	0.97	1.00	1.03	1.05	1.08	1.11	1.14
6							0.85	0.87	0.90	0.92	0.95	0.97	1.00	1.03	1.05	1.08
7								0.83	0.85	0.87	0.90	0.92	0.95	0.97	1.00	1.03
8									0.81	0.83	0.85	0.87	0.90	0.92	0.95	0.97
9										0.79	0.81	0.83	0.85	0.87	0.90	0.92
10											0.77	0.79	0.81	0.83	0.85	0.87
11												0.75	0.77	0.79	0.81	0.83
12													0.73	0.75	0.77	0.79
13														0.71	0.73	0.75
14															0.69	0.71
15																0.67

Table A2. Tabular representation of the present values $V_L(i,t)$ at (i,t) of optimized decisions at each node in a large state L based on recursive Equation (23). Years 4, 8 and 12 represent the transitionary nodes where a switch to a small state could occur. At Year 0, the initial enforcement expenditures are incorporated.

$V_L(i,t)$	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	22.9	23.4	23.8	24.2	24.5	27.5	27.9	28.2	28.5	32.4	33.0	33.4	33.7	39.4	40.4	41.5
1		22.1	22.5	22.9	23.2	26.0	26.4	26.8	27.0	30.8	31.2	31.7	32.0	37.3	38.3	39.4
2			21.4	21.7	22.0	24.7	25.1	25.4	25.6	29.2	29.6	30.0	30.3	35.4	36.3	37.3
3				20.6	20.8	23.4	23.8	24.1	24.3	27.6	28.1	28.4	28.7	33.5	34.5	35.4
4					19.8	22.2	22.5	22.8	23.0	26.2	26.6	27.0	27.2	31.8	32.7	33.5
5						21.0	21.3	21.6	21.8	24.8	25.2	25.6	25.8	30.1	31.0	31.8
6							20.2	20.5	20.7	23.5	23.9	24.2	24.5	28.6	29.4	30.1
7								19.4	19.6	22.3	22.7	23.0	23.2	27.1	27.8	28.6
8									18.6	21.2	21.5	21.8	22.0	25.7	26.4	27.1
9										20.1	20.4	20.6	20.9	24.3	25.0	25.7
10											19.3	19.6	19.8	23.1	23.7	24.3
11												18.6	18.7	21.9	22.5	23.1
12													17.8	20.7	21.3	21.9
13														19.7	20.2	20.7
14															19.1	19.7
15																18.6

Table A3. Tabular representation of the present values $V_S(i,t)$ at (i,t) of optimized decisions at each node in a small state S based on recursive Equation (20).

$V_S(i,t)$	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0					18.6	19.1	19.6	20.1	20.7	21.2	21.8	22.4	23.0	23.6	24.3	24.9
1					17.6	18.1	18.6	19.1	19.6	20.1	20.7	21.2	21.8	22.4	23.0	23.6
2					16.7	17.1	17.6	18.1	18.6	19.1	19.6	20.1	20.7	21.2	21.8	22.4
3					15.8	16.3	16.7	17.1	17.6	18.1	18.6	19.1	19.6	20.1	20.7	21.2
4					15.0	15.4	15.8	16.3	16.7	17.1	17.6	18.1	18.6	19.1	19.6	20.1
5						14.6	15.0	15.4	15.8	16.3	16.7	17.1	17.6	18.1	18.6	19.1
6							14.2	14.6	15.0	15.4	15.8	16.3	16.7	17.1	17.6	18.1
7								13.8	14.2	14.6	15.0	15.4	15.8	16.3	16.7	17.1
8									13.5	13.8	14.2	14.6	15.0	15.4	15.8	16.3
9										13.1	13.5	13.8	14.2	14.6	15.0	15.4
10											12.8	13.1	13.5	13.8	14.2	14.6
11												12.4	12.8	13.1	13.5	13.8
12													12.1	12.4	12.8	13.1
13														11.8	12.1	12.4
14															11.5	11.8
15																11.2

Table A4. Tabular representation of the optimized strategy at (i,t) in a large state L based on recursive Equation (23). Years 4, 8 and 12 represent the transitionary nodes where a switch to a small state could occur. The optimized strategy in a large state is to postpone the planned replacement L^P until year 12 years and incur the risk of a premature corrective replacement W/L^C.

$V_L(i,t)$	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	W/L^C	W/L^C	W/L^C	W/L^C	W/L^C	W/L^C	W/L^C	W/L^C	W/L^C	W/L^C	W/L^C	W/L^C	L^P	L^P	L^P	L^P
1		W/L^C	W/L^C	W/L^C	W/L^C	W/L^C	W/L^C	W/L^C	W/L^C	W/L^C	W/L^C	W/L^C	L^P	L^P	L^P	L^P
2			W/L^C	W/L^C	W/L^C	W/L^C	W/L^C	W/L^C	W/L^C	W/L^C	W/L^C	W/L^C	L^P	L^P	L^P	L^P
3				W/L^C	W/L^C	W/L^C	W/L^C	W/L^C	W/L^C	W/L^C	W/L^C	W/L^C	L^P	L^P	L^P	L^P
4					W/L^C	W/L^C	W/L^C	W/L^C	W/L^C	W/L^C	W/L^C	W/L^C	L^P	L^P	L^P	L^P
5						W/L^C	W/L^C	W/L^C	W/L^C	W/L^C	W/L^C	W/L^C	L^P	L^P	L^P	L^P
6							W/L^C	W/L^C	W/L^C	W/L^C	W/L^C	W/L^C	L^P	L^P	L^P	L^P
7								W/L^C	W/L^C	W/L^C	W/L^C	W/L^C	L^P	L^P	L^P	L^P
8									W/L^C	W/L^C	W/L^C	W/L^C	L^P	L^P	L^P	L^P
9										W/L^C	W/L^C	W/L^C	L^P	L^P	L^P	L^P
10											W/L^C	W/L^C	L^P	L^P	L^P	L^P
11												W/L^C	L^P	L^P	L^P	L^P
12													L^P	L^P	L^P	L^P
13														L^P	L^P	L^P
14															L^P	L^P
15																L^P

Table A5. Tabular representation of the optimized strategy at (i, t) in a small state S based on recursive Equation (20). The optimized strategy is to immediately replace the current bridge with a preventive small replacement S^P when entering a small state. Entering a small state depends on political decision in Year 4, 8 or 12.

$V_S(i, t)$	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0					S^P	S^P	S^P	S^P	S^P	S^P	S^P	S^P	S^P	S^P	S^P	S^P
1					S^P	S^P	S^P	S^P	S^P	S^P	S^P	S^P	S^P	S^P	S^P	S^P
2					S^P	S^P	S^P	S^P	S^P	S^P	S^P	S^P	S^P	S^P	S^P	S^P
3					S^P	S^P	S^P	S^P	S^P	S^P	S^P	S^P	S^P	S^P	S^P	S^P
4					S^P	S^P	S^P	S^P	S^P	S^P	S^P	S^P	S^P	S^P	S^P	S^P
5						S^P	S^P	S^P	S^P	S^P	S^P	S^P	S^P	S^P	S^P	S^P
6							S^P	S^P	S^P	S^P	S^P	S^P	S^P	S^P	S^P	S^P
7								S^P	S^P	S^P	S^P	S^P	S^P	S^P	S^P	S^P
8									S^P	S^P	S^P	S^P	S^P	S^P	S^P	S^P
9										S^P	S^P	S^P	S^P	S^P	S^P	S^P
10											S^P	S^P	S^P	S^P	S^P	S^P
11												S^P	S^P	S^P	S^P	S^P
12													S^P	S^P	S^P	S^P
13														S^P	S^P	S^P
14															S^P	S^P
15																S^P