Improvement of the Richardson-Zaki liquid-solid fluidisation model on the basis of hydraulics (PPT)

Kramer, Onno

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Improvement of the Richardson-Zaki liquid-solid fluidization model on the basis of hydraulics

- Starting point: most popular fluidisation model
- Reference: Richardson-Zaki (1954)
- Model analysis: influence of parameters
- Introduction: hydraulic model components
- Experiments: pilot plant research
- Particles: CaCO3, pellets, garnet sand, crushed calcite
- Data matrix: (grain size, temperature, water flow)
- Validation: data comparison

\[ u_{th} = \frac{v_s}{v_t} \]
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- **Introduction**
  - Objectives
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  - Results and discussion
  - Conclusions
  - Questions
Hydraulic modelling of liquid-solid fluidisation in drinking water treatment processes

Onno Kramer¹, ², ³, ⁴
Eric Baars¹
Peter de Moel³, ⁵
Wim van Vugt²
Johan Padding⁴
Jan Peter van der Hoek¹, ³

¹ Waternet Drinking Water Department
² HU University of Applied Sciences Utrecht, Institute for Life Science and Chemistry,
³ TUD Delft University of Technology, Faculty of Civil Engineering and Geosciences
⁴ TUD Delft University of Technology, Faculty of Mechanical, Maritime and Materials Engineering
⁵ Omnisys Consultancy
Improvement of the Richardson-Zaki liquid-solid fluidisation model on the basis of hydraulics

1.2 million clients
Improvement of the Richardson-Zaki liquid-solid fluidisation model on the basis of hydraulics

- Background: (water cycle)
- Field: (drinking water treatment processes)
- System: (multiphase flows)
- Process: (softening)
- Fluidisation: (liquid-solid = water-calcite pellets)
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- System: (multiphase flows)
- Process: (softening)
- Fluidisation: (liquid-solid = water-calcite pellets)

- Hardness reduction to 1.4 mmol/L
- Reduces solubility of lead (public health) and copper (environment)
- Economic benefits and comfort
  - Reduction of washing powder
  - Increase life time hot water equipment
  - Cleaner laundry, tasteful tea

\[ \text{OH}^- + \text{HCO}_3^- \leftrightarrow \text{CO}_3^{2-} + \text{H}_2\text{O} \]
\[ \text{CO}_3^{2-} + \text{Ca}^{2+} \rightarrow \text{CaCO}_3 \downarrow \]
**Introduction**

- **Background:** (water cycle)
- **Field:** (drinking water treatment processes)
- **System:** (multiphase flows)
- **Process:** (softening)
- **Fluidisation:** (liquid-solid = water-calcite pellets)

**Diagram Description:**
- Seeding
- Pellets
- Hard water
- NaOH
- Soft water
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✓ Objectives:
  • Increasing sustainability
  • Reducing chemical use
  • Improving water quality

✓ Method: improved model based on hydraulics (porosity)

✓ Focus: crystallisation on specific surface area
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$$\varepsilon^n = \frac{v_s}{v_t}$$
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<table>
<thead>
<tr>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Classical Richardson-Zaki equation</td>
</tr>
</tbody>
</table>
| \[
\begin{align*}
    n &= \begin{cases} 
        4.65, & Re_t < 0.2, \\
        4.4 Re_t^{-0.03}, & 0.2 \leq Re_t < 1, \\
        4.4 Re_t^{-0.1}, & 1 \leq Re_t < 500, \\
        2.4 Re_t^{-0.1}, & Re_t \geq 500,
    \end{cases} 
\end{align*}
\] |
| General expression |
| \[
    n = c_1 Re_t^{c_2}
\] |
| Garside & Al-Dibouni equation |
| \[
    \frac{n_L - n}{n - n_T} = \alpha Re_t^\beta
\] |
| Khan & Richardson |
| \[
    \frac{n_L - n}{n - n_T} = \alpha Ar^\beta
\] |

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</table>
| Classical Richardson-Zaki equation | \[ n = \begin{cases} 
  0.2 \leq Re_t < 1, & n = 4.4 Re_t^{0.03} \\
  1 \leq Re_t < 500, & n = 4.4 Re_t^{0.1} \\
  Re_t \geq 500, & n = 2.4 
\end{cases} \] |
| General expression              | \[ n = c_1 Re_t^{c_2} \]                        |
| Garside & Al-Dibouni equation    | \[ \frac{n_L - n}{n - n_T} = \alpha Re_t^\beta \] |
| Khan & Richardson               | \[ \frac{n_L - n}{n - n_T} = \alpha Ar^\beta \] |

Objective

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\[ \frac{v_s}{v_t} \]

Ratio: superficial / terminal settling velocity [-]

Porosity [m³/m³]

\[ \varepsilon \]

Superficial velocity

Terminal velocity

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Granular activated carbon filtration backwash: $\varepsilon \approx 0.45$

Pellet softening fluidisation: $\varepsilon \approx 0.55$
Improvement of the Richardson-Zaki liquid-solid fluidisation model on the basis of hydraulics

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</tr>
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</table>

Garside & Al-Dibouni equation

Khan & Richardson

$n = c_1 Re_t^{\alpha_2}$

$\frac{n_L - n}{n - n_f} = \alpha Re_t^\beta$

$\frac{n_L - n}{n - n_f} = \alpha Ar^\beta$
Improvement of the Richardson-Zaki liquid-solid fluidisation model on the basis of hydraulics

Minimum fluidisation velocity

Carman-Kozeny

\[ c_D = \frac{180}{Re_e} + \frac{2.87}{Re_e^{0.5}} \]

Terminal velocity

Brown-Lawler (improved Schiller-Naumann)

\[ c_D = \frac{24}{Re_t} \left(1 + 0.15Re_t^{-0.681}\right) + \frac{0.407}{1 + \frac{8710}{Re_t}} \]

Interpolation

\[ n = \log \left( \frac{Re_{emf}}{Re_t} \left(1 - \varepsilon_{mf}\right) \right) \]

\[ Re_t = \frac{\rho_d d_p v_t}{\eta} \]

\[ Re_{emf} = \frac{\rho_d d_p v_{mf}}{\eta} \frac{1}{1 - \varepsilon_{mf}} \]
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Carman-Kozeny (at minimum fluidisation)
\[ \varepsilon_n = \frac{\nu_s}{\nu_t} \]

Brown-Lawler (at terminal settling settling)
\[ C_D = \frac{180}{Re_e} + \frac{2.87}{Re_e^{0.1}} \]
\[ C_D = \frac{24}{Re_t} \left( 1 + 0.15 Re_t^{0.681} \right) + \frac{0.407}{1 + \frac{8710}{Re_t}} \]
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✓ 10 sieved fractions
  $(0.4 < d_z < 2.0 \text{ mm})$
✓ 4 temperatures
  $(5, 15, 25, 35 \degree \text{C})$
✓ 25 ascending water flows
  $(0-180 \text{ m/h})$
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✓ Experiments: 76 fluidisation characteristics
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✓ Application: drinking water pellet softening
✓ Model accuracy improvement
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\[
\frac{4.8 - n}{n - 2.4} = 0.015 \, Ar^{0.5}
\]
Improvement of the Richardson-Zaki liquid-solid fluidisation model on the basis of hydraulics

\[ \varepsilon^n = \frac{v_s}{v_t} \]

\[ Re_t = \frac{\rho_f d_p v_t}{\eta} \]

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**Minimum fluidisation prediction error**

**Porosity prediction error**

- Whole range
- mf-180 [m/h]
- 60-90 [m/h]
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- Application: drinking water pellet softening
- Model accuracy improvement
  - minimum fluidisation $>100\% \rightarrow 12\%$
  - porosity $>15\% \rightarrow 3\%$

![Minimum fluidisation prediction error](chart1.png)
![Porosity prediction error](chart2.png)
Improvement of the Richardson-Zaki liquid-solid fluidisation model on the basis of hydraulics

✓ RZ models can be improved based on hydraulics principles i.e. 3 points \((\varepsilon, v)\)
\((0,0) (\varepsilon_{mf}, v_{mf}) (\varepsilon \rightarrow 1, v_t)\)
✓ Porosity can be predicted more accurately
✓ Recommendations:
  • Model enhancement (more general)
  • Identification of irregularly shaped particles
  • Implications for specific surface area (Interfacial Area Density)
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DOI: 10.1016/j.powtec.2018.11.018
Improvement of the Richardson-Zaki liquid-solid fluidisation model on the basis of hydraulics

Thank you for your attention
Personalia
Name: Onno Kramer
Phone.: 06-42147123
E-mail: onno.kramer@waternet.nl
Network: LinkedIn
Publications: TUDelft PureCycle, ResearchGate,

Waternet, Sector Drinking Water, Department, Production

HU University of Applied Sciences Utrecht, Institute for Life Science and Chemistry

Delft University of Technology
Faculty of Civil Engineering and Geosciences, Department Water Management, Section Sanitary Engineering, Research Group Drinking Water
Faculty of Mechanical, Maritime and Materials Engineering, Department Process and Energy, Section Intensified Reaction and Separation Systems
Optional for questions
FLUID-PARTICLE INTERACTIONS AND FLOW CHARACTERISTICS OF FLUIDIZED BEDS AND SETTLING SUSPENSIONS OF SPHERICAL PARTICLES

A.R. KHAN* and J.F. RICHARDSON

Department of Chemical Engineering
University College of Swansea
Singleton Park
Swansea SA2 8PP, UK

(Received October, 1987; in final form August 4, 1988)

The published correlations for the velocity-settling relationship observed during fluidization and sedimentation of uniformly sized spherical particles in solid-liquid systems are compared with published experimental results. It is found that the expression suggested by Richardson and Zaki represents the experimental data well over a wide range of values of Galileo number (10^-5 < Ga < 10^6) and settling (0.4 < e < 1). Methods are given for predicting the constants in the expression as a function of the properties of the system, including wall effects.

KEYWORDS Fluidized beds, Fluid-particle interactions, Spherical particles.

FIGURE 3 Comparison of published correlations with experimental values of index n.
Velocity–Voidage Relationships for Fluidization and Sedimentation in Solid–Liquid Systems

John Garside* and M.R. Al-Dibouni
Department of Chemical Engineering, University College London, London WC1E 7JE

Published experimental results for the velocity–voidage relationship observed during fluidization and sedimentation of uniformly sized spheres in solid–liquid systems are compared and new experimental results are presented. Predictions of various published correlations that are available to describe this relation are compared with these experimental results and the inadequacy of most of the correlations is demonstrated. New correlations are suggested. The most reliable of these is based on a logistic curve and can be represented by the equation

$$\frac{\Delta V}{(\Delta V - U) - U} = 0.065 Re^{-0.127},$$

where $\Delta V = \chi^{1.14}$ and $\beta = 0.812$ for $\chi < 0.85$ or $\beta = 0.625$ for $\chi > 0.85$. $U_0 = \frac{U}{U_1}$, where $U_0$ is the relative average velocity between particles and fluid, $U_1$ is the terminal velocity of a single particle, $Re$ is the particle Reynolds number based on $U_0$, and $\chi$ is the bed voidage.

Figure 8. Variation of exponent $n$ with Reynolds number. (See Table I for explanation of symbols; See Table III for key to different curves.)
A generalized theory of sedimentation


[Paper first received 3 January, and in final form 25 June, 1958]

A theoretical relationship between the concentration and the sedimentation velocity of non-flocculated suspensions of particles is derived. It is shown that the settling velocity relative to that of a single particle in the suspension is \((1 - c)\beta\) where \(\beta\) is a function of particle shape, size distribution and Reynolds number and \(c\) is the volume of solid per unit volume of suspension. The expression is shown to satisfy the experimental results of other workers. An empirical relationship between \(\beta\) and the Reynolds number is suggested.

1. INTRODUCTION

When a body falls through a fluid, it accelerates until it reaches a constant terminal velocity. This velocity is determined by the density of the fluid \(\rho_f\), the density of the body \(\rho_b\), the viscosity of the fluid \(\eta\), the shape and orientation of the body and by some length characterizing the size of the body \(d\). The velocity may also depend, to some extent, upon the size and shape of the containing vessel, but if this is large its influence may be neglected.

The problem becomes more complicated if many bodies are present and the system becomes a sedimenting suspension. When the bodies are more or less evenly dispersed throughout the fluid, their rate of fall is decreased, and it is of considerable practical interest to know the relation between the concentration of bodies and the magnitude of this decrease.

* Now at the City of Liverpool College of Technology.

Many theoretical and empirical relations\(^{1-10}\) have been proposed to solve this problem, but they suffer from various defects, in particular they lack generality. It would be of considerable practical value if an equation could be derived which would cover a wide range of particle shapes, size ranges and rates of fall, and in the following paper an attempt is made to do this.

2. FALL OF SPHERES AT LOW REYNOLDS NUMBER

Consider a mass of equi-velocity particles falling through a pure fluid. If \(c\) is the average volume of one particle, and \(g\) the acceleration due to gravity, then the average force on one particle is \(F = (\rho_b - \rho_f)gc\). If \(c\) is the volume of solid in unit form of suspension, then it may be shown from

\[
\beta = \frac{g}{9} \left( \frac{\rho_b - \rho_f}{\rho_f} \right) \left( \frac{d}{\eta} \right) \left( \frac{1}{\rho_f} \right)
\]

that

\[
\beta = \frac{g}{9} \left( \frac{\rho_b - \rho_f}{\rho_f} \right) \left( \frac{d}{\eta} \right) \left( \frac{1}{\rho_f} \right)
\]

is a function of the Reynolds number. Fig. 2. Variation of the exponent of \((1 - c)\) with Reynolds number
A concentrated suspension of uniform particles settles at a lower rate than one of the particles in isolation in a large expanse of fluid. This phenomenon arises from a combination of factors. Thus, in a suspension there is a significant upflow of displaced fluid, there are changed buoyancy effects and steeper velocity gradients at a given particle velocity relative to the fluid. The relation between sedimentation velocity and concentration in a suspension is similar to that between fluidisation velocity and concentration in a liquid-solid system, and various empirical relations have been suggested. It is now proposed to examine how the constant in one of these relations can be calculated from the slope of the curve of drag coefficient against particle Reynolds number, and to show how calculated and experimental values compare.

**THE EFFECT OF CONCENTRATION ON THE DRAG FORCE VELOCITY RELATION FOR A SPHERE**

For a single isolated particle settling in a fluid

\[
\frac{d}{6} (\rho_p - \rho) g \left( \frac{R'}{\rho_p D} \right) \frac{D^2}{\rho \mu} \frac{d}{4} D^2
\]

from which

\[
R' \left( \frac{\rho_p}{\rho} \right) \frac{d}{6} (\rho_p - \rho) g \left( \frac{R'}{\rho_p D} \right) \frac{D^2}{\rho \mu} \frac{d}{4} D^2 = \frac{2}{3} \frac{d}{\mu} (\rho_p - \rho) g
\]

\[1\]

Suppose that in a suspension of voidage \( \varepsilon \), the force on a particle at a given relative velocity \( U' \) is increased by some factor \( f(\varepsilon) \). Then

\[\pi \left( \frac{R'}{\rho_p D} \right) \frac{D^2}{\rho \mu} \frac{d}{4} D^2 f(\varepsilon) \]

where \( f(\varepsilon) \) takes account of all interparticle effects including the increase in buoyancy force, and \( (R'/\rho_p D) \) is still the friction factor for the isolated particle. Combining eqns (3) and (4)

\[f(\varepsilon) = \frac{2}{3} \left( \frac{\rho_p D}{\rho \mu} \right) Re^2
\]

**Department of Chemical Engineering**

University College, Swansea

Wales

J F RICHARDSON

M A da S JERÓNIMO

Centro de Engenharia Química da Universidade do Porto

Portugal

![Fig 1 Experimental and calculated values of \( n \) as a function of Galileo and particle Reynolds numbers](image-url)
A convenient empirical equation for estimation of the Richardson-Zaki exponent

(Received 1 April 1987; accepted 11 May 1987)

Wilhelm and Kwaak (1948) were the first to publish studies of
the variation of voidage with fluid velocity for fluidized
particles and to show that their results using water as the fluid
(described as particulate fluidization) were correlated by an
equation of the form

\[ \text{Re} = K \varepsilon^n \]  

(1)

Richardson and Zaki (1954) showed some of the logic behind
this choice which can be conveniently written

\[ u = u_0 e^{-r} \]  

(2)

and made a systematic experimental study of how the
exponent, \( n \) (commonly referred to now as the Richardson-
Zaki exponent) varies with \( \text{Re} \). Their results, conveniently
presented in Richardson (1971), for cases where particle size is
small compared with vessel diameter, were described by four
empirical equations covering different Reynolds number
ranges. These equations are a little awkward to use particu-
larly in the regions where they overlap and a continuous
functions covering all values of \( \text{Re} \), is more useful especially
when embodied in a general theory such as that of Foscolo

Inspection of their data suggests it can be fitted by a logistic
curve which is symmetrical and asymptotes to limiting values

\[ \frac{(4.70 - n)}{(n - 2.35)} = 0.175 \text{Re}^{3/4} \]

Fig. 1.
Linear relationship $\log v_s/v_t$ versus $\log c$ like Richardson-Zaki

A generalized theory of sedimentation

Thus equation (16) becomes

$$\frac{F_g}{\eta^2} \propto \left( \frac{U_0 d}{\eta} \right)^n (1 - c)^{c_0}$$

which gives

$$U = U_0 (1 - c)^{c_0}$$


Improvement of the Richardson-Zaki liquid-solid fluidisation model on the basis of hydraulics
Richardson-Zaki (1954) experimental data

Sedimentation and Fluidisation: Part I

By J. F. Richardson, Ph.D.* (Associate Member) and W. N. Zaki, Ph.D.*

Summary

The present work is concerned with the study of sedimentation and liquid-solid fluidisation. In the former, suspended solids are falling under the influence of gravity in a stationary fluid, while in the latter, the particles are kept in suspension by an upward flow of liquid.

The object is to examine experimentally the effect of concentration of suspended particles on their rate of sedimentation, and to find a satisfactory method of correlating the results. The present part of the experimental work has been confined to multiply sized spherical particles, greater than 120 microns in diameter. As reported elsewhere, an attempt has been made to develop an expression, from theoretical considerations, for the rate of settling of suspensions, and the experimental results obtained in the present work are compared with those predicted from this theory. This work has been extended for comparison to liquid-solid fluidization systems.
Drinking water softening (circular economy)

✔ Profit: re-use calcite as a seeding material
  - Cost reduction: 100.000 €/year (0,4%)
  - Sustainability: 40.000 eco-points/year (5%)
  - Valorisation: high market segments: glass/paper/capet...
  - Vision: possibilities introduction of process cycles in industry
  - So much to learn...
    - Legislation
    - Hydraulic
    - LCA calculation
Eureqa®: The A.I.-Powered Modeling Engine

Eureqa automates the process of model building and interpretation, enabling you to extract answers from your data 90% faster.
Symbolic regression: Archimedes number $\rightarrow n_{RZ}$

**Best Solutions of Different Sizes**

<table>
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<tr>
<th>Size</th>
<th>Fit</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>0.205</td>
<td>$n_{RZ} = 3.95 + 6.32e-6 \cdot r + \frac{57}{Ar - 383} - 0.00638 \cdot \sqrt{4r}$</td>
</tr>
<tr>
<td>26</td>
<td>0.202</td>
<td>$n_{RZ} = 3.66 + \frac{232}{Ar} + 1.58e-7 \cdot 4r \cdot \sqrt{r} - 4.45e-5 \cdot 4r - 1.65e-10 \cdot 4r^2$</td>
</tr>
<tr>
<td>28</td>
<td>0.201</td>
<td>$n_{RZ} = 3.77 + 2.28 \cdot 4r \cdot \sqrt{r} + \frac{111995}{Ar^2} - 5.84e-5 \cdot 4r - 2.56e-10 \cdot 4r^2$</td>
</tr>
<tr>
<td>30</td>
<td>0.201</td>
<td>$n_{RZ} = 3.77 + 2.27 \cdot 4r \cdot \sqrt{r} + \frac{111994}{Ar^2} - 5.83e-5 \cdot 4r - 2.56e-10 \cdot 4r^2$</td>
</tr>
<tr>
<td>10</td>
<td>0.219</td>
<td>$n_{RZ} = 4.05 + 8.68e-6 \cdot 4r - 0.00752 \cdot \sqrt{r}$</td>
</tr>
<tr>
<td>13</td>
<td>0.216</td>
<td>$n_{RZ} = 7.12 - 0.394 \cdot \log(3.13e3 + 0.00976 \cdot 4r - 0.00976 \cdot 4r^2)$</td>
</tr>
<tr>
<td>11</td>
<td>0.218</td>
<td>$n_{RZ} = 4.06 + 8.9e-6 \cdot 4r - 0.00763 \cdot \sqrt{r}$</td>
</tr>
</tbody>
</table>

**Solution Details (calculated on validation data)**

- Solution: $n_{RZ} = 3.949 + 6.319e-6 \cdot 4r + 57/(4r - 382.9) - 0.00638 \cdot \sqrt{4r}$
- R² Goodness of Fit: 0.94691871
- Correlation Coefficient: 0.97427472
- Maximum Error: 0.32099426
- Mean Squared Error: 0.0389841242
- Mean Absolute Error: 0.064523855
Symbolic regression: Reynolds terminal number $\rightarrow n_{RZ}$

Best Solutions of Different Sizes

<table>
<thead>
<tr>
<th>Size</th>
<th>Fit</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>0.204</td>
<td>$n_{RZ} = 4.02 + 2.22e320.0148\text{Ret} + 0.00173\text{Retlog(Ret)} - 0.0137\text{Ret}$</td>
</tr>
<tr>
<td>27</td>
<td>0.204</td>
<td>$n_{RZ} = 4.02 + 2.22e320.0148\text{Ret} + 0.00173\text{Retlog(0.0443 + Ret)} - 0.0137\text{Ret}$</td>
</tr>
<tr>
<td>33</td>
<td>0.202</td>
<td>$n_{RZ} = 3.95 + 2.84e320.0148\text{Ret} + 5.94e-0.0055\text{Ret} - 0.0546(0.0024\text{Ret})^{1.5}$</td>
</tr>
<tr>
<td>15</td>
<td>0.213</td>
<td>$n_{RZ} = 4.05 + 0.00186\text{Retlog(Ret)} - 0.0146\text{Ret}$</td>
</tr>
<tr>
<td>11</td>
<td>0.223</td>
<td>$n_{RZ} = 4.11 - 0.0714\text{sqrt(Ret) - 11.1}$</td>
</tr>
<tr>
<td>9</td>
<td>0.227</td>
<td>$n_{RZ} = 4.19 - 0.0756\text{sqrt(Ret)}$</td>
</tr>
<tr>
<td>5</td>
<td>0.343</td>
<td>$n_{RZ} = 3.81 - 0.00293\text{Ret}$</td>
</tr>
<tr>
<td>1</td>
<td>1.000</td>
<td>$n_{RZ} = 3.5$</td>
</tr>
</tbody>
</table>

Solution Details (calculated on validation data)

- Solution: $n_{RZ} = 4.017 + 2.22e320.0148/\text{Ret} + 0.00173/\text{Retlog(\text{Ret}) - 0.0137}/\text{Ret}$
- $R^2$ Goodness of Fit: 0.94951485
- Correlation Coefficient: 0.97517377
- Maximum Error: 0.31294104
- Mean Squared Error: 0.0062786598
- Mean Absolute Error: 0.064299751
CFD opportunities

- Interstitial velocity versus terminal settling velocity
- Tortuosity versus ratio terminal and interstitial velocity
- Influence of the geometric representation (shape) on the specific surface area
- Particle interactions and collisions versus drag
- Relevant forces buoyancy, gravity and friction
- Surface roughness impact
- ...

Any suggestions are welcome.
Please mail me at: o.j.i.kramer@tudelft.nl