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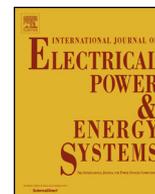
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## Density-based clustering methods for unsupervised separation of partial discharge sources



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### ABSTRACT

The recognition of partial discharge (PD) sources is an important task of the monitoring and diagnostics of high-voltage components. Nowadays, digital PD measuring systems have the capability of extracting features and form scatter plots with such data sets. Part of an unsupervised PD analysis system is to discover clusters within the data sets and link them to particular PD sources. Due to the nature of PD data sets, clusters may appear very close to each other or even merged hindering the separation of sources. Clustering methods based on spatial density such as the density peak clustering (DPC) method and DBSCAN are suitable approaches to discover clusters within PD data sets. However, their accuracy can be reduced due to the proximity among clusters. In this paper, a new method is presented to improve the accuracy of the DPC method. Our method proposes to partition the data set and later pass the resulting subsets to the DPC method. The partitioning is based on the spatial density of data computed by a smoothed density method (SD). SD has the advantage of being fast and not requiring high computational power. As a final step, a routine is applied to group the sub clusters as per the DPC method having a threshold for the data contour distance as a criterion. This method proved higher accuracy to discover clusters in actual PD data sets. However, the threshold for the data contour distance still needs further research.

### 1. Introduction

Partial discharge (PD) measurement is a powerful tool for monitoring and diagnostics of the insulation of high-voltage equipment. Over the decades the technique has evolved significantly but the main challenge still remains the same: to classify, identify and recognize each of the active PD sources in the insulation that might be seen mixed or overlapped in the conventional phase-resolved PD patterns, causing identification problems.

With modern digital PD measuring systems, a collection of features from the pulse shape can be extracted and then used for classification (or separation) and identification purposes. The main challenge is the selection of suitable features from the shape of the PD pulses. In [1–3], current peak  $I_{\text{peak}}$ , charge  $Q$  and energy  $E$  were proposed and tested as suitable clustering parameters under the rationing that those parameters are correlated with the physics of the discharge and that they are more resilient to variations in the acquisition parameters of the digital recording card used for the measurements [4]. Techniques such as the spectral power cluster [5], equivalent time and bandwidth cluster [6], frequency-energy characteristics of the pulses by Wavelets [7],

principal components analysis and t-Distributed stochastic neighbor embedding technique and morphological gradient of cumulative energy [8] have been widely researched for feature extraction and classification of PD sources. Pairwise plotting of these features, if properly tailored, results in data sets that can be clustered and in turn each cluster linked to a PD source.

Literature commonly classifies the clustering techniques into five classes [9–11]: distribution-based algorithms, hierarchical-based algorithms, density-based algorithms and grid-based algorithms, the particular choice of one family or another depending greatly on the shape, density, anomalies and priori knowledge of the data sets [12]. Density-based algorithms stand out as the most suitable family of algorithms for clustering PD data sets due to their remarkable ability of discovering clusters of arbitrary shape, without the previous knowledge of the existing number of clusters. Examples of density-based algorithms are DENCLUE (DENsity ClUstEring) [13], OPTICS (Ordering Points to Identify the Clustering Structure) [14], DBSCAN (Density Based Spatial Clustering of Applications with Noise) [15] and DPC (the density peak clustering) [16]. DBSCAN and DPC are widely known and has been extensively researched to overcome their main drawbacks [17]:

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reduced accuracy with data sets having varied densities and high computational cost with large data sets; a minimum complexity of  $O(N^2)$ . On the other hand, DPC has been prized for being fast and efficient since it finds high-density peaks in a non-iterative [18] manner and only one parameter has to be tuned [19].

Joining others efforts to find improvements for the DPC method [19], this paper takes the aim at tackling the issue derived from the assumption that the cluster centers in the data sets are separated by a relatively large distance. The knowledge of real world data sets extracted from partial discharge measurements brings up awareness of closeness between clusters. Therefore, a method is proposed in which the PD data sets are first split into subsets by means of the smoothed density method (SD) [20], and then each subset is passed to the DPC method. This partition of data sets is done based on the spatial density  $\rho$ . Unlike DPC method, the SD method computes  $\rho$  from a two-dimensional histogram which limits the complexity of the whole algorithm. A final step involves undoing the partition of the data sets for which a solve-a-puzzle-like (SPL) routine is defined. As the subsets are being put together back again, a criterion for distances between subset contours is applied resulting on the ability of the algorithm to discover clusters that are close to each other. In addition, the results that will be discussed in this paper show that this technique proves effective not only to discover clusters even when they have very dissimilar densities and scatters, but also our SPL algorithm makes the whole algorithm little affected by pseudo cluster centers found by the DPC method, also referred to as the “decision graph fraud” [21].

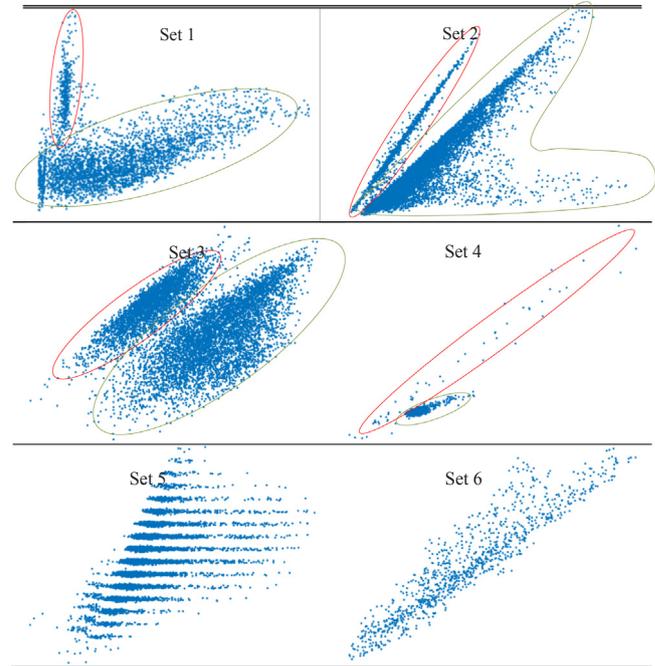
The following parts of this paper are defined as follows: Section 2 describes the characteristics of the six data sets used to test the algorithms. These data sets as well as the Matlab implementation of the algorithm can be found in [22]. Section 3 provides a description of DPC and SD methods and the adjustments to the SD method to split the data sets into subsets. Section 4 defines the SPL routine and the concept of data contour. Finally, the performance of the proposed algorithm is evaluated, and further comments are given in Sections 5 and 6.

## 2. Data sets

For the purpose of this paper, the data sets were brought about from the values of charge (abscissa axis) and energy (ordinate axis) computed from actual PD measurements. The measuring set-up described in [23] was used to record the PD pulses and the values of charge and energy were computed by means of the software tool *PDflex* [22] applying the methods described in [1,2]. The data sets are shown in Table 1. Since the source of the data sets was known, it was possible beforehand to identify and circle (when more than one) the clusters that exist in each of the data set. In addition, the data sets exhibit some particular characteristics such as:

- Distance between clusters:** the distance between clusters can be quite short as in the case of Set 1–3.
- Different scatters:** there can exist a large difference in the scatter of data from each cluster. From the knowledge of the original measurement, it is known that Set 4 has two clusters. The one with the large scatter still represent a PD source and therefore it cannot be discarded or treated as outliers.
- Effect of digital acquisition:** the digital acquisition of the PD pulses can account for a non-continuous (discrete) range of the computed parameters. Set 5 shows this phenomenon. Although it is known that there exists only one cluster, the digitalization produces multiple fake sub clusters.
- Different cluster densities:** clusters with large scatter appear when a relatively low number of PD pulses are measured. In such a case a clustering technique by itself might find several clusters due to the varying density of the data. However, like in the case of Set 6, the desired result is to find only one cluster despite the large scatter.
- Cluster overlapping:** charge and energy of PD pulses are closely

**Table 1**  
Description of the data sets.



related parameters at values close to zero. This means that it is possible that clusters corresponding to different sources tend to merge at values close to zero. Set 2 is an example of this. At higher values the relation between charge and energy depends on the nature of the PD pulses and therefore the clusters divert away.

- Cluster shape:** because of the large range of amplitudes of the PD pulses belonging to a certain source, the values of charge and energy span over a wide range, leading to elongated-shaped clusters pointing to the origin.

## 3. Density-based clustering methods

The density peak clustering technique (DPC) developed by Alex Rodriguez and Alessandro Laio [16] is the method used in this paper. Since 2014 when the method was first published it has received tremendous attention among other reasons because of its relatively simplicity, moderate computation cost and large availability of implementation codes in several programming languages that makes it easy to try out the algorithm.

### 3.1. DPC method

DPC is an approach based on the idea that cluster centers are characterized by a higher density than their surrounding neighbors and by a relatively large distance from points with higher densities [16]. In this description, the density refers to the spatial density  $\rho_i$  and it is defined as the number of data points whose Euclidian distance  $d_{ij}$  to the point  $x_j$  is not larger than a certain threshold  $d_c$ .

$$\rho_i = \sum_j \chi(d_{ij} - d_c)$$

$$\chi(x) = \begin{cases} 1, & x < 0 \\ 0, & x \geq 0 \end{cases} \quad (1)$$

In (1),  $\chi$  is the kernel density function or the weighting function for estimating the influence of a neighboring point. In this definition  $\chi$  is of square type, meaning that all points within a radius  $d_c$  exert the same influence on the point  $x_i$ . The study on the performance of the DPC method reported in [11] has suggested a Gaussian kernel (2) as a better density function. As a result (1) becomes (2).

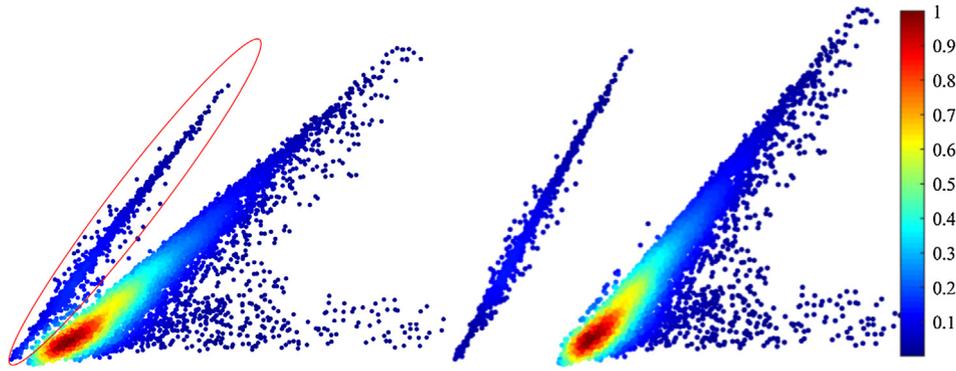


Fig. 1. Set 2: (a) original, (b) having the left cluster arbitrarily separated from the right cluster.

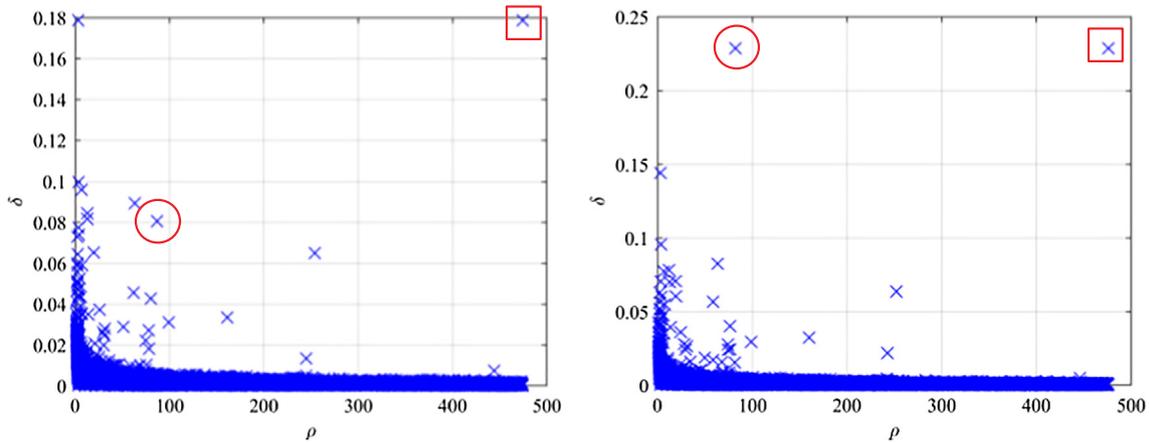


Fig. 2. Decision graphs, (a) for Set 2, (b) for data set of Fig. 1(b).

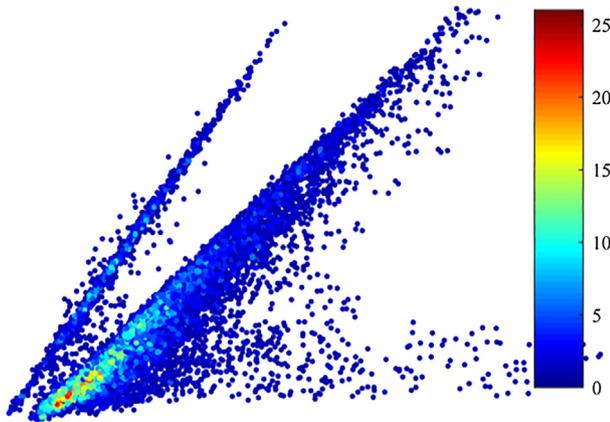


Fig. 3. Density color map for Set 2 as per the histogram method. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

$$\rho_i = \sum_{j \neq i} e^{(-\frac{d_{ij}}{d_c})^2} \quad (2)$$

The second main step in the algorithm is to compute  $\delta_i$  that is the minimum distance between the point  $x_i$  and any other point of higher density.

$$\delta_i = \min_{j: \rho_j > \rho_i} (d_{ij}) \quad (3)$$

$$\delta_i|_{\rho_{\max}} = \max(d_{ij}) \quad (4)$$

Eq. (4) only applies when  $x_j$  is the point with the highest density.

After solving (2), (3) and (4) the decision graph is constructed with the values of  $\rho_i$  as the abscissas and the values of  $\delta_i$  as the ordinates. In the decision graph only those points that have relatively large values of  $\rho_i$  and  $\delta_i$  turn out to be the cluster centers. The last step is to assign each remaining point to the same cluster as its nearest neighbor of higher density.

In the decision graph, the cluster centers are easily identified when the data set fits well the condition that the cluster centers are far away from each other. Besides, if the number of points in the data set is large, the results of the analysis are robust with respect to the choice of  $d_c$ . Both the authors and others [9,11,16] have experimentally set the free parameter  $d_c$  in such a way that 1.6–2% of the data are contained in the resulting radius.

On the other hand, given the characteristics of PD data sets, especially the ones listed in (a) and (b) in Section 2, it might be difficult to determine the cluster centers from the decision graph by its own.

Just to illustrate this difficulty, let's compute the decision graph for Set 2 and compare it to the case when the left cluster in Set 2 (circled in Fig. 2) is arbitrarily moved away from the right cluster as shown in Fig. 1. The color<sup>1</sup> map in Fig. 1 is the representation of the density  $\rho$ .

The original data in Set 2 led to the decision graph of Fig. 2a where only one points (square) is a cluster center candidate. When the Set 2 was modified, the second cluster center (circle), hidden in Fig. 2a, appeared clearly as a second cluster center in Fig. 2b. This occurs because the data set exhibits one of the bottleneck for the correct finding of cluster centers by the DPC method: clusters having large differences of densities and a relative short distance between them. The color map in Fig. 1 highlights that the density of the right cluster is so high that

<sup>1</sup> For interpretation of color in Fig. 1, the reader is referred to the web version of this article.

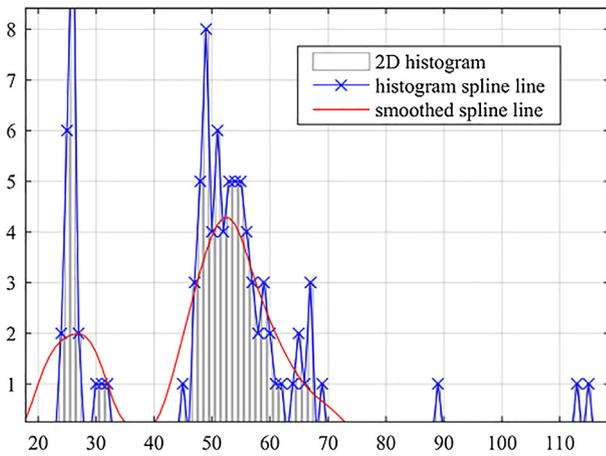


Fig. 4. Example of density with and without smoothing.

masks the density peak of left cluster. In addition, if it happens that the two clusters are very close to each other, then it might be difficult to find the correct number of cluster centers.

Since the characteristic (e) implies that clusters very close to each other will be common in PD data sets, then it is necessary to find a way to overcome the limitations of the decision graph illustrated in Figs. 1 and 2. In this paper, a method to achieve this task is proposed based on the SD method as described next.

### 3.2. SD method

To overcome the intrinsic difficulty of the DPC algorithm to separate clusters with large density differences and close distances, a solution is proposed in which the data set is split into subsets, and then each subset is passed to the DPC algorithm. Each subset comprises data points with density  $\rho_i$  within a certain range. For example, in Fig. 1, subset 1 might be the data points with  $\rho_i$  between 0.8 and 1. The limits of the range of  $\rho_i$  for each subset will be discussed later in Section 3.3.

The calculation of  $\rho$  as per DPC method is costly as it requires the pairwise computation of a distance matrix. In order to improve performance, the smoothed density method proposed by Paul H. C. Eilers and Jelle J. Goeman in [20] is rather preferred to compute the density  $\rho$ .

This technique relies on a two-dimensional histogram instead of a distance matrix and therefore it improves computing speed with large data sets. The two-dimensional histogram approach means that the cluster plot is divided into a grid or bins and the number of points in each bin is the density of the points within such a bin. Although fast, simply diving into bins may result in a non-optimal solution as that of in Fig. 3. In other words, the mere histogram approach does not lead to the smoothed color map shown in Fig. 1.

Let's consider Fig. 4 that corresponds to one column of the two-

dimension density histogram shown as the color map in Fig. 3. It can be noticed that the choppy color map is rendered by the broken spline line (Fig. 4). Applying a smoothing function would render the more optimal density color map of Fig. 1. The SD method proposes to apply a smoothing function first to all the columns of the two-dimensional density histogram individually. The resulting histogram is transposed and again the smoothing function is applied column-wise. This results in a smoothing in the two dimensions of the histogram.

In this method, there are two free parameters: the number of bins in the histogram and a factor  $\lambda$ . The latter is a parameter of the smoothing function. The higher the  $\lambda$  the higher the smoothing and therefore the worse the fit to the original data.

The authors in [20] have suggested 200 bins for the histogram and a value of 20 for  $\lambda$ . Our objective of splitting the data set into subsets makes the analysis little dependent on these parameters and therefore we also have set these values in our algorithms

### 3.3. Grouping data set based on density

Once the smoothed density has been computed, then the points in the data set are split into subsets. Let  $X$  be the vector containing the data set and  $D$  the vector whose elements represent the density  $\rho$  of each point in  $X$  as per the SD method. In order to split into  $n$  subsets, a pair of simple steps has to be followed:

$$qsv = 1 / (n);$$

$$D_{parts} = qsv * \text{ceil}(D / qsv);$$

Now  $D_{parts}$  represents the density of each point in  $X$  with the particularity that there are only  $n$  discrete values.

In Fig. 5 it can be seen that grouping the data based on the now discrete values of density splits the data set into subsets. Splitting too many can result in subsets having a very low number of data points and a large scatter. Too few subsets and the wanted goal of having subsets far away from each other might not be achieved. Extensive trial-and-error tests led to a suitable value of 4 for the number of subsets  $n$ .

## 4. Methodology for unsupervised cluster separation

As described earlier, the DPC algorithm applied to data sets from partial discharge measurements not always leads to satisfactory results. The proposed workaround for this limitation is a series of steps as described next:

- I. Normalization. The data is firstly normalized to unity in both dimensions. This only takes effect for the calculation of proximity of sub clusters.
- II. Grouping data set based on density. In this stage, the normalized data set is split into 4 subsets. The subsets are also referred as decomposition levels.
- III. Passing each decomposition level to the DPC algorithm. The output

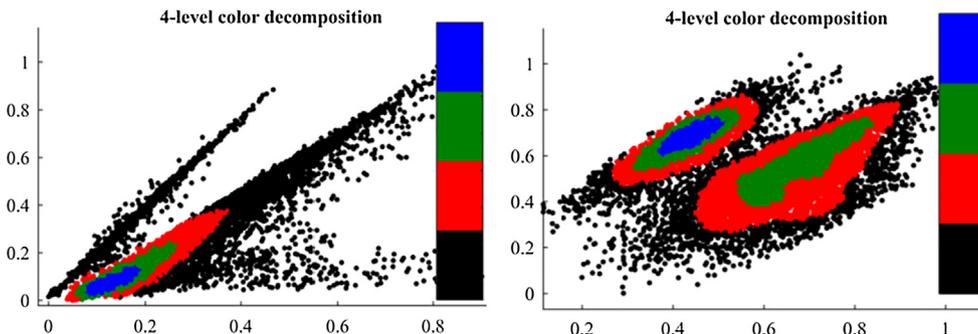


Fig. 5. Grouping of data Set 2 (left) and data Set 3 (right) based on 4-level density.

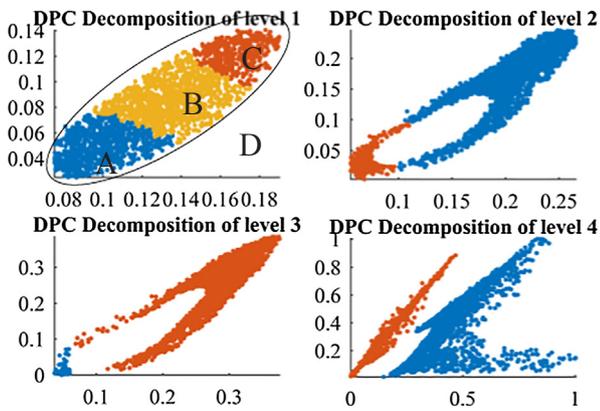


Fig. 6. Results of the DPC algorithm applied to every decomposition level (subsets) of the data Set 2.

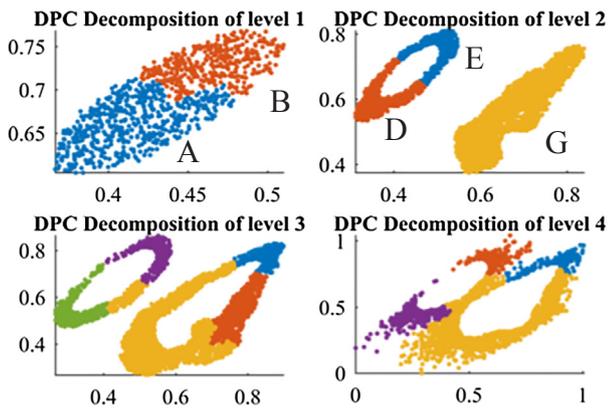


Fig. 7. Results of the DPC algorithm applied to every decomposition level (subsets) of the data Set 3.

of the DPC algorithm is a number of sub clusters.

IV. Grouping back these sub clusters based on their proximity to each other.

Examples of splitting into subsets and then applying the DPC method are shown in Figs. 6 and 7, where each sub cluster is rendered in different color in every decomposition level. For instance, sub clusters A, B and C are the output of DPC method applied to the first decomposition level (subset 1 out of 4) of data of Set 2 (see Table 1).

Step IV involves determining if the two sub clusters (e.g. A and B in Fig. 6) belong to one same parent cluster. This is an iterative method

that finds the points in A that are the closest points to all the points in B. This procedure is repeated but this time finding the closest points in B to all the points in A. If it happens that A and B belong to one same parent cluster then the two sub groups of closest points form contours that are close to each other.

The former procedure may be compared to putting together a puzzle where the player evaluates first if the contours of two pieces match to determine whether they are to put together or not. Here, the distance between two contours has to be lower than a certain threshold so that the respective two sub clusters are considered that belong to one same parent cluster. Since data is normalized, the proximity threshold has been set to 0.025.

The contours for the sub clusters labelled A, B and C in Fig. 6 is illustrated in Fig. 8.

From Fig. 8 it can be seen that A and B belong to one same cluster, but not A and C. However, B and C do belong to one same cluster. Therefore,  $D = (A \cup B) \cup C$ . This new sub cluster D is added to the sub clusters in the decomposition of level 2 and the routine to determine the contours is applied again. This iterative method is applied up to the decomposition of level 4.

Fig. 7 is a good example to show how the method can lead to find two clusters as the final result. In the decomposition level 1, both clusters belong to one parent cluster, then  $C = A \cup B$ . In the decomposition level 2, C (brought from decomposition level 1), D and E belong to one same parent clusters, then  $F = C \cup D \cup E$ , but G does not form contours with any other sub cluster. This way, each sub cluster in the decomposition level 3 will form contours with F or G that are the cores of the two existing clusters in data set 3.

### 5. Results

The results of applying the methodology described in the previous sections are shown in Fig. 9 and the accuracy of our method and the DPC method is contrasted by visual inspection given that the definition of clusters in the data sets is known from Table 1. The methodology in this paper successfully found the two clusters in the cases of sets 1, 2 and 3 and the single cluster in the Set 5. In the case of set 4 despite the existence of two clusters, our methodology only detected one and two clusters in set 6 where there only existed one.

Looking closely at the results of the decomposition levels of Set 4 in Fig. 10 it is observed that all the sub clusters up to decomposition level 3 form one single parent cluster, and the DPC algorithm fails to identify the two sub clusters necessary for the expected result highlighted in Fig. 9: one cluster being the data points within the ellipse and the second cluster all the remaining data points. However, in this case, this second cluster is formed by much less data points and with much larger scatter compared to the data points in the first cluster, which explains

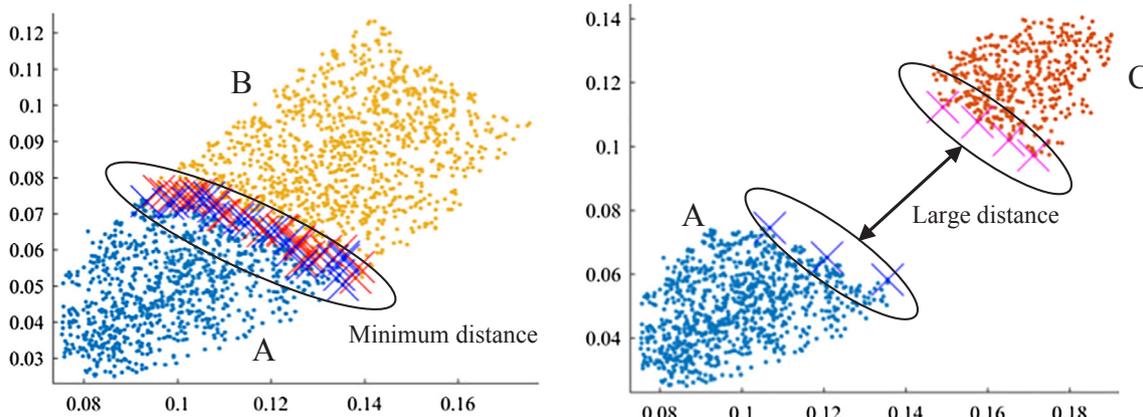


Fig. 8. Calculation of interfaces. Both sub clusters belong to the same parent cluster (left). Both sub clusters do not belong to the same parent cluster (right). Sub clusters from the decomposition level 1 of data set 2 in Fig. 6.

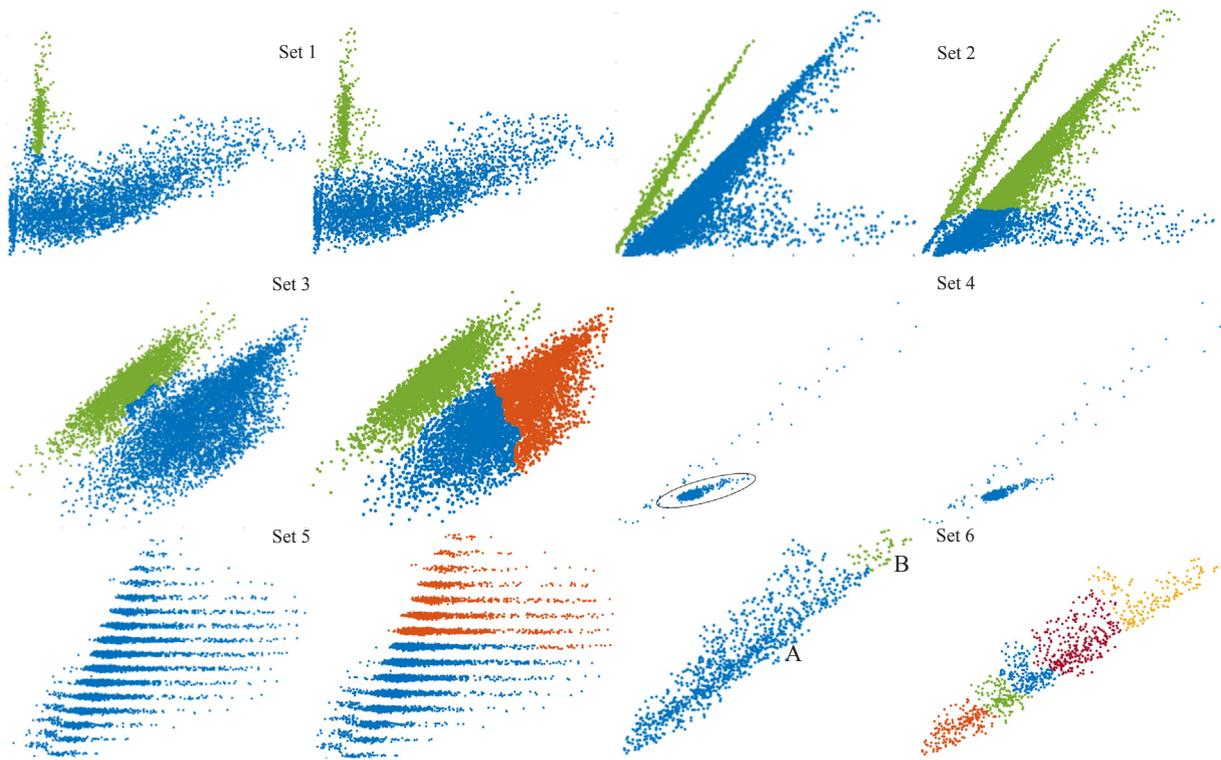


Fig. 9. Comparison of results as per the DPC method (right) and the methods proposed in this paper(left).

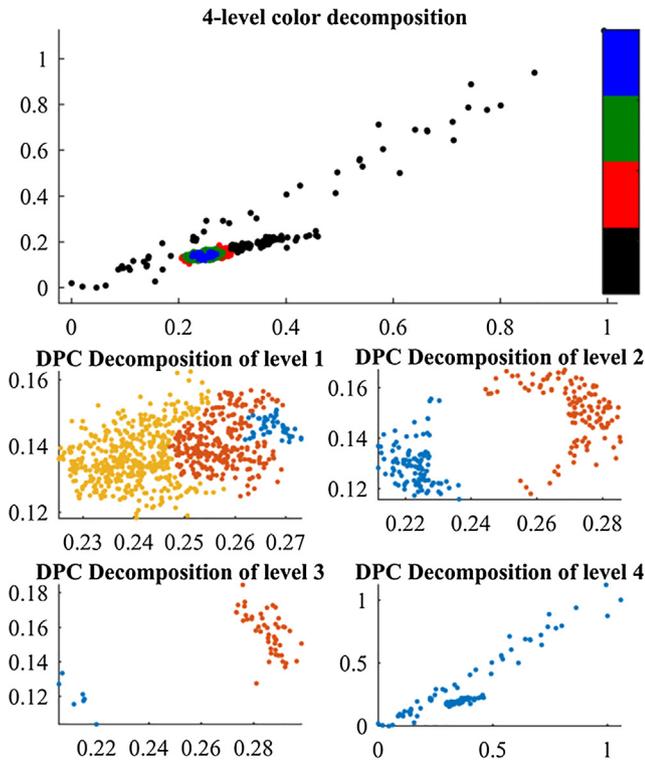


Fig. 10. Decomposition levels for data set 4.

why the DPC algorithm identifies only one cluster.

The scatter and the number of points in the data set also hinder the detection of interfaces between sub clusters. For instance, cluster B found in Set 6 is clearly part of the main cluster A, but it is found as a second cluster because of the large scatter.

Increasing the threshold for the distance between two sub clusters

eventually will result in assigning clusters A and B to the same parent cluster, but also might result in assigning other sub clusters that do not belong to the same parent cluster reducing accuracy.

### 6. Conclusion

The problem of source separation from charge-energy partial discharge plots was approached by combining the smoothed density method (SD) and the density peak clustering method (DPC), followed by a routine to group sub cluster based on a threshold proximity. Six data sets, from actual measurements of partial discharges, were used to test our methods leading to successful separation of the actual sources in most of the cases.

The splitting of the data by means of the SD method presented the advantage that it limits computational load by computing the density  $\rho$  with help of a two-dimensional histogram and not by calculating the Euclidean distances matrix as is required by the DPC method.

The grouping of sub clusters by defining a threshold for the proximity between sub cluster contours proved effective to discover cluster that are close to each other because the partition of data sets inherently has the effect of increasing the distance between subsets, increasing, in turn, the likelihood of the DPC method to find the correct number of clusters.

However, the proposed methodology contains an extra free parameter: the threshold for the contours between two sub clusters that adds to the uncertainty of choosing cluster centers by the decision graph in the DPC method. Nevertheless, the methodology becomes stronger against this free parameter as the number of points in the data set increases.

The “solve-a-puzzle” strategy of our method with threshold proximity as criterion in fact downplays the issue of finding pseudo cluster centers because they can be merged back together if they share contours closely.

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