Distributed Control of Heterogeneous Underactuated Mechanical Systems

Laurens Valk* Tamás Keviczky*
* Delft Center for Systems and Control, Delft University of Technology, 2628 CD, Delft, The Netherlands
(e-mail: laurensvalk@gmail.com, t.keviczky@tudelft.nl)

Abstract: We show how passivity-based control by interconnection and damping assignment (IDA-PBC) can be used as a design procedure to derive distributed control laws for undirected connected networks of underactuated and fully-actuated heterogeneous mechanical systems. With or without leaders, agents are able to reach a stationary formation in the coordinate of interest, even if each agent has different dynamics, provided that each agent satisfies three matching conditions for cooperation. If these are satisfied, we show how existing single-system IDA-PBC solutions can be used to construct distributed control laws, thereby enabling distributed control design for a large class of applications. The procedure is illustrated for a network of flexible-joint robots and a network of heterogeneous inverted pendulum-cart systems.

© 2018, IFAC (International Federation of Automatic Control) Hosting by Elsevier Ltd. All rights reserved.

Keywords: Distributed control, consensus, synchronization, mechanical systems, underactuated systems, passivity-based control by interconnection and damping assignment (IDA-PBC)

1. INTRODUCTION

In a network of cooperative mechanical systems, a typical control objective is to synchronize a subset of generalized coordinates between all systems in the network. More generally, the goal can be to obtain a formation in the coordinates of interest, either with one or more leaders that steer the formation towards a prescribed setpoint, or without leaders, such that the formation comes to rest at an arbitrary point. Each agent (system) uses a control law both to stabilize its own state and to contribute to the formation group objective, relying only on its own state information and information received from its neighbors in the network. Passivity-based control is a well-established control method for networks of fully-actuated nonlinear mechanical systems (see Chopra and Spong (2006); Arcak (2007); Ren and Cao (2011)), but few results are directly applicable if one or more agents are underactuated.

In this paper we show that passivity-based control by interconnection and damping assignment (IDA-PBC), introduced by Ortega et al. (2002), can be used to derive distributed control laws for networks of both underactuated and fully-actuated heterogeneous mechanical systems. If the communication network is undirected and connected, the agents are able to reach a stationary formation in the generalized coordinate of interest, with or without leaders. Additionally, if an IDA-PBC solution is known for each individual agent, we show that under certain conditions independent of the network topology, this solution can be used to construct the distributed control laws.

An early result for the synchronization of a simplified class of underactuated mechanical systems was given by Nair and Leonard (2008), to which our work has parallels by virtue of the close relationship between IDA-PBC and controlled Lagrangians (see Blankenstein et al. (2002)). The current work extends the synchronization objective to a formation objective, it generalizes the application from networks of homogeneous agents to networks of heterogeneous agents, and it extends the leaderless result to networks with leaders that have fixed reference coordinates to steer the group to a desired configuration.

An IDA-PBC approach was used to stabilize synchronization error dynamics in Zhu et al. (2012). While their method reduces the synchronization recovery time after a disturbance on a subsystem, the network must be a ring graph and the result is not a distributed control method, as all agents require knowledge of the absolute reference. A distributed synchronization result for networks of flexible-joint robots was presented by Nuno et al. (2014), which we show to be a special case of the presented distributed IDA-PBC method, and which can be extended to formations to allow non-identical robotic arm poses.

Sections 2 and 3 briefly review the single-agent IDA-PBC problem and Sections 4 and 5 review the necessary concepts from distributed control and graph theory. Section 6 formalizes the distributed IDA-PBC problem, while sections 7–9 give a constructive solution of the problem by providing sufficient conditions for each agent to facilitate cooperation in a network. Section 10 applies the proposed method to networks of flexible-joint robots and networks of heterogeneous systems of underactuation degree one.

2. IDA-PBC FOR A SINGLE MECHANICAL SYSTEM

This section reviews the method of passivity-based control by interconnection and damping assignment (IDA-PBC) when applied to a single mechanical system, as summarized by Acosta et al. (2005). The frictionless, open-loop dynamics of a mechanical system with coordinates $\mathbf{q} \in \mathbb{R}^n$, momenta $\mathbf{p} \in \mathbb{R}^n$, input $\tau \in \mathbb{R}^m$, and output $\mathbf{y} \in \mathbb{R}^m$ are
\[
\dot{q} = \begin{bmatrix} 0_n & I_n \\ -I_n & 0_n \end{bmatrix} \begin{bmatrix} \partial H_d \\ \partial q \end{bmatrix} + \begin{bmatrix} 0_n \times m \end{bmatrix} \tau,
\]
(1)

\[y = F^\top M_1^{-1} p.\]
(2)

\[H = \frac{1}{2} p^\top M_1^{-1} p + V_d,\]
(3)

where \(M(q) = M^\top(q) > 0_n\) is the generalized mass matrix and \(F(q) \in \mathbb{R}^{n \times m}\) is the input matrix (\(\text{rank}(F) = m \leq n\)). The Hamiltonian \(H(q, p) \in \mathbb{R}\) is the sum of the kinetic energy \(\frac{1}{2} p^\top M_1^{-1} p\) and potential energy \(V(q) \in \mathbb{R}\). All vectors are column vectors, including gradients of scalars.

In IDA-PBC for single systems, the control objective is to stabilize the state \((q, p)\) at a desired equilibrium \((q^*, 0)\). This is accomplished by choosing the feedback control \(\tau\) such that (1)–(3) attain the desired ("d") dynamics minimal at the desired equilibrium:

\[\dot{q}^* = \arg\min V_d(q).\]
(7)

The damping matrix satisfies \(K_0 > 0_m\) and the matrix \(J = -J^\top \in \mathbb{R}^{n \times n}\) is free. (We use "J" instead of the commonly used "\(J_0\)" to avoid confusion with other subscripts.)

The setpoint \((q^*, 0)\) is an asymptotically stable equilibrium of the dynamics (4), where the main argument is that \(H_d(6)\) is positive definite near the setpoint and its time derivative along (4) equals \(\frac{\partial}{\partial t} H_d = -y_d^\top K_0 y_d \leq 0\). The complete proof is summarized in Acosta et al. (2005).

The desired dynamics (4)–(6) are obtained by setting them equal to (1)–(3) and solving for \(\tau(q, p)\), which gives the single-agent IDA-PBC feedback law as

\[\tau = (F^\top F)^{-1} F^\top \left( \frac{\partial H}{\partial q} - M_1 \frac{\partial H_d}{\partial q} + J M_1^\top p \right) - K_0 y_d.\]
(8)

For underactuated systems, this law yields the dynamics (4)–(6) only if the kinetic energy matching equation

\[\text{F}^\perp \frac{\partial}{\partial q} \left( p^\top M_1^{-1} p \right) - \text{F}^\perp M_4 M_1^{-1} \frac{\partial}{\partial q} \left( p^\top M_1 p \right) + 2 \text{F}^\perp J M_1^\top p = 0,\]
(9)

and the potential energy matching equation

\[\text{F}^\perp \left( \frac{\partial V_d}{\partial q} - M_1 \frac{\partial V_d}{\partial q} \right) = 0.\]
(10)

both hold, for the annihilator \(\text{F}^\perp\) with \(\text{F}^\perp \text{F} = 0_{(n-m) \times n}\). In fully-actuated systems \(\text{F}\) is full rank and (8) yields (4)–(6) without the need to satisfy matching conditions.

While setpoint tracking primarily requires potential energy shaping of \(V_d\) to satisfy the minimality condition (7), it is usually also necessary to shape the kinetic energy through \(M_4\) and assign gyroscopic forces through \(J\), in order to satisfy the matching conditions (9), (10). Solving this problem is challenging in general. (See Ortega et al. (2017) for an historic overview and recent developments.)

Constructive solutions have been given for special classes of mechanical systems, such as those with only one degree of underactuation in Acosta et al. (2005).

3. DESIRED POTENTIAL ENERGY STRUCTURE

An agent in the network has two non-conflicting control objectives, each pertaining to a subset of its generalized coordinates, partitioned as \(q = (x, \theta) \in \mathbb{R}^{n}\). The coordinates \(x \in \mathbb{R}^\ell\) are to be controlled in cooperation with other agents in the network, while \(\theta \in \mathbb{R}^{n-r}\) are controlled by each agent individually. Before considering a network of systems, we consider how these control goals appear in the single-agent solution, where the goal is to reach the setpoint \(q^* = (x^*, \theta^*)\), for prescribed values \(x^*\) and \(\theta^*\).

In some IDA-PBC solutions, the objectives to reach \(x^*\) and \(\theta^*\) can be alternatively represented using a new coordinate \(z(q) = z(x, \theta) \in \mathbb{R}^r\), chosen such that achieving \(\theta = \theta^*\) and \(z = z^*\) also implies that \(x = x^*\). The choice of \(z\) ensures that the control signal to stabilize \(z\) does not violate the matching conditions, which is crucial for expressing interaction forces between agents in the network later on. Specifically, we use existing IDA-PBC solutions in which the desired potential energy can be written as

\[V_d(q) = V_a(q) + V_c(z(q)).\]
(11)

where \(z(q) \in \mathbb{R}^r\), \(\ell \leq m\), and the cooperation potential \(V_c\) is free in \(z\) as long as \(V_a\) remains positive definite around the setpoint \(q^*\). Then we can write its gradient as

\[\frac{\partial V_d}{\partial q} = \frac{\partial V_a}{\partial q} + \Psi \frac{\partial V_c}{\partial z},\]
(12)

where \(\frac{\partial V_a}{\partial z}\) depends only on \(z\) and \(\Psi\) depends only on \(q\):

\[\Psi(q) = \begin{bmatrix} \frac{\partial z_1}{\partial q} & \cdots & \frac{\partial z_r}{\partial q} \end{bmatrix} \in \mathbb{R}^{n \times \ell}.\]
(13)

In solutions of the form (11), the potential energy condition (10) is implicitly split up in two matching conditions:

\[\text{F}^\perp \left( \frac{\partial V_a}{\partial q} - M_4 M_1^{-1} \frac{\partial V_c}{\partial q} \right) = 0,\]
(14)

\[\text{F}^\perp M_1 M_4^{-1} \Psi = 0_{(n-m) \times \ell}.\]
(15)

Although requiring (14), (15) to hold is more conservative than (10), it ensures that \(V_a(\cdot)\) is free in \(z\), which is crucial in our solution of the distributed IDA-PBC problem.

The term \(V_a\) stabilizes the coordinates \(\theta\) to their fixed setpoint \(\theta^*\), subject to matching condition (14), while \(V_c\) steers the coordinates \(x\) to the desired setpoint, subject to matching condition (15). For example, in a pendulum-cart system where \(q = [x, \theta]^\top\) and \((x, \theta) \in \mathbb{R}\), \(V_c\) stabilizes the pendulum angle \(\theta\) at 0 while \(V_a\) makes the cart position \(x\) converge to the setpoint \(x^*\) by steering \((x, \theta)\) to \((x^*, 0)\).

Examples of IDA-PBC solutions of the form (11)–(15) with explicit descriptions of \(z(q)\) are given in Acosta et al. (2005) and Ryalat and Laila (2016) for systems of underactuation degree one, but solutions are not limited to this class. For example, a fully-actuated point mass with \(q \in \mathbb{R}^3\) might use the term \(V_a(q)\) to stabilize \(\theta = [q_1, q_2]^\top\) at \(\theta^* = [q_1^*, q_2^*]^\top\) and use \(V_c(z(q))\) to steer \(q_2\) to \(q_2^*\). In this case, \(n = m = 3\), \(\ell = 1\), and \(z(q) = x = r_2 \in \mathbb{R}\).
4. NETWORKS OF MECHANICAL SYSTEMS

4.1 Uncontrolled Network Dynamics

Consider a network of $N$ agents, where each agent has the dynamics (1)-(3), given explicitly for each agent $i$, as

$$\begin{align*}
[\dot{q}_i] &= \begin{bmatrix} 0_{n_i} & 1_{n_i} \\ -I_{n_i} & 0_{n_i} \end{bmatrix} + \begin{bmatrix} 0_{n_i \times m_i} \\ F \end{bmatrix} \tau_i, \\
[\dot{p}_i] &= F^T M^{-1} p_i,
\end{align*}$$

$$y_i = F^T M^{-1} p_i,$$

(16)

and

$$H_i = \frac{1}{2} p_i^T M^{-1} p_i + V_i.$$

(17)

As before, $q_i \in \mathbb{R}^{n_i}$, $p_i \in \mathbb{R}^{n_i}$, $\tau_i \in \mathbb{R}^{m_i}$, $F_i(q_i) \in \mathbb{R}^{n_i \times m_i}$, $M_i(q_i) = M_i^T(q_i) > 0$, and $m_i \leq n_i$. The dimensions $n_i$ and $m_i$ may be different for each agent. The dynamics of all agents can be written as one simple mechanical system:

$$\begin{align*}
[\dot{q}] &= \begin{bmatrix} 0_{\bar{n}} & 1_{\bar{n}} \\ -I_{\bar{n}} & 0_{\bar{n}} \end{bmatrix} + \begin{bmatrix} 0_{\bar{n} \times \bar{m}} \\ F \end{bmatrix} \tau,
\end{align*}$$

$$y = F^T M^{-1} \bar{p},$$

$$H = \frac{1}{2} \bar{p}^T M^{-1} \bar{p} + \bar{V},$$

(19)

(20)

(21)

where the corresponding network terms are given by

$$\bar{n} = \sum_{i=1}^N n_i, \quad \bar{m} = \sum_{i=1}^N m_i, \quad \bar{V} = \sum_{i=1}^N V_i,$$

$$q = \begin{bmatrix} q_1 \\ \vdots \\ q_N \end{bmatrix}, \quad p = \begin{bmatrix} p_1 \\ \vdots \\ p_N \end{bmatrix}, \quad \tau = \begin{bmatrix} \tau_1 \\ \vdots \\ \tau_N \end{bmatrix},$$

$$y = \begin{bmatrix} y_1 \\ \vdots \\ y_N \end{bmatrix}, \quad M = \begin{bmatrix} M_1 \\ \vdots \\ M_N \end{bmatrix}, \quad F = \begin{bmatrix} F_1 \\ \vdots \\ F_N \end{bmatrix}.$$
6. DISTRIBUTED IDA-PBC PROBLEM

As in the single-agent case, the IDA-PBC strategy defines the control $\hat{\tau}$ that changes the uncontrolled network dynamics (19)–(21) into the asymptotically stable dynamics

$$
\begin{bmatrix}
\dot{q} \\
\dot{p}
\end{bmatrix} =
\begin{bmatrix}
0_n & M^{-1}M_d \\
-M_d M^{-1} \bar{J} - F_k F^T
\end{bmatrix}
\begin{bmatrix}
\frac{\partial H_d}{\partial q} \\
\frac{\partial H_d}{\partial p}
\end{bmatrix},
$$

(27)

$$
\ddot{y}_d = F^+ \frac{\partial \hat{H}_d}{\partial p} = F^+ M_d^{-1} p,
$$

(28)

$$
\hat{H}_d = \frac{1}{2} \bar{p}^T M_d^{-1} \bar{p} + \hat{V}_d,
$$

(29)

where $M_d > 0_n$, $\hat{V}_d \in \mathbb{R}$, $J = -J^T \in \mathbb{R}^{n \times n}$ and $K_v > 0$ are to be designed to address the control objectives (23)–(26) and the transient response. Similar to (7), we now require

$$
\bar{q}^* = \arg \min \hat{V}_d(\bar{q}).
$$

(30)

The desired dynamics are obtained using the IDA-PBC control law (8) applied to the network of systems, giving

$$
\hat{\tau} = (F^+ F)^{-1} F^+ \left( \frac{\partial H}{\partial q} - M_d M^{-1} \frac{\partial H_d}{\partial \bar{q}} + JM_d^{-1} \bar{p} \right) - K_v \bar{y}_d,
$$

(31)

if the distributed kinetic energy matching condition

$$
\bar{F}^+ \frac{\partial}{\partial \bar{q}} \left( \bar{p}^T M_d^{-1} \bar{p} \right) - \bar{F}^+ M_d M^{-1} \frac{\partial}{\partial \bar{q}} \left( \bar{p}^T M_d^{-1} \bar{p} \right) + 2 \bar{F}^T J \bar{M}_d^{-1} \bar{p} = 0.
$$

(32)

and the distributed potential energy matching condition

$$
\bar{F}^+ \left( \frac{\partial V}{\partial \bar{q}} - M_d M^{-1} \frac{\partial \hat{V}_d}{\partial \bar{q}} \right) = 0,
$$

(33)

both hold.

7. SUFFICIENT CONDITIONS FOR COOPERATION

Despite the large degree of freedom in choosing the stability-preserving interconnection mechanisms, we show that for systems of the class (11)–(15) it is sufficient to shape the potential energy of the interconnections to obtain the desired group objectives (25), (26). The internal objectives (23), (24) can be addressed by choosing

$$
\bar{F}^+ = \begin{bmatrix}
F_{N,i}^+
\end{bmatrix},
\quad
M_d = \begin{bmatrix}
M_{d,1} & \cdots & M_{d,N}
\end{bmatrix},
\quad
J = \begin{bmatrix}
J_1 & \cdots & J_N
\end{bmatrix},
\quad
K_v = \begin{bmatrix}
K_{v,1} & \cdots & K_{v,N}
\end{bmatrix},
$$

(34)

where $F_{N,i}(q_i)$, $M_{d,i}(q_i)$, $J_i(q_i, p_i)$ and $K_{v,i}$ are taken from single-agent IDA-PBC solutions. Substituting these into matching condition (32) yields

$$
\bar{F}_i^+ \frac{\partial}{\partial q_i} \left( p_i^T M_i^{-1} p_i \right) - \bar{F}_i^+ M_i M_i^{-1} \frac{\partial}{\partial q_i} \left( p_i^T M_i^{-1} p_i \right) + 2 \bar{F}_i^+ J_i \bar{M}_i^{-1} p_i = 0 \quad \forall \ i = 1, \ldots, N,
$$

(35)

which are $N$ separate matching conditions, each identical to the single-agent kinetic energy matching condition (9), and solved if each agent has a known IDA-PBC solution. Likewise, inserting the choices (34) into the networked potential energy matching condition (33) yields

$$
\bar{F}_i^+ \left( \frac{\partial V}{\partial q_i} - M_d M^{-1} \frac{\partial \hat{V}_d}{\partial q_i} \right) = 0 \quad \forall \ i = 1, \ldots, N.
$$

(36)

This condition is not trivially solved because the desired potential energy $\hat{V}_d(q)$ depends on the coordinates of all agents. We propose a desired potential energy of the form

$$
\hat{V}_d(q) = \hat{V}_e(z_1(q_1), \ldots, z_N(q_N)) + \sum_{i=1}^{N} V_{s,i}(q_i),
$$

(37)

where the $V_{s,i}(q_i)$ are equal to the internal stabilization component in the single-agent potential energy (11), while $\hat{V}_e$ is a free function in the $z_i(q_i) \in \mathbb{R}^\ell$ variables of all agents. In order to show that (37) solves (36) we first write

$$
\frac{\partial \hat{V}_e}{\partial q_i} = \Psi_i \frac{\partial \hat{V}_e}{\partial z_i},
$$

(38)

in which

$$
\Psi_i(q_i) = \left[ \frac{\partial z_{1,i}}{\partial q_i}, \ldots, \frac{\partial z_{i,i}}{\partial q_i} \right] \in \mathbb{R}^{n_i \times \ell},
$$

(39)

where $z_{k,i}$ is the $k$-th element of the vector $z_i$, each of which depends only on $q_i$. Then (36) becomes

$$
\bar{F}_i^+ \left( \frac{\partial V}{\partial q_i} - M_d M_i M_i^{-1} \frac{\partial \hat{V}_d}{\partial q_i} - M_d M_i M_i^{-1} \Psi_i \frac{\partial \hat{V}_e}{\partial z_i} \right) = 0
\forall \ i = 1, \ldots, N.
$$

(40)

Consequently, if each agent satisfies the separated potential energy matching conditions (14) and (15), that is

$$
\bar{F}_i^+ \left( \frac{\partial V}{\partial q_i} - M_d M_i M_i^{-1} \frac{\partial V_{s,i}}{\partial q_i} \right) = 0 \quad \forall \ i = 1, \ldots, N,
$$

(41)

$$
\bar{F}_i^+ M_d M_i M_i^{-1} \Psi_i = 0_{(n_i - m_i) \times \ell} \quad \forall \ i = 1, \ldots, N,
$$

(42)

then through (40) and (36), the distributed potential energy matching condition (33) holds.

Therefore, the original network dynamics (19)–(21) and the desired dynamics (27)–(29) match independently of the network topology for the choices (34), (37), provided each agent satisfies the matching conditions (9), (14), (15). Because the conditions are local to each agent, no communication is required to guarantee matching, enhancing robustness against communication delays or switching network topologies. Matching still holds if the agents are heterogeneous, whether they have different parameter values, different dynamics, or a different number of coordinates.

8. COUPLING THROUGH POTENTIAL ENERGY

The potential energy of the network $\hat{V}_d$ (37) must be minimal (30) when the local and group objectives (23)–(26) are achieved. Although the matching conditions are decoupled, the systems are coupled through the free cooperative potential energy function $\hat{V}_e$ in (37), which through the control law (31) gives rise to control forces that steer the systems towards their cooperative goal (25), (26).

One possible coupling energy $\hat{V}_e$ is the squared sum of the deviation from the control goals (25), (26), which gives

$$
\hat{V}_e(\bar{z}) = \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} A_{ij} ||z_i - z_j + r_{ij}||^2 + \frac{1}{2} \sum_{i=1}^{N} B_i ||z_i - z_i^*||^2.
$$

(43)

Its gradient with respect to the $z_i$ variables is given by
The total potential energy (37) attains a minimum if its unique minimum satisfying (25), (26). If there are no leaders, \( \mathcal{L} + \mathcal{B} = \mathcal{L} \geq 0 \) and (43) is positive semi-definite around a range of minima, all satisfying (25), (26). The total potential energy (37) attains a minimum if additionally (24) is satisfied, relying on the single-agent solutions. Finally, the total energy (29) attains a minimum if (23) is also satisfied, when all agents are stationary.

9. DISTRIBUTED CONTROL LAW

The control laws for each agent are derived from (31) by substituting the previously made choices given in (34):

\[
\tau_i = \left( F_i^\top F_i \right)^{-1} F_i^\top \left( \frac{\partial H_i}{\partial q_i} - M_{d,i} d_i M_{1,i}^\top \right) \delta_i + J_i M_{1,i}^\top p_i + \delta_i y_{d,i},
\]

where \( y_{d,i} = F_i^\top M_{1,i}^\top p_i \) as in (5). In order to proceed substituting the coupling potential energy \( V_c \) (37), we first write

\[
\frac{\partial H_i}{\partial q_i} = \frac{1}{2} \frac{\partial}{\partial q_i} \left( p_i^\top M_{1,i}^\top p_i \right) + \frac{\partial V_{s,i}}{\partial q_i} + \Psi_i \frac{\partial V_c}{\partial z_i},
\]

at which point the quadratic potential energy gradient (44) is substituted to obtain the control law

\[
\tau_i = \sigma_i - \Phi_i \frac{\partial V_c}{\partial z_i} - K_{v,i} y_{d,i},
\]

where

\[
\sigma_i(p_i) = \left( F_i^\top F_i \right)^{-1} F_i^\top \left( \frac{\partial H_i}{\partial q_i} + J_i M_{1,i}^\top p_i \right) - \left( F_i^\top F_i \right)^{-1} F_i^\top M_{d,i} M_{1,i}^\top \frac{1}{2} p_i^\top M_{1,i}^\top p_i + V_{s,i},
\]

(49)

is equivalent to single-agent IDA-PBC control except for the cooperative component of the potential energy and

\[
\Phi_i(q_i) = \left( F_i^\top F_i \right)^{-1} F_i^\top M_{d,i} M_{1,i}^\top \Psi_i \in \mathbb{R}^{m_i \times \ell},
\]

(50)

is an input matrix that ensures that the potential coupling forces (44) do not violate the matching conditions.

The resulting distributed control law (48) has a stabilization term \( \sigma_i(p_i) \in \mathbb{R}^{m_i} \) and a damping term \( -K_{v,i} y_{d,i} \), each depending only on local information, and a coupling term \( -\Phi_i(q_i) \partial V_c/\partial z_i \) that depends on both local information and information \( z_j \) received from neighboring agents.

10. CASE STUDIES

10.1 Cooperative Flexible-Joint Manipulators

After using an internal control law to compensate for gravity (see Nuño et al. (2014)), the dynamics of a flexible-joint robot \( i \) with joint angles \( \alpha_i \in \mathbb{R}^{m_i} \), motor angles \( \delta_i \in \mathbb{R}^{m_i} \), mass matrix \( N_i(\alpha_i) > 0 \), motor inertia \( \Lambda_i \), joint stiffness \( C_i > 0 \), \( n_i = 2m_i = 2m \), are as (16)–(18) with

\[
q_i = \begin{bmatrix} \alpha_i \\ \delta_i \\ \theta_i \end{bmatrix}, \quad F = \begin{bmatrix} 0_{m_i} \\ I_m \end{bmatrix},
\]

\[
M_i = \begin{bmatrix} N_i & 0_m \\ 0_m & \Lambda_i \end{bmatrix}, \quad V_i = \frac{1}{2} (\delta_i - \alpha_i)^\top C_i (\delta_i - \alpha_i).
\]

The single-agent IDA-PBC solution steers \( x = \alpha \) to the target \( \alpha^* \) and steers \( \theta = \delta - \alpha \) to \( \theta^* = 0 \) without kinetic energy shaping \( (M_i = M \) and \( J = 0) \). While using potential energy shaping only to add energy that steers the motor angles to the desired joint locations:

\[
v_d = V + \frac{1}{2} (z^* - z)^\top P (z^* - z) \quad \text{where} \quad z = \delta \text{ and } P > 0,
\]

which is minimal at the target \( z^* = \delta^* \), \( \theta^* = 0 \).

For a network of flexible-joint robots, we obtain

\[
z_i = \delta_i = F_i^\top q_i, \quad M_{d,i} = M_i, \quad J_i = 0_n,
\]

and \( \ell = m \), which gives, from (39) and (50),

\[
\Psi_i = F_i, \quad \Phi_i = (F_i^\top F_i)^{-1} F_i^\top M_{d,i} M_i \Psi_i = L_i.
\]

For the potential energy we choose \( V_{s,i} = V_i \) such that from (49), \( \sigma_i = 0 \). Then it is easy to verify that the matching conditions (35), (41), (42) hold and that with \( V_c \) as in (43), the control law for each robot (48) becomes

\[
\tau_i = B_i (\delta_i^* - \delta_i) + \sum_{j=1}^{N} A_{ij} (\delta_j - \delta_i - r_{ij}^*) - K_{v,i} \delta_i,
\]

(51)

where \( \delta_i = F_i^\top M_{1,i}^\top p_i \) are the motor velocities. When \( r_{ij} = 0 \), the control law is identical to the non-delayed case given in Nuño et al. (2014), showing how the proposed method systematically gives results without searching extensively for a Lyapunov function to prove its stability. Choosing nonzero \( r_{ij} \) generalizes the result to allow distinct arm poses, facilitating cooperative object grasping.

10.2 Underactuation-Degree One Systems

The conditions for cooperation are also satisfied by the single-agent solution for a class of mechanical systems of underactuation degree one \( (m = n - 1) \), given by Acosta et al. (2005). We refer to the original paper for the precise definitions and assumptions; here we focus primarily on the steps needed for the extension to distributed IDA-PBC. A key assumption is that certain terms, including \( F \), depend on only one coordinate, here taken to be \( q_n \). Acosta et al. (2005) give a constructive procedure to find \( M_d \) and \( J \) to satisfy (9), and give a desired potential energy of the form (11), where \( V_s(q) \) is explicitly given by:

\[
\gamma(q_n) = \int_0^{q_0} s(\mu) \gamma_n(\mu) d\mu, \quad s(q_n) = F_n \frac{\partial V}{\partial q},
\]

(55)

This satisfies (14) since, by substituting (55) into (14):

\[
F_n \frac{\partial V}{\partial q} - F_n M_d M_{1,n}^\top e_n s_n = s - \gamma^\top e_n s_n = 0,
\]

(56)
where $e_k \in \mathbb{R}^n$ is a vector of zeros except its $k$-th entry is 1. The elements of $z \in \mathbb{R}^l$, are, for $j = 1, \ldots, \ell$, $l = m$:

$$z_j = q_j - \int_0^{q_m} \gamma_m(\mu) \, d\mu,$$

which leads to a $\Psi$ matrix (13) given by

$$\Psi = [e_1 \cdots e_m] - \gamma_n^{-1} [\gamma_1 e_m \cdots \gamma_m e_n],$$

which in turn solves condition (15) because

$$F^T M_4 M^{-1} \Psi = \gamma \Psi = 0_{1 \times m},$$

Consequently, all systems considered by Acosta et al. (2005) satisfy the conditions for cooperation (9), (14), (15).

To illustrate the results that can be obtained with the distributed IDA-PBC approach, Fig. 1 shows a simulation of a network consisting of two inverted pendulum-cart systems with different bob lengths $l$, cooperating with a fully-actuated point mass, all translating on a parallel track, to obtain a formation in the horizontal direction. Each system uses the control law (48), where for the pendulum-cart systems the terms $F_i, M_i, J_i, V_i, M_{d,i}$ are taken from the worked example in Acosta et al. (2005), while the point mass has $q_3 = z_3 = x_3 \in \mathbb{R}, \ V_{s,3} = 0, \ M_{d,3} = M_3 > 0$, and $F_3 = 1$. More practical examples demonstrating formations of flexible-joint manipulators and unmanned aerial vehicles are given in Valk (2018).

![Fig. 1. Two inverted pendulums (1, 2) and a point mass (3) exchange information (dashed arrows) to achieve a formation with 0.5 m between each vehicle position $x_i$, where the leader (3) tracks the position $x^*_3 = 2.0 m$.](image)

11. CONCLUSION

We have presented a systematic procedure that yields stable, distributed control laws for undirected networks of heterogeneous underactuated and fully-actuated mechanical systems, achieving stationary formations in the generalized coordinates of interest when each system has a known IDA-PBC solution.

As future work, we aim to generalize the objective to task-space formations, and generalize the agent coupling mechanisms beyond the proposed potential energy method. The matrix $J$ can be used to distribute energy between agents without affecting the stability of the group objective (van der Schaft and Jeltsema (2014)). Another generalization is to exchange passive outputs between agents to relax the damping conditions for individual agents. We also aim to account for communication time delays as done for the special case in Nuño et al. (2014), and account for time-varying group references (Fujimoto et al. 2003)). Finally, we are currently establishing the relation with schemes such as Chopra and Spong (2006), opening up generalizations to directed communication graphs.

REFERENCES


