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# Distributed Control of Heterogeneous Underactuated Mechanical Systems

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**Abstract:** We show how passivity-based control by interconnection and damping assignment (IDA-PBC) can be used as a design procedure to derive distributed control laws for undirected connected networks of underactuated and fully-actuated heterogeneous mechanical systems. With or without leaders, agents are able to reach a stationary formation in the coordinate of interest, even if each agent has different dynamics, provided that each agent satisfies three matching conditions for cooperation. If these are satisfied, we show how existing single-system IDA-PBC solutions can be used to construct distributed control laws, thereby enabling distributed control design for a large class of applications. The procedure is illustrated for a network of flexible-joint robots and a network of heterogeneous inverted pendulum-cart systems.



*Keywords:* Distributed control, consensus, synchronization, mechanical systems, underactuated systems, passivity-based control by interconnection and damping assignment (IDA-PBC)

## 1. INTRODUCTION

In a network of cooperative mechanical systems, a typical control objective is to synchronize a subset of generalized coordinates between all systems in the network. More generally, the goal can be to obtain a formation in the coordinates of interest, either with one or more leaders that steer the formation towards a prescribed setpoint, or without leaders, such that the formation comes to rest at an arbitrary point. Each agent (system) uses a control law both to stabilize its own state and to contribute to the formation group objective, relying only on its own state information and information received from its neighbors in the network. Passivity-based control is a well-established control method for networks of fully-actuated nonlinear mechanical systems (see Chopra and Spong (2006); Arcak (2007); Ren and Cao (2011)), but few results are directly applicable if one or more agents are underactuated.

In this paper we show that passivity-based control by interconnection and damping assignment (IDA-PBC), introduced by Ortega et al. (2002), can be used to derive distributed control laws for networks of both underactuated and fully-actuated heterogeneous mechanical systems. If the communication network is undirected and connected, the agents are able to reach a stationary formation in the generalized coordinate of interest, with or without leaders. Additionally, if an IDA-PBC solution is known for each individual agent, we show that under certain conditions independent of the network topology, this solution can be used to construct the distributed control laws.

An early result for the synchronization of a simplified class of underactuated mechanical systems was given by Nair and Leonard (2008), to which our work has parallels by virtue of the close relationship between IDA-PBC and controlled Lagrangians (see Blankenstein et al. (2002)). The current work extends the synchronization objective to a formation objective, it generalizes the application from networks of homogeneous agents to networks of heterogeneous agents, and it extends the leaderless result to networks with leaders that have fixed reference coordinates to steer the group to a desired configuration.

An IDA-PBC approach was used to stabilize synchronization error dynamics in Zhu et al. (2012). While their method reduces the synchronization recovery time after a disturbance on a subsystem, the network must be a ring graph and the result is not a distributed control method, as all agents require knowledge of the absolute reference. A distributed synchronization result for networks of flexible-joint robots was presented by Nuño et al. (2014), which we show to be a special case of the presented distributed IDA-PBC method, and which can be extended to formations to allow non-identical robotic arm poses.

Sections 2 and 3 briefly review the single-agent IDA-PBC problem and Sections 4 and 5 review the necessary concepts from distributed control and graph theory. Section 6 formalizes the distributed IDA-PBC problem, while sections 7–9 give a constructive solution of the problem by providing sufficient conditions for each agent to facilitate cooperation in a network. Section 10 applies the proposed method to networks of flexible-joint robots and networks of heterogeneous systems of underactuation degree one.

## 2. IDA-PBC FOR A SINGLE MECHANICAL SYSTEM

This section reviews the method of passivity-based control by interconnection and damping assignment (IDA-PBC) when applied to a single mechanical system, as summarized by Acosta et al. (2005). The frictionless, open-loop dynamics of a mechanical system with coordinates $\boldsymbol{q} \in \mathbb{R}^n$, momenta $\boldsymbol{p} \in \mathbb{R}^n$, input $\boldsymbol{\tau} \in \mathbb{R}^m$, and output $\boldsymbol{y} \in \mathbb{R}^m$ are

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$$\begin{bmatrix} \dot{\boldsymbol{q}} \\ \dot{\boldsymbol{p}} \end{bmatrix} = \begin{bmatrix} \mathbf{0}_n & \mathbf{I}_n \\ -\mathbf{I}_n & \mathbf{0}_n \end{bmatrix} \begin{bmatrix} \dfrac{\partial H}{\partial \boldsymbol{q}} \\ \dfrac{\partial H}{\partial \boldsymbol{p}} \end{bmatrix} + \begin{bmatrix} \mathbf{0}_{n\times m} \\ \mathbf{F} \end{bmatrix} \boldsymbol{\tau}, \tag{1}$$

$$\boldsymbol{y} = \mathbf{F}^\top \mathbf{M}^{-1} \boldsymbol{p}, \tag{2}$$

$$H = \tfrac{1}{2} \boldsymbol{p}^\top \mathbf{M}^{-1} \boldsymbol{p} + V, \tag{3}$$

where $\mathbf{M}(\boldsymbol{q}) = \mathbf{M}^\top(\boldsymbol{q}) > \mathbf{0}_n$ is the generalized mass matrix and $\mathbf{F}(\boldsymbol{q}) \in \mathbb{R}^{n\times m}$ is the input matrix (rank$(\mathbf{F}) = m \leq n$). The Hamiltonian $H(\boldsymbol{q}, \boldsymbol{p}) \in \mathbb{R}$ is the sum of the kinetic energy $\frac{1}{2}\boldsymbol{p}^\top \mathbf{M}^{-1}(\boldsymbol{q})\boldsymbol{p}$ and potential energy $V(\boldsymbol{q}) \in \mathbb{R}$. All vectors are column vectors, including gradients of scalars.

In IDA-PBC for single systems, the control objective is to stabilize the state $(\boldsymbol{q}, \boldsymbol{p})$ at a desired equilibrium $(\boldsymbol{q}^*, \boldsymbol{0})$. This is accomplished by choosing the feedback control $\boldsymbol{\tau}$ such that (1)–(3) attain the desired ("d") dynamics

$$\begin{bmatrix} \dot{\boldsymbol{q}} \\ \dot{\boldsymbol{p}} \end{bmatrix} = \begin{bmatrix} \mathbf{0}_n & \mathbf{M}^{-1}\mathbf{M}_\mathrm{d} \\ -\mathbf{M}_\mathrm{d}\mathbf{M}^{-1} & \mathbf{J} - \mathbf{F}\mathbf{K}_v\mathbf{F}^\top \end{bmatrix} \begin{bmatrix} \dfrac{\partial H_\mathrm{d}}{\partial \boldsymbol{q}} \\ \dfrac{\partial H_\mathrm{d}}{\partial \boldsymbol{p}} \end{bmatrix}, \tag{4}$$

$$\boldsymbol{y}_\mathrm{d} = \mathbf{F}^\top \mathbf{M}_\mathrm{d}^{-1} \boldsymbol{p}, \tag{5}$$

$$H_\mathrm{d} = \tfrac{1}{2} \boldsymbol{p}^\top \mathbf{M}_\mathrm{d}^{-1} \boldsymbol{p} + V_\mathrm{d}, \tag{6}$$

where the desired mass matrix $\mathbf{M}_\mathrm{d}(\boldsymbol{q}) = \mathbf{M}_\mathrm{d}^\top(\boldsymbol{q})$ is positive definite near $\boldsymbol{q}^*$ and the desired potential energy $V_\mathrm{d}(\boldsymbol{q})$ is chosen such that the desired Hamiltonian $H_\mathrm{d}$ is locally minimal at the desired equilibrium:

$$\boldsymbol{q}^* = \arg\min V_\mathrm{d}(\boldsymbol{q}). \tag{7}$$

The damping matrix satisfies $\mathbf{K}_v > \mathbf{0}_m$ and the matrix $\mathbf{J} = -\mathbf{J}^\top \in \mathbb{R}^{n\times n}$ is free. (We use "$\mathbf{J}$" instead of the commonly used "$\mathbf{J}_2$" to avoid confusion with other subscripts.) The setpoint $(\boldsymbol{q}^*, \boldsymbol{0})$ is an asymptotically stable equilibrium of the dynamics (4), where the main argument is that $H_\mathrm{d}$ (6) is positive definite near the setpoint and its time derivative along (4) equals $\frac{\mathrm{d}}{\mathrm{d}t} H_\mathrm{d} = -\boldsymbol{y}_\mathrm{d}^\top \mathbf{K}_v \boldsymbol{y}_\mathrm{d} \leq 0$. The complete proof is summarized in Acosta et al. (2005).

The desired dynamics (4)–(6) are obtained by setting them equal to (1)–(3) and solving for $\boldsymbol{\tau}(\boldsymbol{q}, \boldsymbol{p})$, which gives the single-agent IDA-PBC feedback law as

$$\boldsymbol{\tau} = \left(\mathbf{F}^\top\mathbf{F}\right)^{-1}\mathbf{F}^\top\left(\frac{\partial H}{\partial \boldsymbol{q}} - \mathbf{M}_\mathrm{d}\mathbf{M}^{-1}\frac{\partial H_\mathrm{d}}{\partial \boldsymbol{q}} + \mathbf{J}\mathbf{M}_\mathrm{d}^{-1}\boldsymbol{p}\right) - \mathbf{K}_v \boldsymbol{y}_\mathrm{d}. \tag{8}$$

For underactuated systems, this law yields the dynamics (4)–(6) only if the kinetic energy matching equation

$$\mathbf{F}^\perp \frac{\partial}{\partial \boldsymbol{q}}\left(\boldsymbol{p}^\top \mathbf{M}^{-1} \boldsymbol{p}\right) - \mathbf{F}^\perp \mathbf{M}_\mathrm{d}\mathbf{M}^{-1} \frac{\partial}{\partial \boldsymbol{q}}\left(\boldsymbol{p}^\top \mathbf{M}_\mathrm{d}^{-1} \boldsymbol{p}\right) + 2\mathbf{F}^\perp \mathbf{J}\mathbf{M}_\mathrm{d}^{-1}\boldsymbol{p} = \boldsymbol{0}, \tag{9}$$

and the potential energy matching equation

$$\mathbf{F}^\perp \left(\frac{\partial V_i}{\partial \boldsymbol{q}} - \mathbf{M}_\mathrm{d}\mathbf{M}^{-1}\frac{\partial V_\mathrm{d}}{\partial \boldsymbol{q}}\right) = \boldsymbol{0}, \tag{10}$$

both hold, for the annihilator $\mathbf{F}^\perp$ with $\mathbf{F}^\perp\mathbf{F} = \mathbf{0}_{(n-m)\times m}$. In fully-actuated systems $\mathbf{F}$ is full rank and (8) yields (4)–(6) without the need to satisfy matching conditions.

While setpoint tracking primarily requires potential energy shaping of $V_\mathrm{d}$ to satisfy the minimality condition (7), it is usually also necessary to shape the kinetic energy through $\mathbf{M}_\mathrm{d}$ and assign gyroscopic forces through $\mathbf{J}$, in order to satisfy the matching conditions (9), (10). Solving this problem is challenging in general. (See Ortega et al. (2017) for an historic overview and recent developments.) Constructive solutions have been given for special classes of mechanical systems, such as those with only one degree of underactuation in Acosta et al. (2005).

## 3. DESIRED POTENTIAL ENERGY STRUCTURE

An agent in the network has two non-conflicting control objectives, each pertaining to a subset of its generalized coordinates, partitioned as $\boldsymbol{q} = (\boldsymbol{x}, \boldsymbol{\theta}) \in \mathbb{R}^n$. The coordinates $\boldsymbol{x} \in \mathbb{R}^\ell$ are to be controlled in cooperation with other agents in the network, while $\boldsymbol{\theta} \in \mathbb{R}^{n-\ell}$ are controlled by each agent individually. Before considering a network of systems, we consider how these control goals appear in the single-agent solution, where the goal is to reach the setpoint $\boldsymbol{q}^* = (\boldsymbol{x}^*, \boldsymbol{\theta}^*)$, for prescribed values $\boldsymbol{x}^*$ and $\boldsymbol{\theta}^*$.

In some IDA-PBC solutions, the objectives to reach $\boldsymbol{x}^*$ and $\boldsymbol{\theta}^*$ can be alternatively represented using a new coordinate $\boldsymbol{z}(\boldsymbol{q}) = \boldsymbol{z}(\boldsymbol{x}, \boldsymbol{\theta}) \in \mathbb{R}^\ell$, chosen such that achieving $\boldsymbol{\theta} = \boldsymbol{\theta}^*$ and $\boldsymbol{z} = \boldsymbol{z}^*$ also implies that $\boldsymbol{x} = \boldsymbol{x}^*$. The choice of $\boldsymbol{z}$ ensures that the control signal to stabilize $\boldsymbol{z}$ does not violate the matching conditions, which is crucial for expressing interaction forces between agents in the network later on. Specifically, we use existing IDA-PBC solutions in which the desired potential energy can be written as

$$V_\mathrm{d}(\boldsymbol{q}) = V_\mathrm{s}(\boldsymbol{q}) + V_\mathrm{c}(\boldsymbol{z}(\boldsymbol{q})), \tag{11}$$

where $\boldsymbol{z}(\boldsymbol{q}) \in \mathbb{R}^\ell$, $\ell \leq m$, and the cooperation potential $V_\mathrm{c}$ is free in $\boldsymbol{z}$ as long as $V_\mathrm{d}$ remains positive definite around the setpoint $\boldsymbol{q}^*$. Then we can write its gradient as

$$\frac{\partial V_\mathrm{d}}{\partial \boldsymbol{q}} = \frac{\partial V_\mathrm{s}}{\partial \boldsymbol{q}} + \boldsymbol{\Psi}\frac{\partial V_\mathrm{c}}{\partial \boldsymbol{z}}, \tag{12}$$

where $\frac{\partial V_\mathrm{c}}{\partial \boldsymbol{z}}$ depends only on $\boldsymbol{z}$ and $\boldsymbol{\Psi}$ depends only on $\boldsymbol{q}$:

$$\boldsymbol{\Psi}(\boldsymbol{q}) = \begin{bmatrix} \dfrac{\partial z_1}{\partial \boldsymbol{q}} & \cdots & \dfrac{\partial z_\ell}{\partial \boldsymbol{q}} \end{bmatrix} \in \mathbb{R}^{n\times\ell}. \tag{13}$$

In solutions of the form (11), the potential energy condition (10) is implicitly split up in two matching conditions:

$$\mathbf{F}^\perp \left(\frac{\partial V}{\partial \boldsymbol{q}} - \mathbf{M}_\mathrm{d}\mathbf{M}^{-1}\frac{\partial V_\mathrm{s}}{\partial \boldsymbol{q}}\right) = \boldsymbol{0}, \tag{14}$$

$$\mathbf{F}^\perp \mathbf{M}_\mathrm{d}\mathbf{M}^{-1}\boldsymbol{\Psi} = \mathbf{0}_{(n-m)\times\ell}. \tag{15}$$

Although requiring (14), (15) to hold is more conservative than (10), it ensures that $V_\mathrm{c}(\cdot)$ is free in $\boldsymbol{z}$, which is crucial in our solution of the distributed IDA-PBC problem.

The term $V_\mathrm{s}$ stabilizes the coordinates $\boldsymbol{\theta}$ to their fixed setpoint $\boldsymbol{\theta}^*$, subject to matching condition (14), while $V_\mathrm{c}$ steers the coordinates $\boldsymbol{x}$ to the desired setpoint, subject to matching condition (15). For example, in a pendulum-cart system where $\boldsymbol{q} = [x\ \ \theta]^\top$ and $z(x, \theta) \in \mathbb{R}$, $V_\mathrm{s}$ stabilizes the pendulum angle $\theta$ at 0 while $V_\mathrm{c}$ makes the cart position $x$ converge to the setpoint $x^*$ by steering $z(x, \theta)$ to $z(x^*, 0)$.

Examples of IDA-PBC solutions of the form (11)–(15) with explicit descriptions of $\boldsymbol{z}(\boldsymbol{q})$ are given in Acosta et al. (2005) and Ryalat and Laila (2016) for systems of underactuation degree one, but solutions are not limited to this class. For example, a fully-actuated point mass with $\boldsymbol{q} \in \mathbb{R}^3$ might use the term $V_\mathrm{s}(\boldsymbol{q})$ to stabilize $\boldsymbol{\theta} = [q_1\ \ q_3]^\top$ at $\boldsymbol{\theta}^* = [q_1^*\ \ q_3^*]^\top$ and use $V_\mathrm{c}(z(\boldsymbol{q}))$ to steer $q_2$ to $q_2^*$. In this case, $n = m = 3$, $\ell = 1$, and $z(\boldsymbol{q}) = x = q_2 \in \mathbb{R}$.

## 4. NETWORKS OF MECHANICAL SYSTEMS

### 4.1 Uncontrolled Network Dynamics

Consider a network of $N$ agents, where each agent has the dynamics (1)–(3), given explicitly for each agent $i$ as

$$\begin{bmatrix}\dot{\boldsymbol{q}}_i\\ \dot{\boldsymbol{p}}_i\end{bmatrix} = \begin{bmatrix}\mathbf{0}_{n_i} & \mathbf{I}_{n_i}\\ -\mathbf{I}_{n_i} & \mathbf{0}_{n_i}\end{bmatrix}\begin{bmatrix}\dfrac{\partial H_i}{\partial \boldsymbol{q}_i}\\ \dfrac{\partial H_i}{\partial \boldsymbol{p}_i}\end{bmatrix} + \begin{bmatrix}\mathbf{0}_{n_i\times m_i}\\ \mathbf{F}_i\end{bmatrix}\boldsymbol{\tau}_i, \tag{16}$$

$$\boldsymbol{y}_i = \mathbf{F}_i^\top \mathbf{M}_i^{-1}\boldsymbol{p}_i, \tag{17}$$

$$H_i = \tfrac{1}{2}\boldsymbol{p}_i^\top \mathbf{M}_i^{-1}\boldsymbol{p}_i + V_i. \tag{18}$$

As before, $\boldsymbol{q}_i \in \mathbb{R}^{n_i}$, $\boldsymbol{p}_i \in \mathbb{R}^{n_i}$, $\boldsymbol{\tau}_i \in \mathbb{R}^{m_i}$, $\mathbf{F}_i(\boldsymbol{q}_i) \in \mathbb{R}^{n_i\times m_i}$, $\mathbf{M}_i(\boldsymbol{q}_i) = \mathbf{M}_i^\top(\boldsymbol{q}_i) > \mathbf{0}_{n_i}$, and $m_i \leq n_i$. The dimensions $n_i$ and $m_i$ may be different for each agent. The dynamics of all agents can be written as one simple mechanical system:

$$\begin{bmatrix}\dot{\bar{\boldsymbol{q}}}\\ \dot{\bar{\boldsymbol{p}}}\end{bmatrix} = \begin{bmatrix}\mathbf{0}_{\bar{n}} & \mathbf{I}_{\bar{n}}\\ -\mathbf{I}_{\bar{n}} & \mathbf{0}_{\bar{n}}\end{bmatrix}\begin{bmatrix}\dfrac{\partial \bar{H}}{\partial \bar{\boldsymbol{q}}}\\ \dfrac{\partial \bar{H}}{\partial \bar{\boldsymbol{p}}}\end{bmatrix} + \begin{bmatrix}\mathbf{0}_{\bar{n}\times\bar{m}}\\ \bar{\mathbf{F}}\end{bmatrix}\bar{\boldsymbol{\tau}}, \tag{19}$$

$$\bar{\boldsymbol{y}} = \bar{\mathbf{F}}^\top\bar{\mathbf{M}}^{-1}\bar{\boldsymbol{p}}, \tag{20}$$

$$\bar{H} = \tfrac{1}{2}\bar{\boldsymbol{p}}^\top\bar{\mathbf{M}}^{-1}\bar{\boldsymbol{p}} + \bar{V}, \tag{21}$$

where the corresponding network terms are given by

$$\begin{aligned}
&\bar{n} = \sum_{i=1}^N n_i, \quad \bar{m} = \sum_{i=1}^N m_i, \quad \bar{V} = \sum_{i=1}^N V_i,\\
&\bar{\boldsymbol{q}} = \begin{bmatrix}\boldsymbol{q}_1\\ \vdots\\ \boldsymbol{q}_N\end{bmatrix}, \quad \bar{\boldsymbol{p}} = \begin{bmatrix}\boldsymbol{p}_1\\ \vdots\\ \boldsymbol{p}_N\end{bmatrix}, \quad \bar{\boldsymbol{\tau}} = \begin{bmatrix}\boldsymbol{\tau}_1\\ \vdots\\ \boldsymbol{\tau}_N\end{bmatrix},\\
&\bar{\boldsymbol{y}} = \begin{bmatrix}\boldsymbol{y}_1\\ \vdots\\ \boldsymbol{y}_N\end{bmatrix}, \quad \bar{\mathbf{M}} = \begin{bmatrix}\mathbf{M}_1 & & \\ & \ddots & \\ & & \mathbf{M}_N\end{bmatrix}, \quad \bar{\mathbf{F}} = \begin{bmatrix}\mathbf{F}_1 & & \\ & \ddots & \\ & & \mathbf{F}_N\end{bmatrix}.
\end{aligned} \tag{22}$$

There is no physical contact between the systems, but interaction arises due to their control signals. Generally, the control law $\boldsymbol{\tau}_i$ of agent $i$ can be a function of its own state and of information it receives from other agents.

### 4.2 Modeling Communication on Graphs

Communication between agents can be modeled using properties from graph theory (see Ren and Cao (2011)). Each agent $i$ is a node of a graph. Agent $i$ can send information to agent $j$ if there exists an edge $(i, j)$ between nodes $i$ and $j$ with a weight $\mathcal{A}_{ij} > 0$. In this paper we consider only undirected graphs, where information flow is bidirectional: if the edge $(i, j)$ exists then $(j, i)$ also exists, and $\mathcal{A}_{ij} = \mathcal{A}_{ji} > 0$. In this case agent $i$ and $j$ are neighbors. Self edges are not allowed: $\mathcal{A}_{ii} = 0$. If there are no edges between nodes $i$ and $j$ then $\mathcal{A}_{ij} = \mathcal{A}_{ji} = 0$ and the two agents cannot exchange information.

An edge sequence of the form $(i, k), (k, j), \ldots, (z, y), (y, a)$ is called a path. A graph is connected if there is a path between every pair of nodes. The graph can be compactly described by the Laplacian matrix $\boldsymbol{\mathcal{L}} \in \mathbb{R}^{N\times N}$, defined by $\mathcal{L}_{ii} = \sum_{j=1}^N \mathcal{A}_{ij}$ and $\mathcal{L}_{ij} = -\mathcal{A}_{ij}$ if $i \neq j$. If the graph is connected, then $\boldsymbol{\mathcal{L}}$ has one zero eigenvalue and its remaining eigenvalues are positive, such that $\boldsymbol{\mathcal{L}} \geq \mathbf{0}_N$.

### 4.3 Local and Group Objectives

The overall objective is to have each agent $i$ stabilize its own coordinates $\boldsymbol{\theta}_i \in \mathbb{R}^{n_i-\ell}$ at $\boldsymbol{\theta}_i^*$ while achieving a desired stationary formation between the agents in the coordinates $\boldsymbol{x}_i \in \mathbb{R}^\ell$. As in the single-agent case, the latter objective can be represented as a formation in $\boldsymbol{z}_i \in \mathbb{R}^\ell$. A formation is a configuration where each pair of neighboring agents $i$ and $j$ reaches a desired difference $\boldsymbol{r}_{ij}^* = \boldsymbol{z}_j^* - \boldsymbol{z}_i^*$. We show that this can be accomplished if each agent communicates only the variable $\boldsymbol{z}_i$ with its neighbors.

An agent can be a leader or a follower. If agent $i$ is a follower, it knows only the desired inter-agent differences $\boldsymbol{r}_{ij}^*$ and we define $\mathcal{B}_i = 0$. One or more leaders may also know their target $\boldsymbol{z}_i^*$, compatible with the distances $\boldsymbol{r}_{ij}^*$, in which case $\mathcal{B}_i > 0$. Denoting $\boldsymbol{\mathcal{B}} = \mathrm{diag}(\mathcal{B}_1, \ldots, \mathcal{B}_N)$, then $\boldsymbol{\mathcal{L}} + \boldsymbol{\mathcal{B}} > \mathbf{0}_N$ for a connected graph with at least one leader (Ren and Cao (2011)). In summary, the objectives become:

$$\lim_{t\to\infty} ||\dot{\boldsymbol{q}}_i(t)|| = 0 \quad \forall\, i = 1, \ldots, N, \tag{23}$$

$$\lim_{t\to\infty} ||\boldsymbol{\theta}_i(t) - \boldsymbol{\theta}_i^*|| = 0 \quad \forall\, i = 1, \ldots, N, \tag{24}$$

$$\lim_{t\to\infty} ||\boldsymbol{z}_i(t) - \boldsymbol{z}_j(t) + \boldsymbol{r}_{ij}^*|| = 0 \quad \forall\, i, j \mid \mathcal{A}_{ij} > 0, \tag{25}$$

$$\lim_{t\to\infty} ||\boldsymbol{z}_i(t) - \boldsymbol{z}_i^*|| = 0 \quad \forall\, i \mid \mathcal{B}_i > 0. \tag{26}$$

The corresponding desired equilibrium is denoted $\bar{\boldsymbol{q}}^*$. If there is at least one leader, the formation must come to standstill at a unique point compatible with the targets $\boldsymbol{z}_i^*$. If there are no leaders, the formation comes to standstill at an arbitrary point. If all $\boldsymbol{r}_{ij}^*$ are zero, then the formation goal simplifies to synchronization, sometimes called consensus or agreement, again with or without leaders.

## 5. TOP-DOWN DISTRIBUTED CONTROL

Typical passivity-based distributed control methods follow bottom-up design approaches, which consider how passive systems can be interconnected to preserve passivity. Weighted sums of the energy functions of each system are used as candidate Lyapunov functions in order to assess closed-loop stability (Chopra and Spong (2006); Arcak (2007); Ren and Cao (2011)). This approach is especially successful for networks of fully-actuated systems like robot manipulators, where internal control laws render each system passive with respect to an output that ensures synchronization of both generalized coordinates and velocities between systems. This approach does not easily generalize to underactuated systems, complicating the procedure of finding internal control laws and stable interconnections.

This paper uses a top-down approach instead, starting from a class of stable desired dynamics for the whole network, and deriving the distributed control laws and interconnection conditions to preserve stability. Any remaining degrees of freedom can be used to address the transient response of the network and its subsystems. This approach is well-suited for networks of fully-actuated and underactuated systems, and combinations thereof. Whereas choosing a desired class of dynamics may appear more restrictive than allowing arbitrary dynamics and Lyapunov functions, the structure of the solution reveals both the potential force and damping mechanisms commonly found in a bottom-up approach, but also non-trivial gyroscopic coupling forces while preserving stability.

## 6. DISTRIBUTED IDA-PBC PROBLEM

As in the single-agent case, the IDA-PBC strategy defines the control $\bar{\boldsymbol{\tau}}$ that changes the uncontrolled network dynamics (19)–(21) into the asymptotically stable dynamics

$$\begin{bmatrix}\dot{\bar{\boldsymbol{q}}}\\ \dot{\bar{\boldsymbol{p}}}\end{bmatrix} = \begin{bmatrix}\mathbf{0}_{\bar{n}} & \bar{\mathbf{M}}^{-1}\bar{\mathbf{M}}_{\mathrm{d}}\\ -\bar{\mathbf{M}}_{\mathrm{d}}\bar{\mathbf{M}}^{-1} & \bar{\mathbf{J}}-\bar{\mathbf{F}}\bar{\mathbf{K}}_v\bar{\mathbf{F}}^\top\end{bmatrix}\begin{bmatrix}\dfrac{\partial \bar{H}_{\mathrm{d}}}{\partial \bar{\boldsymbol{q}}}\\ \dfrac{\partial \bar{H}_{\mathrm{d}}}{\partial \bar{\boldsymbol{p}}}\end{bmatrix}, \tag{27}$$

$$\bar{\boldsymbol{y}}_{\mathrm{d}} = \bar{\mathbf{F}}^\top\frac{\partial \bar{H}_{\mathrm{d}}}{\partial \bar{\boldsymbol{p}}} = \bar{\mathbf{F}}^\top\bar{\mathbf{M}}_{\mathrm{d}}^{-1}\bar{\boldsymbol{p}}, \tag{28}$$

$$\bar{H}_{\mathrm{d}} = \tfrac{1}{2}\bar{\boldsymbol{p}}^\top\bar{\mathbf{M}}_{\mathrm{d}}^{-1}\bar{\boldsymbol{p}} + \bar{V}_{\mathrm{d}}, \tag{29}$$

where $\bar{\mathbf{M}}_{\mathrm{d}} > \mathbf{0}_{\bar{n}}$, $\bar{V}_{\mathrm{d}} \in \mathbb{R}$, $\bar{\mathbf{J}} = -\bar{\mathbf{J}}^\top \in \mathbb{R}^{\bar{n}\times\bar{n}}$ and $\bar{\mathbf{K}}_v > 0$ are to be designed to address the control objectives (23)–(26) and the transient response. Similar to (7), we now require

$$\bar{\boldsymbol{q}}^* = \arg\min \bar{V}_{\mathrm{d}}(\bar{\boldsymbol{q}}). \tag{30}$$

The desired dynamics are obtained using the IDA-PBC control law (8) applied to the network of systems, giving

$$\bar{\boldsymbol{\tau}} = \left(\bar{\mathbf{F}}^\top\bar{\mathbf{F}}\right)^{-1}\bar{\mathbf{F}}^\top\left(\frac{\partial \bar{H}}{\partial \bar{\boldsymbol{q}}} - \bar{\mathbf{M}}_{\mathrm{d}}\bar{\mathbf{M}}^{-1}\frac{\partial \bar{H}_{\mathrm{d}}}{\partial \bar{\boldsymbol{q}}} + \bar{\mathbf{J}}\bar{\mathbf{M}}_{\mathrm{d}}^{-1}\bar{\boldsymbol{p}}\right) - \bar{\mathbf{K}}_v\bar{\boldsymbol{y}}_{\mathrm{d}} \tag{31}$$

if the distributed kinetic energy matching condition

$$\bar{\mathbf{F}}^\perp\frac{\partial}{\partial \bar{\boldsymbol{q}}}\left(\bar{\boldsymbol{p}}^\top\bar{\mathbf{M}}^{-1}\bar{\boldsymbol{p}}\right) - \bar{\mathbf{F}}^\perp\bar{\mathbf{M}}_{\mathrm{d}}\bar{\mathbf{M}}^{-1}\frac{\partial}{\partial \bar{\boldsymbol{q}}}\left(\bar{\boldsymbol{p}}^\top\bar{\mathbf{M}}_{\mathrm{d}}^{-1}\bar{\boldsymbol{p}}\right) + 2\bar{\mathbf{F}}^\perp\bar{\mathbf{J}}\bar{\mathbf{M}}_{\mathrm{d}}^{-1}\bar{\boldsymbol{p}} = \boldsymbol{0}, \tag{32}$$

and the distributed potential energy matching condition

$$\bar{\mathbf{F}}^\perp\left(\frac{\partial \bar{V}}{\partial \bar{\boldsymbol{q}}} - \bar{\mathbf{M}}_{\mathrm{d}}\bar{\mathbf{M}}^{-1}\frac{\partial \bar{V}_{\mathrm{d}}}{\partial \bar{\boldsymbol{q}}}\right) = \boldsymbol{0}, \tag{33}$$

both hold.

## 7. SUFFICIENT CONDITIONS FOR COOPERATION

Despite the large degree of freedom in choosing the stability-preserving interconnection mechanisms, we show that for systems of the class (11)–(15) it is sufficient to shape the potential energy of the interconnections to obtain the desired group objectives (25), (26). The internal objectives (23), (24) can be addressed by choosing

$$\bar{\mathbf{F}}^\perp = \begin{bmatrix}\mathbf{F}_1^\perp & & \\ & \ddots & \\ & & \mathbf{F}_N^\perp\end{bmatrix}, \quad \bar{\mathbf{M}}_{\mathrm{d}} = \begin{bmatrix}\mathbf{M}_{\mathrm{d},1} & & \\ & \ddots & \\ & & \mathbf{M}_{\mathrm{d},N}\end{bmatrix},$$
$$\bar{\mathbf{J}} = \begin{bmatrix}\mathbf{J}_1 & & \\ & \ddots & \\ & & \mathbf{J}_N\end{bmatrix}, \quad \bar{\mathbf{K}}_v = \begin{bmatrix}\mathbf{K}_{v,1} & & \\ & \ddots & \\ & & \mathbf{K}_{v,N}\end{bmatrix}, \tag{34}$$

where $\mathbf{F}_i^\perp(\boldsymbol{q}_i)$, $\mathbf{M}_{\mathrm{d},i}(\boldsymbol{q}_i)$, $\mathbf{J}_i(\boldsymbol{q}_i,\boldsymbol{p}_i)$ and $\mathbf{K}_{v,i}$ are taken from single-agent IDA-PBC solutions. Substituting these into matching condition (32) yields

$$\mathbf{F}_i^\perp\frac{\partial}{\partial \boldsymbol{q}_i}\left(\boldsymbol{p}_i^\top\mathbf{M}_i^{-1}\boldsymbol{p}_i\right) - \mathbf{F}_i^\perp\mathbf{M}_{\mathrm{d},i}\mathbf{M}_i^{-1}\frac{\partial}{\partial \boldsymbol{q}_i}\left(\boldsymbol{p}_i^\top\mathbf{M}_{\mathrm{d},i}^{-1}\boldsymbol{p}_i\right) + 2\mathbf{F}_i^\perp\mathbf{J}_i\mathbf{M}_{\mathrm{d},i}^{-1}\boldsymbol{p}_i = \boldsymbol{0} \quad \forall\, i = 1,\ldots,N, \tag{35}$$

which are $N$ separate matching conditions, each identical to the single-agent kinetic energy matching condition (9), and solved if each agent has a known IDA-PBC solution. Likewise, inserting the choices (34) into the networked potential energy matching condition (33) yields

$$\mathbf{F}_i^\perp\left(\frac{\partial V_i}{\partial \boldsymbol{q}_i} - \mathbf{M}_{\mathrm{d},i}\mathbf{M}_i^{-1}\frac{\partial \bar{V}_{\mathrm{d}}}{\partial \boldsymbol{q}_i}\right) = \boldsymbol{0} \quad \forall\, i = 1,\ldots,N. \tag{36}$$

This condition is not trivially solved because the desired potential energy $\bar{V}_{\mathrm{d}}(\boldsymbol{q})$ depends on the coordinates of all agents. We propose a desired potential energy of the form

$$\bar{V}_{\mathrm{d}}(\bar{\boldsymbol{q}}) = \bar{V}_{\mathrm{c}}(\boldsymbol{z}_1(\boldsymbol{q}_1),\ldots,\boldsymbol{z}_N(\boldsymbol{q}_N)) + \sum_{i=1}^N V_{\mathrm{s},i}(\boldsymbol{q}_i), \tag{37}$$

where the $V_{\mathrm{s},i}(\boldsymbol{q}_i)$ are equal to the internal stabilization component in the single-agent potential energy (11), while $\bar{V}_{\mathrm{c}}$ is a free function in the $\boldsymbol{z}_i(\boldsymbol{q}_i) \in \mathbb{R}^\ell$ variables of all agents. In order to show that (37) solves (36) we first write

$$\frac{\partial \bar{V}_{\mathrm{c}}}{\partial \boldsymbol{q}_i} = \boldsymbol{\Psi}_i\frac{\partial \bar{V}_{\mathrm{c}}}{\partial \boldsymbol{z}_i}, \tag{38}$$

in which

$$\boldsymbol{\Psi}_i(\boldsymbol{q}_i) = \begin{bmatrix}\dfrac{\partial \boldsymbol{z}_{1,i}}{\partial \boldsymbol{q}_i} & \cdots & \dfrac{\partial \boldsymbol{z}_{\ell,i}}{\partial \boldsymbol{q}_i}\end{bmatrix} \in \mathbb{R}^{n_i\times\ell}, \tag{39}$$

where $z_{k,i}$ is the $k$-th element of the vector $\boldsymbol{z}_i$, each of which depends only on $\boldsymbol{q}_i$. Then (36) becomes

$$\mathbf{F}_i^\perp\left(\frac{\partial V_i}{\partial \boldsymbol{q}_i} - \mathbf{M}_{\mathrm{d},i}\mathbf{M}_i^{-1}\frac{\partial V_{\mathrm{s},i}}{\partial \boldsymbol{q}_i} - \mathbf{M}_{\mathrm{d},i}\mathbf{M}_i^{-1}\boldsymbol{\Psi}_i\frac{\partial \bar{V}_{\mathrm{c}}}{\partial \boldsymbol{z}_i}\right) = \boldsymbol{0} \quad \forall\, i = 1,\ldots,N. \tag{40}$$

Consequently, if each agent satisfies the separated potential energy matching conditions (14) and (15), that is

$$\mathbf{F}_i^\perp\left(\frac{\partial V_i}{\partial \boldsymbol{q}_i} - \mathbf{M}_{\mathrm{d},i}\mathbf{M}_i^{-1}\frac{\partial V_{\mathrm{s},i}}{\partial \boldsymbol{q}_i}\right) = \boldsymbol{0} \quad \forall\, i = 1,\ldots,N, \tag{41}$$

$$\mathbf{F}_i^\perp\mathbf{M}_{\mathrm{d},i}\mathbf{M}_i^{-1}\boldsymbol{\Psi}_i = \mathbf{0}_{(n_i-m_i)\times\ell} \quad \forall\, i = 1,\ldots,N, \tag{42}$$

then through (40) and (36), the distributed potential energy matching condition (33) holds.

Therefore, the original network dynamics (19)–(21) and the desired dynamics (27)–(29) match independently of the network topology for the choices (34), (37), provided each agent satisfies the matching conditions (9), (14), (15). Because the conditions are local to each agent, no communication is required to guarantee matching, enhancing robustness against communication delays or switching network topologies. Matching still holds if the agents are heterogeneous, whether they have different parameter values, different dynamics, or a different number of coordinates.

## 8. COUPLING THROUGH POTENTIAL ENERGY

The potential energy of the network $\bar{V}_{\mathrm{d}}$ (37) must be minimal (30) when the local and group objectives (23)–(26) are achieved. Although the matching conditions are decoupled, the systems are coupled through the free cooperative potential energy function $\bar{V}_{\mathrm{c}}$ in (37), which through the control law (31) gives rise to control forces that steer the systems towards their cooperative goal (25), (26).

One possible coupling energy $\bar{V}_{\mathrm{c}}$ is the squared sum of the deviation from the control goals (25), (26), which gives

$$\bar{V}_{\mathrm{c}}(\bar{\boldsymbol{z}}) = \frac{1}{4}\sum_{i=1}^N\sum_{j=1}^N \mathcal{A}_{ij}||\boldsymbol{z}_i - \boldsymbol{z}_j + \boldsymbol{r}_{ij}^*||^2 + \frac{1}{2}\sum_{i=1}^N \mathcal{B}_i||\boldsymbol{z}_i - \boldsymbol{z}_i^*||^2. \tag{43}$$

Its gradient with respect to the $\boldsymbol{z}_i$ variables is given by

$$\frac{\partial \bar{V}_c}{\partial \boldsymbol{z}_i} = \mathcal{B}_i(\boldsymbol{z}_i - \boldsymbol{z}_i^*) + \sum_{j=1}^{N} \mathcal{A}_{ij}(\boldsymbol{z}_i - \boldsymbol{z}_j + \boldsymbol{r}_{ij}^*). \tag{44}$$

For appropriate constants $\boldsymbol{c}_1 \in \mathbb{R}^{N\ell}$ and $c_0 \in \mathbb{R}$, the potential (43) can be written as the quadratic form

$$\bar{V}_c = \tfrac{1}{2}\bar{\boldsymbol{z}}^\top(\bar{\boldsymbol{\mathcal{L}}} + \bar{\boldsymbol{\mathcal{B}}})\bar{\boldsymbol{z}} + \boldsymbol{c}_1^\top \bar{\boldsymbol{z}} + c_0, \tag{45}$$

with $\bar{\boldsymbol{\mathcal{L}}} = \boldsymbol{\mathcal{L}} \otimes \mathbf{I}_\ell$ and $\bar{\boldsymbol{\mathcal{B}}} = \boldsymbol{\mathcal{B}} \otimes \mathbf{I}_\ell$, where $\otimes$ denotes the Kronecker product. The eigenvalues of $(\bar{\boldsymbol{\mathcal{L}}} + \bar{\boldsymbol{\mathcal{B}}})$ are equal to those of $(\boldsymbol{\mathcal{L}} + \boldsymbol{\mathcal{B}})$, each value with multiplicity $\ell$ (Bellman (1960)). Hence, if the graph is connected and there is at least one leader, $\bar{\boldsymbol{\mathcal{L}}} + \bar{\boldsymbol{\mathcal{B}}} > \mathbf{0}_{N\ell}$, such that the coupling potential energy (43) is positive definite around its unique minimum satisfying (25), (26). If there are no leaders, $\bar{\boldsymbol{\mathcal{L}}} + \bar{\boldsymbol{\mathcal{B}}} = \bar{\boldsymbol{\mathcal{L}}} \geq \mathbf{0}_{N\ell}$ and (43) is positive semi-definite around a range of minima, all satisfying (25), (26). The total potential energy (37) attains a minimum if additionally (24) is satisfied, relying on the single-agent solutions. Finally, the total energy (29) attains a minimum if (23) is also satisfied, when all agents are stationary.

## 9. DISTRIBUTED CONTROL LAW

The control laws for each agent are derived from (31) by substituting the previously made choices given in (34):

$$\begin{aligned}\boldsymbol{\tau}_i = \left(\mathbf{F}_i^\top \mathbf{F}_i\right)^{-1}\mathbf{F}_i^\top \left(\frac{\partial H_i}{\partial \boldsymbol{q}_i} - \mathbf{M}_{\mathrm{d},i}\mathbf{M}_i^{-1}\frac{\partial \bar{H}_\mathrm{d}}{\partial \boldsymbol{q}_i} + \mathbf{J}_i \mathbf{M}_{\mathrm{d},i}^{-1}\boldsymbol{p}_i\right)\\ - \mathbf{K}_{v,i}\boldsymbol{y}_{\mathrm{d},i},\end{aligned} \tag{46}$$

where $\boldsymbol{y}_{\mathrm{d},i} = \mathbf{F}_i^\top \mathbf{M}_{\mathrm{d},i}^{-1}\boldsymbol{p}_i$ as in (5). In order to proceed substituting the coupling potential $\bar{V}_c$ (37), we first write

$$\frac{\partial \bar{H}_\mathrm{d}}{\partial \boldsymbol{q}_i} = \frac{1}{2}\frac{\partial}{\partial \boldsymbol{q}_i}\left(\boldsymbol{p}_i^\top \mathbf{M}_{\mathrm{d},i}^{-1}\boldsymbol{p}_i\right) + \frac{\partial V_{\mathrm{s},i}}{\partial \boldsymbol{q}_i} + \boldsymbol{\Psi}_i\frac{\partial \bar{V}_c}{\partial \boldsymbol{z}_i}, \tag{47}$$

at which point the quadratic potential energy gradient (44) is substituted to obtain the control law

$$\begin{aligned}\boldsymbol{\tau}_i &= \boldsymbol{\sigma}_i - \boldsymbol{\Phi}_i\frac{\partial \bar{V}_c}{\partial \boldsymbol{z}_i} - \mathbf{K}_{v,i}\boldsymbol{y}_{\mathrm{d},i}\\ &= \boldsymbol{\sigma}_i + \boldsymbol{\Phi}_i\left(\mathcal{B}_i(\boldsymbol{z}_i^* - \boldsymbol{z}_i) + \sum_{j=1}^{N}\mathcal{A}_{ij}(\boldsymbol{z}_j - \boldsymbol{z}_i - \boldsymbol{r}_{ij}^*)\right)\\ &\quad - \mathbf{K}_{v,i}\boldsymbol{y}_{\mathrm{d},i},\end{aligned} \tag{48}$$

where

$$\begin{aligned}\boldsymbol{\sigma}_i(\boldsymbol{q}_i,\boldsymbol{p}_i) &= \left(\mathbf{F}_i^\top \mathbf{F}_i\right)^{-1}\mathbf{F}_i^\top\left(\frac{\partial H_i}{\partial \boldsymbol{q}_i} + \mathbf{J}_i\mathbf{M}_{\mathrm{d},i}^{-1}\boldsymbol{p}_i\right)\\ &\quad - \left(\mathbf{F}_i^\top \mathbf{F}_i\right)^{-1}\mathbf{F}_i^\top \mathbf{M}_{\mathrm{d},i}\mathbf{M}_i^{-1}\frac{\partial}{\partial \boldsymbol{q}_i}\left(\frac{1}{2}\boldsymbol{p}_i^\top \mathbf{M}_{\mathrm{d},i}^{-1}\boldsymbol{p}_i + V_{\mathrm{s},i}\right)\end{aligned} \tag{49}$$

is equivalent to single-agent IDA-PBC control except for the cooperative component of the potential energy and

$$\boldsymbol{\Phi}_i(\boldsymbol{q}_i) = \left(\mathbf{F}_i^\top \mathbf{F}_i\right)^{-1}\mathbf{F}_i^\top \mathbf{M}_{\mathrm{d},i}\mathbf{M}_i^{-1}\boldsymbol{\Psi}_i \in \mathbb{R}^{m_i\times\ell} \tag{50}$$

is an input matrix that ensures that the potential coupling forces (44) do not violate the matching conditions.

The resulting distributed control law (48) has a stabilization term $\boldsymbol{\sigma}_i(\boldsymbol{q}_i,\boldsymbol{p}_i) \in \mathbb{R}^{m_i}$ and a damping term $-\mathbf{K}_{v,i}\boldsymbol{y}_{\mathrm{d},i}$, each depending only on local information, and a coupling term $-\boldsymbol{\Phi}_i(\boldsymbol{q}_i)\partial\bar{V}_c/\partial \boldsymbol{z}_i$ that depends on both local information and information $\boldsymbol{z}_j$ received from neighboring agents.

## 10. CASE STUDIES

### 10.1 Cooperative Flexible-Joint Manipulators

After using an internal control law to compensate for gravity (see Nuño et al. (2014)), the dynamics of a flexible-joint robot $i$ with joint angles $\boldsymbol{\alpha}_i \in \mathbb{R}^m$, motor angles $\boldsymbol{\delta}_i \in \mathbb{R}^m$, mass matrix $\mathbf{N}_i(\boldsymbol{\alpha}_i) > \mathbf{0}_m$, motor inertia $\boldsymbol{\Lambda}_i > \mathbf{0}_m$, joint stiffness $\mathbf{C}_i > \mathbf{0}_m$, $n_i = 2m_i = 2m$, are as (16)–(18) with

$$\boldsymbol{q}_i = \begin{bmatrix}\boldsymbol{\alpha}_i\\ \boldsymbol{\delta}_i\end{bmatrix}, \quad \mathbf{F} = \begin{bmatrix}\mathbf{0}_m\\ \mathbf{I}_m\end{bmatrix},$$

$$\mathbf{M}_i = \begin{bmatrix}\mathbf{N}_i & \mathbf{0}_m\\ \mathbf{0}_m & \boldsymbol{\Lambda}_i\end{bmatrix}, \quad V_i = \frac{1}{2}(\boldsymbol{\delta}_i - \boldsymbol{\alpha}_i)^\top \mathbf{C}_i(\boldsymbol{\delta}_i - \boldsymbol{\alpha}_i). \tag{51}$$

The single-agent IDA-PBC solution steers $\boldsymbol{x} = \boldsymbol{\alpha}$ to the target $\boldsymbol{\alpha}^*$ and steers $\boldsymbol{\theta} = \boldsymbol{\delta} - \boldsymbol{\alpha}$ to $\boldsymbol{\theta}^* = \boldsymbol{0}$ without kinetic energy shaping ($\mathbf{M}_\mathrm{d} = \mathbf{M}$ and $\mathbf{J} = \mathbf{0}_n$), while using potential energy shaping only to add energy that steers the motor angles to the desired joint locations: $V_\mathrm{d} = V + \frac{1}{2}(\boldsymbol{z} - \boldsymbol{z}^*)^\top \mathbf{P}(\boldsymbol{z} - \boldsymbol{z}^*)$ where $\boldsymbol{z} = \boldsymbol{\delta}$ and $\mathbf{P} > \mathbf{0}_m$, which is minimal at the target $\boldsymbol{z}^* = \boldsymbol{\delta}^*$, $\boldsymbol{\theta}^* = \boldsymbol{0}$.

For a network of flexible-joint robots, we obtain

$$\boldsymbol{z}_i = \boldsymbol{\delta}_i = \mathbf{F}^\top \boldsymbol{q}_i, \quad \mathbf{M}_{\mathrm{d},i} = \mathbf{M}_i, \quad \mathbf{J}_i = \mathbf{0}_n, \tag{52}$$

and $\ell = m$, which gives, from (39) and (50),

$$\boldsymbol{\Psi}_i = \mathbf{F}, \quad \boldsymbol{\Phi}_i = (\mathbf{F}^\top\mathbf{F})^{-1}\mathbf{F}^\top \mathbf{M}_{\mathrm{d},i}\mathbf{M}_i\boldsymbol{\Psi}_i = \mathbf{I}_m. \tag{53}$$

For the potential energy we choose $V_{\mathrm{s},i} = V_i$ such that from (49), $\boldsymbol{\sigma}_i = \boldsymbol{0}$. Then it is easy to verify that the matching conditions (35), (41), (42) hold and that with $\bar{V}_c$ as in (43), the control law for each robot (48) becomes

$$\boldsymbol{\tau}_i = \mathcal{B}_i(\boldsymbol{\delta}_i^* - \boldsymbol{\delta}_i) + \sum_{j=1}^{N}\mathcal{A}_{ij}(\boldsymbol{\delta}_j - \boldsymbol{\delta}_i - \boldsymbol{r}_{ij}^*) - \mathbf{K}_{v,i}\dot{\boldsymbol{\delta}}_i, \tag{54}$$

where $\dot{\boldsymbol{\delta}}_i = \mathbf{F}_i^\top \mathbf{M}_i^{-1}\boldsymbol{p}_i$ are the motor velocities. When $\boldsymbol{r}_{ij}^* = \boldsymbol{0}$, the control law is identical to the non-delayed case given in Nuño et al. (2014), showing how the proposed method systematically gives results without searching extensively for a Lyapunov function to prove its stability. Choosing nonzero $\boldsymbol{r}_{ij}^*$ generalizes the result to allow distinct arm poses, facilitating cooperative object grasping.

### 10.2 Underactuation-Degree One Systems

The conditions for cooperation are also satisfied by the single-agent solution for a class of mechanical systems of underactuation degree one ($m = n - 1$), given by Acosta et al. (2005). We refer to the original paper for the precise definitions and assumptions; here we focus primarily on the steps needed for the extension to distributed IDA-PBC. A key assumption is that certain terms, including $\mathbf{F}$, depend on only one coordinate, here taken to be $q_n$. Acosta et al. (2005) give a constructive procedure to find $\mathbf{M}_\mathrm{d}$ and $\mathbf{J}$ to satisfy (9), and give a desired potential energy of the form (11), where $V_\mathrm{s}(\boldsymbol{q})$ is explicitly given as

$$\begin{aligned}V_\mathrm{s}(q_n) &= \int_0^{q_n}\frac{s(\mu)}{\gamma_n(\mu)}\mathrm{d}\mu, \quad s(q_n) = \mathbf{F}^\perp\frac{\partial V}{\partial \boldsymbol{q}},\\ \boldsymbol{\gamma}(q_n) &= [\gamma_1 \ \cdots \ \gamma_n]^\top = \mathbf{M}^{-1}\mathbf{M}_\mathrm{d}\left(\mathbf{F}^\perp\right)^\top.\end{aligned} \tag{55}$$

This satisfies (14) since, by substituting (55) into (14):

$$\mathbf{F}^\perp\frac{\partial V}{\partial \boldsymbol{q}} - \mathbf{F}^\perp \mathbf{M}_\mathrm{d}\mathbf{M}^{-1}\boldsymbol{e}_n\frac{s}{\gamma_n} = s - \boldsymbol{\gamma}^\top \boldsymbol{e}_n\frac{s}{\gamma_n} = 0, \tag{56}$$

where $\boldsymbol{e}_k \in \mathbb{R}^n$ is a vector of zeros except its $k$-th entry is 1. The elements of $\boldsymbol{z} \in \mathbb{R}^\ell$, are, for $j = 1, \ldots \ell$, $\ell = m$:

$$z_j = q_j - \int_0^{q_n} \frac{\gamma_j(\mu)}{\gamma_n(\mu)} \mathrm{d}\mu, \tag{57}$$

which leads to a $\boldsymbol{\Psi}$ matrix (13) given by

$$\boldsymbol{\Psi} = \left[\boldsymbol{e}_1 \, \cdots \, \boldsymbol{e}_m\right] - \gamma_n^{-1} \left[\gamma_1 \boldsymbol{e}_n \, \cdots \, \gamma_m \boldsymbol{e}_n\right], \tag{58}$$

which in turn solves condition (15) because

$$\mathbf{F}^\perp \mathbf{M}_\mathrm{d} \mathbf{M}^{-1} \boldsymbol{\Psi} = \boldsymbol{\gamma}^\top \boldsymbol{\Psi} = \mathbf{0}_{1\times m}. \tag{59}$$

Consequently, all systems considered by Acosta et al. (2005) satisfy the conditions for cooperation (9), (14), (15).

To illustrate the results that can be obtained with the distributed IDA-PBC approach, Fig. 1 shows a simulation of a network consisting of two inverted pendulum-cart systems with different bob lengths $l$, cooperating with a fully-actuated point mass, all translating on a parallel track, to obtain a formation in the horizontal direction. Each system uses the control law (48), where for the pendulum-cart systems the terms $\mathbf{F}_i$, $\mathbf{M}_i$, $\mathbf{J}_i$, $V_i$, $\mathbf{M}_{\mathrm{d},i}$ are taken from the worked example in Acosta et al. (2005), while the point mass has $q_3 = z_3 = x_3 \in \mathbb{R}$, $V_{\mathrm{s},3} = 0$, $M_{\mathrm{d},3} = M_3 > 0$, and $F_3 = 1$ [1]. More practical examples demonstrating formations of flexible-joint manipulators and unmanned areal vehicles are given in Valk (2018).

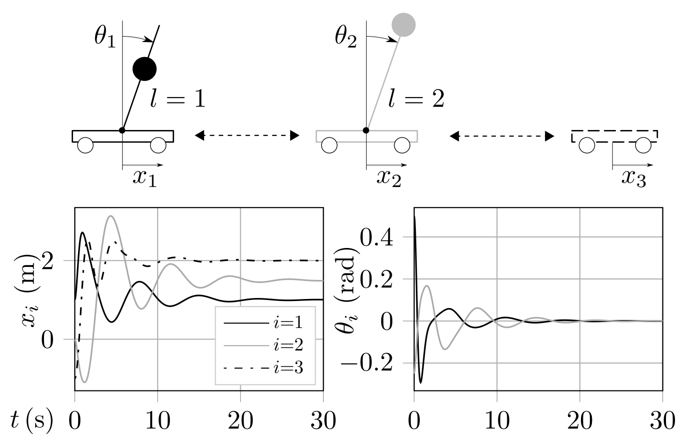


Fig. 1. Two inverted pendulums (1, 2) and a point mass (3) exchange information (dashed arrows) to achieve a formation with 0.5 m between each vehicle position $x_i$, where the leader (3) tracks the position $\boldsymbol{x}_3^* = 2\,\mathrm{m}$.

## 11. CONCLUSION

We have presented a systematic procedure that yields stable, distributed control laws for undirected networks of heterogeneous underactuated and fully-actuated mechanical systems, achieving stationary formations in the generalized coordinates of interest when each system has a known IDA-PBC solution.

As future work, we aim to generalize the objective to task-space formations, and generalize the agent coupling mechanisms beyond the proposed potential energy method. The matrix $\bar{\mathbf{J}}$ can be used to distribute energy between agents without affecting the stability of the group objective (van der Schaft and Jeltsema (2014)). Another generalization is to exchange passive outputs between agents to relax the damping conditions for individual agents. We also aim to account for communication time delays as done for the special case in Nuño et al. (2014), and account for time-varying group references (Fujimoto et al. (2003)). Finally, we are currently establishing the relation with schemes such as Chopra and Spong (2006), opening up generalizations to directed communication graphs.

[1] Simulation details, parameters, and the source code are available at `https://github.com/laurensvalk/underactuated-systems`.

## REFERENCES

Acosta, J.A., Ortega, R., Astolfi, A., and Mahindrakar, A.D. (2005). Interconnection and damping assignment passivity-based control of mechanical systems with underactuation degree one. *IEEE Transactions on Automatic Control*, 50(12), 1936–1955.

Arcak, M. (2007). Passivity as a design tool for group coordination. *IEEE Transactions on Automatic Control*, 52(8), 1380–1390.

Bellman, R. (1960). *Introduction to Matrix Analysis.* McGraw-Hill.

Blankenstein, G., Ortega, R., and van der Schaft, A. (2002). The matching conditions of controlled Lagrangians and IDA-passivity based control. *International Journal of Control*, 75(9), 645–665.

Chopra, N. and Spong, M.W. (2006). Passivity-based control of multi-agent systems. In *Advances in Robot Control*, 107–134. Springer.

Fujimoto, K., Sakurama, K., and Sugie, T. (2003). Trajectory tracking control of port-controlled Hamiltonian systems via generalized canonical transformations. *Automatica*, 39(12), 2059–2069.

Nair, S. and Leonard, N.E. (2008). Stable synchronization of mechanical system networks. *SIAM Journal on Control and Optimization*, 47(2), 661–683.

Nuño, E., Valle, D., Sarras, I., and Basañez, L. (2014). Leader–follower and leaderless consensus in networks of flexible-joint manipulators. *European Journal of Control*, 20(5), 249–258.

Ortega, R., Donaire, A., and Romero, J.G. (2017). Passivity-based control of mechanical systems. In *Feedback Stabilization of Controlled Dynamical Systems*, 167–199. Springer.

Ortega, R., Spong, M.W., Gómez-Estern, F., and Blankenstein, G. (2002). Stabilization of a class of underactuated mechanical systems via interconnection and damping assignment. *IEEE Transactions on Automatic Control*, 47(8), 1218–1233.

Ren, W. and Cao, Y. (2011). *Distributed Coordination of Multi-agent Networks: Emergent Problems, Models, and Issues.* Springer Science & Business Media.

Ryalat, M. and Laila, D.S. (2016). A simplified IDA-PBC design for underactuated mechanical systems with applications. *European Journal of Control*, 27, 1–16.

Valk, L. (2018). *Distributed Control of Underactuated and Heterogeneous Mechanical Systems.* MSc thesis. Delft University of Technology. `http://resolver.tudelft.nl/uuid:f689e074-d744-4f63-865e-3c59a7e69dfd`.

van der Schaft, A. and Jeltsema, D. (2014). Port-Hamiltonian systems theory. *Foundations and Trends® in Systems and Control*, 1(2-3), 173–378.

Zhu, D., Zhou, D., Zhou, J., and Teo, K.L. (2012). Synchronization control for a class of underactuated mechanical systems via energy shaping. *Journal of Dynamic Systems, Measurement, and Control*, 134(4), 410071.