Railway timetable rescheduling for multiple simultaneous disruptions

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Abstract Unexpected disruptions occur in the railways on a daily basis, which are typically handled manually by experienced traffic controllers with the support of predefined contingency plans. When several disruptions occur simultaneously, it is rather hard for traffic controllers to make traffic management decisions, because 1) the predefined contingency plans corresponding to these disruptions may conflict with each other and 2) no predefined contingency plan considering the combined effects of all disruptions is available. This paper proposes a Mixed Integer Linear Programming (MILP) model that reschedules the timetable automatically in case of multiple simultaneous disruptions occurring at different geographic locations. This multiple-disruptions rescheduling model considers the interactions between service adjustments made for different disruptions. The combined multiple-disruptions rescheduling model is applied every time an extra disruption occurs by considering all ongoing disruptions. Also, a sequential single-disruption rescheduling model is considered to handle each new disruption with the last solution as reference. A case study is performed by assuming two simultaneous disruptions occurring in part of the Dutch railways with 38 stations and 10 train lines operating half-hourly in each direction. By setting 3 minutes as the computation time limit in the considered disruption scenario, the combined approach resulted in less cancelled train services and train delays compared to the sequential approach.

Keywords Railways · Timetable rescheduling · Multiple disruptions

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1 Introduction

On the Dutch railways, the number of unplanned disruptions occurring each year has increased from 1846 in 2011 to 4085 in 2017 (data source: *www.rijdendetreinen.nl*). This indicates that multiple disruptions are likely to happen during operations, which is particularly true during winter (Trap et al, 2017). Due to the complexity of disruptions, lots of contingency plans are designed beforehand to enable quick responses in practice, while each of these contingency plans corresponds to only one disruption at a specific location. When disruptions occur simultaneously at different locations, contingency plans corresponding to different disruptions may conflict with each other. Thus, traffic controllers have to make traffic management decisions based on their own experiences, which can eventually recover the original schedule although not in an optimal and time-efficient way (Ghaemi et al, 2017b).

Therefore, it is necessary to propose an efficient way of handling multiple simultaneous disruptions. This has been seldom dealt with in the literature so far. Although several contributions have been made on timetable rescheduling during disruptions, the focus is on one single disruption only (Louwerse and Huisman, 2014; Zhan et al, 2015; Binder et al, 2017; Ghaemi et al, 2017a, 2018). Veelenturf et al (2015) propose a model said to be applicable to multiple track blockages, however no experimental evidence has been given. Van Aken et al (2017) design alternative timetables for the case of multiple simultaneous planned disruptions (i.e. possessions for maintenance), focusing on full-day possessions, where transitions between the original timetable and the rescheduled timetable, and vice versa, are not needed to be considered. For shorter disruptions, such transitions have to be taken into account.

In this paper, we consider unplanned disruptions that cause complete track blockages in open track sections. Our focus is on rescheduling the timetable in case of multiple simultaneous complete track blockages where each disruption is connected to another by some train line. Here, train services need to be adjusted to the multiple time-space disruption windows that have overlapping periods but are located in different sections and may start/end at different time instants. The main challenge is that the service adjustments towards one disruption window may influence the ones towards another disruption window, and vice versa.

We put forward a multiple-disruption rescheduling model on the basis of the single-disruption rescheduling model proposed by Zhu and Goverde (2018). The single-disruption rescheduling model applies delaying, reodering, cancelling, flexible stopping and flexible short-turning and considers platform capacity and trains turning at terminals. These characteristics are all kept in the multiple-disruption rescheduling model. Flexible stopping means that for each train the scheduled stops could be skipped and extra stops could be added. Flexible short-turning means that each train is given a full choice of short-turn station candidates corresponding to the served or passed stations where the infrastructure layouts enable short-turning.

This paper contributes to advance the current state of the art on railway timetable rescheduling by introducing a model that is able to tackle multiple simultaneous disruptions. The proposed model also leads to a major contribution to current state of practice, by providing a tool for supporting complex decisions for situations where predefined contingency plans are not supportive as in the case of multiple simultaneous disruptions.

In Section 2, we give a general overview of two approaches to handle multiple disruptions: a sequential single-disruption rescheduling model and a combined multiple-disruption rescheduling model. To show the differences between these two rescheduling models, Section 3 introduces the single-disruption rescheduling model first, then Section 4 explains how the multiple-disruption rescheduling model is extended from the single-disruption rescheduling model, and what kind of constraints are additionally established. A case study is given in Section 5 while Section 6 concludes the paper.

2 General framework

In this paper, multiple simultaneous disruptions are defined as two or more disruptions that

- have overlapping periods,
- occur at different geographic locations,
- may start/end in different time instants, and
- are pairwise connected by at least one train line.

They can be handled by two approaches. One is the sequential approach that uses the single-disruption rescheduling model to solve each disruption sequentially. Another is the combined approach that applies the multiple-disruption model to handle each extra disruption with all ongoing disruptions taken into account.

2.1 The sequential approach

The schematic layout of the sequential approach is shown in Figure 1 where the single-disruption rescheduling model is applied every time a new disruption emerges. As this model can deal with one disruption only at one time, it uses the last solution as reference when handling the disruption that starts second or later. This means that 1) the train services that are previously decided to be cancelled will remain cancelled; 2) the train departures/arrivals that are previously decided to be delayed can no longer occur before those time instants, as early departures/arrivals are not allowed; and 3) the short-turnings between the trains that do not run through the new track blockage will remain.

2.2 The combined approach

The schematic layout of the combined approach is shown in Figure 2 where the single-disruption rescheduling model is applied for the 1st disruption only and the multiple-disruption rescheduling model is applied every time an extra disruption emerges. When handling the disruption that starts second or later, the multiple-disruption rescheduling model makes service adjustments by taking all ongoing disruptions into account as well as the train arrivals and departures that have already been realized according to the previous rescheduled timetable.

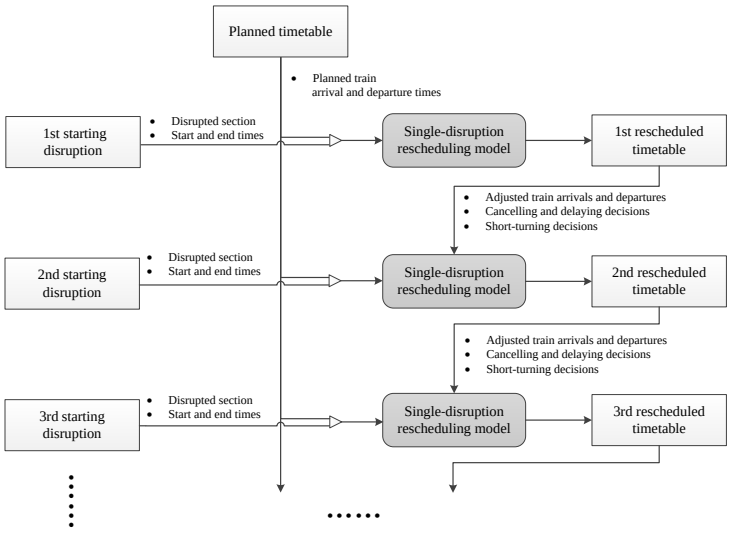

Fig. 1 Dealing with multiple simultaneous disruptions by the sequential approach

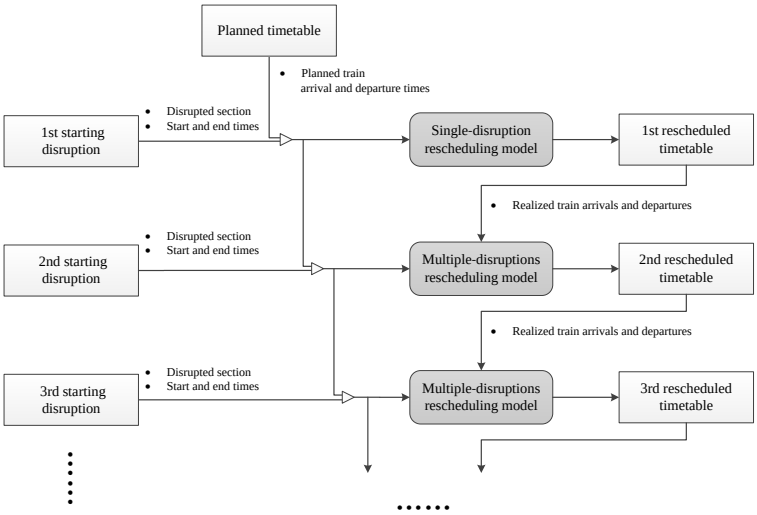

Fig. 2 Dealing with multiple simultaneous disruptions by the combined approach

2.3 Objective function and decision variables

The multiple-disruption rescheduling model and the single-disruption rescheduling model have the same objective of minimizing train service deviations from the planned timetable,

$$\text{minimize} \sum_{e \in E_{\text{ar}}} wc_e + \sum_{e \in E_{\text{ar}} \cup E_{\text{de}}} d_e, \tag{1}$$

where c_e is a binary variable deciding whether or not an arrival/departure event e is cancelled (if yes, $c_e = 1$), d_e represents the delay of event e, and w is a fixed penalty for each cancelled service. A service refers to a train run between two geographic adjacent stations. E_{ar} (E_{de}) is the set of arrival (departure) events. Each event has several attributes including the original scheduled time o_e, the corresponding train line tl_e, train tr_e, station st_e and operation direction dr_e.

For each event e, its rescheduled time x_e is a decision variable, of which the last adjusted value will be represented as r_e when handling a later disruption. For the event e that has a short-turning possibility (i.e. $e \in E_{\text{ar}}^{\text{turn}} \cup E_{\text{de}}^{\text{turn}}$), a binary decision variable y_e is needed to decide whether or not the train corresponding to e chooses the station corresponding to e as the short-turn station (if yes, $y_e = 1$). It is necessary to have this binary variable, since a train may have multiple short-turn station candidates to choose. Here, $E_{\text{ar}}^{\text{turn}}$ ($E_{\text{de}}^{\text{turn}}$) is the set of arrival (departure) events corresponding to A_{turn} that includes all short-turn activities. A short-turn activity $a \in A_{\text{turn}}$ is established from an arrival event to departure event that belong to the same train line and occur at the same station, but operate in opposite directions. Another binary variable m_a is needed, which decides whether or not a short-turn activity a is maintained (if yes, $m_a = 1$). The aforementioned decision variables are used in both rescheduling models. Their notation is shown in Table 1. Note that the decision variables for reordering, flexible stopping, platform capacity and turning trains at terminals are not presented in this paper, but can be found in Zhu and Goverde (2018). These decision variables as well as the corresponding constraints are the same in both the single-disruption rescheduling model and the multiple-disruption rescheduling model.

Besides short-turn activities that are established between different trains, running, dwell and pass-through activities are also constructed for each single train. A running activity is from a departure event to an arrival event that belong to the same train at geographic adjacent stations. The departure event occurs at the upstream station relative to the station where the arrival event occurs. A dwell (pass-through) activity is from an arrival event to a departure event that belongs to the same train, occurs at the same station, and with the departure event occurring later (at the same time as the arrival event). Dwell and pass-through activities together constitute station activities. The descriptions of these activities can be found in Table 2 that shows the notation.

Table 1 Decision variables

Symbol	Description
c_e	The binary variable deciding whether or not event e is cancelled. If yes, $c_e = 1$.
d_e	The delay of event e
x_e	The rescheduled time of event e
y_e	The binary variable deciding whether or not the train corresponding to e chooses the station corresponding to e as the short-turn station $tr_e, \forall e \in E_{\text{ar}}^{\text{turn}} \cup E_{\text{de}}^{\text{turn}}$. If yes, $y_e = 1$.
m_a	The binary variable deciding whether or not a short-turn activity $a \in A_{\text{turn}}$ is maintained. If yes, $m_a = 1$.

Table 2 Notation

Symbol	Description
o_e	The original scheduled time of event e
tl_e	The corresponding train line of event e
tr_e	The corresponding train of event e
st_e	The corresponding station of event e
dr_e	The operation direction of event e
n	The nth disruption that currently emerges
r_e	The previous rescheduled time of event e
t^i_{start}	The start time of the ith disruption, $1 \leq i \leq n$
t^i_{end}	The end time of the ith disruption, $1 \leq i \leq n$
$st^{i,tr}_{\text{en}}$	The entry station of the ith disrupted section regarding to train tr
$st^{i,tr}_{\text{ex}}$	The exit station of the ith disrupted section regarding to train tr
$tail(a)$	The tail of activity a
$head(a)$	The head of activity a
D	The maximum delay allowed to an event
L_a	The minimum duration of an activity a
M_1	A positive large number whose value is set to 1440
M_2	A positive large number whose value is set to twice of M_1
a_{run}	Running activity: $a_{\text{run}} = (e, e'), e \in E_{\text{de}}, e' \in E_{\text{ar}}, tr_e = tr_{e'}, st_e$ is upstream neighbouring to $st_{e'}$
a_{dwell}	Dwell activity: $a_{\text{dwell}} = (e, e'), e \in E_{\text{ar}}, e' \in E_{\text{de}}, tr_e = tr_{e'}, st_e = st_{e'}, o_e < o_{e'}$
a_{pass}	Pass-through activity: $a_{\text{pass}} = (e, e'), e \in E_{\text{ar}}, e' \in E_{\text{de}}, tr_e = tr_{e'}, st_e = st_{e'}$ $o_e = o_{e'}$
a_{turn}	Short-turn activity: $a_{\text{turn}} = (e, e'), e \in E_{\text{ar}},\ e' \in E_{\text{de}}, tr_e \neq tr_{e'}, dr_e \neq dr_{e'}$, $st_e = st_{e'}, tl_e = tl_{e'}$
A_{run}	Set of run activities
A_{dwell}	Set of dwell activities
A_{pass}	Set of pass-through activities
A_{station}	Set of station activities: $A_{\text{station}} = A_{\text{dwell}} \cup A_{\text{pass}}$
A^i_{turn}	Set of short-turn activities serving the ith disruption, $i \in Dis$
A_{turn}	Set of short-turn activities: $\bigcup_{i \in Dis} A^i_{\text{turn}} = A_{\text{turn}}$
E_{ar}	Set of arrival events
E_{de}	Set of departure events
E^{NMdelay}	Set of events that don't have upper limit on their delays
$E^{i,\text{turn}}_{\text{ar}}$	Set of arrival events corresponding to the short-turn activities serving the ith disruption: $E^{i,\text{turn}}_{\text{ar}} = \bigcup_{a \in A^i_{\text{turn}}} \{tail\,(a)\}, i \in Dis$
$E^{\text{turn}}_{\text{ar}}$	Set of arrival events corresponding to all short-turn activities: $\bigcup_{i \in Dis} E^{i,\text{turn}}_{\text{ar}} = A^{\text{turn}}_{\text{ar}}$
$E^{i,\text{turn}}_{\text{de}}$	Set of departure events corresponding to the short-turn activities serving the ith disruption: $E^{i,\text{turn}}_{\text{de}} = \bigcup_{a \in A^i_{\text{turn}}} \{head\,(a)\}, i \in Dis$
$E^{\text{turn}}_{\text{de}}$	Set of departure events corresponding to all short-turn activities: $\bigcup_{i \in Dis} E^{i,\text{turn}}_{\text{de}} = A^{\text{turn}}_{\text{de}}$
$ST^{tr_e}_{\text{en}}$	Set of entry stations of all disrupted sections regarding to train tr
$ST^{tr_e}_{\text{ex}}$	Set of exit stations of all disrupted sections regarding to train tr

3 The single-disruption rescheduling model

In Zhu and Goverde (2018), timetable rescheduling under one single disruption is formulated into a MILP model where delaying, reordering, cancelling, flexible stopping and flexible short-turning are applied and platform capacity and turning

trains at terminals are considered. This section introduces this single-disruption model by showing part of its constraints that will help to understand the multiple-disruption rescheduling model.

3.1 Constraints for cancelling and delaying train services

For an event e, the relation between its rescheduled time x_e, the cancelling decision c_e and the delaying decision (d_e) is formulated by

$$M_1 c_e \leq x_e - o_e \leq M_1, \qquad e \in E_{\text{ar}} \cup E_{\text{de}}, \tag{2}$$

$$x_e - o_e = d_e + M_1 c_e, \qquad e \in E_{\text{ar}} \cup E_{\text{de}}, \tag{3}$$

$$d_e \geq 0, \qquad e \in E_{\text{ar}} \cup E_{\text{de}}, \tag{4}$$

$$d_e \leq D, \qquad e \in (E_{\text{ar}} \cup E_{\text{de}}) \setminus E^{\text{NMdelay}}, \tag{5}$$

where o_e is the original scheduled time of e, E_{ar} (E_{de}) is the set of arrival (departure) events, and E^{NMdelay} is the set of events that do not have an upper limit on their delays. Those events relate to trains which are already running in the network when the disruption occurs. Thus, they can no longer be completely cancelled, but only delayed or short-turned. In case they are unable to be short-turned due to insufficient station/rolling stock capacity, they have to be delayed (i.e. wait) at least to the end of the disruption. This is why their corresponding events are not imposed with an upper limit on the delays. Constraint (2) means that for any event e, its rescheduled time x_e cannot be earlier than its original scheduled time o_e, and is set to $o_e + M_1$ in case it is cancelled. The binary variable c_e decides whether or not an event e is cancelled; if yes, $c_e = 1$. Constraint (3) means that for a cancelled event e, its delay d_e is set to 0, because each cancelled service has been given a fixed penalty in the objective; for a kept event, its delay is the time difference between the rescheduled time and the original scheduled time. Constraint (4) forces the delay of any event to be non-negative, and (5) ensures that an event that does not belong to E^{NMdelay} can only be delayed by at most D minutes.

Each event that originally occurs earlier than the disruption start time (i.e. t_{start}) cannot be cancelled, and should run as planned. This is realized by

$$c_e = 0, \qquad e \in E_{\text{ar}} \cup E_{\text{de}}, o_e < t_{\text{start}}, \tag{6}$$

$$x_e - o_e = 0, \qquad e \in E_{\text{ar}} \cup E_{\text{de}}, o_e < t_{\text{start}}. \tag{7}$$

A train can no longer access a blocked open track section during the time a disruption occurs. This is represented by

$$x_e \geq t_{\text{end}}(1 - c_e), \qquad e \in E_{\text{de}}, st_e = st_{\text{en}}^{tr_e}, t_{\text{start}} \leq o_e < t_{\text{end}}, \tag{8}$$

where $st_{\text{en}}^{tr_e}$ represents the entry station of the disrupted section regarding to the train tr_e corresponding to e, and t_{end} is the end time of disruption. Constraint (8) means that a departure that originally occurs at the entry station of the disrupted section is either cancelled or delayed to the end of the disruption. If this departure

is cancelled, so will its corresponding arrival event in the running activity. This is realized by

$$c_{e'} - c_e = 0, \qquad (e, e') \in A_{\text{run}}, \tag{9}$$

which means that any two events e and e' that constitute a running activity must be cancelled or kept simultaneously to guarantee the operation consistency between them.

Similarly for any two events e and e' that constitute the same dwell/pass-through activity (i.e. station activity), the operation consistency should always be kept, if neither of them has short-turning possibility. This is formulated by

$$c_{e'} - c_e = 0, \qquad (e, e') \in A_{\text{station}}, e \notin E_{\text{ar}}^{\text{turn}}, e' \notin E_{\text{de}}^{\text{turn}}, \tag{10}$$

where $E_{\text{ar}}^{\text{turn}}$ ($E_{\text{de}}^{\text{turn}}$) is the set of arrival (departure) events corresponding to any short-turn activity. Note that all scheduled short-turn activities are pre-constructed and are the input of the rescheduling model. For events that originally occur at stations whose track layouts do not permit short-turning, no short-turn activities are established.

3.2 Constraints for flexible short-turning of train services

For any two events e and e' that constitute a station activity and one of them has short-turning possibility, the operation consistency between them could be broken, considering one event is short-turned while the other event is cancelled. Note that in the single disruption case, at most one of e and e' from $(e, e') \in A_{\text{station}}$ has short-turning possibility. This means that the situation of $(e, e') \in A_{\text{station}}, e \in E_{\text{ar}}^{\text{turn}}$ and $e' \in E_{\text{de}}^{\text{turn}}$ will never happen under the single disruption case. An example is given to explain this by Figures 3 and 4. In Figure 3, the solid lines represent the original scheduled services. In Figure 4, the solid lines represent the rescheduled services, while the dashed (dotted) lines represent the original scheduled services that are cancelled (delayed) in the rescheduled service plan. Figure 3 shows the original schedule of two trains where e_1 (e_1') represents the arrival (departure) event of the yellow train at station D and e_2 (e_2') represents the arrival (departure) event of the blue train at station D. Thus, we have $(e_1, e_1') \in A_{\text{station}}$ and $(e_2, e_2') \in A_{\text{station}}$. When a complete track blockage occurs between stations E and F (see Figure 4), at the top of section E-F the yellow train is short-turned to the blue train at station F, whereas at the bottom of section E-F the yellow train is delayed to be served by the short-turned blue train at station D (assuming station E lacks capacity). In this case, at the bottom of section E-F, the short-turning will never happen from e_1 to e_2', which means these two events do not have short-turning possibilities. Thus, we have $(e_1, e_1') \in A_{\text{station}}, e_1 \notin E_{\text{ar}}^{\text{turn}}, e_1' \in E_{\text{de}}^{\text{turn}}$ and $(e_2, e_2') \in A_{\text{station}}, e_2 \in E_{\text{ar}}^{\text{turn}}, e_2' \notin E_{\text{de}}^{\text{turn}}$.

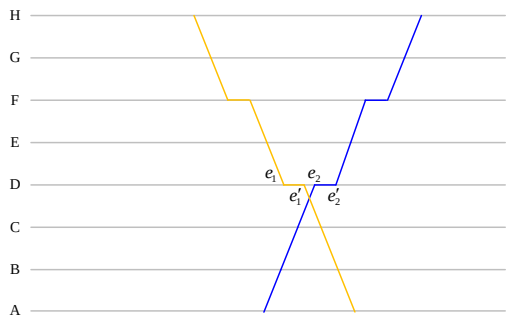

Fig. 3 The original service plan where $(e_1, e_1') \in A_{\text{station}}$ and $(e_2, e_2') \in A_{\text{station}}$

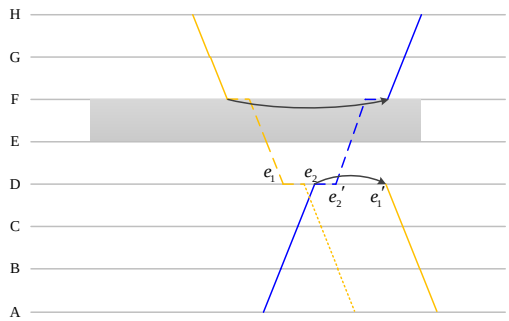

Fig. 4 The rescheduled service plan when one disruption occurs, where $(e_1, e_1') \in A_{\text{station}}, e_1 \notin E_{\text{ar}}^{\text{turn}}, e_1' \in E_{\text{de}}^{\text{turn}}$ and $(e_2, e_2') \in A_{\text{station}}, e_2 \in E_{\text{ar}}^{\text{turn}}, e_2' \notin E_{\text{de}}^{\text{turn}}$

To decide the operation consistency of two events e and e' that constitute the same station activity and one of them has short-turning possibility, the following constraints are established:

$$c_e \leq c_{e'}, \qquad (e, e') \in A_{\text{station}}, e \in E_{\text{ar}}^{\text{turn}}, e' \notin E_{\text{de}}^{\text{turn}}, \tag{11}$$

$$c_{e'} \leq c_e + y_e, \qquad (e, e') \in A_{\text{station}}, e \in E_{\text{ar}}^{\text{turn}}, e' \notin E_{\text{de}}^{\text{turn}}, \tag{12}$$

$$c_{e'} \geq y_e, \qquad (e, e') \in A_{\text{station}}, e \in E_{\text{ar}}^{\text{turn}}, e' \notin E_{\text{de}}^{\text{turn}}, \tag{13}$$

$$c_{e'} \leq c_e, \qquad (e, e') \in A_{\text{station}}, e \notin E_{\text{ar}}^{\text{turn}}, e' \in E_{\text{de}}^{\text{turn}}, \tag{14}$$

$$c_e \leq c_{e'} + y_{e'}, \qquad (e, e') \in A_{\text{station}}, e \notin E_{\text{ar}}^{\text{turn}}, e' \in E_{\text{de}}^{\text{turn}}, \tag{15}$$

$$c_e \geq y_{e'}, \qquad (e, e') \in A_{\text{station}}, e \notin E_{\text{ar}}^{\text{turn}}, e' \in E_{\text{de}}^{\text{turn}}, \tag{16}$$

where y_e is a binary variable deciding whether or not the train corresponding to e chooses the station corresponding to e as the short-turn station (if yes, $y_e = 1$). Constraints (11) and (12) together ensure that for an arrival event e that has short-turning possibility (i.e. $e \in E_{\text{ar}}^{\text{turn}}$), it should be cancelled or kept simultaneously with its corresponding departure event in the station activity, if the train

corresponding to e does not choose the station corresponding to e as the short-turn station (i.e. $y_e = 0$). Otherwise (i.e. $y_e = 1$), the departure event corresponding to e in the station activity must be cancelled (13). Similarly, constraints (14) and (15) together ensure that for a departure event e' that has short-turning possibility (i.e. $e' \in E_{\text{de}}^{\text{turn}}$), it should be cancelled or kept simultaneously with its corresponding arrival event in the station activity, if the train corresponding to e' does not choose the station corresponding to e' as the short-turn station (i.e. $y_{e'} = 0$). Otherwise (i.e. $y_{e'} = 1$), the arrival event corresponding to e' in the station activity must be cancelled (16).

Because of flexible short-turning, each train is provided with multiple short-turn station candidates, whereas a train can only be short-turned at most one station on each side of the disrupted section, if its operation in the disrupted section is cancelled. To realize this, we establish

$$\sum_{e:tr_e=tr} y_e = c_{e'}, \quad tr \in TR_{\text{turn}}, e \in E_{\text{ar}}^{\text{turn}}, e' \in E_{\text{de}}, tr_{e'} = tr, st_{e'} = st_{\text{en}}^{tr_{e'}}, \tag{17}$$

$$\sum_{e':tr_{e'}=tr} y_{e'} = c_e, \; tr \in TR_{\text{turn}}, e' \in E_{\text{de}}^{\text{turn}}, e \in E_{\text{ar}}, tr_e = tr, st_e = st_{\text{ex}}^{tr_e}, \tag{18}$$

where TR_{turn} is the set of trains that correspond to the events in $E_{\text{ar}}^{\text{turn}} \cup E_{\text{de}}^{\text{turn}}$, and $st_{\text{en}}^{tr_{e'}}$ ($st_{\text{ex}}^{tr_e}$) represents the entry (exit) station of the disrupted section regarding to the train corresponding to e' (e). Note that for any two trains tr_a and tr_b that serve the same train line but operate in opposite directions, there must be $st_{\text{en}}^{tr_a} \neq st_{\text{en}}^{tr_b}$, $st_{\text{en}}^{tr_a} = st_{\text{ex}}^{tr_b}$ and $st_{\text{en}}^{tr_b} = st_{\text{ex}}^{tr_a}$.

Constraints (17) and (18) determine the short-turn station for a train at each side of the disrupted section. Next, it is necessary to decide which short-turn activity will be maintained at each short-turn station chosen for this train, as there are multiple short-turn activities as candidates. To this end, we establish

$$\sum_{\substack{a \in A_{\text{turn}}, \\ tail(a)=e}} m_a = c_{e'} - c_e, \qquad (e, e') \in A_{\text{station}}, e \in E_{\text{ar}}^{\text{turn}}, e' \notin E_{\text{de}}^{\text{turn}}, \tag{19}$$

$$\sum_{\substack{a \in A_{\text{turn}}, \\ head(a)=e'}} m_a = c_e - c_{e'}, \qquad (e, e') \in A_{\text{station}}, e \notin E_{\text{ar}}^{\text{turn}}, e' \in E_{\text{de}}^{\text{turn}}, . \tag{20}$$

where m_a is the binary variable deciding whether or not a short-turn activity a will be maintained; if yes $m_a = 1$. Constraint (19) means that for an arrival event $e \in E_{\text{ar}}^{\text{turn}}$, one and only one of its corresponding short-turn activities would be maintained, if e is kept and its corresponding departure event in the station activity is cancelled. Constraint (20) means that for a departure event $e' \in E_{\text{de}}^{\text{turn}}$, one and only one of its corresponding short-turn activities would be maintained, if e' is kept and its corresponding arrival event in the station activity is cancelled.

If a short-turn activity is maintained, the minimum short-turn duration must be respected, which is formulated by

$$M_1 c_e + 2D(1 - m_a) + x_{e'} - x_e \geq m_a L_a, \qquad a = (e, e') \in A_{\text{turn}}, \tag{21}$$

where A_{turn} is the set including all short-turn activities, and L_a represents the minimum duration required for short-turn activity a.

Note that the constraints of flexible stopping, reordering, platform capacity and turning trains at terminals (i.e. OD turning) in the single-disruption model can be found in Zhu and Goverde (2018). The objective (1) of the single-disruption rescheduling model is that minimizes the deviations (i.e. cancelling and delaying) from the planned timetable.

4 The multiple-disruption rescheduling model

Among constraints (2) - (21) that serve the single-disruption rescheduling model, (6) - (8), (17) and (18) need further extensions to be used in the multiple-disruption rescheduling model, while the remainders are kept exactly the same. In this section, we explain why and how these extensions are made, and also introduce additional constraints that are particularly needed for the multiple-disruption rescheduling model.

4.1 Constraints for cancelling and delaying train services

When a disruption occurs at the time that the previous occurring disruption(s) is still ongoing, an event e whose previous rescheduled time r_e is earlier than the current disruption start time t^n_{start} can no longer be cancelled and its current rescheduled time x_e should respect its previous one r_e. Here, n represents the nth disruption that currently emerges ($n \geq 2$). Considering these, we extend (6) and (7) that are applicable to only the first occurring disruption to (22) and (23) that are applicable for each extra emerging disruption:

$$c_e = 0, \qquad e \in E_{\text{ar}} \cup E_{\text{de}}, r_e < t^n_{\text{start}}, n \geq 2, \tag{22}$$

$$x_e - r_e = 0, \qquad e \in E_{\text{ar}} \cup E_{\text{de}}, r_e < t^n_{\text{start}}, n \geq 2. \tag{23}$$

In case of multiple disruptions, a train cannot run into any disrupted sections during the corresponding disruption periods. As the existing constraint (8) can only forbid a train running into one disrupted section, we extend it to

$$x_e \geq t^i_{\text{end}}(1 - c_e), \qquad e \in E_{\text{de}}, st_e = st^{i,tr_e}_{\text{en}}, t^i_{\text{start}} \leq o_e < t^i_{\text{end}}, 1 \leq i \leq n, \tag{24}$$

where t^i_{start} (t^i_{end}) represents the start (end) time of the ith disruption, st^{i,tr_e}_{en} refers to the entry station of the ith disrupted section regarding to the train corresponding to e. To be more specific, constraint (24) means that a departure originally occurring at the entry station of the ith disrupted section during the ith disruption period should either be cancelled or delayed to the end of that disruption.

Thus, the constraints for cancelling and delaying train services are (2) - (5), (9) and (10), and (22) - (24).

4.2 Constraints for flexible short-turning of train services

In Section 3, we have explained that in the single-disruption case, for any two events e and e' with $(e, e') \in A_{\text{station}}$, at most one of them has a short-turning possibility. However under multiple disruptions, both of them may have short-turning possibilities. An example is given to explain this by Figures 5 - 6 where the solid lines represent the rescheduled services, while the dashed (dotted) lines represent the original scheduled services that are cancelled (delayed) in the rescheduled service plan. In Figures 5 and 6, we keep the disruption from Figure 4 and additionally assume another disruption occurring later between stations B and C. The differences between Figures 5 and 6 is the short-turning at station D. In Figure 5, the short-turning from e_1 to e_2' is established, whereas in Figure 6 the short-turning from e_2 to e_1' is established instead. In fact, the short-turning at station D shown in either Figure 5 or Figure 6 could happen. This means that events e_1, e_1', e_2 and e_2' all have short-turning possibilities. In this case, we have $(e_1, e_1') \in A_{\text{station}}, e_1 \in E_{\text{ar}}^{\text{turn}}, e_1' \in E_{\text{de}}^{\text{turn}}$ and $(e_2, e_2') \in A_{\text{station}}, e_2 \in E_{\text{ar}}^{\text{turn}}, e_2' \in E_{\text{de}}^{\text{turn}}$. Note that Figures 5 and 6 do not show the rolling stock connections at station E or station C, which are actually considered in the multiple-disruption rescheduling model.

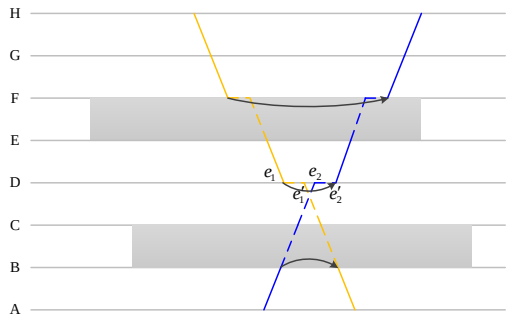

Fig. 5 One possible rescheduled service plan with two disruptions, where $(e_1, e_1') \in A_{\text{station}}, e_1 \in E_{\text{ar}}^{\text{turn}}, e_1' \in E_{\text{de}}^{\text{turn}}$ and $(e_2, e_2') \in A_{\text{station}}, e_2 \in E_{\text{ar}}^{\text{turn}}, e_2' \in E_{\text{de}}^{\text{turn}}$

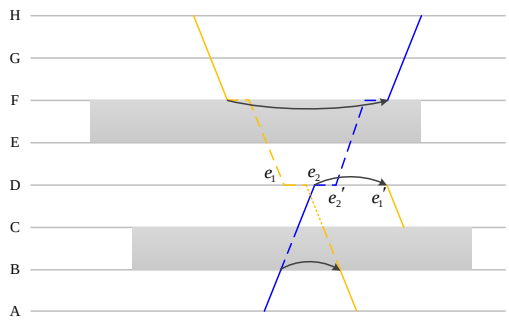

Fig. 6 One possible rescheduled service plan with two disruptions, where $(e_1, e_1') \in A_{\text{station}}, e_1 \in E_{\text{ar}}^{\text{turn}}, e_1' \in E_{\text{de}}^{\text{turn}}$ and $(e_2, e_2') \in A_{\text{station}}, e_2 \in E_{\text{ar}}^{\text{turn}}, e_2' \in E_{\text{de}}^{\text{turn}}$

Considering that under multiple disruptions there are events e and e' with $(e, e') \in A_{\text{station}}$ and both of them have short-turning possibilities, we need additional constraints to decide whether to break the operation consistency between them. The added constraints are:

$$c_e - c_{e'} = y_{e'} - y_e, \qquad (e, e') \in A_{\text{station}}, e \in E_{\text{ar}}^{\text{turn}}, e' \in E_{\text{de}}^{\text{turn}}, \tag{25}$$

$$y_e + y_{e'} \leq 1, \qquad (e, e') \in A_{\text{station}}, e \in E_{\text{ar}}^{\text{turn}}, e' \in E_{\text{de}}^{\text{turn}}, \tag{26}$$

$$c_{e'} \geq y_e, \qquad (e, e') \in A_{\text{station}}, e \in E_{\text{ar}}^{\text{turn}}, e' \in E_{\text{de}}^{\text{turn}}, \tag{27}$$

$$c_{e'} \leq 1 - y_{e'}, \qquad (e, e') \in A_{\text{station}}, e \in E_{\text{ar}}^{\text{turn}}, e' \in E_{\text{de}}^{\text{turn}}, \tag{28}$$

$$c_e \geq y_{e'}, \qquad (e, e') \in A_{\text{station}}, e \in E_{\text{ar}}^{\text{turn}}, e' \in E_{\text{de}}^{\text{turn}}, \tag{29}$$

$$c_e \leq 1 - y_e, \qquad (e, e') \in A_{\text{station}}, e \in E_{\text{ar}}^{\text{turn}}, e' \in E_{\text{de}}^{\text{turn}}, . \tag{30}$$

Constraints (25) - (30) together ensure that 1) if y_e and $y_{e'}$ are both equal to 0, then e and e' must be kept/cancelled simultaneously; 2) at most one of y_e and $y_{e'}$ can be 1, and for the one with value 1 its corresponding event will be kept due to short-turning, whereas for the one with value 0 its corresponding event will be cancelled. Recall that binary variable y_e decides whether or not the train corresponding to e chooses the station corresponding to e as the short-turn station.

The existing (17) and (18) ensure that at most one station can be chosen for a train at each side of a disrupted section as the short-turn station. When multiple sections are disrupted, a train could be short-turned at each side of each disrupted section. In such a case, (17) and (18) are no longer applicable, which need to be extended as follows:

$$\sum_{e:tr_e=tr} y_e \geq c_{e'}, \quad tr \in TR_{\text{turn}}^i, e \in E_{\text{ar}}^{i,\text{turn}}, e' \in E_{\text{de}}, tr_{e'} = tr, st_{e'} = st_{\text{en}}^{i,tr_{e'}}, 1 \leq i \leq n, \tag{31}$$

$$\sum_{e':tr_{e'}=tr} y_{e'} \geq c_e, \; tr \in TR_{\text{turn}}^i, e' \in E_{\text{de}}^{i,\text{turn}}, e \in E_{\text{ar}}, tr_e = tr, st_e = st_{\text{ex}}^{i,tr_e}, 1 \leq i \leq n, \tag{32}$$

where $E_{\text{ar}}^{i,\text{turn}}$ ($E_{\text{de}}^{i,\text{turn}}$) is the set of arrival (departure) events relevant to the short-turn activities corresponding to the ith disruption, TR_{turn}^i is the set of trains corresponding to the events in $E_{\text{ar}}^{i,\text{turn}} \cup E_{\text{de}}^{i,\text{turn}}$, and $st_{\text{en}}^{i,tr_{e'}}$ (st_{ex}^{i,tr_e}) represents the entry (exit) station of the ith disrupted section regarding to the train corresponding to e' (e). Constraints (31) and (32) ensure that at least one station can be chosen for a train at each side of the ith disrupted section as the short-turn station, if the operation of this train in the ith disrupted section is cancelled. In (31) and (32), we use "$\geq$" instead of "$=$" because the short-turn activities relevant to one train could correspond to different disruptions. In other words, it is possible that an event $e \in E_{\text{ar}}^{i,\text{turn}} \cap E_{\text{ar}}^{j,\text{turn}}$ (or $e' \in E_{\text{de}}^{i,\text{turn}} \cap E_{\text{de}}^{j,\text{turn}}$), while $i \neq j, 1 \leq i, j \leq n$. For example in Figure 5 or Figure 6, the short-turning activity from the blue train to the yellow train at station B corresponds to the first disruption (i.e. blocked section E-F) as an earlier short-turning and also corresponds to the second disruption (i.e. blocked section B-C) apparently. Thus, the arrival event of the blue train at station B must belong to both $E_{\text{ar}}^{1,\text{turn}}$ and $E_{\text{ar}}^{2,\text{turn}}$.

At each sides of all disrupted sections, the number of short-turn stations chosen for a train cannot be larger than the number of its departure (arrival) events that originally occur at the entry (exit) stations of these disrupted sections but were cancelled. To this end, we establish

$$\sum_{e:tr_e=tr} y_e \leq \sum_{e'} c_{e'}, \quad tr \in TR_{\text{turn}}, e \in E_{\text{ar}}^{\text{turn}}, e' \in E_{\text{de}}, tr_{e'} = tr, st_{e'} \in ST_{\text{en}}^{tr_{e'}}, 1 \leq i \leq n, \tag{33}$$

$$\sum_{e':tr_{e'}=tr} y_{e'} \leq \sum_{e} c_e, \; tr \in TR_{\text{turn}}, e' \in E_{\text{de}}^{\text{turn}}, e \in E_{\text{ar}}, tr_e = tr, st_e \in ST_{\text{ex}}^{tr_e}, 1 \leq i \leq n, \tag{34}$$

where $ST_{\text{en}}^{tr_{e'}} = \bigcup_{i=\{1,\cdots,n\}} st_{\text{en}}^{i,tr_{e'}}$, $ST_{\text{ex}}^{tr_e} = \bigcup_{i=\{1,\cdots,n\}} st_{\text{ex}}^{i,tr_e}$, $E_{\text{ar}}^{\text{turn}} = \bigcup_{i\in\{1,\cdots,n\}} E_{\text{ar}}^{i,\text{turn}}$, and $E_{\text{de}}^{\text{turn}} = \bigcup_{i\in\{1,\cdots,n\}} E_{\text{de}}^{i,\text{turn}}$. Constraint (33) ensures that at the top of all disrupted sections, the number of short-turn stations chosen for train tr is no larger than the number of its corresponding departure events that originally occur at the entry stations of disrupted sections but cancelled. Constraint (34) ensures that at the bottom of all disrupted sections, the number of short-turn stations chosen for train tr is no larger than the number of its corresponding arrival events that originally occur at the exit stations of these disrupted sections but cancelled.

Considering (19) and (20) are only applicable to events e and e' that constitute the same station activity while only one of them has short-turning possibility, we additionally establish (35) and (36) for events e and e' that constitute the same station activity and both of them have short-turning possibilities.

$$\sum_{\substack{a\in A_{\text{turn}},\\ tail(a)=e}} m_a = c_{e'} - c_e + y_{e'}, \qquad (e, e') \in A_{\text{station}}, e \in E_{\text{ar}}^{\text{turn}}, e' \in E_{\text{de}}^{\text{turn}}, \tag{35}$$

$$\sum_{\substack{a\in A_{\text{turn}},\\ head(a)=e'}} m_a = c_e - c_{e'} + y_e, \qquad (e, e') \in A_{\text{station}}, e \in E_{\text{ar}}^{\text{turn}}, e' \in E_{\text{de}}^{\text{turn}}. \tag{36}$$

Constraint (35) ensures that at most one of the short-turn activities corresponding to arrival event e will be maintained, and (36) ensures that at most one of the short-turn activities corresponding to departure event e' will be maintained. In (35), it happens that $c_{e'} = 0$ and $c_e = 1$, which makes $c_{e'} - c_e = -1$ while the left term of this equality must be non-negative. Considering this, $y_{e'}$ is added at the right side, whose value must be 1 in this case due to constraints (25) and (27). Similar reasoning is applied for adding y_e at the right side of (36).

Thus, the constrains for flexible short-turning train services are (11) - (16), (19) - (21), and (25) - (36).

To summarize, for the multiple-disruption rescheduling model, the used constraints include (2) - (5), (9) - (16), and (19) - (36). Besides, constraints of flexible stopping, reordering, platform capacity and turning trains at terminals (i.e. OD turning) for the single-disruption model are all kept for the multiple-disruption model, which can be found in Zhu and Goverde (2018). The objective (1) minimizes the deviations (i.e. cancelling and delaying) from the planned timetable.

5 Case study

The considered network is part of the Dutch railways, which is shown in Figure 7, where red crossings indicate the disrupted areas assumed in the case study.

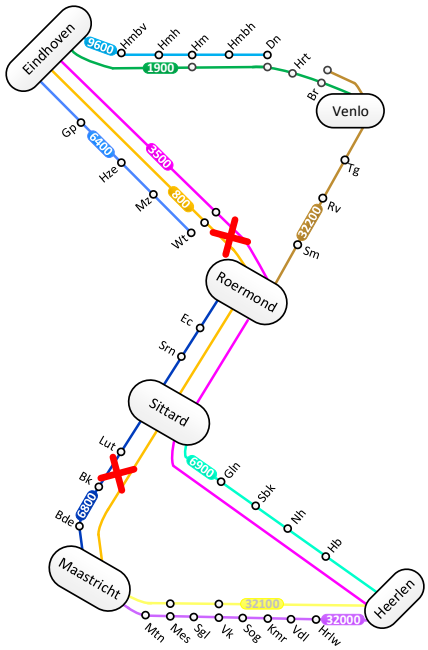

Fig. 7 The train lines operating in the considered network

There are 38 stations located in this network with 10 train lines operating half-hourly in each direction. In the proposed model, trains turning at their terminals (i.e. OD turnings) to serve the opposite operations are taken into account. Table 3 lists the terminals of the train lines that are located in the considered network, while the terminals outside the considered network are neglected.

Table 3 Train lines in the considered network

Train line	Type	Terminals in the considered network
800	Intercity	Maastricht (Mt)
1900	Intercity	Venlo (Vl)
3500	Intercity	Heerlen (Hrl)
6400	Sprinter	Eindhoven (Ehv) and Wt
6800	Sprinter	Rm
6900	Sprinter	Sittard (Std) and Hrl
9600	Sprinter	Ehv and Dn
32000	Sprinter	—
32100	Intercity	Mt and Hrl
32200	Sprinter	Roermond (Rm)

5.1 Disruption scenario and parameter settings

According to the definition of multiple simultaneous disruptions given in Section 2, we assume two disruptions as follows:

- a 1st disruption between stations Bk and Lut from 8:06 to 10:06, and
- a 2nd disruption between stations Roermond (Rm) and Wt from 8:12 to 10:16.

Train lines 6800 and 3500 are affected by the 1st and 2nd disruptions, respectively, while train line 800 is affected by both disruptions. We apply both the sequential approach (see Figure 1) and the combined approach (see Figure 2), to deal with the multiple simultaneous disruption scenario. In both cases, we don't allow the original scheduled stops to be skipped. This is because passenger demand is not taken into account, while the decision of whether an original scheduled stop should be skipped or not needs to be made according to passenger demand. However, we allow extra stops to be added, considering that a train may dwell at a station where it originally passes through to wait for the platform capacity to be released in the downstream station where it will be short-turned.

We set the minimum duration required for short-turning or OD turning to 300 s, the minimum arrival/departure headway at open track sections to 180 s, the minimum headway between a departure and an arrival that correspond to the same station track to 180 s, the minimum dwell time at a station to 30 s, the maximum delay for each departure/arrival to 25 min, and the penalty of cancelling a service to 100 min.

Considering the time requirement for computation, we set 180 s as the upper time limit to get a solution from either the single-disruption rescheduling model or the multiple disruptions one. Both models are solved by the optimization software GUROBI release 7.0.1 on a desktop with Intel Xeon CPU E5-1620 v3 at 3.50 GHz and 16 GB memory.

5.2 Results

The 1st rescheduled timetables corresponding to the 1st disruption obtained by the sequential approach and the combined approach is the same, which are shown in Figure 8. The solid lines represent the rescheduled services, and the dotted (dashed) lines represent the original scheduled services that are delayed (cancelled) in the rescheduled timetable. From Figure 8 we can see that at the top of the disrupted section Bk-Lut, four dark blue trains (train line 6800) are short-turned at station Lut while four yellow trains (train line 800) are short-turned earlier at station Std. Station Lut has two tracks only and both of them are alongside platforms. Thus at station Lut, a minimum headway has to be respected between the arrival of a train and the departure of another train that previously arrives at station Lut from the same direction. Under this circumstance, if a yellow train (train line 800) was short-turned at station Lut, it would be delayed at station Lut and the opposite operations serviced by it would be delayed a lot. In such a case, although there are two services cancelled less, the resulting delays are more than the penalty on cancelling two services, which is why the model short-turns four yellow trains (train line 800) earlier at station Std instead. Similarly, station Bk also has only two tracks and both of them are alongside platforms. As a result, four

dark blue trains (train line 6800) have to be delayed at station Bde to respect the minimum headway between their arrivals and the departures of previous arriving yellow trains (train line 800) at station Bk.

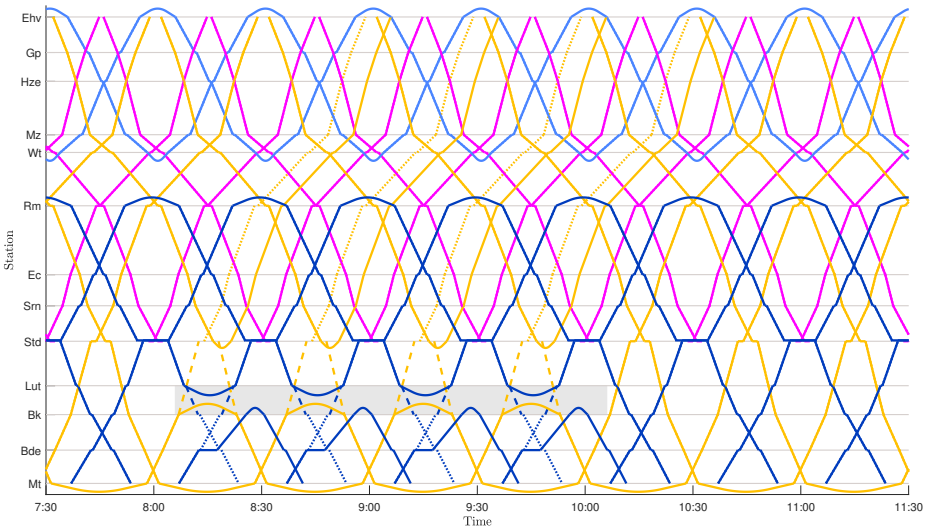

Fig. 8 The 1st rescheduled timetable obtained by the sequential/combined approach: from Eindhoven (Ehv) to Maastricht (Mt)

The 2nd rescheduled timetable obtained by the sequential approach is shown in Figure 9 where the red triangles represent extra stops. Compared to Figure 8, there are more yellow train services (train line 800) cancelled between stations Std and Rm in Figure 9. This is because yellow trains (train line 800) have to be short-turned at station Rm due to the emerging disruption (disrupted section Rm-Wt), which however may be inoperable due to their short-turnings at station Std. At the top of the disrupted section Rm-Wt, four pink trains (train line 3500) additionally dwell at station Mz. This is because station Wt has four tracks while only two of them are alongside platforms. Thus, each of these four pink trains (train line 3500) has to wait at station Mz to ensure the headway between its arrival and the departure of a short-turned light blue train (train line 6400) at station Wt where a yellow train (train line 800) is still occupying another platform at that time. At station Wt, the departures of four upstream yellow trains (train line 800) are delayed more than necessary. This is because in the sequential approach, the delaying decisions made for the previous disruption are kept. Hence, the adjusted arrival and departure times from the previous step are now the reference timetable, while early arrivals/departures are not allowed, which now is with respect to this timetable. Also at station Wt, the arrival of a downstream pink train (train line 3500) is delayed to respect the minimum headway between this arrival and the departure of a previous delayed arriving pink train (train line 3500), since a light blue train (train line 6400) is occupying a platform at that time.

The 2nd rescheduled timetable obtained by the combined approach is shown in Figure 10 where two yellow trains (train line 800) are short-turned at station Lut

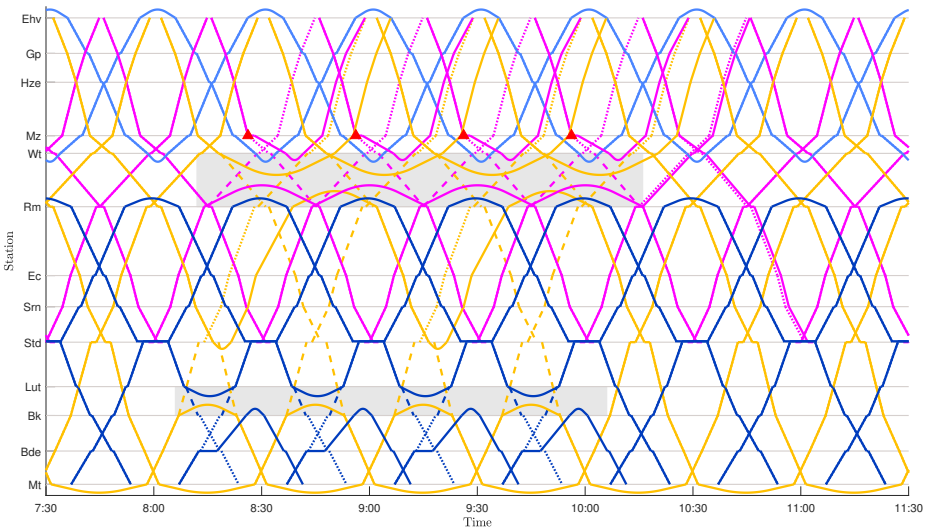

Fig. 9 The 2nd rescheduled timetable obtained by the sequential approach: from Eindhoven (Ehv) to Maastricht (Mt)

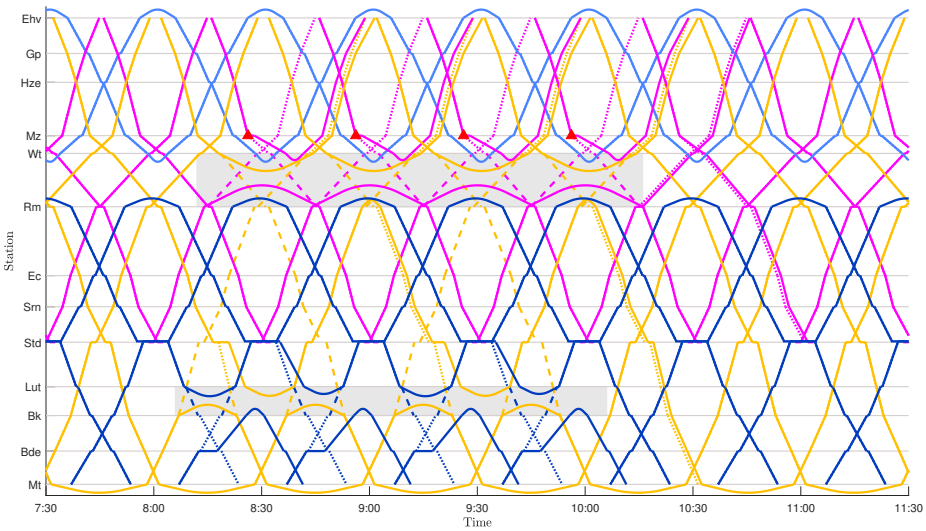

Fig. 10 The 2nd rescheduled timetable obtained by the combined approach: from Eindhoven (Ehv) to Maastricht (Mt)

instead of station Std in Figure 9. With the combined approach (Figure 10), two upstream yellow trains (train line 800) between Std and Rm and four upstream yellow trains (train line 800) between Wt and Ehv are less delayed than when using the sequential approach (Figure 9). Less delays in the combined approach are obtained because of the use of the multiple-disruption rescheduling model that differently from the single-disruption rescheduling model does not rely on previously taken decisions, thus having a wider search space to find a better solution.

Table 4 shows the general results of the 2nd rescheduled timetables obtained by both approaches. We can see that the sequential approach found an optimal solution in 87 seconds, while the combined approach only found a suboptimal solution with an optimality gap of 0.67% under the required computation time limit (i.e. 180 s). Even so, the 2nd rescheduled timetable obtained by the combined approach still resulted in 4 less cancelled services and 266 minutes less train delays (i.e. arrival and departure delays), compared to the one obtained by the sequential approach. Recall that a service refers to a train run between two geographic adjacent stations.

Table 4 Results of the 2nd rescheduled timetables obtained by both approaches under the required time limit

Approach	Objective value [min]	Total cancelled services	Total train delays [min]	Computation time [sec]	Optimality gap
Sequential	6314	52	1114	87	0.00%
Combined	5678	48	878	180	0.67%

Without the computation time limit, the combined approach found an optimal solution in 884 seconds. The resulting objective value is 5640, the total cancelled services are 44, and the total train delays are 1240 minutes. Compared to the suboptimal solution obtained by the combined approach, the optimal solution results in 4 less train services cancelled, but 362 minutes more train delays. We found that the consideration of station capacity is the main reason to find an optimal solution relatively slow, since lots of binary variables are used for considering station capacity. By excluding station capacity constraints, the multiple-disruption rescheduling model can find an optimal solution in only 11 seconds for the concerned scenario.

6 Conclusions and future research

We proposed a multiple-disruption rescheduling model that reschedules all train services together each time an extra disruption occurs. Such a model contributes to advance the current state of the art on railway disruption management that mostly focuses on solving a single disruption. A contribution to current practice is also provided since the proposed model could support dispatchers during complex situations not effectively handled by predefined contingency plans. The proposed combined multiple-disruption rescheduling model has been compared to a sequential single-disruption rescheduling model that solves disruptions one by one with previous rescheduling decisions as reference. In terms of solution quality, the combined approach outperforms the sequential approach.

A case study is performed to part of the Dutch railways with 38 stations and 10 train lines operating half-hourly in each direction. Complete track blockages are assumed to occur in two different sections in the considered network, which have overlapping periods and both last 2 hours approximately, but start and end at different time instants. By setting 180 s as the computation time limit, the sequential approach generated an optimal solution with 4 more cancelled train services and 236 minutes more train delays than the suboptimal solution (with optimality gap

of 0.67%) obtained by the combined approach. Without computation time limit, the combined approach needed 884 seconds to find an optimal solution. This long computation time originates from station capacity constraints where lots of binary variables are used. To speed up the computation, a different way of including station capacity might be explored without infringing the solution quality. Besides, more experiments will be designed to explore the impact of different time periods, amount, and locations of multiple disruptions.

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