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Modelling the iono-acoustic wave field for proton beam range verification

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Abstract—In proton therapy, cancer patients are irradiated with high energy protons. For a successful treatment it is important that the location with the highest energy deposition, the so-called Bragg-peak, is located inside the tumor and not in the healthy surrounding tissue. Here, we investigate if the iono-acoustic wave field generated by the protons can be used to monitor the Bragg-peak location during treatment. To this end we present a new numerical method to model the pressure field generated by a clinical proton pencil beam. To compute the field, we convolve a 3-D Greens function, representing the impulse response of the medium, with a volume density of injection rate source. This source describes the expansion of the medium due to a local temperature increase caused by the energy deposited by the protons. To image the proton dose distribution, we first measure the pressure field synthetically by a matrix transducer positioned below the Bragg peak in a plane parallel to the beam. Next, we use these measurements to solve the linear inverse problem iteratively. To regularize the inversion, we take the temporal behavior of the dose deposition as prior knowledge. For the presented example, where the pencil beam has a proton range of 63 mm we are able to reconstruct the location of the Bragg peak within 4 mm accuracy.

Index Terms—iono-acoustics, Bragg peak localization, imaging

I. INTRODUCTION

In proton therapy, high-energy charged particles are used to treat cancer. When these particles traverse the body, they deposit their energy via Coulomb interactions within the medium. Since part of this energy is transformed into heat, a thermo-elastic expansion of the medium takes place. This expansion, in turn, leads to the formation of a transient pressure wave field. This pressure field propagates outwards, away from the heated region.

Although the thermo-acoustic effect was already discovered in 1880 by Alexander Graham Bell [1], the first clinical applications were recognized much later. Only in 1988 it was shown that proton beams interacting with water and soft tissue could be detected by using the iono-acoustic effect. [2] Seven years later, in 1995, the real clinical breakthrough took place when Hayakawa demonstrated the detection of acoustic waves in vivo during proton therapy treatment of a hepatic patient. [3] In those days, it was already recognized that for a successful clinical application it is essential to measure the dose distribution in 3-D.

In the present numerical study, we investigate the applicability of a model-based inversion algorithm to image the proton dose distribution from the iono-acoustic wave field. To regularize the linear inverse problem we incorporate prior knowledge about the temporal behavior of the proton beam. Note that we aim for 3-D imaging of clinical relevant proton dose distributions. Consequently, our numerical model is based on the proton beam of the recently opened proton accelerator of the proton therapy center HollandPTC in Delft, the Netherlands [4].

II. THEORY

When protons traverse through the body they interact with the tissue. This interaction leads to a local energy deposition and hence a local temperature increase of the tissue. In case the interaction is short, the absorbed dose \( D(\vec{r}, t) = D_e(\vec{r}) D_t(t) \) can be separated into two parts; one that is a function of the position \( \vec{r} \) and the other one that is function of time \( t \). Taking this into account, the resulting volume source density of injection rate \( q(\vec{r}, t) \) that creates the pressure field will read

\[
q(\vec{r}, t) = \frac{\Gamma_0}{\rho_0 c_0^2} E(\vec{r}, t) = \frac{\Gamma_0}{c_0^2} D_e(\vec{r}) D_t(t) ,
\]

where \( \Gamma_0, \rho_0, c_0 \) for the Gruneisen parameter, the mass density and the speed of sound, respectively, and where \( E(\vec{r}, t) \) is the deposited energy density per unit time.

At HollandPTC, an isochronous cyclotron is used to generate the proton beam. The resulting temporal dependence of the proton dose may be described by a sequence of \( N_{ic} \) proton spills

\[
D_t(t) = \left( \sqrt{2\pi \sigma_t N_{ic}} \right)^{-1} \sum_{n=0}^{N_{ic}-1} e^{-\frac{1}{2}(t-n\tau)/\sigma_t^2} ,
\]

where \( \sigma_t \) is the Gaussian width of the proton spills, and \( \tau \) is the temporal spacing between proton spills. Transforming
TABLE I
BEAM AND MEDIUM PARAMETERS

<table>
<thead>
<tr>
<th>variable</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_2$</td>
<td>1 ns</td>
</tr>
<tr>
<td>$\tau$</td>
<td>13 ns</td>
</tr>
<tr>
<td>$N_{ic} = \frac{\tau_{pulse}}{\tau}$</td>
<td>492</td>
</tr>
<tr>
<td>$I = N_{proton} \times q_{proton} / \tau_{pulse}$</td>
<td>30 nA</td>
</tr>
<tr>
<td>$\tau_{pulse}$</td>
<td>6.4 $\mu$s</td>
</tr>
<tr>
<td>$N_{proton}$</td>
<td>$1.2 \times 10^6$</td>
</tr>
<tr>
<td>proton range</td>
<td>63 mm</td>
</tr>
<tr>
<td>$c_0$</td>
<td>1520 m/s</td>
</tr>
<tr>
<td>$\rho_0$</td>
<td>996 kg/m$^3$</td>
</tr>
<tr>
<td>$\Gamma_0$</td>
<td>0.8</td>
</tr>
</tbody>
</table>

equation (2) to the temporal Fourier domain with angular frequency $\omega$ yields

$$D_\omega (\omega) = \left( \sqrt{2\pi} N_{ic} \right)^{-1} \sum_{n=0}^{N_{ic}-1} e^{-in\tau\omega-\omega^2\sigma_t^2/2} \left( \sqrt{2\pi} N_{ic} \right)^{-1} \frac{1-e^{-in\tau\omega}}{1-e^{-i\omega\tau}} e^{\omega^2\sigma_t^2/2}.$$ 

Note that the caret symbol $\hat{}$ is used for quantities defined in the temporal Fourier domain.

For homogeneous lossless media the resulting pressure field is governed by the wave equation

$$\left( \nabla^2 + \frac{\omega^2}{c_0^2} \right) \hat{p} (\vec{r}, \omega) = -i\omega \rho_0 \hat{q} (\vec{r}, \omega),$$

where $\hat{p} (\vec{r}, \omega)$ is the pressure field generated by the source $\hat{q} (\vec{r}, \omega)$. Based on equations (1) to (4), the wave equation can be solved for $\hat{p} (\vec{r}, \omega)$ according to [6]

$$\hat{p} (\vec{r}, \omega) = i\omega \rho_0 \hat{q}_\omega (\omega) \int G (\vec{r} - \vec{r'}, \omega) q_r (\vec{r'}) dV (\vec{r'}),$$

where the impulse response of the medium is defined via the Greens’ function

$$\hat{G} (\vec{r} - \vec{r'}, \omega) = \frac{e^{-i\frac{\omega}{c_0} |\vec{r} - \vec{r'}|}}{4\pi |\vec{r} - \vec{r'}|}.$$ 

Due to the low temporal frequency of the resulting pressure field, scattering will be neglected but may easily be included in the future. [7]

For imaging by model based inversion, equation (5) will be solved for the unknown $q_r (\vec{r'})$. Here we take as input $\hat{p} (\vec{r}, \omega)$ and assume to have prior knowledge about the temporal behavior of the proton beam via $\hat{q}_\omega (\omega)$. To solve the resulting linear inverse problem we use a conjugate gradient minimization scheme.

III. RESULTS

To test our approach we use the proton treatment facility of HollandPTC as a starting point. [4] The most relevant parameters for our simulations are presented in Table I. In particular, we would like to point out that we aimed for a clinical relevant setting.

For our simulations, we assumed the medium to be homogeneous with material properties similar to water. The spatial domain was discretised using $(N_x \times N_y \times N_z) = (116 \times 112 \times 80)$ cubic elements with dimensions $\Delta_x = \Delta_y = \Delta_z = 1.24$ mm. The temporal signal was of length 0.44 ms and discretised in steps of $\Delta t = 2.5$ $\mu$s.

The original proton dose distribution is shown in Fig. 1. The dose distribution has a pencil beam structure with a Gaussian shaped in the radial direction with a Gaussian width $\sigma_{r,i} = 4.5$ mm. The maximum of the proton dose distribution

Fig. 1. The original proton dose distribution in water using the beam parameters of Table I. The pencil beam clearly spreads out at the location of the Bragg peak. Top: iso-surface of the distribution. Bottom: planar cross sections of the distribution in the $(yz)$-, $(xz)$-, and $(xy)$-planes. All planes are taken through the original Bragg peak location.

Fig. 2. Snap shot of the iono-acoustic pressure field at $t = 0.35$ $\mu$s in the $(yz)$-, $(xz)$-, and $(xy)$-planes on a dB scale. All three planes are taken through the Bragg peak.
lies at \((x, y, z) = (86 \text{ mm}, 63 \text{ mm}, 45 \text{ mm})\).

A snapshot of the resulting pressure field at \(t = 0.35 \mu s\) is shown in Fig. 2. The wave field is shown in the \((yz)-, (xz)-,\) and \((xy)\)-planes. All three planes include the location of the Bragg peak. The results show that the wave field is mainly generated in the region with the highest energy deposition; the location of the Bragg peak. Note that the amplitude and temporal behavior of the resulting pressure field not only depend on the amplitude but also on the spatial distribution and temporal profile of the dose distribution. [8]

To image the dose distribution, a set of 900 point receivers is homogeneous distributed in the plane \(z = 0\). An example of a synthetically measured A-scan is displayed in Fig. 3, both in time and frequency domain. Fig. 4 shows all the 900 A-scans in the temporal Fourier domain. These results show that the dominant frequency is around 40 kHz and that there is little spectral content left above 100 kHz. For the reconstruction, we only use the spectral components marked by the 17 black lines in Fig. 4. The result after 32 iterations is shown in Fig. 5. This distribution is less sharp than the original distribution and the distance between the maximum in the original and in the reconstructed dose distribution is 3.9 mm.

**IV. Conclusion**

The iono-acoustic wave field for a proton beam with clinically relevant parameters have been modeled using Greens functions. Imaging the proton dose distribution is feasible by solving the linear inverse problem iteratively. To regularize this inverse problem, we took the temporal profile of the proton dose distribution as prior knowledge. The resulting reconstruction is similar to the original dose distribution and the error in the location of the Bragg peak is 3.9 mm. Although not shown, it should be noted that the amplitude and temporal behavior of the resulting pressure field and hence the accuracy of the reconstruction strongly depends on the original dose distribution. For this particular example the Bragg peak location lay 63 mm into the medium. It is expected that the error can be reduced significantly, e.g. by optimizing the positions of the receivers or by taking more prior about the beam properties into account.

**References**


