

Adaptive synchronization in networks with heterogeneous uncertain Kuramoto-like units

Azzollini, Ilario A.; Baldi, Simone; Kosmatopoulos, Elias B.

DOI

[10.23919/ECC.2018.8550539](https://doi.org/10.23919/ECC.2018.8550539)

Publication date

2018

Document Version

Accepted author manuscript

Published in

Proceedings of 2018 European Control Conference (ECC2018)

Citation (APA)

Azzollini, I. A., Baldi, S., & Kosmatopoulos, E. B. (2018). Adaptive synchronization in networks with heterogeneous uncertain Kuramoto-like units. In Proceedings of 2018 European Control Conference (ECC2018) (pp. 2417-2422). [8550539] Piscataway, NJ, USA: IEEE.
<https://doi.org/10.23919/ECC.2018.8550539>

Important note

To cite this publication, please use the final published version (if applicable).
Please check the document version above.

Copyright

Other than for strictly personal use, it is not permitted to download, forward or distribute the text or part of it, without the consent of the author(s) and/or copyright holder(s), unless the work is under an open content license such as Creative Commons.

Takedown policy

Please contact us and provide details if you believe this document breaches copyrights.
We will remove access to the work immediately and investigate your claim.

Adaptive synchronization in networks with heterogeneous uncertain Kuramoto-like units

Ilario A. Azzollini, Simone Baldi, and Elias B. Kosmatopoulos

Abstract—We analyze adaptive synchronization capabilities in networks with Kuramoto-like units whose dynamical features are unknown and thus synchronization protocols must exhibit co-evolution capabilities. In the presence of heterogeneous and uncertain units, synchronization should be enabled by appropriate adaptive protocols that counteract the effect of heterogeneity. An interaction protocol is presented that is used by the units to communicate with each other: the protocol is based on a distributed disagreement measure. The aim of the protocol is to adapt feedback and coupling gains, so as to guarantee the emergence of a synchronous solution. The adaptive strategy is distributed, i.e. each unit self-determines the strength of its gains by using only neighboring measurements. Convergence of the synchronization error to zero is shown via Lyapunov analysis, and numerical examples demonstrate the effectiveness of the proposed protocol.

Index Terms—Adaptive synchronization, Kuramoto-like model, uncertain systems.

I. INTRODUCTION

Synchronization is a collective phenomenon occurring in systems of interacting units, and is ubiquitous in nature, society and technology [1]. Literature has distinguished among two types of synchronization: in the first one, synchronization towards the same evolution which is unknown a priori emerges from the negotiation process taking place on the network (this is sometimes referred to as leaderless synchronization [2]); in the second one, the network is steered in some desired and a priori known solution using a limited set of leader nodes (this is sometimes referred to as leader-follower synchronization or pinning control [3], [4]). In the 80's Kuramoto proposed an exactly solvable model of collective synchronization, which became known as the Kuramoto model [5]. This model has been shown to capture various synchronization phenomena in biological and man-made dynamical systems of coupled oscillators, spanning from flocks of birds and schools of fishes [6], blinks in groups of fireflies [1], the utility power grid [7], to countless other synchronization phenomena [8].

Synchronization research has been first focusing on non-evolving (or non-adaptive) networks of phase oscillators (see

[9] and references therein): it was found that synchronization can emerge in the presence of simple static coupling where neighboring nodes adjust their dynamics proportionally to the mismatch between some output function of their states [10], [11], [12], [13]. Most synchronization models have shown that synchronization is favored if the coupling strength is large enough and the spectrum of variety of the oscillators is narrow [1] (almost homogeneous oscillators). In this spirit, the authors in [14] provided a threshold of the couplings that brings from incoherence to synchrony: synchronization occurs when the coupling strength dominates the worst-case dissimilarity over the network. Summarizing, these studies enlighten the crucial role played by the connectivity (interaction topology) and structural properties (parameters and coupling/feedback gains of each unit) in the emergence of synchronized states [15].

However, real-world networks have uncertain and heterogeneous parameters which might even change with time. If uncertainties are large, adaptive-gain approaches are needed to achieve synchronization [16], [17]. In particular, researchers have later been focusing on networks characterized by evolving, adapting couplings which vary in time according to different environmental conditions, leading to the study of evolving (or adaptive) networks [18]. In [19] a simple model of adaptive Kuramoto network is given in which adaptation is taken into account by mechanisms of homophily (reinforcing interactions with correlated units) and homeostasis (preserving the overall connection strength). In [20] a set of adaptive strategies for synchronization and consensus of complex networks of dynamical systems is presented: the main limitation of these approaches is that they address networks composed of identical oscillators. The authors in [21] devise an adaptive scheme to achieve phase synchronization by suppressing the negative effect of the heterogeneity in the network, while in [22] protocols are designed to adaptively interact with system dynamics and preserve the sum of all incoming pairwise coupling strengths. In [23] a co-evolutionary rewiring strategy that depends only on the phase differences of neighboring oscillators is studied for Kuramoto agents. However, synchronization of evolving Kuramoto networks is usually shown numerically but not analytically proven.

In this work we analyze synchronization capabilities in Kuramoto networks whose dynamical features are unknown and thus synchronization protocols must exhibit co-evolution capabilities. With respect to the three ingredients indicated in [24], in this work we consider (*i*) units described by a Kuramoto-like model (like the standard Kuramoto model,

I. Azzollini and S. Baldi are with the Delft Center for Systems and Control, Delft University of Technology, The Netherlands. E. Kosmatopoulos is with the Department of Electrical and Computer Engineering, Democritus University of Thrace, Xanthi, 67100 Greece, and with the Informatics & Telematics Institute, Center for Research and Technology Hellas (ITI-CERTH), Thessaloniki 57001, Greece (emails: i.a.azzollini@student.tudelft.nl, s.baldi@tudelft.nl and kosmatop@iti.gr).

The research leading to these results has been partially funded by the European Commission H2020-SEC-2016-2017-1, Border Security: autonomous systems and control systems, under contract #740593 (ROBORDER) and H2020-ICT-2014-1, FIRE+ (Future Internet Research & Experimentation), under contract #645220 (RAWFIE).

with an additional inertia term); (ii) adaptive interaction protocol based on a disagreement measure with all neighbors and (iii) an undirected graph. Convergence of the synchronization error to zero is shown via Lyapunov analysis. Finally, as opposed to state-of-the-art approaches based on a distributed observer [25], [26], the proposed approach does not have to construct observer states, which simplifies the adaptation procedure.

The rest of the paper is organized as follows. In Sect. II we give the problem formulation. The disagreement-based protocol is given in Sect. III. Numerical examples are in Sect. IV, and Sect. V concludes the work.

Notation: The notation in this paper is standard. Matrices are denoted by capital letters, e.g. X , while vectors and scalars by small letters, e.g. x . The transpose of a matrix or of a vector is indicated with X^T and x^T respectively. A vector signal $x \in \mathbb{R}^n$ is said to belong to \mathcal{L}_2 class ($x \in \mathcal{L}_2$), if $\int_0^t \|x(\tau)\|^2 d\tau < \infty, \forall t \geq 0$. A vector signal $x \in \mathbb{R}^n$ is said to belong to \mathcal{L}_∞ class ($x \in \mathcal{L}_\infty$), if $\max_{t \geq 0} \|x(t)\| < \infty, \forall t \geq 0$. A time-invariant undirected communication graph of order N is completely defined by the pair $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{1, \dots, N\}$ is a finite nonempty set of nodes, and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is a set of corresponding non-ordered pair of nodes, called edges. The adjacency matrix of a weighted undirected graph $\mathcal{K} = [k_{ij}]$ is defined as $k_{ii} = 0$ and $k_{ij} = k_{ji} > 0$ if $(i, j) \in \mathcal{E}$, where $i \neq j$. The adjacency matrix $\mathcal{A} = [a_{ij}]$ of an unweighted undirected graph is defined as $a_{ii} = 0$ and $a_{ij} = a_{ji} = 1$ if $(i, j) \in \mathcal{E}$, where $i \neq j$. The Laplacian matrix of the unweighted graph is defined as $\mathcal{L} = [l_{ij}]$, where $l_{ii} = \sum_j a_{ij}$ and $l_{ij} = -a_{ij}$, if $i \neq j$. An undirected graph \mathcal{G} is said to be connected if, taken any arbitrary pair of nodes (i, j) where $i, j \in \mathcal{V}$, there is a path that leads from i to j . In this work we indicate with N the number of nodes (or agents) in the network. The all-ones N -vector is defined as $\mathbf{1}_N = \text{col}(1, 1, \dots, 1)$. In the same way we define the all-zeros $(N \times n)$ -vector $\mathbf{0}_{(N \times n) \times 1} = \text{col}(0, 0, \dots, 0)$.

II. PROBLEM FORMULATION

The network (1) of heterogeneous coupled oscillators with unknown dynamics is considered in this work. Time index t may be omitted when obvious.

$$m_i \ddot{\theta}_i + d_i \dot{\theta}_i = \tau_i - \sum_{j=1}^N k_{ij} \sin(\theta_i - \theta_j), \quad i \in \mathcal{V} \quad (1)$$

The meaning of the parameters in (1) can be examined via the mechanical analogy of mass points in Figure 1. After

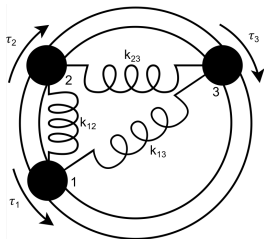


Fig. 1. Mechanical analogy of a network of three coupled oscillators.

neglecting any collision, each point, or agent, will move on the circle describing an angle (or phase, by analogy) θ_i and an angular velocity (or frequency, by analogy) $\dot{\theta}_i$, under the effect of an external driving torque τ_i , an elastic restoring torque $k_{ij} \sin(\theta_i - \theta_j)$ (with $k_{ij} = k_{ji}$), and a viscous damping torque $d_i \dot{\theta}_i$ that is opposite to the direction of motion. All inertial coefficients m_i , damping coefficients d_i and stiffness coefficients k_{ij} have positive but *unknown* value. The external driving torque has two components

$$\tau_i = \omega_i + u_i, \quad i \in \mathcal{V} \quad (2)$$

where u_i is the actual control torque and ω_i is a term proportional to the natural angular velocity (or frequency) of the agent i , that is the angular velocity it would have if there were no couplings. After defining the state $x_i = [\theta_i, \dot{\theta}_i]^T$, (1) can be rewritten as

$$\dot{x}_i = \underbrace{\begin{bmatrix} 0 & 1 \\ 0 & -\frac{d_i}{m_i} \end{bmatrix}}_{A_i} x_i + \underbrace{\begin{bmatrix} 0 \\ \frac{1}{m_i} \end{bmatrix}}_{b_i} \left(u_i + \omega_i - \sum_{j=1}^N k_{ij} \sin(\theta_i - \theta_j) \right). \quad (3)$$

The following connectivity assumption is made.

Assumption 1: The graph \mathcal{G} of the network is undirected and connected.

Problem 1: [Adaptive state synchronization] Consider a network of unknown oscillators (1) satisfying Assumption 1. Find a distributed strategy (i.e. exploiting only state measurements from neighbors) for the control input u_i such that, without any knowledge of the parameters m_i , d_i and k_{ij} , the network state synchronizes to the same behavior, i.e. $x_i - x_j \rightarrow 0, \forall i, j$.

Two results are now given which are instrumental to solving the problem above.

Proposition 1: [Homogeneization via reference model] For the following reference model

$$\dot{x}_m = \underbrace{\begin{bmatrix} 0 & 1 \\ a_{21}^* & a_{22}^* \end{bmatrix}}_{A_0} x_m + \underbrace{\begin{bmatrix} 0 \\ \frac{1}{m^*} \end{bmatrix}}_{b_0} u \quad (4)$$

with $x_m \in \mathbb{R}^{2 \times 1}$, there exist a family of vectors $k_i^* \in \mathbb{R}^{2 \times 1}$ and a family of scalars $l_i^* > 0$ such that

$$\begin{cases} A_i + b_i k_i^{*T} = A_0 \\ l_i^* b_i = b_0 \end{cases} \quad (5)$$

Furthermore, there exists an ideal controller

$$u_i^* = k_i^{*T} x_i + l_i^* f^T \sum_{j=1}^N a_{ij} (x_i - x_j) + c_i^* + \sum_{j=1}^N g_{ij}^* a_{ij} \sin(\theta_i - \theta_j) \quad (6)$$

with $c_i^* = -\omega_i$, $g_{ij}^* = k_{ij}$ and $f \in \mathbb{R}^{2 \times 1}$ to be designed, which leads to the following dynamics

$$\dot{x}_i = A_0 x_i + b_0 f^T \sum_{j=1}^N a_{ij} (x_i - x_j) \quad (7)$$

Proof: The proof directly follows from applying the control input (6) to agent (3), and using (5).

The following result, allows us to design f to achieve synchronization for the homogeneous dynamics in (7).

Proposition 2: [Homogeneous network synchronization] The homogeneous network (7) synchronizes if

$$A_0 + \lambda_i b_0 f^T \text{ is Hurwitz, } \forall i \in \mathcal{V} / \{1\} \quad (8)$$

where λ_i 's, $i \in \mathcal{V} / \{1\}$, are the non-zero eigenvalues of the Laplacian, or equivalently if

$$P(A_0 + \lambda_i b_0 f^T) + (A_0 + \lambda_i b_0 f^T)^T P < \mathbf{0}, \forall i \in \mathcal{V} / \{1\} \quad (9)$$

where $P \in \mathbb{R}^{2 \times 2}$ is a symmetric positive definite matrix.

Proof: Proposition 2 is well known result in synchronization literature [27], and the proof is not given for lack of space. The interested reader is also referred to the companion paper [28] for more details.

Remark 1: Since A_i , b_i , ω_i , k_{ij} are unknown, the ideal control (6) cannot be implemented to solve Problem 1. Therefore, some adaptation mechanisms must be devised to estimate the unknown ideal gains in Proposition 1.

The aim of the adaptation mechanism in the following section is to make the heterogeneous network converge to the behavior of the homogeneous network in Proposition 2, estimating the unknown gains by exploiting only measurements from neighbors.

III. DISTRIBUTED DISAGREEMENT-BASED ADAPTIVE SYNCHRONIZATION

The following synchronizing protocol is proposed

$$\begin{aligned} u_i(t) = & k_i^T(t)x_i + l_i(t)f^T \sum_{j=1}^N a_{ij}(x_i(t) - x_j(t)) + c_i(t) + \\ & + \sum_{j=1}^N g_{ij}(t)a_{ij} \sin(\theta_i(t) - \theta_j(t)) \end{aligned} \quad (10)$$

where k_i , l_i , c_i , g_{ij} , are the (time-dependent) estimates of k_i^* , l_i^* , c_i^* , g_{ij}^* , respectively. The following synchronization result holds.

Theorem 1: Under Assumption 1, the heterogeneous Kuramoto network (3), controlled using the synchronizing protocol (10) and the following adaptive laws

$$\begin{aligned} \dot{k}_i^T &= -\gamma \left(\sum_{j=1}^N a_{ij}(e_i - e_j) \right)^T P b_0 x_i^T \\ \dot{l}_i &= -\gamma \left(\sum_{j=1}^N a_{ij}(e_i - e_j) \right)^T P b_0 f^T e_i \\ \dot{c}_i &= -\gamma \left(\sum_{j=1}^N a_{ij}(e_i - e_j) \right)^T P b_0 \\ \dot{g}_{ij} &= -\gamma \left(\sum_{j=1}^N a_{ij}(e_i - e_j) \right)^T P b_0 \sin(\theta_i - \theta_j) \end{aligned} \quad (11)$$

with adaptive gain $\gamma > 0$, and e_i being the local synchronization error

$$e_i = \sum_{j=1}^N a_{ij}(x_i - x_j), \quad (12)$$

reaches synchronization provided that the matrix P and the vector f are chosen such that condition (9) holds.

Proof: The closed-loop network formed by (3) and (10) is given by

$$\begin{aligned} \dot{x}_i = & (A_i + b_i k_i^T)x_i + l_i b_i f^T \sum_{j=1}^N a_{ij}(x_i - x_j) + b_i c_i + \\ & + b_i \sum_{j=1}^N g_{ij} a_{ij} \sin(\theta_i - \theta_j) \end{aligned} \quad (13)$$

which can be rewritten as a function of the estimation errors,

$$\begin{aligned} \dot{x}_i = & (A_0 + b_i \tilde{k}_i^T(t))x_i + (b_0 + \tilde{l}_i(t)b_i)f^T \sum_{j=1}^N a_{ij}(x_i - x_j) + \\ & + b_i \tilde{c}_i(t) + b_i \sum_{j=1}^N \tilde{g}_{ij}(t)a_{ij} \sin(\theta_i - \theta_j) \end{aligned} \quad (14)$$

where $\tilde{k}_i(t) = k_i(t) - k_i^*$, $\tilde{l}_i(t) = l_i(t) - l_i^*$, $\tilde{c}_i(t) = c_i(t) - c_i^*$, $\tilde{g}_{ij}(t) = g_{ij}(t) - g_{ij}^*$. By defining for compactness

$$\begin{aligned} B_k(t) &= \text{diag}(b_1 \tilde{k}_1^T(t), \dots, b_N \tilde{k}_N^T(t)) \\ B_l(t) &= \text{diag}(\tilde{l}_1(t)b_1 f^T, \dots, \tilde{l}_N(t)b_N f^T) \\ B_c(t) &= \text{diag}(b_1 \tilde{c}_1(t), \dots, b_N \tilde{c}_N(t)) \\ B_g(t) &= \text{diag}(b_1 \sum_{j=1}^N \tilde{g}_{1j}(t)a_{1j} \sin(\theta_1 - \theta_j), \dots, \\ & \dots, b_N \sum_{j=1}^N \tilde{g}_{Nj}(t)a_{Nj} \sin(\theta_N - \theta_j)) \end{aligned} \quad (15)$$

the closed-loop for the overall network can be written as

$$\begin{aligned} \dot{x} = & (I_N \otimes A_0 + B_k(t))x + (I_N \otimes b_0 f^T + B_l(t))e + \\ & + B_c(t) + B_g(t) \end{aligned} \quad (16)$$

where $x = [x_1^T, x_2^T, \dots, x_N^T]^T$ and $e = [e_1^T, e_2^T, \dots, e_N^T]^T$. Note that $e_i \in \mathbb{R}^{2 \times 1}$ and the error for the overall network can be written as $e = (\mathcal{L} \otimes I_2)x$. Since the graph is undirected and connected, there exists a unitary matrix $\mathcal{U} = [\frac{1}{\sqrt{N}} \mathbf{1}_N \ \mathcal{U}_2]$ with $\mathcal{U}_2 \in \mathbb{R}^{N \times (N-1)}$ such that $\mathcal{U}^T \mathcal{L} \mathcal{U} = \text{diag}(0, \lambda_2, \dots, \lambda_N) \triangleq \Lambda$. This can be used to define $\bar{e} = (\mathcal{U} \otimes I_2)e$. Moreover let $\bar{e} = [\bar{e}_1^T, \bar{e}_2^T, \dots, \bar{e}_N^T]^T$, it is easily checked that

$$\begin{aligned} \bar{e}_1 &= \left(\frac{1}{\sqrt{N}} \mathbf{1}_N \otimes I_2 \right) e \\ &= \left(\frac{1}{\sqrt{N}} \mathbf{1}_N \otimes I_2 \right) (\mathcal{L} \otimes I_2)x = \mathbf{0}_{(N \times 2) \times 1}. \end{aligned}$$

We can now write the overall error dynamics as

$$\begin{aligned} \dot{e} = & [(I_N \otimes A_0) + (\mathcal{L} \otimes b_0 f^T)]e + \\ & + (\mathcal{L} \otimes I_2)(B_k(t)x + B_l(t)e + B_c(t) + B_g(t)). \end{aligned} \quad (17)$$

The adaptive laws (11) arise from considering the Lyapunov function candidate $V = V_1 + V_2 + V_3 + V_4 + V_5$, where

$$\begin{aligned} V_1 &= e^T (I_N \otimes P)e, \\ V_2 &= \sum_{i=1}^N \frac{\tilde{k}_i^T(t) \gamma^{-1} \tilde{k}_i(t)}{|l_i^*|}, \quad V_3 = \sum_{i=1}^N \frac{\tilde{l}_i(t) \gamma^{-1} \tilde{l}_i^T(t)}{|l_i^*|}, \\ V_4 &= \sum_{i=1}^N \frac{\tilde{c}_i(t) \gamma^{-1} \tilde{c}_i^T(t)}{|l_i^*|}, \quad V_5 = \sum_{i=1}^N \frac{\tilde{g}_{ij}(t) \gamma^{-1} \tilde{g}_{ij}^T(t)}{|l_i^*|}. \end{aligned} \quad (18)$$

Then we have

$$\begin{aligned} \dot{V}_1 &= [2e^T (I_N \otimes P)] \dot{e} \\ &= 2e^T (I_N \otimes P) [(I_N \otimes A_0) + (\mathcal{L} \otimes b_0 f^T)] e + \\ &\quad + 2e^T (I_N \otimes P) [(\mathcal{L} \otimes I_2) (B_k x + B_l e + B_c + B_g)] \\ &= 2\bar{e}^T (I_N \otimes P A_0 + \Lambda \otimes P b_0 f^T) \bar{e} + \\ &\quad + 2e^T (\mathcal{L} \otimes P) (B_k x + B_l e + B_c + B_g) \\ &= \sum_{i=2}^N \bar{e}_i^T [P(A_0 + \lambda_i b_0 f^T) + (A_0 + \lambda_i b_0 f^T)^T P] \bar{e}_i + \\ &\quad + 2 \sum_{i=1}^N \tilde{k}_i^T(t) x_i b_i^T P \left(\sum_{j=1}^N a_{ij} (e_i - e_j) \right) + \\ &\quad + 2 \sum_{i=1}^N \tilde{l}_i(t) e_i^T f b_i^T P \left(\sum_{j=1}^N a_{ij} (e_i - e_j) \right) + \\ &\quad + 2 \sum_{i=1}^N \tilde{c}_i(t) b_i^T P \left(\sum_{j=1}^N a_{ij} (e_i - e_j) \right) + \\ &\quad + 2 \sum_{i=1}^N \left(\sum_{j=1}^N \tilde{g}_{ij}(t) \sin(\theta_i - \theta_j) \right) b_i^T P \left(\sum_{j=1}^N a_{ij} (e_i - e_j) \right). \end{aligned} \quad (19)$$

Moreover, by using (11) we have

$$\dot{V}_2 = -2 \sum_{i=1}^N \frac{\gamma^{-1}}{|l_i^*|} \tilde{k}_i^T(t) x_i b_i^T P \left(\sum_{j=1}^N a_{ij} (e_i - e_j) \right) \quad (20)$$

and similarly for \dot{V}_3 , \dot{V}_4 and \dot{V}_5 . This leads to

$$\dot{V} = \sum_{i=2}^N \bar{e}_i^T [P(A_0 + \lambda_i b_0 f^T) + (A_0 + \lambda_i b_0 f^T)^T P] \bar{e}_i \quad (21)$$

which is negative semi-definite provided that condition (9) holds. Using standard Lyapunov arguments we can prove boundedness of all closed-loop signals and convergence of e to 0. In fact, since $V > 0$ and $\dot{V} \leq 0$, it follows that $V(t)$ has a limit, i.e.,

$$\lim_{t \rightarrow \infty} V(e(t), \tilde{\Omega}(t)) = V_\infty < \infty \quad (22)$$

where we have collected all parametric errors in $\tilde{\Omega}$. The finite limit implies $V, e, \tilde{\Omega} \in \mathcal{L}_\infty$. In addition, by integrating \dot{V} it follows that for some $Q > 0$

$$\int_0^\infty e^T(\tau) Q e(\tau) d\tau \leq V(e(0), \tilde{\Omega}(0)) - V_\infty \quad (23)$$

from which we establish that $e \in \mathcal{L}_2$. Finally, since \dot{V} is uniformly continuous in time (this is satisfied because \dot{V}

is finite), the Barbalat's lemma implies $\dot{V} \rightarrow 0$ as $t \rightarrow \infty$ and hence $e \rightarrow 0$, from which we derive $x_i \rightarrow x_j, \forall i, j$. This concludes the proof.

Remark 2: Theorem 1 provides a leaderless synchronization protocol driving the synchronization error (12) to zero. The final synchronization state to which the network will converge depends on the network initial conditions and cannot be in general imposed a priori. To steer the network in some desired and a priori known state, it is needed to include in the network (3) a leader node with dynamics as in (4) and without any adaptive law. This can be done in two ways: the most direct approach is to keep using the proposed adaptive laws (11), as shown in Sect. IV. An alternative approach, not elaborated here for lack of space, is to exploit a Lyapunov function depending on the pinning matrix [29].

Remark 3: In order to implement (11), and in particular the term $\sum_{j=1}^N a_{ij} (e_i - e_j)$, it is required to communicate among neighbors the extra variable e_i , which is also local information: note that this is also equivalent to communicating x_i to the neighbors of the neighbors (2-hop communication). Communication of extra local variables is often at the core of many synchronization protocols: for example, synchronization based on distributed observer [30], [31] requires communication of extra local variables representing the observer states. Inspired by this idea, let us consider the same synchronizing protocol (10), but this time with the following adaptive version of the distributed observer

$$\begin{aligned} \dot{\chi}_i &= A_0 \chi_i + \mu \left(b_0 f^T \sum_{j=1}^N a_{ij} (\chi_i - \chi_j) \right) \\ \dot{k}_i^T &= -\gamma (x_i - \chi_i)^T P b_0 x_i^T \\ \dot{l}_i &= -\gamma (x_i - \chi_i)^T P b_0 f^T \sum_{j=1}^N a_{ij} (x_i - x_j) \\ \dot{c}_i &= -\gamma (x_i - \chi_i)^T P b_0 \\ \dot{g}_{ij} &= -\gamma (x_i - \chi_i)^T P b_0 \sin(\theta_i - \theta_j) \end{aligned} \quad (24)$$

with adaptive gain $\gamma > 0$ and distributed observer gain $\mu > 0$. The following intuition lies behind (24): a virtual homogeneous network in the form (7) can be constructed in a distributed way, having the same graph as the heterogeneous network. This is the first equation in (24). Since Proposition 2 guarantees synchronization of the virtual homogeneous network, the adaptation laws in (24) can now force each agent in the heterogeneous Kuramoto network to behave as its corresponding agent in the homogeneous network ($x_i - \chi_i \rightarrow 0$), therefore also achieving synchronization (the proof is not given for lack of space). Actually, (24) resembles, with minor modifications, the synchronization protocol adopted in literature for the so-called Euler-Lagrange agents [25], [26]. Now, comparing (24) with (11), we see that the proposed disagreement-based protocol is essentially simpler, because it does not require to construct in a distributed manner the observer variables χ_i .

Remark 4: In some applications it is of interest to synchronize the only frequency, while the phase may not synchronize. One possibility to achieve this via (11) is to

introduce a phase error in the form $\theta_i - \theta_j = h(\omega_i - \omega_j)$, with $h > 0$ a design parameter: this resembles the idea of velocity-dependent time headway in platooning [32].

IV. NUMERICAL EXAMPLES

Simulations using protocol (11) are carried out in the following, considering the weighted graph shown in Figure 2. The parameters and initial conditions for each heterogeneous Kuramoto agent (3) are reported in Table I. Please recall that the agent parameters are unknown to the designer, i.e. the values of Table I are used for simulations but not for control design.

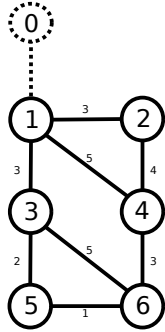


Fig. 2. The undirected weighted communication graph.

TABLE I

PARAMETERS AND INITIAL CONDITIONS FOR THE KURAMOTO AGENTS

	m_i	d_i	ω_i	$\theta_i(0)$	$\dot{\theta}_i(0)$
agent #1	1.1	0.1	5	0	0.6
agent #2	1.3	0.15	10	π	0.5
agent #3	1.2	0.2	15	$\pi/2$	0.4
agent #4	1.8	0.21	20	$(5/4)\pi$	0.3
agent #5	1.5	0.25	25	$\pi/4$	0.2
agent #6	1	0.3	30	$(3/2)\pi$	0.1

The reference model is chosen as

$$\dot{x}_m = \underbrace{\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}}_{A_0} x_m + \underbrace{\begin{bmatrix} 0 \\ 0.8 \end{bmatrix}}_{b_0} u, \quad x_m = \begin{bmatrix} \theta_m \\ \dot{\theta}_m \end{bmatrix} \quad (25)$$

which also represents agent 0 in Figure 2, with chosen initial conditions $x_m(0) = [0, 1]$. The vector f and the matrix P are taken as

$$P = \begin{bmatrix} 1.5824 & 0.5824 \\ 0.5824 & 1.2607 \end{bmatrix}, \quad f^T = \begin{bmatrix} -1 & -1 \end{bmatrix}, \quad (26)$$

which satisfy condition (9). Finally, the adaptive gain is taken $\gamma = 1$, and all estimated control gains k_i, l_i, c_i, g_{ij} , are initialized to 0.

The adaptive synchronization resulting from (11) is shown in Figure 3. Synchronization is achieved and, due to heterogeneity, note that each agent has different control inputs u_i that reach different steady-state values.

Finally, Figure 4 shows the adaptive control gains of (11) for all the systems. Overall, the protocol (11) shows synchronization capabilities in the presence of both uncertainty and

heterogeneity, and without the need to construct a distributed observer.

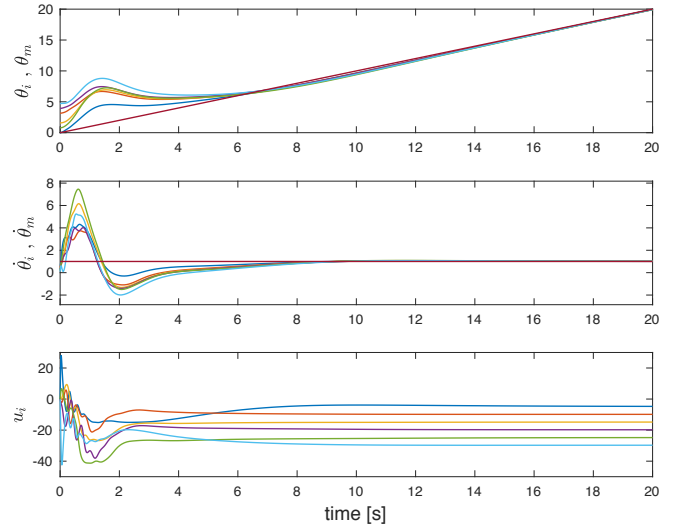


Fig. 3. Protocol (11): synchronization of the states of each agent i to the leader reference state $[\theta_m, \dot{\theta}_m]$. The control inputs u_i are shown at the bottom.

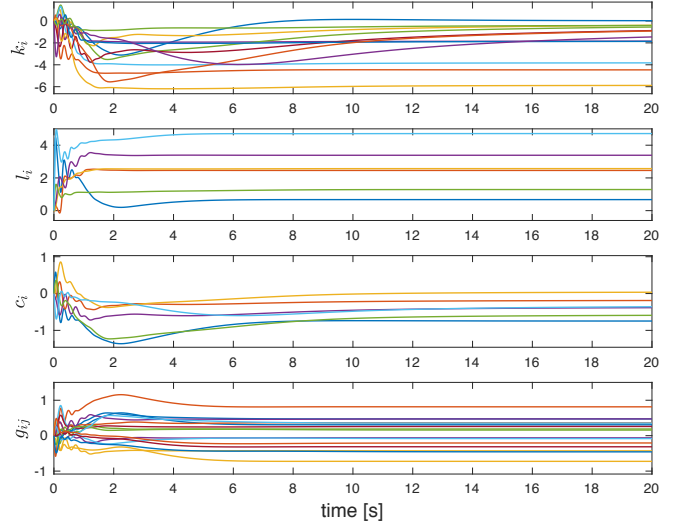


Fig. 4. Protocol (11): adaptive gains.

V. CONCLUSIONS

We analyzed the synchronization capabilities in heterogeneous networks with Kuramoto-like agents whose dynamics are unknown and thus synchronization protocols must exhibit co-evolution capabilities. An adaptive synchronization protocol was presented, based on a distributed disagreement measure. Convergence of the synchronization error to zero was shown via Lyapunov analysis, and numerical examples demonstrated the effectiveness of the proposed protocols. We have also shown that the proposed protocol simplifies some observer-based adaptive protocol (e.g. like the one used in Euler-Lagrange agents), since it does not require to construct any observer state.

Future work could include saturation constraints so as to model different features of the units: for example, in Kuramoto models of microgrids [33], [34], sources and consumers can only deliver or drain power, respectively. Beyond the microgrid example, future work should be devoted to the following points. The proposed adaptive protocol achieve synchronization by ‘cancelling-out’ nonlinearities in a sort of adaptive feedback linearization scheme. However, it has been shown that feedback linearization does not lead in general to optimal control inputs [35]: it would be of interest to develop a new adaptive protocol that, while still achieving synchronization, exploits the nonlinearities instead of cancelling them. Finally, a relevant extension could be to consider switching topology: this can be achieved using adaptive switched tools, as in [36].

REFERENCES

- [1] S. Strogatz, *Sync: The emerging Science of spontaneous Order*. Hyperion, New York, 2003.
- [2] P. Wieland, R. Sepulchre, and F. Allgower, “An internal model principle is necessary and sufficient for linear output synchronization,” *Automatica*, vol. 47, no. 5, pp. 1068 – 1074, 2011.
- [3] X. Wang, Y. Hong, J. Huang, and Z. P. Jiang, “A distributed control approach to a robust output regulation problem for multi-agent linear systems,” *IEEE Transactions on Automatic Control*, vol. 55, no. 12, pp. 2891–2895, 2010.
- [4] F. Sorrentino, M. di Bernardo, F. Garofalo, and G. Chen, “Controllability of complex networks via pinning,” *Physical Review E*, vol. 75, pp. 1–6, 2007.
- [5] Y. Kuramoto, *Chemical Oscillations, Waves, and Turbulence*. Springer, 1984.
- [6] I. D. Couzin, J. Krause, N. R. Franks, and S. A. Levin, “Effective leadership and decision-making in animal groups on the move,” *Nature*, vol. 433, pp. 513 – 516, 2005.
- [7] A. R. Bergen and D. J. Hill, “A structure preserving model for power system stability analysis,” *IEEE transactions on power apparatus and systems*, vol. 100, pp. 25–35, 1981.
- [8] M. Okaniwa and H. Ishii, “An averaging method for synchronization in Kuramoto models,” *3rd IFAC Workshop on Distributed Estimation and Control in Networked Systems*, vol. 45, no. 26, pp. 282 – 287, 2012.
- [9] F. Dorfler and F. Bullo, “Synchronization in complex networks of phase oscillators: A survey,” *Automatica*, vol. 50, no. 6, pp. 1539 – 1564, 2014.
- [10] A. Jadbabaie, J. Lin, and A. S. Morse, “Coordination of groups of mobile autonomous agents using nearest neighbor rules,” *IEEE Transactions on Automatic Control*, vol. 48, no. 6, pp. 988–1001, 2003.
- [11] R. Olfati-Saber, J. A. Fax, and R. M. Murray, “Consensus and cooperation in networked multi-agent systems,” *Proceedings of the IEEE*, vol. 95, no. 1, pp. 215–233, 2007.
- [12] X. F. Wang and G. Chen, “Synchronization in scale-free dynamical networks: robustness and fragility,” *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, vol. 49, no. 1, pp. 54–62, 2002.
- [13] P. DeLellis, M. di Bernardo, and D. Liuzza, “Convergence and synchronization in heterogeneous networks of smooth and piecewise smooth systems,” *Automatica*, vol. 56, pp. 1 – 11, 2015.
- [14] F. Dorfler, M. Chertkov, and F. Bullo, “Synchronization in complex oscillator networks and smart grids,” *Proceedings of the National Academy of Sciences*, vol. 110, no. 6, pp. 2005–2010, 2013.
- [15] P. S. Skardal and A. Arenas, “Control of coupled oscillator networks with application to microgrid technologies,” *Science Advances*, vol. 1, no. 7, pp. 1–6, 2015.
- [16] S. Baldi, “Cooperative output regulation of heterogeneous unknown systems via passification-based adaptation,” *IEEE Control Systems Letters*, vol. 2, no. 1, pp. 151–156, 2018.
- [17] S. Baldi and P. Frasca, “Adaptive synchronization of unknown heterogeneous agents: an adaptive virtual model reference approach,” *Journal of the Franklin Institute*, 2018.
- [18] T. Gross and B. Blasius, “Adaptive coevolutionary networks: a review,” *Journal of The Royal Society Interface*, vol. 5, no. 20, pp. 259–271, 2008.
- [19] S. Assenza, R. Gutierrez, J. Gomez-Gardenes, V. Latora, and S. Boccaletti, “Emergence of structural patterns out of synchronization in networks with competitive interactions,” *Scientific Reports*, vol. 1, no. 99, pp. 1 – 8, 2011.
- [20] P. DeLellis, M. diBernardo, and F. Garofalo, “Novel decentralized adaptive strategies for the synchronization of complex networks,” *Automatica*, vol. 45, no. 5, pp. 1312 – 1318, 2009.
- [21] Q. Ren, M. He, X. Yu, Q. Long, and J. Zhao, “The adaptive coupling scheme and the heterogeneity in intrinsic frequency and degree distributions of the complex networks,” *Physics Letters A*, vol. 378, no. 3, pp. 139 – 146, 2014.
- [22] S.-Y. Ha, S. E. Noh, and J. Park, “Synchronization of Kuramoto oscillators with adaptive couplings,” *SIAM Journal on Applied Dynamical Systems*, vol. 15, no. 1, pp. 162–194, 2016.
- [23] L. Papadopoulos, J. Z. Kim, J. Kurths, and D. S. Bassett, “Development of structural correlations and synchronization from adaptive rewiring in networks of Kuramoto oscillators,” *Chaos*, vol. 27, no. 7, p. 073115, 2017.
- [24] P. DeLellis, M. di Bernardo, T. E. Goroehowski, and G. Russo, “Synchronization and control of complex networks via contraction, adaptation and evolution,” *IEEE Circuits and Systems Magazine*, vol. 10, no. 3, pp. 64–82, 2010.
- [25] Z. Feng, G. Hu, W. Ren, W. E. Dixon, and J. Mei, “Distributed coordination of multiple unknown euler-lagrange systems,” *IEEE Transactions on Control of Network Systems*, vol. PP, no. 99, pp. 1–1, 2016.
- [26] J. Mei, W. Ren, B. Li, and G. Ma, “Distributed containment control for multiple unknown second-order nonlinear systems with application to networked lagrangian systems,” *IEEE Transactions on Neural Networks and Learning Systems*, vol. 26, no. 9, pp. 1885–1899, 2015.
- [27] Z. Li, Z. Duan, G. Chen, and L. Huang, “Consensus of multiagent systems and synchronization of complex networks: A unified viewpoint,” *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 57, no. 1, pp. 213–224, 2010.
- [28] S. Baldi, I. A. Azzollini, and E. Kosmatopoulos, “A distributed disagreement-based protocol for synchronization of uncertain heterogeneous agents,” in *2018 European Control Conference (ECC)*, 2018.
- [29] T. E. Gibson, “Adaptation and synchronization over a network: Asymptotic error convergence and pinning,” in *2016 IEEE 55th Conference on Decision and Control (CDC)*, 2016, pp. 2969–2974.
- [30] M. Lu and L. Liu, “Distributed feedforward approach to cooperative output regulation subject to communication delays and switching networks,” *IEEE Transactions on Automatic Control*, vol. 62, no. 4, pp. 1999–2005, 2017.
- [31] H. Cai, F. L. Lewis, G. Hu, and J. Huang, “The adaptive distributed observer approach to the cooperative output regulation of linear multi-agent systems,” *Automatica*, vol. 75, pp. 299 – 305, 2017.
- [32] Y. A. Harfouch, S. Yuan, and S. Baldi, “An adaptive switched control approach to heterogeneous platooning with inter-vehicle communication losses,” *IEEE Transactions on Control of Network Systems*, vol. PP, no. 99, pp. 1–1, 2017.
- [33] G. Filatrella, A. H. Nielsen, and N. F. Pedersen, “Analysis of a power grid using a Kuramoto-like model,” *The European Physical Journal B*, vol. 61, no. 4, pp. 485–491, 2008.
- [34] J. Giraldo, E. Mojica-Nava, and N. Quijano, “Synchronization of dynamical networks with a communication infrastructure: A smart grid application,” in *52nd IEEE Conference on Decision and Control*, 2013, pp. 4638–4643.
- [35] E. D. Sontag, “A universal construction of artstein’s theorem on nonlinear stabilization,” *Systems & Control Letters*, vol. 13, no. 2, pp. 117 – 123, 1989.
- [36] S. Yuan, B. D. Schutter, and S. Baldi, “Adaptive asymptotic tracking control of uncertain time-driven switched linear systems,” *IEEE Transactions on Automatic Control*, vol. 62, no. 11, pp. 5802–5807, 2017.