Empirical Analysis of the Macroscopic Characteristics of Bicycle Flow during the Queue Discharge Process at a Signalized Intersection

Bernat Goñi-Ros¹, Yufei Yuan¹, Winnie Daamen¹, and Serge P. Hoogendoorn¹

Abstract

Signalized intersections are one of the most common types of bottleneck in urban cycling networks. Gaining knowledge on the macroscopic characteristics of bicycle flow during the queue discharge process is crucial for developing ways to reduce the delay experienced by cyclists at intersections. This paper aims to determine these characteristics (including jam density, shockwave speed, and discharge flow), and to unveil possible relationships between them, particularly whether and to what extent discharge flow is correlated with jam density and shockwave speed (which is of high relevance from a traffic management viewpoint). To this end, the study analyzes high-resolution bicycle trajectories derived from video footage on a one-direction cycle path leading to an intersection in Amsterdam (the Netherlands). Linear regression analysis is used to investigate the relationships between macroscopic variables. The results indicate that jam density, shockwave speed, and discharge flow vary considerably across traffic-signal cycles, which highlights the stochastic nature of bicycle flow. Furthermore, the results show that discharge flow is strongly positively correlated with jam density and shockwave speed. It is hypothesized that there is a causal relationship between these variables, which would imply that traffic engineers can increase discharge flows (thus reducing delay) at signalized intersections if they find effective ways to increase jam densities and shockwave speeds.

Bicycle use is increasing in cities all around the world owing to the advantages that cycling offers to travelers and to the implementation of policies promoting this transportation mode (1). Although this trend has important benefits for society from the environmental, accessibility, and public health viewpoints (2), it also comes with challenges, such as safety issues and congestion at bottlenecks. One of the most common types of bottleneck in urban cycling networks is the signalized intersection, where bicycle flow is interrupted (so cyclists need to stand in a queue) during red-signal phases. Long queues may not dissolve completely during one green-signal phase, thus some cyclists may need to wait for more than one red-signal phase to access an intersection. In order to mitigate this problem, it is necessary to maximize the queue outflow during green-signal phases. Some cities are currently testing innovative street-layout concepts and/or bicycle traffic management measures to achieve this goal (3). For that, knowledge concerning the characteristics of bicycle flows and, more specifically, the characteristics of the bicycle queue-formation and discharge processes, is crucial. Key characteristics (from a macroscopic perspective) are jam density, shockwave speed, and queue discharge flow. The jam density of a standing-still queue can be defined as the average number of cyclists per unit of space, and is an outcome of the queue-formation process. Knowing typical jam density values one can estimate queue lengths. When the traffic light turns green, a shockwave between two traffic states (i.e., cyclists who are standing still, and cyclists who are starting to move) emerges. The speed at which this shockwave propagates upstream, which is determined by the gaps cyclists keep when they start to move, is called shockwave speed (or wave speed). Knowing typical wave

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speed values one can estimate the time that any cyclist will start moving given its location within the queue. The queue discharge flow (or simply discharge flow) is generally defined as the average number of cyclists that pass the stop line per unit of time. Importantly, the discharge flow determines the delay experienced by cyclists at the intersection (given the red- and green-phase lengths, and a certain demand profile).

Although knowledge concerning the characteristics of the queue discharge process is very important for the development of effective ways to reduce the delay and discomfort experienced by cyclists at intersections, currently there is only a partial understanding of these characteristics. To the authors’ knowledge, empirical measurements of wave speeds in bicycle queues have not been reported in the literature. And although several studies present empirical measurements of jam density and discharge flow, the reported values vary considerably: jam densities range between 0.27 and 0.65 bicycles/m² (4–8), whereas discharge flows range between 1900 and 4300 bicycles/h/m (7–12). Multiple causes may explain this high variation. Regarding jam density, the variation may be as a result of differences in bicycle dimensions (13, 14), presence of other types of vehicles (e.g., scooters, e-bikes), and cultural differences in perception of personal space, as well as differences in the methods used to calculate jam density. Also, it has been shown that jam density is influenced by queue size (15) and cycle path width (8). Moreover, variation in jam density may be partially intrinsic to the queue-formation process (15). On the other hand, it has been shown that variation in discharge flow measurements can partially be the result of differences in time of the day (which is related to trip purpose) (12), cycle path width (11, 12, 16) and grade (17), as well as methods used to calculate discharge flow (18). Also, it seems that the capacity of uninterrupted cycle paths is influenced by the presence of vehicles other than bicycles (18, 19), so it is reasonable to believe that the discharge flow at intersections may also depend on that. However, another reason explaining the high variation in discharge flows reported in the literature may be a possible dependency on jam density (which itself also shows a considerable amount of variation, as mentioned above) and/or wave speed (which may also be highly variable). First-order traffic flow theory (20) states that \( q_d = \omega \cdot (k_j - k_c) \), where \( q_d \), \( \omega \), \( k_j \) and \( k_c \) denote discharge flow, wave speed, jam density, and critical density, respectively. Critical density is defined as the maximum density achievable under uncongested traffic conditions, and is usually estimated as the average number of cyclists per unit of space observed right downstream of the stop line during the queue discharge process (7). Note that the equation presented above already assumes that \( q_d, k_j \) and \( \omega \) are interrelated.

The main objective of this study is to determine the macroscopic characteristics of bicycle flow during the queue discharge process at signalized intersections (including jam density, wave speed, and discharge flow) based on empirical data, and to unveil possible relationships between these characteristics, particularly whether and to what extent discharge flow is correlated with jam density and wave speed. As mentioned above, discharge flow is a very relevant macroscopic variable from a traffic management viewpoint, as it determines the delay experienced by cyclists at an intersection. For this purpose, the study analyzes bicycle trajectories on a one-way cycle path leading to a signalized intersection in Amsterdam (the Netherlands). The trajectories were derived from top-view video images. Jam density, wave speed, discharge flow, and critical density were measured for 57 queue discharge periods. Linear regression analysis was used to investigate the relationships between these macroscopic flow characteristics. Thus this paper takes into account and investigates the theoretical relationship \( q_d = \omega \cdot (k_j - k_c) \), but it also establishes simpler relationships in order to confirm and highlight correlations between macroscopic characteristics based on the empirical observations. Also, note that the paper focuses on the analysis of the queue discharge process only when: (a) the queue is formed mainly by bicycles; and (b) cyclists stay within the cycle path and are not disturbed by any traffic agents external to the queue while moving. An analysis of the queue discharge process with mixed traffic and/or complex traffic interactions is beyond the scope of this paper.

The rest of this paper is structured as follows. The next section describes the site and explains the procedure used to derive the trajectory data, followed by a description of the methods used to calculate the macroscopic variables and explore the relationships between them. The results of the analysis are then presented and their possible implications for traffic management discussed. The final section contains the conclusions of this study.

**Data Collection**

Bicycle trajectories along a cycle path leading to a signalized intersection in Amsterdam were derived from top-view video images. The cycle path is 2 m wide and unidirectional; it is segregated from car traffic and is used by both bicycles and scooters. Traffic demand in this path is relatively high in peak hours. Access to the intersection is regulated by a traffic light. In order to obtain the video footage, a 10-m pole with two video cameras on top (21) was placed next to it. The camera views can be seen in Figure 1, a and b. The front camera recorded the last 9 m of the cycle path upstream of the stop line, whereas the back camera recorded the previous 11 m
Therefore, the two cameras combined covered 20 m of cycle path upstream of the traffic light. The frame rate varied between 5 and 10 fps during the recordings. Bicycle traffic was recorded between 12:45 and 19:00 h on June 6, 2016. By looking at the video footage, queue discharge periods were selected that met the following criteria: (a) at least seven bicycles/scooters are standing still inside the path before the traffic light turns green; (b) there are no more than two scooters within the queue; (c) all the bicycles/scooters move toward the stop line and pass it without getting out of the cycle path; (d) no pedestrian crosses the cycle path during the discharge process; (e) the discharge process is not affected by downstream traffic conditions; and (f) all the bicycles/scooters pass the stop line before the end of the green phase. The threshold established by criterion (a) aims to ensure that the selected queues are long enough to allow for a proper analysis of the macroscopic flow characteristics during the discharge process. Criterion (b) defines a threshold to guarantee that the selected queues are formed mainly by bicycles (instead of scooters). Criterion (c) ensures that, in the selected periods, no cyclist leaves the cycle path during the queue discharge process. This is important because sometimes cyclists do not respect the traffic rules and move to the road or to the sidewalk before crossing the stop line (see the site in Figure 1a). This irregular behavior may have a significant influence on the macroscopic characteristics of bicycle flow. Criteria (d), (e), and (f) ensure that the queue discharge process is not disturbed by the traffic signal nor by agents external to the bicycle queue.

The total number of selected discharge periods was 57. The trajectories of all bicycles and scooters forming part of the queue were derived for every period (see example in Figure 1c). The complete procedure to derive the trajectories for a given period is as follows:

1. **Video clip decomposition.** The video clip corresponding to the selected period is decomposed into its individual frames.
2. **Manual cyclist tracking.** TrajectoryViewer, a software package developed at TU Delft, is used to manually track over time (i.e., in each video frame) the cyclists that form part of the queue. In this step, space coordinates \((x, y)\) are expressed in number of pixels, and time \((t)\) is given by the frame number. The vehicle type is also recorded. The main input required by TrajectoryViewer are the individual frames of a given video clip.
3. **Height transformation.** Space coordinates are transformed in order to project the head positions to the ground. This is done using linear functions that give the \(x\) and \(y\) coordinates (in pixels) of the

\[x_{\text{ground}} = \frac{x_{\text{pixel}}}{w_{\text{pixel}}} \times w_{\text{ground}}
\]

\[y_{\text{ground}} = \frac{y_{\text{pixel}}}{h_{\text{pixel}}} \times h_{\text{ground}}
\]

*Figure 1.* View of the two cameras installed on the 10-m pole at 13:34:23 h, when the traffic light was red and 12 bicycles were standing still forming a queue, and derived trajectories: (a) front camera; (b) back camera; (c) derived trajectories.
ground position directly below the head of any cyclist in the video frames. These functions are derived by manually selecting several head points in the frames and identifying their projected points on the ground, and then applying linear regression. The linear functions are different for the front and back cameras.

4. Orthorectification. The $x$ and $y$ coordinates of the ground points are transformed in order to obtain cyclist trajectories as seen from a top view (see Figure 1c) instead of a side view. This is achieved by performing a wedge transformation of the coordinates, which corrects for the distortion caused by the cameras not pointing vertically downwards to the cycle path. The parameters of the wedge transformation function are determined using ImageTracker (22). Also, in this step, $x$ and $y$ coordinates are scaled into a unified Cartesian coordinate system (in meters). The scaling parameters are determined using ImageTracker. Orthorectification is necessary to ensure that distances between all points of a given trajectory are equivalent to real distances between cyclist heads (projected to the ground).

5. Time coding. The time units of each trajectory point are changed from frame number to seconds. To do this, it is necessary to first assign a time instant in seconds to every frame in accordance with the video file metadata.

6. Trajectory merging. Trajectories from the two cameras that correspond to the same cyclist are merged into consistent full trajectories. This is done by coupling pairs of trajectories that are very close in space and time within the cycle path area where the two camera views overlap.

Data Analysis Methods

This section presents the way macroscopic flow characteristics were calculated for each selected discharge period, and describes the methodology used to establish relationships between these characteristics.

Measured Quantities

An example of the trajectories during one of the selected discharge periods is shown in Figure 2a. The three following quantities were measured for every selected period: jam density ($k_j$), wave speed ($\omega$), discharge flow ($q_d$), critical density ($k_c$) and critical speed ($\omega_c$). This was done as follows. First, a count area was defined between the stop line (see stop line in Figure 1a) and a parallel line located 0.4 m downstream of the stop line (which we called line $a$, see Figure 2b). The count area stretches across the full width of the cycle path (2 m). Then, in every period, two indices ($i$ and $j$) were assigned to every cyclist within the queue (Figure 2b). Index $i$ gives the rank of each cyclist regarding their distance to line $a$ at standstill ($i = 1$ corresponds to the closest cyclist to that line). Note that the $x$-coordinate of line $a$ is constant ($x_a$).

Index $j$ gives the rank of each cyclist regarding the time they pass line $a$ ($j = 1$ corresponds to the cyclist that crosses it first). Note that a certain cyclist can have different $i$ and $j$ indices. For instance, in Figure 2b, the third closest cyclist to line $a$ at standstill ($i = 3$) is the second cyclist to pass it ($j = 2$). Once cyclists were assigned these indices, the macroscopic quantities were calculated as follows (see also Figure 2b).

The jam density ($k_j$) was calculated by dividing the number of bicycles in the queue ($N$) minus one by the distance at standstill between the bicycles that are closest and farthest away from line $a$ at standstill ($i = 1$ and $i = N$, respectively), which is denoted by $L$ (see Figure 2b). The jam density is standardized by dividing it by the cycle path width ($W$).

$$k_j = \frac{N - 1}{L \cdot W} \tag{1}$$

where:

$$L = d_N(t_0) - d_1(t_0) \tag{2}$$

In Equation 2, $d_i$ denotes the distance from line $a$ (i.e., $d_i = x_a - x_i$); and $t_0$ denotes the first time instant for which the positions of all cyclists are known. Note that at $t_0$ all cyclists are standing still. In this definition of jam density, cyclist $i = 1$ is not counted as being inside area $L \cdot W$ (this is why the numerator of the right-hand term of Equation 1 is $N - 1$). This is done for consistency with the definitions of discharge flow, critical density, and critical speed, which exclude the first vehicle to pass line $a$ (see Equations 3–5).

The wave speed ($\omega$) was estimated by fitting a line (using linear regression) to the last points in the $td$ plane before bicycles $j = 1, \ldots, N$ start moving, which are denoted by $(t_{jm}, d_j(t_{jm}))$, where $t_{jm}$ is the last time instant before cyclist $j$ starts moving. The intercept of this line is fixed to $d_1(t_0)$, and the $t$ coordinates are transformed to $t_{jm}$. The wave speed ($\omega$) is defined as the slope of the fitted line, which can be expressed as $d = d_1(t_0) + \omega \cdot (t - t_{jm})$ (see Figure 2b).

Discharge flow ($q_d$), critical density ($k_c$) and critical speed ($\omega_c$) are defined as the average flow, density and speed within the count area (right downstream of the stop line), and they were calculated using Edie’s definitions (23) (see also Figure 2b). Discharge flow was defined as a flow through an area instead of a line to ensure consistency with the definitions of critical density and speed. Note that $q_d$ and $k_c$ were standardized by
Discharge flow \( q_d \) was calculated as follows:

\[
q_d = \frac{\sum_{j=2}^{N} x_j}{\Delta x \cdot \Delta t \cdot W}
\]

where: \( \Delta x \) is the distance in the \( x \)-dimension between the stop line and line \( a \) (0.4 m); \( \Delta t \) is the period between the time instants when cyclists \( j = 1 \) and \( j = N \) cross line \( a \) (i.e., when \( d_1 \) and \( d_N \) become zero, respectively); and \( x_j \) is the distance that bicycle \( j \) travels through the count area during period \( \Delta t \).

Note that, since the time instant when cyclist \( j = 1 \) crosses line \( a \) is used to define the period \( \Delta t \) (see Figure 2b), cyclist \( j = 1 \) is not counted when calculating \( q_d \). This is done to exclude the period with zero flow between the beginning of the green-signal phase and the passing time of the first cyclist. The length of this period may vary substantially because of variation in the reaction times of cyclists \( j = 1 \) to the green signal, so including it may add error to the discharge flow measurements. For consistency, the first cyclist is not counted when calculating the critical density \( k_c \) and the critical speed \( v_c \) either. These quantities were calculated as follows (see also Figure 2b):

\[
k_c = \frac{\sum_{j=2}^{N} \tau_j}{\Delta x \cdot \Delta t \cdot W}
\]

and
\[ v_c = \frac{\sum_{j=2}^{N} \chi_j}{\sum_{j=2}^{N} \tau_j} = \frac{q_d}{k_c} \]  

where \( \tau_j \) denotes the travel time of cyclist \( j \) within the count area during period \( \Delta t \).

**Data Analysis**

The empirical measurements of macroscopic flow characteristics (jam density, wave speed, discharge flow, critical density, and critical speed) described above were analyzed in two different ways. First, the variability of the measurements across the 57 selected periods was analyzed. Histograms were created and descriptive statistics calculated (such as the average and the percentiles) in order to gain insight into the shapes of their distributions. Note that the analyzed periods are only a selection of all the observed periods (the selection criteria were explained earlier). Therefore, the histograms and descriptive statistics do not represent all observed periods but only those periods for which it is appropriate to measure the macroscopic flow characteristics given the scope of this research project.

Second, the relationships between different macroscopic flow characteristics were analyzed. The main goal was to determine whether and to what extent discharge flow (which can be seen as an outcome of the queue discharge process) is correlated with jam density and wave speed (which are features of that process). After visual inspection of the data points (see, for example, Figure 3), linear regression analysis was selected to explore these relationships. Nevertheless, this approach is considered as a first step in the analysis of the relationships between macroscopic flow characteristics. The use of more complex regression techniques may provide additional insights, but that will be addressed in further research. In this paper, five linear regression models were estimated based on the empirical measurements:

- Model 1: \( q_d = \alpha_1 + \beta_1 \cdot k_j \);
- Model 2: \( q_d = \alpha_2 + \beta_2 \cdot k_j + \gamma_2 \cdot \omega \);
- Model 3: \( k_c = \alpha_3 + \beta_3 \cdot k_j + \gamma_3 \cdot \omega \);
- Model 4: \( \omega = \alpha_4 + \beta_4 \cdot k_j \);
- Model 5: \( q_d = \beta_5 \cdot (k_j - k_c) \).

\( F \)-tests and \( t \)-tests were used to determine the significance of the estimated models and their parameters (a 5% significance level was used for all models and parameters). Models 1 and 2 were estimated to investigate the relationship between discharge flow \( (q_d) \), jam density \( (k_j) \), and wave speed \( (\omega) \). An incremental approach was used to gain insight into the individual correlation of \( q_d \) with \( k_j \) and \( \omega \). For this reason, two separate linear regression models were estimated: one with only \( k_j \) as regressor (Model 1), and one with both \( k_j \) and \( \omega \) as regressors (Model 2). Model 3 was estimated to investigate the relationship between critical density \( (k_c) \) on one hand, and jam density \( (k_j) \) and wave speed \( (\omega) \) on the other. Model 4 was estimated to determine whether and to what extent jam density and wave speed are linearly correlated.
Finally, Model 5 was estimated to determine the wave speed according to first-order traffic flow theory—which states that parameter \( \beta_2 \) is analogous to the wave speed (20)—and to see whether the empirical data match this theory.

Models 1–5 were first estimated using raw scores (i.e., the \( k_j \), \( \omega \), \( q_j \) and \( k_j \) measurements expressed in their own units). Then, the models were also estimated using standard scores (Z-scores) in order to obtain their standardized coefficients. These coefficients indicate how many standard deviations the regressand is expected to change per standard deviation increase in the regressors, according to a given regression model. In the case of these models, standardization of the coefficients is useful because the variables involved are measured in different units.

**Results**

**Descriptive Statistics**

The total number of queue discharge periods analyzed in this study is 57. The average queue size in these periods is 12.12 cyclists, and in 75% of them the queue consists of 10 or more cyclists (see Table 1). In 44 out of 57 periods (77.2%), there are no scooters in the queue; in 11 periods (19.3%), there is one scooter, and in the remaining 2 periods (3.5%), there are two. The maximum percentage of scooters is 14.29% (see Table 1), corresponding to a period in which 2 out of 14 cyclists are riding a scooter.

The measurements of the macroscopic quantities defined earlier vary substantially across different queue discharge periods. Figure 4, a–c, shows the histograms of these measurements, and Table 1 shows various descriptive statistics. Jam density measurements (\( k_j \)) range between 0.31 and 0.65 cyclists/m², with an average equal to 0.45 cyclists/m² and a standard deviation equal to 0.08 cyclists/m². Visual inspection of the video images indicates that some of the factors contributing to this high variation in jam density may be the presence of social groups, and the preference of some cyclists to stand on the right side of the cycle path, where they can put their right foot on the sidewalk (see Figure 1b). Also, some cyclists seem to avoid standing on the pedestrian crossing area. Wave speeds range between 1.86 and 7.67 m/s (6.69 and 27.63 km/h); the average is 3.98 m/s (14.32 km/h), and the standard deviation is 1.25 m/s (4.51 km/h). Discharge flow (\( q_d \)) measurements range between 0.34 and 0.79 cyclists/s/m (1227 and 2829 cyclists/h/m) (see Table 1). The average discharge flow is 0.60 cyclists/s/m (2148 cyclists/h/m) and the standard deviation is 0.10 cyclists/s/m (351 cyclists/h/m). Regarding the critical density (\( k_c \)), the average is 0.26 cyc/m² and the standard deviation is 0.07 cyc/m². The average critical speed is 2.43 m/s (8.75 km/h) and the standard deviation is 0.46 m/s (1.66 km/h).

Table 1 and Figure 4f also show the distribution of start-up times, which correspond to the length of the time period between the beginning of the green phase and the last time instant before the first vehicle to pass line \( a \) starts moving (\( t_1^a \), see earlier). The average start-up time is 0.70 s, and in 25% of the analyzed periods it is equal to or greater than 1.02 s. In four cases (7%), it is negative, meaning that cyclist \( j = 1 \) starts moving before the traffic light turns green.

**Relationships between Macroscopic Characteristics**

The regression analysis results show that there is a positive linear relationship between discharge flow (\( q_d \)) and jam density (\( k_j \)) (see Table 2, Model 1, and Figure 3a). According to this model, \( k_j \) accounts for 45% of the variance in \( q_d \) (\( R^2 = 0.453 \)). If wave speed (\( \omega \)) is added to jam density as regressor, a multiple linear model with a better fit is achieved (see Table 2, Model 2b). According to this model, \( k_j \) and \( \omega \) combined account for 71% of the variance in \( q_d \) (\( R^2 \) equals 0.709), 26 percentage points more than \( k_j \) alone. Note that this model was estimated with the intercept (\( \alpha_2 \)) fixed to zero, since the t-tests suggest that the intercept is not significantly different than this value (see Table 2, Model 2a). Furthermore, there seems to be a positive linear relationship between critical density (\( k_c \)), jam density, and wave speed. The latter two variables account for 31% of the variance in \( k_c \) (see Table 2, Model 3b). Note that this model was also estimated.

### Table 1. Descriptive Statistics (57 Observations)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Avg</th>
<th>StD</th>
<th>p10</th>
<th>p25</th>
<th>p50</th>
<th>p75</th>
<th>p90</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Queue size (cyclists)</td>
<td>12.12</td>
<td>2.99</td>
<td>8.20</td>
<td>10</td>
<td>12</td>
<td>13.25</td>
<td>16.80</td>
<td>7</td>
<td>19</td>
</tr>
<tr>
<td>Percentage of scooters (%)</td>
<td>2.08</td>
<td>4.05</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>9.09</td>
<td>0.00</td>
<td>14.29</td>
</tr>
<tr>
<td>Jam density (cyc/m²)</td>
<td>0.45</td>
<td>0.08</td>
<td>0.34</td>
<td>0.40</td>
<td>0.46</td>
<td>0.50</td>
<td>0.55</td>
<td>0.31</td>
<td>0.65</td>
</tr>
<tr>
<td>Wave speed (m/s)</td>
<td>3.98</td>
<td>1.25</td>
<td>2.43</td>
<td>2.99</td>
<td>3.81</td>
<td>4.71</td>
<td>5.68</td>
<td>1.86</td>
<td>7.67</td>
</tr>
<tr>
<td>Discharge flow (cyc/s/m)</td>
<td>0.60</td>
<td>0.10</td>
<td>0.46</td>
<td>0.53</td>
<td>0.60</td>
<td>0.68</td>
<td>0.73</td>
<td>0.34</td>
<td>0.79</td>
</tr>
<tr>
<td>Critical density (cyc/m²)</td>
<td>0.26</td>
<td>0.07</td>
<td>0.19</td>
<td>0.20</td>
<td>0.25</td>
<td>0.30</td>
<td>0.35</td>
<td>0.15</td>
<td>0.48</td>
</tr>
<tr>
<td>Critical speed (m/s)</td>
<td>2.43</td>
<td>0.46</td>
<td>1.86</td>
<td>2.09</td>
<td>2.46</td>
<td>2.80</td>
<td>2.94</td>
<td>1.34</td>
<td>3.45</td>
</tr>
<tr>
<td>Start-up time (s)</td>
<td>0.70</td>
<td>0.75</td>
<td>0.04</td>
<td>0.34</td>
<td>0.59</td>
<td>1.02</td>
<td>1.33</td>
<td>-1.55</td>
<td>3.55</td>
</tr>
</tbody>
</table>
Figure 4. Frequency distributions of the measured quantities: (a) jam density; (b) wave speed; (c) queue discharge flow; (d) critical density; (e) critical speed; (f) start-up time.
with the intercept \((\alpha_1)\) fixed to zero. Interestingly, wave speed is found to be negatively linearly correlated to jam density. However, this relationship does not seem to be very strong, as \(k_j\) only accounts for 10% of the variance in \(v\) (see Table 2, Model 4, and Figure 3b).

Finally, there is a positive linear relationship between discharge flow \((q_d)\) and the difference between jam density and critical density \((k_j - k_c)\). This difference accounts for 48% of the variance in \(q_d\) (see Table 2, Model 4, and Figure 3b).

According to first-order traffic flow theory, the slope of this linear relation \((\beta_5)\) should correspond to the wave speed \((20)\). However, the value of \(\beta_5\) (2.69 m/s) is lower than the average observed wave speed (3.98 m/s), thus this theoretical model may tend to underestimate the discharge flow. One of the reasons for this may be related to the definitions of the macroscopic quantities. First-order traffic flow theory assumes infinite acceleration, which implies that after the traffic light turns green, bicycle traffic at the count area is immediately homogeneous. In reality, however, acceleration is not infinite. The first group of cyclists to cross the stop line are not moving at constant speed (see, for instance, Figure 2a), thus traffic at the count area is not immediately homogeneous. As this first group of cyclists is included in the calculation of flow and density at the count area, higher critical densities and lower discharge flows may be obtained than those assumed by first-order traffic flow theory. This implies that \(\beta_5\) cannot be interpreted directly as the wave speed.

### Possible Implications for Traffic Management

Although regression analysis can only establish correlation (not causation) between variables, it is hypothesized that discharge flow is actually influenced by jam density and wave speed (and possibly also by other factors not accounted for in this study, such as the way cyclists stop and go.

<table>
<thead>
<tr>
<th>Table 2. Model Estimation Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>(F)-stat</td>
</tr>
<tr>
<td>Model 1: (q_d (\text{cyc/m}) = \alpha_1 + \beta_1 \cdot k_j (\text{cyc/m}^2))</td>
</tr>
<tr>
<td>45.6</td>
</tr>
<tr>
<td>(\alpha_1)</td>
</tr>
<tr>
<td>Model 2a: (q_d (\text{cyc/m}) = \alpha_2 + \beta_2 \cdot k_j (\text{cyc/m}^2) + \gamma_2 \cdot \omega (\text{m/s}))</td>
</tr>
<tr>
<td>74.8</td>
</tr>
<tr>
<td>(\alpha_2)</td>
</tr>
<tr>
<td>Model 2b: (q_d (\text{cyc/m}) = \beta_2 \cdot k_j (\text{cyc/m}^2) + \gamma_2 \cdot \omega (\text{m/s}))</td>
</tr>
<tr>
<td>70.0</td>
</tr>
<tr>
<td>(\beta_2)</td>
</tr>
<tr>
<td>Model 3a: (k_c (\text{cyc/m}^2) = \alpha_3 + \beta_3 \cdot k_j (\text{cyc/m}^2) + \gamma_3 \cdot \omega (\text{m/s}))</td>
</tr>
<tr>
<td>16.7</td>
</tr>
<tr>
<td>(\alpha_3)</td>
</tr>
<tr>
<td>Model 3b: (k_c (\text{cyc/m}^2) = \beta_3 \cdot k_j (\text{cyc/m}^2) + \gamma_3 \cdot \omega (\text{m/s}))</td>
</tr>
<tr>
<td>12.1</td>
</tr>
<tr>
<td>(\beta_3)</td>
</tr>
<tr>
<td>Model 4: (\omega (\text{m/s}) = \alpha_4 + \beta_4 \cdot k_j (\text{cyc/m}^2))</td>
</tr>
<tr>
<td>5.93</td>
</tr>
<tr>
<td>(\alpha_4)</td>
</tr>
<tr>
<td>Model 5: (q_d (\text{cyc/m}) = \beta_5 \cdot (k_j (\text{cyc/m}^2) - k_c (\text{cyc/m}^2)))</td>
</tr>
<tr>
<td>51.78</td>
</tr>
</tbody>
</table>
accelerate from standstill, and passing maneuvers during the queue discharge process). This hypothesis is assumed true in first-order traffic flow theory, as explained previously. It is argued that the hypothesis is plausible from a traffic-engineering viewpoint, at least within reasonable ranges of $k_j$ and $\omega$: if cyclists stand closer to each other in the queue (i.e., if $k_j$ is higher) and/or they start moving earlier (i.e., if $\omega$ increases), they will probably cross the stop line earlier, which implies that the discharge flow ($q_d$) will increase. This hypothesis needs to be experimentally tested. However, a relevant implication follows from assuming it is true: traffic engineers can increase the discharge flow at signalized intersections (and thus reduce delay) if they find ways to influence the behavior of cyclists in such a way that queues become denser and wave speeds higher.

In this respect, Model 2b suggests that, within the range of observed values, if both $k_j$ and $\omega$ increase by one standard deviation, the discharge flow increases by $(0.848 + 0.558) \cdot 100 = 140.6\%$ of its standard deviation, that is, 0.141 bicycles/s/m (508 bicycles/h/m). This can be concluded from the values of the standardized coefficients of $k_j$ and $\omega$ (see Table 2, Model 2b). Let us analyze an imaginary case to illustrate the hypothetical influence of these two quantities on the discharge flow according to Model 2b. Assuming a 2-m wide cycle path and a traffic-light cycle with a 20-s long effective green time, if a queue has the average $k_j$ and $\omega$ observed in our measurements, the maximum number of cyclists that can access the intersection during the green phase is $2m \cdot 20s \cdot (0.959m/s \cdot 0.45cyc/m^2 + 0.041cyc/m^2 \cdot 3.98m/s) = 23.8$ cyclists. If there are more cyclists in the queue, these need to wait one more red-signal phase. Instead, if the jam density and wave speed increase both by one standard deviation, this number increases to $2m \cdot 20s \cdot (0.959m/s \cdot (0.45cyc/m^2 + 0.08cyc/m^2) + 0.041cyc/m^2 \cdot (3.98m/s + 1.25m/s)) = 28.9$ cyclists (a 22% increase). It is important to remark that Model 2b is not a validated predictive model. Nevertheless, these simple calculations give an indication of the potential benefits that could be obtained by increasing the jam densities and wave speeds of queues at signalized intersections.

Conclusions and Outlook

A proper understanding of the bicycle queue discharge process is crucial for developing ways to reduce delay at signalized intersections. The main goal of this study was to determine the macroscopic characteristics of bicycle flow during the queue discharge process, and to unveil possible relationships between them. To this end, the study analyzed bicycle trajectories from a one-way cycle path leading to a signalized intersection in Amsterdam. Jam density, wave speed, and discharge flow were measured for 57 queue discharge periods. Linear relationships between these macroscopic flow variables were established using regression analysis.

The results show that jam density, wave speed, and discharge flow vary considerably across traffic-signal cycles, although they are within the range of values reported in the literature. Further research is needed to establish the causes of this variability; however, its existence is a key finding that highlights the stochastic nature of bicycle flow. An important implication is that traffic engineers should take stochasticity into account when predicting bicycle flow dynamics at intersections. Moreover, the results confirm that discharge flow is positively linearly correlated with both jam density and wave speed, at least within the range of observed values. Also, discharge flow is positively linearly correlated to the difference between jam density and critical density, as expected from first-order traffic flow theory. However, it was found that this theory is not able to describe bicycle flow dynamics at intersections very accurately.

The data set analyzed in this paper is of limited size (57 discharge periods at only one intersection). An analysis of additional trajectory data, preferably from other sites (including intersections with two-directional cycle paths), is necessary to obtain more conclusive results. More specifically, data from other sites are crucial to identify the factors contributing to the variability of jam density, wave speed, and discharge flow at signalized intersections, as well as to establish more accurately the relationships between these variables. In this regard, it is necessary to investigate whether these relationships may be non-linear and also whether additional variables may need to be considered.

Although regression analysis cannot establish causation between variables (only correlation), it is hypothesized that discharge flow is actually influenced by jam density and wave speed, as generally assumed by traffic flow theory. Further experimental research may validate this hypothesis; nevertheless, two relevant implications follow from assuming it is true. The first one is that variability in jam density and wave speed in real bicycle queues may partially explain the differences in discharge flow reported in the literature. The second implication is that there is often room to fit additional cyclists in queues, and cyclists are able to start moving earlier, while increasing the discharge flow. This points to two potentially effective strategies to increase outflow at signalized intersections, and thus to reduce the delay experienced by cyclists (especially if queues are too long to dissolve completely during one green-signal phase): (a) increasing jam densities; and (b) increasing wave speeds.

However, more research is needed to determine whether it is possible to implement these strategies through specific traffic-engineering measures. To increase
jam densities, one possibility could be to guide cyclists to certain locations during the queue-formation process by indicating ideal paths and/or stopping locations within the waiting area. A further possibility could involve installing static or dynamic information panels asking cyclists to stand close to each other. These panels could also be used to encourage cyclists to start moving early when the traffic light turns green, thus increasing wave speeds. In addition, it may be possible to increase wave speeds by installing traffic lights displaying the time left before the beginning of the green-signal phase; this may induce cyclists to get ready and start moving earlier. The potential effectiveness of these measures is not clear at this point and should be evaluated through field tests.

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Author Contributions
The authors confirm contribution to the paper as follows: study conception and design: all authors; data collection: B. Goñi-Ros, Y. Yuan; study supervision: S. P. Hoogendoorn. All authors reviewed the results and approved the final version of the manuscript.

References


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