Integrating Dynamic Signaling Commands Under Fixed-Block Signaling Systems into Train Dispatching Optimization Problems

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Abstract
In railway operations, when a disruption occurs, train dispatchers aim to adjust the affected schedule and to minimize negative consequences during and after the disruption. As one of the most important components of the railway system, railway signals are used to guarantee the safety of train services. We study the train dispatching problem with consideration of railway signaling commands under the fixed-block signaling system. In such a system, signaling commands dynamically depend on the movement of the preceding trains in the network. We clarify the impact of the signaling commands on train schedules, which has so far been neglected in the literature on railway train dispatching, and we propose an innovative set of signaling constraints to describe the impact. The determination of the signal indicators is presented using “if-then” constraints, which are further transformed into linear inequalities by applying two transformation properties. Activation of the train speed limits that result from the signaling commands is the core purpose of the signaling constraints, and this is implemented by using the signal indicators. Moreover, we formulate the Greenwave (GW) policy, which requires that trains always proceed under green signals, and we further investigate the impact of the GW policy on delays. In numerical experiments, the proposed signaling constraints are employed within a time-instant optimization problem, which is a mixed-integer linear programming (MILP) problem. The experimental results demonstrate the effectiveness of the proposed signaling constraints and show the impact of the signaling commands and GW policy on the train dispatching solution.

Railways are crucial to address the ever-increasing mobility of people and goods, due to their positive characteristics of high capacity, high speeds, and eco-friendliness. A negative characteristic of railway services, however, is their limited reliability and punctuality, which hinders attainment of the modal share sought by policy makers and researchers. Train movements on a railway network are regulated by timetables. In daily operations, perturbations (caused by bad weather, infrastructure failures, extra passenger flows, etc.) unavoidably occur, which may affect the normal operations and cause a deviation from the planned timetable. In such cases, the task of a train dispatcher (controller) is to take proper measures to adjust the schedules affected, in order to reduce the negative consequences (delays). This is the train dispatching problem. Due to the high interdependency between trains utilizing the available capacity, train dispatching work is usually complex, especially when the railway network is operated close to saturation, in densely urbanized zones, or during peak hours. An ineffective train dispatching decision could result in a snowball effect with consecutive delays, downgrading the reliability and punctuality of train services. Fast and effective decisions for the train dispatching problem are always desired.

Railway signaling is one of the most important parts of the railway system. There is a wide variety of railway signals and many signaling systems with different

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principles all over the world, for example, the moving-block signaling systems commonly used in high-speed railway networks and the fixed-block signaling systems commonly used by conventional railways. The core function of the signaling systems is same, however—indicating the state of the block section(s) ahead for the train driver in order to guarantee the safety of train services. A signaling command can be indicated by a single aspect or by multiple aspects. In the United States and in many European countries, the signaling command provides an additional requirement for train operations, namely indicating the maximum allowed speed to the driver. The driver has to control the train to prevent it from exceeding this speed to ensure safety. Otherwise, a worse situation may happen, where the available distance is not enough to stop the train.

An extensive body of studies is available in the literature that addresses the train dispatching problem, having different focuses, such as: considering multiple classes of running traffic (1), passenger connections (2), speed management (3), and maintenance plans (4), and using different approaches (e.g., linear/nonlinear optimization and heuristics). However, a gap still exists with regard to the signaling commands. Train speed limits that result from signaling commands have been neglected in the literature (5), although they are indeed required in real train operations. To the best of our knowledge, no study is yet available for generating optimal train dispatching solutions that integrate precisely the actual signal aspect shown to train drivers, and that guarantee no violation of the signaling commands, based on the fixed-block signaling system. The reality of train operations and the gap in the scientific literature have motivated us to include the signaling commands while addressing the train dispatching problem.

We therefore study the train dispatching problem with consideration of railway signaling commands, focusing on railway networks with a fixed-block signaling system, as is common in the United States and in many European countries. As the signaling commands dynamically depend on the movement of the preceding trains in the network, we use binary variables (namely signal indicators) to indicate the signaling commands. These signal indicators are determined by a set of “if-then” constraints, which could be further transformed into linear inequalities by applying two transformation properties. These constraints could be generalized to other signaling systems. Train speed limits that result from the signaling commands are restricted in the signaling constraints by employing the signal indicators. In addition, we formulate the Greenwave (GW) policy and explore its impact on the train dispatching solution (i.e., the train delays). Basically, the aim of the GW policy is to require trains to follow their scheduled speed profile exactly, thus avoiding the need for speed profile adjustments. In the numerical experiments, a time-instant optimization approach (mixed-integer linear programming, MILP) proposed in our previous work (6) is used to apply the proposed signaling constraints, aiming at delivering a train dispatching solution with minimization of train delays. The experimental results demonstrate the effectiveness of the proposed model, including signaling constraints, and show the impact of the signaling commands and GW policy on the train dispatching solution.

Literature Review

An extensive body of literature is available for the railway train dispatching problem; interested readers might refer to the surveys (5, 7, 8). This section briefly reviews the state of the art for the train dispatching problem, especially focusing on the studies that use operations research-based techniques and on how signaling issues are treated.

Advances in scheduling theory have made it possible to solve real-life train dispatching instances in which not only departure/arrival times (9), but also train orders, routes, speeds, and further operational freedom were considered as variables (10–14).

Several operations research-based techniques are now available for addressing the train dispatching problem. A particularly popular model is the alternative graph-based model, which uses a combination of job shop and alternative graph techniques (9). This alternative graph-based formulation method considers microscopic details and is further employed in many studies, for example, dealing with the train rerouting problem by developing meta-heuristics, including a tabu search algorithm (11) and a variable neighborhood search algorithm (15), and investigating the impact of the details and the number of operational constraints on the applicability of models, in terms of solution quality and computational efficiency (16).

Another stream of studies addresses the train dispatching problem at a macroscopic level, which allows for faster resolution and a larger geographical scope. Schöbel (17) proposed an event-activity based integer programming model to solve the delay management problem. The model was further extended to address a discrete time/cost trade-off problem of maintaining service quality and reducing passengers’ inconvenience (18), and by including headways and capacity constraints and testing multiple pre-processing heuristics in order to fix integer variables and to speed up the computation (19). In the proposed problems, connections are decided to be maintained or dropped by minimizing the number of missed connections, while minimizing the delays of all events.

Other approaches have also been proposed to solve this problem. Luan et al. (4) employed the flag
variables-based formulation method to address the integration of train scheduling and preventive maintenance planning. Pellegrini et al. developed a heuristic algorithm, named RECIFE-MILP, based on an extended version of the MILP formulation they had proposed (20, 21). Samà et al. (22) further investigated how to select the most promising train routes among all possible alternatives, through developing an ant colony optimization meta-heuristic. Some dispatching decision support systems have been developed by researchers, and a few of them have been used in practice. Mazzarello and Ottaviani (23) developed a traffic management support system that is able to optimize traffic flow in large railway networks equipped with either fixed- or moving-block signaling systems. D’Ariano et al. (24) developed an advanced decision support system, known as ROMA (railway traffic optimization by means of alternative graphs), for dispatching trains based on microscopic details. Decomposition technologies were used for handling large areas in this system. It is verified that the system is able to find feasible and efficient schedules quickly. An exact approach and a master-slave solution algorithm (based on decomposition) were presented by Lamorgese and Mannino (25), based on which a dispatching decision support system was developed. This system has been in operation in Norway since February 2014 and represents one of the first operative applications of mathematical optimization to train dispatching.

Most approaches in the literature neglect the impact of signaling issues while rescheduling the traffic. A recent study (3) takes the impact of signaling into account, focusing on high-speed traffic based on the quasi-moving-block system. The authors proposed an alternative graph-based optimization problem to reschedule the high-speed traffic, which integrates the modeling of traffic management measures and the supervision of speeds, braking, and headways. The current paper fills the knowledge gap regarding the signaling issues in the train dispatching problem, which are mostly neglected in the literature. The main contribution is thus the consideration of different signaling commands, not only “Clear” and “Stop” (commonly satisfied by a track capacity constraint), but also “Approach (Limited)” (mostly neglected in the literature), depending dynamically on the traffic state. The proposed approach is suitable for inclusion in optimization schemes, in time-instant (or time-continuous, in the terminology of [5]) formulations, and can be extended to virtually any other fixed-block signaling system.

**Problem Description**

This section introduces fixed-block signaling systems, followed by the relevant signaling commands that should be respected during train operations. We then describe the formulation method of the time-instant optimization approach used in our previous study (6) for addressing the train dispatching problem. This formulation method will be used to construct the signaling constraints later on.

**Railway Signaling Systems**

A railway signaling system is used to direct railway traffic and to keep trains clear of each other at all times (26, 27). A railway signal shows whether the track is clear ahead and also indicates to train drivers how far ahead the track is clear. Figure 1 illustrates three fixed-block signaling systems that are basic and widespread all over the world. In such fixed-block signaling systems, each track is divided into a sequence of block sections, and each block section is protected by a fixed signal placed at its entrance and displayed to the driver of an approaching train.

Figure 1a illustrates a two-aspect signaling system, which has a basic signal with a red and a green aspect. A green aspect indicates that the block section is accessible for trains, and a red aspect rejects the access of trains. As stated in (28), this two-aspect signaling system works well for trains with speeds less than 50 km/h; however, for a train traveling at a higher speed (e.g., over 50 km/h), the train driver needs a warning of a red aspect ahead to allow time to stop. Therefore, multi-aspect signals have appeared, as shown in Figure 1b and c, for a three-aspect signal and a four-aspect signal respectively. The three-aspect signal has a red, a yellow, and a green aspect. If a block section is occupied by a train, the signal placed at its entrance will display a red aspect, and the two signals behind will show a yellow aspect and a green aspect respectively. The yellow aspect provides an advance warning of the red aspect ahead, and the driver then knows that there is only one clear block section ahead. The green aspect indicates that there are at least two clear block sections ahead, and the driver can maintain the design speed until facing a yellow aspect. The four-aspect signaling system in Figure 1c works similarly to the three-aspect signaling system, except that two advance warnings are provided before a red aspect: a single-yellow and a double-yellow aspect. One purpose of doing this is to provide an earlier warning for higher-speed trains, and another purpose is to allow better track occupancy by shortening the length of the block sections.

In the United States and in most European countries, the signal indicates the maximum allowed speed to the driver, and the driver has to control the train to avoid exceeding this speed to ensure safety (otherwise, the driver will be penalized). The blue curves in Figure 1 indicate the maximum speed profiles of train $f_1$ at the
current moment, in the case of different signal aspects at sight distance.

Let us consider the three-aspect signaling system as an example to explain the impact of signal aspects on train speeds. Figure 2 presents the time-space-speed graphs for a train $f_1$ on five consecutive block sections. Different scenarios are illustrated, depending on the movement of the preceding train $f_2$. The orange horizontal line marked for each train on each block section directs the signal aspect faced by the train at the corresponding sight distance. In the case of always facing a green aspect, the driver can proceed with the design speed ($V_{\text{max,green}}$), as illustrated in the planned scenario of Figure 2a. The pre-planned train paths are further indicated by the dashed black lines in the cases of disruptions, illustrated in Figure 2b–f, and the train paths after adjusting are indicated by the solid black lines. In the upper portion of each subfigure, we sketch the train paths and the signal aspects (in color) displayed as a function of the time; in the lower portion, we present the maximum allowed speed for train $f_1$. In the case of facing a yellow aspect at the sight distance of block section $s_2$, the driver of train $f_1$ has to decrease the train speed from the design speed ($V_{\text{max,green}}$) to the approach speed ($V_{\text{max,yellow}}$), which is maintained until the following signal becomes visible. When the train reaches the next sight point, there are five possible scenarios, as shown in Figure 2b–f, which are labeled as scenario C1,..., C5 respectively:

- In scenario C1, the signal placed at the sight distance of block section $s_3$ shows a green aspect, and the driver is allowed to accelerate the train from the approach speed to the design speed.
- In scenario C2, the signal stays yellow, and the train is controlled to enter the next block section at the approach speed.
- In scenario C3, the signal stays red until the train completely stops, and the signal becomes green only after a certain waiting time.
- In scenario C4, the signal is yellow and then switches to green in the sight time (i.e., the running time of a train over the sight distance on a block section). The driver possibly accelerates the train to the design speed after a small reaction time.
- In scenario C5, the signal is red and then switches to yellow (or green directly) during the braking phase. In this scenario, the driver stops braking.
and possibly accelerates the train to the approach speed (or the design speed) after a small reaction time.

We can conclude that the maximum allowed speed of a train on a block section depends on the signaling command (i.e., the signal aspect displayed), and the signaling command dynamically depends on the movement of the preceding train in the railway network. Note that hereafter we make an assumption of considering the most conservative case in terms of safety—the movement of each train on each block section respects the signaling command shown at the moment that the train is reaching sight distance, that is, the possibility of scenarios C4 and C5 in Figure 2 is ignored.

The Time-Instant Formulation Method for the Train Dispatching Problem

In a time-instant (or time-continuous, in the terminology of [5]) optimization approach for addressing the train dispatching problem, we use arrival time variables \( a \) and departure time variables \( d \) to describe train movements on block sections. More specifically, \( a_{f,s} \) indicates the arrival time of train \( f \) at block section \( s \), and \( d_{f,s} \) indicates the departure time of train \( f \) from block section \( s \). The arrival and departure safety headway time intervals \( g_{f,s} \) and \( h_{f,s} \) can be either pre-determined as parameters (6) or considered as variables (10). For determining the section blocking time, the occupancy time of block section \( s \) for the arrival of train \( f \) is formulated as

\[
\sigma_{f,s} = a_{f,s} - g_{f,s}, \quad \forall f \in F, s \in E_f, \tag{1}
\]

and the release time of block section \( s \) for the departure of train \( f \) is formulated as

\[
\delta_{f,s} = d_{f,s} + h_{f,s}, \quad \forall f \in F, s \in E_f, \tag{2}
\]

where \( F \) is the set of trains, \( E_f \) is the set of block sections that train \( f \) may use, and \( \sigma_{f,s} \) and \( \delta_{f,s} \) indicate the starting and ending time of blocking section \( s \) for train \( f \).

Figure 3 illustrates the movement of train \( f \) on block section \( s \) by using arrival and departure time variables. More specifically, train \( f \) arrives at time \( a_{f,s} = 4 \) and departs at time \( d_{f,s} = 7 \). As we have the safety headway times \( g_{f,s} = 2 \) and \( h_{f,s} = 1 \), block section \( s \) is blocked for train \( f \) from time \( \sigma_{f,s} = 2 \) to time \( \delta_{f,s} = 8 \).

To generate a conflict-free train dispatching solution, the block section capacity constraint is proposed by avoiding the overlap between any pair of trains on the same block section, formulated as follows:

\[
\sigma_{f_1,s} + (1 - \theta_{f_1,f_2,s}) \cdot M \geq \delta_{f_1,s}, \quad \forall f_1 \in F, f_2 \in F, s \in E_{f_1} \cap E_{f_2} \tag{3}
\]

\[
\sigma_{f_1,s} + (1 - \theta_{f_1,f_2,s}) \cdot M \geq \delta_{f_1,s}, \quad \forall f_1 \in F, f_2 \in F, s \in E_{f_1} \cap E_{f_2} \tag{4}
\]

where \( \theta_{f_1,f_2,s} \) is a binary train order variable, with \( \theta_{f_1,f_2,s} = 1 \) if train \( f_2 \) arrives at block section \( s \) or block section \( \hat{s} \) after train \( f_1 \), and otherwise \( \theta_{f_1,f_2,s} = 0 \), and \( M \) is a sufficiently large positive number. Note that we indicate bi-directional block section on a single-track segment as \( s \) and \( \hat{s} \), which refer to one physical block section in the opposite direction. Thus, the model can be applied to single-track, double-track, or \( N \)-track networks. Interested readers may refer to references (6), (10), and (14) for more details.

Mathematical Formulation of the Signaling Commands

This section formulates the signaling constraints to implement the signaling commands, which are innovative in comparison with previous studies. We first clarify the impact of the signaling commands on the train schedules, as revealed by the train travel times. Then, we formulate signaling constraints by applying signal indicators to represent signaling commands, which dynamically depend on the condition of the block sections ahead, that is, the relative position of the preceding train in the network. The time-instant formulation method is considered for constructing the signaling constraints. Moreover, we consider the three-aspect signaling system in Figure 1b as an example to formulate the signaling constraints. For the four-aspect signaling system and other signaling systems that are not detailed in this paper, a similar approach can be followed.

Impact of Signaling Commands on Train Schedules: Additional Train Travel Time

A non-green signal requires a reduction in train speed. To implement the speed reduction, a direct way is to restrict train speed. However, few train dispatching models consider the train speed as variable; as a result, we cannot add restrictions to the train speed. As the reduction in train speed can be reflected by an increased travel time of a train on a block section, we can formulate the reduction in train speed by requiring additional train travel time. Therefore, this section interprets the impact of the signaling commands on train schedules, as revealed by train travel time.

Figure 4 is a three-layer figure that illustrates the possible scenarios of a case that train \( f \) traverses two adjacent block sections \( s_1 \) and \( s_2 \). The sight point and the end point of block sections \( s_1 \) and \( s_2 \) are labeled as P1..., P4 respectively, as shown in Layer (a) of Figure 4. A speed-time graph is used to show the relation between the speed...
Figure 2. Time-space-speed graphs of the possible scenarios for train operations.
reduction and the additional travel time. In Layer (b), seven speed-time graphs for train \( f \) from the position \( P1 \) to \( P4 \) are sketched. A single or double capital letter(s) is used to indicate the signal aspects shown at position \( P2 \) and \( P4 \) respectively, for example, “GG” means two green aspects. The X-axis indicates the time at which train \( f \) arrives at the positions \( P1,..., P4 \). An integrated graph of these separate speed-time graphs is provided in Layer (c), in which the effects of the signaling commands (speed reduction) on the train travel times are graphically interpreted. In this layer, the X-axis indicates the time at which train \( f \) arrives at a position under a given scenario, for example, the time point labeled “\( P4_{YG} \)” is the arrival time of train \( f \) at position \( P4 \), if the two signals at positions \( P2 \) and \( P4 \) show the yellow and green aspects, respectively.

In Layer (b) of Figure 4, the colored area of the graphs (1–6) indicates the distance from position \( P1 \) to \( P4 \), that is, the distance traveled by a train equals its speed multiplied by the elapsed time; thus these areas should be equal to each other. As sketched in graphs (3) and (7), train \( f \) faces the red aspect directly after the green aspect, which should be prevented in the three-aspect signaling system for safety purposes; therefore, these two scenarios will not occur. The other five graphs correspond to the scenarios \( C1,..., C3 \) presented in Figure 2. Recall that we make an assumption of considering the worst case; so the possibilities of scenarios \( C4 \) and \( C5 \) in Figure 2 are neglected.

Moreover, we use two binary variables (speed indicators) \( \lambda_{f,s}^{\text{red}} \) and \( \lambda_{f,s}^{\text{yellow}} \) to indicate whether train \( f \) results in a red aspect and a yellow aspect for train \( f \) on block section \( s \). More specifically, \( \lambda_{f,s}^{\text{red}} = 1 \) if train \( f \) is occupying the block section one ahead of block section \( s \), which makes train \( f \) face the red aspect on block section \( s \), otherwise, \( \lambda_{f,s}^{\text{red}} = 0 \). Thus, the scenarios presented in Layer (b) of Figure 4 can be represented by using these speed indicators, as marked.

Let us focus on Layer (c) of Figure 4. Note that \( \tau_{f,s} \) indicates the free-flow travel time of train \( f \) on block section \( s \) under the normal condition (i.e., a green aspect), and \( \Delta_{f,s}^{Y}, \Delta_{f,s}^{YG}, \) and \( \Delta_{f,s}^{YR} \) indicate the additional/decreased travel time caused by the train speed reduction (i.e., by the “Approach” and “Stop” signaling commands). If the signal placed at position \( P2 \) is green, as in graphs (1) and (2) in Layer (b), no train speed reduction is required. If the signal placed at position \( P2 \) is yellow, as in graphs (4) to (6) in Layer (b), then train \( f \) reduces its speed by following the yellow curve, and its travel time on block section \( s_2 \) increases. The amount of the increased travel time further depends on the display of the signal placed at position \( P4 \):

- If the signal at position \( P4 \) is green, i.e., graph (4) in Layer (b), then train \( f \) is allowed to accelerate by following the green curve, and the additional travel time (compared with the free-flow travel time) in this case is shorter than that in the case of facing a yellow aspect, which is calculated by \( \Delta_{f,s_2}^{Y} - \Delta_{f,s_2}^{YG} \).
- If the signal at position \( P4 \) is yellow, i.e., graph (5) in Layer (b), then train \( f \) maintains its speed by following the yellow line, and the additional travel time on block section \( s_2 \) is denoted as \( \Delta_{f,s_2}^{Y} \), compared with the free-flow travel time in the normal scenario of facing green aspects. We consider that a second yellow aspect allows the train to pass at the approaching speed.
- In the remaining case that train \( f \) faces a red aspect at position \( P4 \), corresponding to graph (6) in Layer (b), train \( f \) has to decelerate by following the red curve until it stops completely, and the additional travel time is measured by \( \Delta_{f,s_2}^{Y} + \Delta_{f,s_2}^{YR} \).

**Formulations of the Signaling Constraints**

This section formulates the signaling constraints, dynamically determining the signal indicators and implementing the reduction in train speed by requiring additional
travel time. As discussed, for each train on each block section, the signaling command received by the driver depends on the movement of the preceding train in the network. Therefore, we need to identify the signaling commands by determining the condition (i.e., being occupied or released) of the block section(s) ahead.

Figure 5 illustrates how to identify the “Stop” signaling command; for the “Approach” command, a similar approach can be followed. In Figure 5a, train $f_1$ meets a red aspect in block section $s_1$ at the sight distance, because train $f_2$ is occupying block section $s_2$ at that moment. In such cases, the blocking time of train $f_1$ on block section $s_1$ and the blocking time of train $f_2$ on block section $s_2$ are “overlapping” in the sight time as shown in the timetable; we then have $l_{\text{red}}^{f_1,f_2,s_1} = 1$. For other cases shown in Figure 5b, train $f_1$ faces a non-red aspect in block section $s_1$, and no “overlap” happens in the sight time between the blocking time of train $f_1$ and train $f_2$. As a result, we have $l_{\text{red}}^{f_1,f_2,s_1} = 0$.

To determine the red signal indicator $l_{\text{red}}^{f_1,f_2,s_1}$, let us consider the binary variables $y_{f_1,f_2,s_1}$ and $y_{f_1,f_2,s_1}^\text{adj}$ to satisfy the conditions

\[
\begin{align*}
[d_{f_1,s_1} \leq \sigma_{f_2,s_2}] & \iff [y_{f_1,f_2,s_1} = 1], \\
\forall f_1 \in F, f_2 \in F, f_1 \neq f_2, s_1 \in E_{f_1}, s_2 \in E_{f_2} \cap E_{adj}^{s_1}, & \quad (5) \\
[\delta_{f_2,s_2} \leq d_{f_1,s_1} - \sigma_{f_1,s_1}] & \iff [y_{f_1,f_2,s_1}^\text{adj} = 1], \\
\forall f_1 \in F, f_2 \in F, f_1 \neq f_2, s_1 \in E_{f_1}, s_2 \in E_{f_2} \cap E_{adj}^{s_1} & \quad (6)
\end{align*}
\]
if train $f_1$ and train $f_2$ run in the same direction, or
\[
[ d_{f_1,s_1} \leq \sigma_{f_1,s_2} ] \Leftrightarrow \gamma_{f_1,s_1}^1 = 1,
\]
\[
\forall f_1 \in F, f_2 \in F, f_1 \neq f_2, s_1 \in E_f, s_2 \in E_f \cap E_{s_2}^{adj},
\]
\[
[ \delta_{f_2,s_2} \leq d_{f_1,s_1} - \epsilon_{f_1,s_1}^\text{sight} ] \Leftrightarrow \gamma_{f_1,s_1}^2 = 1,
\]
\[
\forall f_1 \in F, f_2 \in F, f_1 \neq f_2, s_1 \in E_f, s_2 \in E_f \cap E_{s_1}^{adj}
\] (7)

if train $f_1$ and train $f_2$ run in opposite directions. Note that $\epsilon_{f_1,s_1}^\text{sight}$ indicates the running time of train $f_1$ over the sight distance on block section $s_1$, and $E_{s_1}^{adj}$ indicates the set of the adjacent block sections of block section $s_1$. As a result, we have $\gamma_{f_1,s_1}^1 = 0$ and $\gamma_{f_1,s_1}^2 = 0$ for the case in Figure 5a, and we have either $\gamma_{f_1,s_1}^1 = 1$ or $\gamma_{f_1,s_1}^2 = 1$ for the case in Figure 5b. Then, the red signal indicator $\lambda_{f_1,s_1}^{\text{red}}$ can be formulated as
\[
\lambda_{f_1,s_1}^{\text{red}} = (1 - \gamma_{f_1,s_1}^1) \cdot (1 - \gamma_{f_1,s_1}^2),
\]
\[
\forall f_1 \in F, f_2 \in F, f_1 \neq f_2, s \in E_f.
\] (9)

By applying the transformation properties proposed in (29), the if-then constraints Equations 5 to 8 and the nonlinear constraint (Equation 9) can be reformulated as linear inequalities. Moreover, formulations similar to Equations 5 to 9 can also be constructed for the yellow signal indicator $\lambda_{f_1,s_1}^{\text{yellow}}$. For the sake of compactness, we do not provide those details here.

According to the above analysis of the additional train travel time, which reflects the train speed reduction required by the signaling commands, the travel time constraint is given as follows:
\[
d_{f,s_1} - a_{f,s_2} \geq w_{f,s_1}^{\text{min}} + \tau_{f,s_2} + \Delta_{f,s_1}^Y \sum_{f \in F} \lambda_{f,s_1}^{\text{yellow}} + \Delta_{f,s_1}^{\text{YR}} \sum_{f \in F} \lambda_{f,s_1}^{\text{red}} - \\
\Delta_{f,s_1}^{\text{YG}} \cdot (1 - \sum_{f \in F} \lambda_{f,s_1}^{\text{red}}) \cdot (1 - \sum_{f \in F} \lambda_{f,s_1}^{\text{yellow}}),
\]
\[
\forall f \in F, s_1 \in E_f, s_2 \in E_f \cap E_{s_1}^{adj}.
\] (10)

where $w_{f,s_2}^{\text{min}}$ indicates the minimum dwell time of train $f$ on block section $s_2$. Equation 10 ensures that the actual travel time of train $f$ on block section $s$ is not less than the sum of the minimum dwell time, the free-flow travel time, and the additional travel time caused by the non-"Clear" signaling commands.

Moreover, the GW result would not violate the speed limits that result from the signaling commands, and the GW policy is proven to be effective in saving energy consumption and dealing with delays (30). We can employ the GW policy by setting all signal indicators to zero, as follows:
\[
\sum_{f \in F} \sum_{s \in E_f} \sum_{f \neq f} (\lambda_{f,s_1}^{\text{yellow}} + \lambda_{f,s_1}^{\text{red}}) = 0.
\] (11)

As a result, all trains will always only encounter green aspects when traversing the network.

As the signaling constraints proposed here can be transformed into linear inequalities, we can still use the solution approach of the original models to solve the dispatching problem that integrates the signaling commands. If we consider the train dispatching model proposed in (6), which is an MILP model, a standard MILP solver can be used. For more details, interested readers may refer to the description in reference (6).

**Numerical Experiments**

We conducted the experiments on a rail network with two main tracks, composed of 48 nodes and 53 block sections, as depicted in Figure 6. The two tracks in different directions are independent, so only one direction is considered. We consider 1.5 hours of traffic with 20 trains in three train categories: eight intercity, eight sprinter, and four freight trains. Sprinter trains stop at all stations; intercity and freight trains stop only at the origin and destination stations. We consider 20 delay cases of the primary delays following a three-parameter Weibull distribution, as investigated in (31). We adopt the CPLEX optimization studio 12.6.3 with default settings to solve MILP problems. The following experiments are
performed on a computer with an Intel®Core™ i7 @ 2.00 GHz processor and 16 GB RAM.

In Figure 7a and b, the average results of the 10 delay cases are given, in terms of the train delay times, the train travel times, and the number of the red and yellow signals faced by the trains. The results in Figure 7a are normalized in the range 0 to 1. As shown, when integrating the signaling constraints, the total delay time and the total travel time increase by 16% and 2% respectively, and the number of yellow aspects that the trains meet decreases by 47%. As the speed limits required by the signaling commands are activated, the model optimizes the train departure and arrival times by avoiding trains facing non-green aspects (as much as possible). However, some trains still face yellow aspects on some block sections, that is, 11 times on average. In such cases, the speed reduction (i.e., the additional travel time) is enforced in our model; therefore, the train travel time becomes...
larger, which further results in the increased total delay time and travel time. Considering the signaling constraints enables us to avoid the full stop caused by the red signal, as the times that the trains confront the red signal are reduced to zero. By applying the GW policy, where the trains can always proceed under green signals, the total delay time further increases by 13%, compared with the results with the signaling constraints; however, the total travel time decreases to become similar to the results without the signaling constraints. The computation time for obtaining the results with consideration of the signaling constraints is longer, but still within 300 seconds.

We further explore more details regarding the signals. In Figure 7c and d, we present the results of all the 10 delay cases by summing up the number of the red/yellow signals that are faced by the trains. Figure 7c illustrates the distribution of the red and yellow signals on the network, of which the layout originates from Figure 6. A darker color implies that a larger number of red/yellow signals occurred at the position. As shown, the red and yellow signals are mostly faced by trains before entering the merging area of the two lines and also in the merging area. This results from the increased train interactions in these areas. Figure 7d presents the probability of the yellow signals becoming green and the risk of being red, which is obtained by analyzing the overlapping time of the two blocking times, as interpreted in Figure 5. The bars indicate the numbers of the green, yellow, and red signals, and the exact values are labeled. The Y-axis of the yellow line (within the yellow bar) indicates the number of the yellow signals, and the X-axis is the probability of becoming green or red. The black dashed line is a benchmark line, indicating that the probability to become green equals the risk to be red. When neglecting the signaling constraints, lots yellow signals occurred have big chance to become green. By considering the signaling constraints, the total number of the yellow signals decreases from 212 to 111, and the number of the yellow signals that have larger probability to become green is reduced. Considering the signaling constraints enables us to reduce the number of the yellow signals faced by the trains, i.e., taking the chance of letting the yellow signals become green as much as possible.

We can conclude that the consideration of the signaling commands leads to larger delay times and larger travel times, but increases the realism of the dispatching solution obtained and the safety of the train services. The GW policy hardly affects the train travel time, but results in a significant increase of train delays.

Conclusions
This paper integrates dynamic signaling commands under the fixed-block signaling system into train dispatching optimization problems. We have investigated the impact of the signaling commands on train schedules, and we have implemented this impact in our model by presenting the signaling constraints. In these constraints, the signaling commands (represented by the signal indicators) are dynamically determined by the train movements, and the train speed limits are activated according to the signal indicators. In our experimental results, consideration of the signaling commands results in larger delay times (16%) and longer travel times, but reduces the number of the non-green signals that are met and increases the realism of the train dispatching solution and the safety of the train services, as the real operational requirements (speed limits) caused by the signaling commands are included. In fact, the 16% gap can be thought of as an approximation error of currently available fixed speed dispatching models neglecting signaling aspects. Moreover, we consider the Greenwave (GW) policy to dispatch trains, which results in longer train delay times; however, the total train travel time does not change much, and it is similar to the result that neglects the signaling impact.

Our future research will focus on the following main extensions. First, as the proposed approach for constructing the signaling constraints can be extended to other signaling systems, we will study different signaling systems and compare their performance, in terms of delay, energy, capacity, robustness, etc., from the planning and operational perspectives. Second, when applying the GW policy, the energy consumption is expected to be reduced. We will evaluate the energy consumption by applying the model we proposed in reference (10), instead of that in reference (6). Finally, in order to extend the applicability of the model to large-scale networks, we will focus on developing heuristic algorithms and distributed optimization methods for improving the computational efficiency of the model.

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Author Contributions
The authors confirm contribution to the paper as follows: study conception and design: Xiaojie Luan, Bart De Schutter, Francesco Corman, and Gabriel Lodewijks; data collection: Xiaojie Luan, Francesco Corman; analysis and interpretation of results: Xiaojie Luan, Bart De Schutter, and Francesco Corman; draft manuscript preparation: Xiaojie Luan, Bart De Schutter, Francesco Corman, and Gabriel Lodewijks. All authors reviewed the results and approved the final version of the manuscript.
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