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A Simultaneous Adaptation Law for a Class of Nonlinearly-Parametrized Switched Systems

Spandan Roy and Simone Baldi

Abstract— **This paper proposes a new adaptive control method for a class of nonlinearly-parametrized switched systems that includes Monod kinetics and Euler-Lagrange systems with nonlinear in parameters form as special cases. As compared to the adaptive switched frameworks proposed in literature, the proposed adaptation framework has the distinguishing feature of updating the gains of the active and inactive subsystems simultaneously: by doing this it avoids high gains for the active subsystems, or vanishing gains for the inactive ones. The design is studied analytically and its performance is validated in simulation with a robotic manipulator example.**

I. Introduction

Switched systems represent an important class of hybrid systems consisting of subsystems with continuous dynamics together with a logic that orchestrates the switching action between them [1]–[9]. While some adaptive control approaches have been proposed to deal with the relevant problem of having parametric uncertainties in the subsystem dynamics ([10]–[13] for linear and [14]–[18] for nonlinear subsystems), only few approaches, namely [17], [18], address some classes of uncertain switched systems whose subsystem dynamics have nonlinear in parameters (NLIP) form.

Unfortunately, such classes are quite restrictive in the sense explained hereafter. The procedure used in [17], [18] to upper bound the uncertain system dynamics relies on the parameter separation-based method pioneered in [19]. Such procedure requires to find two scalar functions (one dependent on the states, one dependent on the uncertain parameters) whose construction necessarily requires structural and parametric knowledge of the system dynamics (see Example 1 in Section II). In addition, by considering continuously differentiable dynamics, such classes do not cover a large number of practically relevant non-smooth dynamics.

In consideration of the above discussions, in this work we consider a class of nonlinearly-parametrized switched systems, with the following properties:

- no assumption is imposed on the smoothness of the system dynamics;
- the upper bound structure does not require structural/parametric knowledge of the system dynamics;
- the class includes (non-smooth) Monod kinetics and Euler-Lagrange (EL) systems in NLIP form as special cases.

In literature on adaptive control of switched systems, usually only the gains of the active subsystem are updated: however, this leads to several problems such as having monotonic high gains for the active subsystems or having exponentially vanishing gains for the inactive subsystems (cf. [12], [17], [18] and the discussion in Remark 5). In this work, a new adaptive control method is formulated whose distinguishing feature is of updating the gains of the active and inactive subsystems simultaneously: by doing this it avoids the aforementioned problems.

The rest of the paper is organized as follows: Section II describes the objectives of this work; Section III details the proposed control framework, with stability analysis carried out in Section IV; a simulation study is provided in Section V, while Section VI presents the concluding remarks.

The following notations are used throughout the paper: $\lambda_{\min}(\bullet)$, $\lambda_{\max}(\bullet)$ and $||\bullet||$ represent minimum eigenvalue, maximum eigenvalue and Euclidean norm of $(\bullet)$ respectively; $\mathbf{I}$ denotes identity matrix with appropriate dimension; $(\bullet)^{\dagger}$ denotes generalized inverse of $(\bullet)$.

II. System Dynamics and Problem Formulation

Consider the following class of switched systems having N nonlinear subsystem dynamics in line with [20], [21],

$$\ddot{\mathbf{q}} = \mathbf{f}_{\sigma}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{b}_{\sigma}(\mathbf{q}, \dot{\mathbf{q}})\boldsymbol{\tau}_{\sigma}, \quad \sigma(t) \in \Omega \tag{1}$$

where $\mathbf{q}, \dot{\mathbf{q}} \in \mathbb{R}^n$ denote system states and $\sigma(t) : [0\ \infty) \mapsto \Omega$ is a piecewise constant function of time, called the switching signal, taking values in $\Omega = \{1, 2, \cdots, N\}$; for each σ, $\mathbf{f}_{\sigma} : \mathbb{R}^{2n} \mapsto \mathbb{R}^n$ and $\mathbf{b}_{\sigma} : \mathbb{R}^{2n} \mapsto \mathbb{R}^{n\times m}$ are the system dynamics terms with $m \geq n$ and $\boldsymbol{\tau}_{\sigma} \in \mathbb{R}^m$ is the control input. For each subsystem, $\mathbf{f}_{\sigma}$ is considered to be NLIP, with the following property:

Property 1: Define $\mathbf{x} \triangleq col[\mathbf{q}\ \ \dot{\mathbf{q}}]$. The system dynamics term $\mathbf{f}_{\sigma}(\mathbf{x})$ can be upper bounded as:

$$||\mathbf{f}_{\sigma}(\mathbf{x})|| \leq \theta_{0\sigma} + \theta_{1\sigma}||\mathbf{x}|| + \cdots + \theta_{\nu\sigma}||\mathbf{x}||^{\nu} \triangleq \mathbf{Y}_{\sigma}^T(||\mathbf{x}||)\boldsymbol{\Theta}_{\sigma}, \tag{2}$$

where $\boldsymbol{\Theta}_{\sigma} = [\theta_{0\sigma}\ \theta_{1\sigma}\ \cdots\ \theta_{\nu\sigma}]^T$ is a vector of constant parameters with $\theta_{i\sigma} \in \mathbb{R}^+$, $i = 0, 1, \cdots, \nu$ and $\mathbf{Y}_{\sigma}(||\mathbf{x}||) = [\mathbf{1}\ ||\mathbf{x}||\ ||\mathbf{x}||^2\ \cdots\ ||\mathbf{x}||^{\nu}]^T$ is the regressor.

Some remarks are given to explain the relevance of (1)-(2) as compared to the state of the art.

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Remark 1: Property 1 holds for many practical NLIP systems such as Monod kinetic [22], EL systems [20] etc. For such systems, the existing LIP-based adaptive control solutions [14]–[16] are inapplicable. Determination of ν in (2) does not require structural knowledge of the dynamics, as ν can be determined from the first law of physics. For example, EL dynamics with Coriolis and centrifugal terms satisfy (2) with $\nu = 2$, irrespective of the structures of system dynamics (e.g., robotic systems, humanoids, ship dynamics, pneumatic muscle, active suspension system [20], [21], [23]). Also, Monod kinetics [23] satisfy (2) with $\nu = 2$.

Remark 2: It is noteworthy that no assumption on smoothness of $\mathbf{f}_\sigma$ is necessary for (2) to hold. This is not the case for the NLIP methods [17], [18], which assume $\mathbf{f}_\sigma$ to be continuously differentiable with $\mathbf{f}_\sigma(\mathbf{0}) = \mathbf{0}$. The first condition fails to hold for many practical systems due to unavoidable friction effects, e.g. Coulomb friction (e.g., robotic manipulators), Stribeck friction (high precision systems [24]) etc. The condition $\mathbf{f}_\sigma(\mathbf{0}) = \mathbf{0}$ implies that the effects of time-dependent bounded external disturbances are ignored. In (2), such disturbances can easily be accounted through $\theta_{0\sigma}$.

Remark 3: Different upper bound structures have been proposed in literature. Most notably, [17]–[19] consider

$$||\mathbf{f}_\sigma(\mathbf{x})|| \leq \varphi_\sigma(\mathbf{x})\phi_\sigma(\theta_\sigma), \tag{3}$$

where $\varphi_\sigma(\mathbf{x}) \geq 1$, $\phi_\sigma(\theta_\sigma) \geq 1$ are two $\mathcal{C}^\infty$ scalar functions and θ_σ denote the set of unknown system parameters. According to (3), for a polynomial $||\mathbf{f}_\sigma||$ of order ν, one should select $\varphi_\sigma(\mathbf{x})$ to be a polynomial function of at least $(\nu+1)^{th}$ (resp. $(\nu+2)^{th}$) degree in order to satisfy (3) when ν is an odd (resp. even) number. Moreover, as φ_σ is a scalar function, some parametric knowledge of the system dynamics is necessarily required to design a suitable φ_σ to satisfy (3) globally for all $\mathbf{x}$. Two clarifying examples follow:

Example 1: Consider the two spring-connected pendulum from [17]

$$\ddot{x}_1 = ((m_1 gr)/J_1 - (hr^2)/4J_1)\sin(x_1) + (hr^2(l-b))/2J_1 + u_1/J_1 + (hr^2\sin(x_2))/4J_1, \tag{4a}$$

$$\ddot{x}_2 = ((m_2 gr)/J_2 - (hr^2)/4J_2)\sin(x_2) + (hr^2(l-b))/2J_2 + u_2/J_2 + (hr^2\sin(\dot{x}_1))/4J_2, \tag{4b}$$

where the meaning of all parameters in (4) can be found in [17]. *Employing the knowledge of the parameters h, r, l, b and* J_1, J_2, the choice made in [17] for φ_σ to satisfy (3) is $\varphi_1 = 1+\dot{x}_1^2+\dot{x}_2^2+3(x_1+\dot{x}_1)^2+(2x_2+\dot{x}_2)^2(1+e^{x_2})+v_{21}^2$, $\varphi_2 = 1+x_1^2+\dot{x}_1^2+\dot{x}_2^2+2(x_1+\dot{x}_1)^2+(1+x_2^2)(2x_2+\dot{x}_2)^2+v_{22}^2$ (v_{21}, v_{22} are adaptive control inputs designed as polynomials of state and estimates of θ_σ, with at least degree one). The interested readers can verify that it is not easy to select a φ_σ that does not use any parametric knowledge. On the other hand, it can be easily verified that $||\mathbf{f}_\sigma||$ in (4) can be upper bounded as in (2) with $\nu = 1$, i.e., *a polynomial with degree one*, and without using any knowledge of the parameters.

Example 2: The situation of Example 1 occurs even with simpler dynamics. For example, according to the upper bound (3), the function $\mathbf{f}_\sigma(\mathbf{x}) = f(x) = \theta^* x^2$ cannot be globally upper bounded by $\varphi_\sigma = (1 + a_0 x^2 + a_1 x^4)$ and $\phi_\sigma = (1+\theta^2)$ for all x, unless $a_0, a_1 \in \mathbb{R}^+$ are designed with some knowledge of θ^* (i.e. parametric knowledge). Note that, as highlighted by the functions in Example 1, the use of odd powers in φ_σ is harmful in general. In fact, inserting a term $a_2 x^3$ in φ may violate the condition $\varphi_\sigma \geq 1$ for negative values of x. Also, the absolute function cannot be used in (3) because it would violate the $\mathcal{C}^\infty$ property of φ_σ.

In this work, $\mathbf{f}_\sigma$ is considered to be uncertain in the sense that $\theta_{i\sigma}$'s in (2) are *completely unknown*. On the other hand, $\mathbf{b}_\sigma$ is considered to be uncertain in the sense that only some nominal knowledge is available, according to the following assumption:

Assumption 1: Let $\hat{\mathbf{b}}_\sigma(\mathbf{x})$ be the nominal value of $\mathbf{b}(\mathbf{x})$. Assume there exists a known scalar $\bar{E}_\sigma$ such that for $\mathbf{E}_\sigma \triangleq (\mathbf{b}_\sigma \hat{\mathbf{b}}_\sigma^\dagger - \mathbf{I})$ the following holds

$$||\mathbf{E}_\sigma|| \leq \bar{E}_\sigma < 1, \quad \forall \sigma \in \Omega. \tag{5}$$

Remark 4: Using the knowledge of $\hat{\mathbf{b}}(\mathbf{x})$, the existence of $\bar{E}_\sigma$ defines the allowable amount of uncertainty in $\mathbf{b}(\mathbf{x})$.

The following class of switching signals is considered in (1):

Definition 1: Average Dwell Time (ADT) [2]: For a switching signal $\sigma(t)$ and each $t_2 \geq t_1 \geq 0$, let $N_\sigma(t_1, t_2)$ denote the number of discontinuities in the interval $[t_1, t_2)$. Then $\sigma(t)$ has an ADT ϑ if for a given scalar $N_0 > 0$

$$N_\sigma(t_1, t_2) \leq N_0 + (t_2 - t_1)/\vartheta, \quad \forall t_2 \geq t_1 \geq 0$$

where N_0 is termed as chatter bound.

For convenience of notation, we will use $\mathcal{N}(p)$ to denote the set of inactive subsystems, when subsystem $\sigma(t) = p$ is active.

III. Controller Design

Let us consider the tracking problem for a desired trajectory $\mathbf{q}_\sigma^d(t)$ according to the following commonly-adopted assumption [20], [21]:

Assumption 2: The desired trajectories are selected such that $\mathbf{q}_\sigma^d, \dot{\mathbf{q}}_\sigma^d, \ddot{\mathbf{q}}_\sigma^d \in \mathcal{L}_\infty$ and $\mathbf{q}, \dot{\mathbf{q}}$ are available as feedback.

Let $\mathbf{e}(t) \triangleq \mathbf{q}(t) - \mathbf{q}_{\sigma(t)}^d(t)$ be the tracking error, $\boldsymbol{\xi}(t) \triangleq col[\mathbf{e}(t)\ \dot{\mathbf{e}}(t)]$ and $\mathbf{r}_\sigma$ be the filtered tracking error variable defined as

$$\mathbf{r}_\sigma \triangleq \mathbf{B}^T \mathbf{P}_\sigma \boldsymbol{\xi}, \qquad \sigma \in \Omega \tag{6}$$

where $\mathbf{P}_\sigma > \mathbf{0}$ is the solution to the Lyapunov equation $\mathbf{A}_\sigma^T \mathbf{P}_\sigma + \mathbf{P}_\sigma \mathbf{A}_\sigma = -\mathbf{Q}_\sigma$ for some $\mathbf{Q}_\sigma > \mathbf{0}$, $\mathbf{A}_\sigma \triangleq \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{K}_{1\sigma} & -\mathbf{K}_{2\sigma} \end{bmatrix}$ and $\mathbf{B} \triangleq \begin{bmatrix} \mathbf{0} \\ \mathbf{I} \end{bmatrix}$. Here, $\mathbf{K}_{1\sigma}$ and $\mathbf{K}_{2\sigma}$ are two user-defined positive definite gain matrices and their positive definiteness guarantees $\mathbf{A}_\sigma$ is Hurwitz.

The control law is designed as

$$\boldsymbol{\tau}_\sigma = \hat{\mathbf{b}}_\sigma^\dagger(-\boldsymbol{\Lambda}_\sigma \boldsymbol{\xi} - \rho_\sigma \mathbf{r}_\sigma + \ddot{\mathbf{q}}_\sigma^d), \tag{7}$$

where $\boldsymbol{\Lambda}_\sigma \triangleq [\mathbf{K}_{1\sigma}\ \mathbf{K}_{2\sigma}]$ and the design of ρ_σ will be discussed later. Substituting (7) in (1) yields

$$\ddot{\mathbf{e}} = -\boldsymbol{\Lambda}_\sigma \boldsymbol{\xi} - \rho_\sigma \mathbf{r}_\sigma - \mathbf{E}_\sigma \rho_\sigma \mathbf{r}_\sigma + \boldsymbol{\Psi}_\sigma, \tag{8}$$

where $\mathbf{\Psi}_\sigma \triangleq \mathbf{f}_\sigma + \mathbf{E}_\sigma(\ddot{\mathbf{q}}^d_\sigma - \mathbf{\Lambda}_\sigma \boldsymbol{\xi})$ is treated as the overall uncertainty. Hence, using Property 1 and Assumption 2, one can verify that $\exists \theta^*_{i\sigma} \in \mathbb{R}^+ \; i = 0, \cdots, \nu \; \forall \sigma \in \Omega$

$$||\mathbf{\Psi}_\sigma|| \leq \theta^*_{0\sigma} + \theta^*_{1\sigma}||\boldsymbol{\xi}|| + \cdots + \theta^*_{\nu\sigma}||\boldsymbol{\xi}||^\nu \triangleq \mathbf{Y}^T_\sigma(||\boldsymbol{\xi}||)\mathbf{\Theta}^*_\sigma, \tag{9}$$

where $\theta^*_{i\sigma}$'s are unknown scalars and $\mathbf{\Theta}^*_\sigma = [\theta^*_{0\sigma} \; \theta^*_{1\sigma} \; \theta^*_{2\sigma} \; \cdots \; \theta^*_{\nu\sigma}]^T$. The gain ρ_σ in (7) is designed as

$$\rho_\sigma = \frac{1}{1 - \bar{E}_\sigma}\{(\hat{\theta}_{0\sigma} + \gamma_{0\sigma}) + (\hat{\theta}_{1\sigma} + \gamma_{1\sigma})||\boldsymbol{\xi}|| + \cdots$$
$$\cdots + (\hat{\theta}_{\nu\sigma} + \gamma_{\nu\sigma})||\boldsymbol{\xi}||^\nu\} \triangleq \mathbf{Y}^T_\sigma(||\boldsymbol{\xi}||)(\hat{\mathbf{\Theta}}_\sigma + \mathbf{\Gamma}_\sigma), \tag{10}$$

where $\hat{\mathbf{\Theta}}_\sigma \triangleq [\hat{\theta}_{0\sigma} \; \hat{\theta}_{1\sigma} \; \hat{\theta}_{2\sigma} \; \cdots \; \hat{\theta}_{\nu\sigma}]^T$ is the estimate of $\mathbf{\Theta}^*_\sigma$; $\mathbf{\Gamma}_\sigma \triangleq [\gamma_{0\sigma} \; \gamma_{1\sigma} \; \gamma_{2\sigma} \; \cdots \; \gamma_{\nu\sigma}]^T$ is a dynamic auxiliary gain whose adaptation laws must be properly designed for closed-loop stability. To this purpose, the gains $\hat{\theta}_{i\sigma}, \gamma_{i\sigma}$ are adapted using the following laws:

$$\dot{\hat{\theta}}_{ip} = \eta||\boldsymbol{\xi}||^{i+1} - \alpha_i\hat{\theta}_{ip},$$
$$\dot{\hat{\theta}}_{i\bar{p}} = \eta||\boldsymbol{\xi}||^{i+1} - \alpha_i\hat{\theta}_{i\bar{p}}, \tag{11a}$$
$$\dot{\gamma}_{ip} = -\left(\beta_i + \eta\epsilon_i\hat{\theta}_{ip}||\boldsymbol{\xi}||^{i+2}\right)\gamma_{ip} + \beta_i\underline{\epsilon}_i,$$
$$\dot{\gamma}_{i\bar{p}} = -\left(\beta_i + \eta\epsilon_i\hat{\theta}_{i\bar{p}}||\boldsymbol{\xi}||^{i+2}\right)\gamma_{i\bar{p}} + \beta_i\underline{\epsilon}_i, \tag{11b}$$
$$\hat{\theta}_{ip}(t_0), \; \hat{\theta}_{i\bar{p}}(t_0) > 0, \; \gamma_{ip}(t_0), \gamma_{i\bar{p}}(t_0) > \underline{\epsilon}_i, \tag{11c}$$

where $\eta \triangleq \max_{\sigma\in\Omega}(\lambda_{\max}(\mathbf{P}_\sigma))$, $\bar{p} \in \mathcal{N}(p)$, $\alpha_i, \beta_i, \epsilon_i, \underline{\epsilon}_i \in \mathbb{R}^+ \; i = 0, \cdots, \nu$ are static design scalars and t_0 is the initial time. From (11a)-(11b) and the initial conditions (11c), it can be verified that $\exists \underline{\gamma}_{i\sigma} \in \mathbb{R}^+$ such that

$$\hat{\theta}_{i\sigma}(t) \geq 0 \text{ and } \gamma_{i\sigma}(t) \geq \underline{\gamma}_{i\sigma} \quad \forall t \geq t_0. \tag{12}$$

***Remark** 5:* In state-of-the-art methods, the gains for inactive subsystems are usually not updated (i.e., constant). While this appears as a natural choice, such a choice may be not robust in the sense of [25]. More specifically, in order to provide robust adaptation via leakage, [12] has shown that the gains for the inactive subsystems should decrease exponentially. Clearly, if a subsystem remains inactive for sufficiently long time, its gains will become very small, leading to a new transient whenever the subsystem is activated again. Contrary to the constant or to the exponentially decreasing policies, (11a) updates the adaptive gains for both active and inactive subsystems simultaneously, while the term $\eta||\boldsymbol{\xi}||^{i+1}$ prevents $\hat{\theta}_{i\bar{p}}$, $\bar{p} \in \mathcal{N}(p)$ from becoming very small. Also, note that differently from state-of-the-art methods [17], [18], (11a)-(11b) do not require to monotonically increase the gains of the active subsystems, thus preventing issues stemming from high gains (cf. [25, §8.4]).

IV. Stability Analysis of The Proposed Robust Adaptive Controller

We define $\varrho_{M\sigma} \triangleq \lambda_{\max}(\mathbf{P}_\sigma)$, $\varrho_{m\sigma} \triangleq \lambda_{\min}(\mathbf{P}_\sigma)$, $\bar{\varrho}_M \triangleq \max_{\sigma\in\Omega}(\varrho_{M\sigma})$ and $\underline{\varrho}_m \triangleq \min_{\sigma\in\Omega}(\varrho_{m\sigma})$. Following Definition 1 of ADT [2], the switching law is proposed as

$$\vartheta > \vartheta^* = \ln\mu/\kappa, \tag{13}$$

where $\mu \triangleq \bar{\varrho}_M/\underline{\varrho}_m$; κ is a scalar defined as $0 < \kappa < \varrho$ where $\varrho_p \triangleq (\lambda_{\min}(\mathbf{Q}_\sigma)/\lambda_{\max}(\mathbf{P}_\sigma))$, $\varrho \triangleq \min_{\sigma\in\Omega}(\varrho_p)$.

***Theorem** 1:* Under Assumptions 1-2, the closed-loop trajectories of system (1) employing the control laws (7) and (10) with adaptive law (11) and switching law (13) are Uniformly Ultimately Bounded (UUB) if the gains α_i and β_i are designed as $\alpha_i > \max_{\sigma\in\Omega}(\varrho_\sigma/2)$ and $\beta_i > \max_{\sigma\in\Omega}(\varrho_\sigma/2)$. Further, an ultimate bound b on the tracking error $\boldsymbol{\xi}$ can be found as

$$b \in \left[0, \sqrt{(2\bar{\varrho}_M^{(N_0+1)}\mathcal{B})/\underline{\varrho}_m^{(N_0+2)}}\right], \tag{14}$$

$$\mathcal{B} \triangleq \max\left\{\frac{\sum_{p=1}^N\sum_{i=0}^\nu\left(\frac{\alpha_i^2}{4\bar{\alpha}_{ip}} + \frac{\varrho_p}{2}\right)\theta^{*\,2}_{ip} + \frac{(\beta_i\underline{\epsilon}_{ip})^2}{4\bar{\beta}_{ip}}}{(\varrho - \kappa)}, \frac{\underline{\varrho}_m}{2\underline{\gamma}_m^4\epsilon_m^2}\right\},$$

where $\bar{\alpha}_{i\sigma} \triangleq (\alpha_i - \frac{\varrho_\sigma}{2})$, $\bar{\beta}_{i\sigma} \triangleq (\beta_i - \frac{\varrho_\sigma}{2})$, $\underline{\gamma}_m \triangleq \min_{\sigma\in\Omega, i=0,1,\cdots,\nu}(\underline{\gamma}_{i\sigma})$ and $\epsilon_m \triangleq \min_{i=0,1,\cdots,\nu}(\epsilon_i)$.

Proof: Stability relies on the Lyapunov candidate:

$$V(t) = (1/2)\boldsymbol{\xi}^T(t)\mathbf{P}_{\sigma(t)}\boldsymbol{\xi}(t) + (1/2)\sum_{p=1}^N\sum_{i=0}^\nu\{(\hat{\theta}_{ip}(t) - \theta^*_{ip})^2 + \gamma^2_{ip}(t)\}, \tag{15}$$

Note that $\mathbf{\Lambda}_\sigma\boldsymbol{\xi} = \mathbf{K}_{1\sigma}\mathbf{e} + \mathbf{K}_{2\sigma}\dot{\mathbf{e}}$. Using this relation, the error dynamics obtained in (8) becomes

$$\dot{\boldsymbol{\xi}} = \mathbf{A}_\sigma\boldsymbol{\xi} + \mathbf{B}\left(\mathbf{\Psi}_\sigma - \rho_\sigma\mathbf{r}_\sigma - \mathbf{E}_\sigma\rho_\sigma\mathbf{r}_\sigma\right). \tag{16}$$

Note that $V(t)$ might be discontinuous at the switching instants and only remains continuous during the time interval of two consecutive switchings. The active subsystem is $\sigma(t^-_{l+1})$ when $t \in [t_l \quad t_{l+1})$ and $\sigma(t_{l+1})$ when $t \in [t_{l+1} \quad t_{l+2})$. Without the loss of generality, the behaviour of the Lyapunov function is studied at the switching instant $t_{l+1}, \; l \in \mathbb{N}^+$. We have before and after switching

$$V(t^-_{l+1}) = (1/2)\boldsymbol{\xi}^T(t^-_{l+1})\mathbf{P}_{\sigma(t^-_{l+1})}\boldsymbol{\xi}(t^-_{l+1}) + (1/2)\sum_{p=1}^N\sum_{i=0}^\nu\{(\hat{\theta}_{ip}(t^-_{l+1}) - \theta^*_{ip})^2 + \gamma^2_{ip}(t^-_{l+1})\},$$
$$V(t_{l+1}) = (1/2)\boldsymbol{\xi}^T(t_{l+1})\mathbf{P}_{\sigma(t_{l+1})}\boldsymbol{\xi}(t_{l+1}) + (1/2)\sum_{p=1}^N\sum_{i=0}^\nu\{(\hat{\theta}_{ip}(t_{l+1}) - \theta^*_{ip})^2 + \gamma^2_{ip}(t_{l+1})\},$$

respectively. Thanks to the continuity of the tracking error $\boldsymbol{\xi}$ in (16) and of the gains $\hat{\theta}_i$'s and γ_i's in (11), we have $\boldsymbol{\xi}(t^-_{l+1}) = \boldsymbol{\xi}(t_{l+1})$, $(\hat{\theta}_{ip}(t^-_{l+1}) - \theta^*_{ip}) = (\hat{\theta}_{ip}(t_{l+1}) - \theta^*_{ip})$ and $\gamma_{ip}(t^-_{l+1}) = \gamma_{ip}(t_{l+1})$. Further, owing to the facts $\boldsymbol{\xi}^T(t)\mathbf{P}_{\sigma(t)}\boldsymbol{\xi}(t) \leq \bar{\varrho}_M\boldsymbol{\xi}^T(t)\boldsymbol{\xi}(t)$ and $\boldsymbol{\xi}^T(t)\mathbf{P}_{\sigma(t)}\boldsymbol{\xi}(t) \geq \underline{\varrho}_m\boldsymbol{\xi}^T(t)\boldsymbol{\xi}(t)$, one has

$$V(t_{l+1}) - V(t^-_{l+1}) = \frac{1}{2}\boldsymbol{\xi}^T(t_{l+1})(\mathbf{P}_{\sigma(t_{l+1})} - \mathbf{P}_{\sigma(t^-_{l+1})})\boldsymbol{\xi}(t_{l+1})$$
$$\leq \frac{\bar{\varrho}_M - \underline{\varrho}_m}{2\underline{\varrho}_m}\boldsymbol{\xi}^T(t_{l+1})\mathbf{P}_{\sigma(t^-_{l+1})}\boldsymbol{\xi}(t_{l+1}) \leq \frac{\bar{\varrho}_M - \underline{\varrho}_m}{\underline{\varrho}_m}V(t^-_{l+1})$$
$$\Rightarrow V(t_{l+1}) \leq \mu V(t^-_{l+1}), \tag{17}$$

with $\mu = \bar{\varrho}_M/\underline{\varrho}_m \geq 1$. At this point, the behaviour of $V(t)$ between two consecutive switching instants, i.e., when $t \in [t_l \;\; t_{l+1})$ can be studied.

Using (6), (16) and the Lyapunov equation $\mathbf{A}_\sigma^T\mathbf{P}_\sigma + \mathbf{P}_\sigma\mathbf{A}_\sigma = -\mathbf{Q}_\sigma$, the time derivative of (15) yields

$$\begin{aligned}\dot{V}(t) &= (1/2)\boldsymbol{\xi}^T(t)(\mathbf{A}^T_{\sigma(t^-_{l+1})}\mathbf{P}_{\sigma(t^-_{l+1})} + \mathbf{P}_{\sigma(t^-_{l+1})}\mathbf{A}_{\sigma(t^-_{l+1})})\boldsymbol{\xi}(t) \\ &+ \boldsymbol{\xi}^T(t)\mathbf{P}_{\sigma(t^-_{l+1})}\mathbf{B}\left(\boldsymbol{\Psi}_{\sigma(t^-_{l+1})} - (\mathbf{I}+\mathbf{E}_{\sigma(t^-_{l+1})})\rho_{\sigma(t^-_{l+1})}\mathbf{r}_{\sigma(t^-_{l+1})}\right) \\ &+ \sum_{p=1}^{N}\sum_{i=0}^{\nu}\left\{(\hat{\theta}_{ip}(t)-\theta^*_{ip})\dot{\hat{\theta}}_{ip}(t) + \gamma_{ip}(t)\dot{\gamma}_{ip}(t)\right\} \\ &\leq -(1/2)\boldsymbol{\xi}^T(t)\mathbf{Q}_{\sigma(t^-_{l+1})}\boldsymbol{\xi}(t) + ||\boldsymbol{\Psi}_{\sigma(t^-_{l+1})}||||\mathbf{r}_{\sigma(t^-_{l+1})}|| \\ &\quad - (1-\bar{E}_{\sigma(t^-_{l+1})})\rho_{\sigma(t^-_{l+1})}||\mathbf{r}_{\sigma(t^-_{l+1})}||^2 \\ &\quad + \sum_{p=1}^{N}\sum_{i=0}^{\nu}\left\{(\hat{\theta}_{ip}(t)-\theta^*_{ip})\dot{\hat{\theta}}_{ip}(t) + \gamma_{ip}(t)\dot{\gamma}_{ip}(t)\right\}. \qquad (18)\end{aligned}$$

Owing to (12) and Assumption 1, one has $\rho_{\sigma(t)} \geq 0 \;\forall t$. Further, from (6) we have $||\mathbf{r}_\sigma|| \leq \lambda_{\max}(\mathbf{P}_\sigma)||\boldsymbol{\xi}||$. Using these observations and (9), (18) is simplified as

$$\begin{aligned}\dot{V}(t) \leq &-(1/2)\boldsymbol{\xi}^T(t)\mathbf{Q}_{\sigma(t^-_{l+1})}\boldsymbol{\xi}(t) \\ &+ \mathbf{Y}^T_{\sigma(t^-_{l+1})}\boldsymbol{\Theta}^*_{\sigma(t^-_{l+1})}\lambda_{\max}(\mathbf{P}_{\sigma(t^-_{l+1})})||\boldsymbol{\xi}(t)|| \\ &+ \sum_{p=1}^{N}\sum_{i=0}^{\nu}\{(\hat{\theta}_{ip}(t)-\theta^*_{ip})\dot{\hat{\theta}}_{ip}(t) + \gamma_{ip}(t)\dot{\gamma}_{ip}(t)\}. \quad (19)\end{aligned}$$

Using (11a) we have

$$\begin{aligned}&\sum_{p=1}^{N}\sum_{i=0}^{\nu}(\hat{\theta}_{ip}-\theta^*_{ip})\dot{\hat{\theta}}_{ip} = \sum_{p=1}^{N}\sum_{i=0}^{\nu}(\hat{\theta}_{ip}-\theta^*_{ip})(\eta||\boldsymbol{\xi}||^{i+1} - \alpha_i\hat{\theta}_{ip}) \\ &= \sum_{p=1}^{N}\{\sum_{i=0}^{\nu}\eta\hat{\theta}_{ip}||\boldsymbol{\xi}||^{i+1} + \alpha_i\hat{\theta}_{ip}\theta^*_{ip} - \alpha_i\hat{\theta}^2_{ip}\} - \mathbf{Y}^T_p\boldsymbol{\Theta}^*_p\eta||\boldsymbol{\xi}||. \\ &\qquad (20)\end{aligned}$$

Similarly (11b) leads to

$$\begin{aligned}\gamma_{ip}\dot{\gamma}_{ip} &= -\left(\beta_i + \eta\epsilon_i\hat{\theta}_{ip}||\boldsymbol{\xi}||^{i+2}\right)\gamma^2_{ip} + \beta_i\underline{\epsilon}_i\gamma_{ip} \\ &\leq -\beta_i\gamma^2_{ip} - \eta\underline{\gamma}^2_{ip}\epsilon_i\hat{\theta}_{ip}||\boldsymbol{\xi}||^{i+2} + \beta_i\underline{\epsilon}_i\gamma_{ip}, \qquad (21)\end{aligned}$$

where the last inequality comes from (12), as $\gamma_{i\sigma} \geq \underline{\gamma}_{i\sigma} \;\; \forall t \geq t_0$. By design $\eta = \max_{\sigma\in\Omega}(\lambda_{\max}(\mathbf{P}_\sigma))$ we have $\{\mathbf{Y}^T_{\sigma(t^-_{l+1})}\boldsymbol{\Theta}^*_{\sigma(t^-_{l+1})}\lambda_{\max}(\mathbf{P}_{\sigma(t^-_{l+1})})||\boldsymbol{\xi}|| - \sum_{p=1}^{N}\mathbf{Y}^T_p\boldsymbol{\Theta}^*_p\eta||\boldsymbol{\xi}||\} \leq 0$. Using this relation and substituting (20) and (21) in (19) yields

$$\begin{aligned}\dot{V}(t) \leq &-\frac{1}{2}\lambda_{\min}(\mathbf{Q}_{\sigma(t^-_{l+1})})||\boldsymbol{\xi}(t)||^2 + \sum_{p=1}^{N}\sum_{i=0}^{\nu}\{\alpha_i\hat{\theta}_{ip}(t)\theta^*_{ip} \\ &-\eta\hat{\theta}_{ip}||\boldsymbol{\xi}(t)||^{i+1}(\underline{\gamma}^2_{ip}\epsilon_i||\boldsymbol{\xi}(t)||-1) - \alpha_i\hat{\theta}^2_{ip}(t) - \beta_i\gamma^2_{ip}(t) \\ &+ \beta_i\underline{\epsilon}_i\gamma_{ip}(t)\}. \qquad (22)\end{aligned}$$

Since $\hat{\theta}_{ip} \geq 0$ by design (12), one obtains

$$V \leq \frac{1}{2}\lambda_{\max}(\mathbf{P}_\sigma)||\boldsymbol{\xi}||^2 + \frac{1}{2}\sum_{p=1}^{N}\sum_{i=0}^{\nu}\hat{\theta}^2_{ip} + {\theta^*_{ip}}^2 + \gamma^2_{ip}. \quad (23)$$

Hence, using (23), the condition (22) is further simplified to

$$\begin{aligned}\dot{V}(t) \leq &-\varrho V(t) + \sum_{p=1}^{N}\sum_{i=0}^{\nu}\{\alpha_i\hat{\theta}_{ip}(t)\theta^*_{ip} - \bar{\alpha}_{ip}\hat{\theta}^2_{ip}(t) \\ &-\eta\hat{\theta}_{ip}||\boldsymbol{\xi}(t)||^{i+1}(\underline{\gamma}^2_{ip}\epsilon_i||\boldsymbol{\xi}(t)||-1) + (\varrho_p/2){\theta^*_{ip}}^2 \\ &-\bar{\beta}_{ip}\gamma^2_{ip}(t) + \beta_i\underline{\epsilon}_i\gamma_{ip}(t)\}, \qquad (24)\end{aligned}$$

where $\bar{\alpha}_{ip} > 0$ and $\bar{\beta}_{ip} > 0$ by design (from (14)). Again, the following rearrangements can be made

$$\begin{aligned}\alpha_i\hat{\theta}_{ip}\theta^*_{ip} - \bar{\alpha}_{ip}\hat{\theta}^2_{ip} &= -\bar{\alpha}_{ip}\left(\hat{\theta}_{ip} - \frac{\alpha_i\theta^*_{ip}}{2\bar{\alpha}_{ip}}\right)^2 + \frac{\left(\alpha_i\theta^*_{ip}\right)^2}{4\bar{\alpha}_{ip}}, \\ \beta_i\underline{\epsilon}_i\gamma_{ip} - \bar{\beta}_{ip}\gamma^2_{ip} &= -\bar{\beta}_{ip}\left(\gamma_{ip} - \frac{\beta_i\underline{\epsilon}_i}{2\bar{\beta}_{ip}}\right)^2 + \frac{(\beta_i\underline{\epsilon}_i)^2}{4\bar{\beta}_{ip}}. \quad (25)\end{aligned}$$

We had defined earlier $0 < \kappa < \varrho$. Then, using (25), $\dot{V}(t)$ from (24) gets simplified to

$$\begin{aligned}\dot{V}(t) \leq &-\kappa V(t) - (\varrho-\kappa)V(t) + \sum_{p=1}^{N}\sum_{i=0}^{\nu}\left(\frac{\alpha_i^2}{4\bar{\alpha}_{ip}} + \frac{\varrho_p}{2}\right){\theta^*_{ip}}^2 \\ &-\eta\hat{\theta}_{ip}||\boldsymbol{\xi}(t)||^{i+1}(\underline{\gamma}^2_{ip}\epsilon_i||\boldsymbol{\xi}(t)||-1) + (\beta_i\underline{\epsilon}_i)^2/(4\bar{\beta}_{ip}). \quad (26)\end{aligned}$$

Again, the definition of the Lyapunov function (15) yields

$$V(t) \geq (1/2)\lambda_{\min}(\mathbf{P}_{\sigma(t)})||\boldsymbol{\xi}||^2 \geq (\underline{\varrho}_m/2)||\boldsymbol{\xi}||^2. \quad (27)$$

Hence, applying (27) to (26) and considering the structure of $\mathcal{B}$ in (14), the behaviour of $V(t)$ between the two consecutive switching intervals, i.e., $t \in [t_l \;\; t_{l+1})$, is studied for two possible scenarios:

- $V(t) \geq \mathcal{B}$, we have $\dot{V}(t) \leq -\kappa V(t)$ from (26) implying exponential decrease of $V(t)$;
- when $V(t) < \mathcal{B}$, $V(t)$ may increase.

With these possibilities, two cases with initial conditions are further selected as: Case (i) $V(t_l) \geq \mathcal{B}$ and Case (ii) $V(t_l) < \mathcal{B}$.

Case (i): $V(t_l) \geq \mathcal{B}$

Let T_1 denote the time instant when $V(t)$ enters into the bound $\mathcal{B}$ and $N_1(t)$ denotes the number of intervals a subsystem p, $p \in \Omega$ remains active for $t \in [t_l \;\; t_l + T_1)$. Accordingly, for $t \in [t_l \;\; t_l + T_1)$, using (17), (26) and $N_\sigma(t_l, t)$ from Definition 1 we have

$$\begin{aligned}V(t) &\leq \exp\left(-\kappa(t - t_{N_1(t)-1})\right)V(t_{N_1(t)-1}) \\ &\leq \mu\exp\left(-\kappa(t - t_{N_1(t)-1})\right)V(t^-_{N_1(t)-1}) \\ &\leq \mu\exp\left(-\kappa(t - t_{N_1(t)-1})\right) \\ &\quad\times\mu\exp\left(-\kappa(t_{N_1(t)-1} - t_{N_1(t)-2})\right)V(t^-_{N_1(t)-2}) \\ &\quad\vdots \\ &\leq \mu^{N_\sigma(t_0,t)}\exp\left(-\kappa(t-t_0)\right)V(t_0) \\ &= c\left(\exp\left(-\kappa + \ln\mu/\vartheta\right)\right)V(t_0), \qquad (28)\end{aligned}$$

where $c \triangleq \exp\left(N_0\ln\mu\right)$ is a constant. Substituting the ADT condition $\vartheta > \ln\mu/\kappa$ in (28) yields $V(t) < cV(t_0)$ for $t \in [t_l \;\; t_l + T_1)$. Moreover, as $V(t_l + T_1) < \mathcal{B}$, one has $V(t_{N_1(t)+1}) < \mu\mathcal{B}$ from (17) at the next switching instant

$t_{N_1(t)+1}$ after $t_l + T_1$. This implies that $V(t)$ may be larger than $\mathcal{B}$ from the instant $t_{N_1(t)+1}$, leading to further analysis.

We assume $V(t) \geq \mathcal{B}$ for $t \in [t_{N_1(t)+1} \quad t_l + T_2)$, where T_2 denotes the time before next switching. Let $N_2(t)$ represent the number of all switching intervals for $t \in [t_{N_1(t)+1} \quad t_l + T_2)$. Then, substituting $V(t_l)$ with $V(t_{N_1(t)+1})$ in (28) and following the similar procedure for analysis as (28), we have $V(t) \leq cV(t_{N_1(t)+1}) < c\mu\mathcal{B}$ for $t \in [t_{N_1(t)+1} \quad t_l + T_2)$. Since $V(t_l + T_2) < \mathcal{B}$, we have $V(t_{N_1(t)+N_2(t)+2}) < \mu_{\sigma(t^-_{N_1(t)+N_2(t)+2})}\mathcal{B}$ at the next switching instant $t_{N_1(t)+N_2(t)+2}$ after $t_l + T_2$. Following similar lines of proof recursively, one can conclude that $V(t) < c\mu\mathcal{B}$ for $t \in [t_l + T_1 \quad \infty)$. This implies that once $V(t)$ enters the interval $[0, \mathcal{B}]$, it cannot exceed the bound $c\mu\mathcal{B}$ any time later with the ADT switching law (13).

Case (ii): $V(t) < \mathcal{B}$

It can be easily verified that the same argument mentioned above for Case (i) also holds for Case (ii).

Thus, observing the stability notions of the Cases (i) and (ii), it can be concluded that the closed-loop system remains UUB globally. Further, based on this analysis, we have

$$V(t) \leq \max\left(cV(t_0), c\mu\mathcal{B}\right), \quad \forall t \geq t_0. \tag{29}$$

Using (27) and (29) we have

$$||\boldsymbol{\xi}||^2 \leq (2/\underline{\varrho}_m) \max\left(cV(t_0), c\mu\mathcal{B}\right), \quad \forall t \geq t_0. \tag{30}$$

Therefore, using (30), an ultimate bound b on the tracking error $\boldsymbol{\xi}$ can be found as (14). ∎

Remark 6: The existence of $\bar{\alpha}_{i\sigma} > 0$ and $\bar{\beta}_{i\sigma} > 0$ to establish (25) justifies the reason for the selection of $\alpha_i > \max_{\sigma\in\Omega}(\varrho_\sigma/2)$ and $\beta_i > \max_{\sigma\in\Omega}(\varrho_\sigma/2)$.

Remark 7: The proposed method is a robust adaptive design in the sense of [25], i.e. it can cope with external disturbances and unmodelled dynamics. As a trade-off, it cannot guarantee asymptotic convergence of the tracking error. On the other hand, the adaptation method in [17], [18] has the merit of attaining asymptotic convergence of the tracking error, at the price of considering the ideal case, i.e. ignoring external disturbances and unmodelled dynamics. A robust adaptation method for nonlinearly-parametrized switched systems with asymptotic tracking error in the ideal case is, to the best of the authors' knowledge, still missing.

V. Simulation Results

Consider the following switched EL dynamics with two (non-smooth) subsystems, where each subsystem represents a 2-link manipulator with different system parameters:

$$\mathbf{M}_\sigma(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}_\sigma(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{G}_\sigma(\mathbf{q}) + \mathbf{F}_\sigma(\dot{\mathbf{q}}) + \mathbf{d}_\sigma = \boldsymbol{\tau}_\sigma, \tag{31}$$

$$\mathbf{M}_\sigma = \begin{bmatrix} M_{\sigma_{11}} & M_{\sigma_{12}} \\ M_{\sigma_{12}} & M_{\sigma_{22}} \end{bmatrix}, \mathbf{q} = \begin{bmatrix} q_l \\ q_u \end{bmatrix},$$

$$M_{\sigma_{11}} = (m_{\sigma_l} + m_{\sigma_u})l_{\sigma_u}^2 + m_{\sigma_u} l_{\sigma_l}(l_{\sigma_l} + 2l_{\sigma_u}\cos(q_u)),$$

$$M_{\sigma_{12}} = m_{\sigma_u} l_{\sigma_u}(l_{\sigma_u} + l_{\sigma_l}\cos(q_u)), M_{\sigma_{22}} = m_{\sigma_u} l_{\sigma_u}^2,$$

$$\mathbf{C}_\sigma = \begin{bmatrix} -m_{\sigma_u} l_{\sigma_l} l_{\sigma_u}\sin(q_u)\dot{q}_u & -m_{\sigma_u} l_{\sigma_l} l_{\sigma_u}\sin(q_u)(\dot{q}_l + \dot{q}_u) \\ 0 & m_{\sigma_u} l_{\sigma_l} l_{\sigma_u}\sin(q_u)\dot{q}_u \end{bmatrix},$$

$$\mathbf{G}_\sigma = \begin{bmatrix} m_{\sigma_l} l_{\sigma_l} g\cos(q_l) + m_{\sigma_u} g(l_{\sigma_u}\cos(q_l + q_u) + l_{\sigma_l}\cos(q_l)) \\ m_{\sigma_u} g l_{\sigma_u}\cos(q_l + q_u) \end{bmatrix},$$

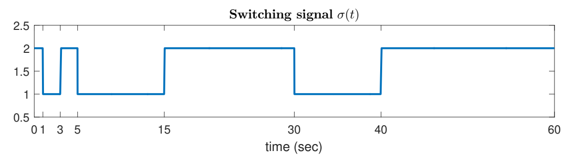

Fig. 1: The switching signal.

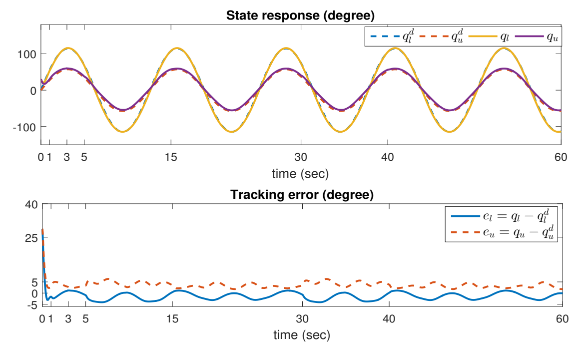

Fig. 2: Tracking performance of the proposed controller.

$$\mathbf{F}_\sigma = \begin{bmatrix} f_{\sigma_{vl}}\,\mathrm{sgn}(\dot{q}_l) \\ f_{\sigma_{vu}}\,\mathrm{sgn}(\dot{q}_u) \end{bmatrix}, \mathbf{d}_\sigma = \begin{bmatrix} 0.05\cos(0.05t) \\ 0.05\cos(0.05t) \end{bmatrix},$$

where 'sgn' is standard 'signum' function defining static Coulomb friction. The system dynamics (31), when represented in the form (1), becomes NLIP (due to the inversion of $\mathbf{M}_\sigma$) and has $\nu = 2$ following Property 1 [20]. Here (m_{p_l}, l_{p_l}, q_l) and (m_{p_u}, l_{p_u}, q_u) denote the mass, length and position of link 1 and 2 respectively for subsystem p with $p = \{1, 2\}$. The actual (and unknown) parametric values of the manipulator subsystems are taken as

1. $m_{1_l} = m_{1_u} = 1.2$kg, $l_{1_l} = l_{1_u} = 1$m and $f_{1_{vl}} = f_{1_{vu}} = 0.5$;
2. $m_{2_l} = m_{2_u} = 2.4$kg, $l_{2_l} = l_{2_u} = 2$m and $f_{2_{vl}} = f_{2_{vu}} = 0.6$,

with $g = 9.8\text{m/sec}^2$ for both subsystems. The objective is to track a desired trajectory defined as $\{q_l^d, q_u^d\} = \{2\sin(0.5t), \sin(0.5t)\}$rad. Selection of $\mathbf{K}_{11} = 170\mathbf{I}, \mathbf{K}_{12} = 120\mathbf{I}, \mathbf{K}_{21} = 25\mathbf{I}, \mathbf{K}_{22} = 12\mathbf{I}, \mathbf{Q}_1 = \mathbf{Q}_2 = 0.2\mathbf{I}$, $\kappa = 0.9\varrho$ yields the ADT $\vartheta^* = 6.26$sec according to (13). Therefore, a switching law $\sigma(t)$ is designed as in Fig. 1 (note that the fast switchings for $1-3$sec and $3-5$sec are compensated by slower switching later on). To have a $\hat{\mathbf{b}}_p$ in (7), we select the nominal parameter as $m_{1_l} = m_{1_u} = 1.0$kg, $l_{1_l} = l_{1_u} = 0.9$m and $m_{2_l} = m_{2_u} = 2.0$kg, $l_{2_l} = l_{2_u} = 1.9$m, while $\mathbf{C}_\sigma, \mathbf{F}_\sigma, \mathbf{G}_\sigma$ and $\mathbf{d}_\sigma$ are considered to be completely unknown. It is possible to show that (5) is satisfied with $\bar{E}_1 = \bar{E}_2 = 0.3$. Other control parameters are designed as $\alpha_i = \beta_i = 1, \epsilon_i = \underline{\epsilon}_i = 0.2$ with $i = 0, 1, 2$. The initial conditions are selected as $\hat{\theta}_{ip}(0) = \hat{\theta}_{i\bar{p}}(0) = \gamma_{ip}(0) = \gamma_{i\bar{p}}(0) = 0.3$ and $q_l(0) = q_u(0) = 0.5$rad, respectively.

The performance of the proposed controlled system is depicted in Fig. 2 in terms of state responses and tracking errors (reported in degree for better comprehension). In line with Remark 5, it can be noted from Figs. 3-4 that, the gains

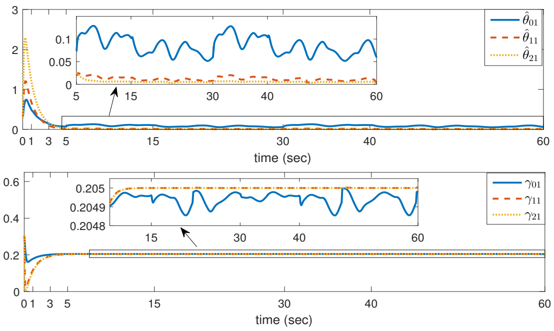

Fig. 3: Gains for subsystem 1.

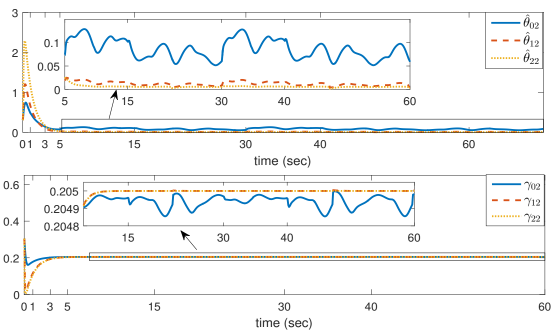

Fig. 4: Gains for subsystem 2.

$\hat{\theta}_{i\bar{p}}$ of the inactive subsystems do not decrease exponentially for the entire switch-off period (e.g., for $t \in [15,\ 30)$ and $t \in [30,\ 40)$ for subsystems 1 and 2, respectively).

VI. Conclusions

A new adaptive control framework was presented for a class of nonlinearly-parametrized switched systems. The class under consideration comprises Monod and Euler-Lagrange dynamics (with possibly non-smooth terms) as a special case. A highlighting feature of the proposed framework was to simultaneously update the gains of the active and inactive subsystems, avoiding high gains for the former and vanishing gains for the latter. Robust stability analysis was provided in terms of uniformly ultimately boundedness and the performance of the controller was verified using a robotic manipulator simulation example.

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