A Numerical Approach for the Evaluation of the Local Stress Ratio in Fatigue-Driven Delamination Analysis

Raimondo, Antonio; Bisagni, Chiara

DOI
10.2514/6.2019-1545

Publication date
2019

Document Version
Final published version

Published in
AIAA Scitech 2019 Forum

AIAA SciTech Forum
7-11 January 2019, San Diego, California
AIAA Scitech 2019 Forum

10.2514/6.2019-1545

A Numerical Approach for the Evaluation of the Local Stress Ratio in Fatigue-Driven Delamination Analysis

Antonio Raimondo[1] and Chiara Bisagni[2]
Delft University of Technology, Delft, 2629HS, Netherlands

An approach based on the cohesive zone model for analyzing fatigue-driven delamination in composite structures under cyclic loading is presented. The proposed technique, called "Min-Max Load Approach", is able to dynamically capture the local stress ratio during the evolution of damage. The possibility to know the local stress ratio is relevant in all the situations where its value is different from the applied load ratio and cannot be determined a priori. In a single Finite Element analysis, two identical models are analyzed with two different constant loads, the minimum and the maximum load during the fatigue cycle. The implemented methodology allows the two models to interact with each other, by exchanging information to correctly calculate the crack growth rate. At first, the approach has been validated in simulations of mode I and mixed-mode propagation by using Double Cantilever Beam and Mixed-Mode Bending. Then, to prove the effectiveness of the developed methodology, a modified version of the Mixed-Mode Bending test has been numerically investigated. In this test, the mode I and mode II components of the load are decoupled and applied independently, resulting in a local stress ratio different from the applied load ratio.

I. Nomenclature

a	=	Crack length
b_{0I}	=	Semi-empirical fatigue delamination growth law exponent
C	=	Paris' law constant
d	=	Damage variable
d_s	=	Quasi-static damage variable
d_f	=	Fatigue damage variable
d_i^J	=	Damage variable at integration point J at cycle i
G_C	=	Critical energy release rate
G_{max}	=	Maximum energy release rate in fatigue cycle
G_{min}	=	Minimum energy release rate in fatigue cycle
G_{th}	=	Energy release rate fatigue threshold
h	=	Semi-empirical fatigue delamination growth law coefficient
K	=	Cohesive stiffness
l_{CZ}	=	Length of cohesive zone
m	=	Paris' law exponent
N	=	Number of cycles
R	=	Applied load ratio, P_{min}/P_{max}
R_{Local}	=	Local stress ratio, $\sqrt{G_{min}/G_{max}}$
α	=	Function of the fracture toughness in the semi-empirical fatigue delamination growth law
Δ^0	=	Cohesive displacement at damage initiation
Δ^f	=	Cohesive displacement at failure
Δd_{max}	=	Maximum variation of the damage variable during a cycle jump
ΔG	=	Variation of energy release rate during the load cycle
δ^I	=	Mode I displacement in modified MMB specimen
δ^{II}	=	Mode II displacement in modified MMB specimen
η	=	Benzeggagh-Kenane material parameter

[1] Post-doctoral Researcher, Faculty of Aerospace Engineering.
[2] Professor, Faculty of Aerospace Engineering, AIAA Associate Fellow.

λ	=	Cohesive displacement
τ^0	=	Cohesive interface strength
ϕ	=	Mixed-Mode ratio

II. Introduction

Fatigue-driven delamination is one of the most critical type of damage in laminated fiber-composites, and it is significant in term of skin-stiffener separation in the case of stiffened structures. Usually it is difficult to detect the internal delamination, and it can bring severe loss in mechanical properties of the component. Furthermore, it can rapidly grow under service loading condition, leading to the sudden collapse of the structure.

The estimation of fatigue life of a composite structure remains still an unsolved challenge, due to the complexity of the mechanisms involved in the phenomenon. Most of the existing methodologies for the prediction of delamination growth under fatigue loading make use of the Paris' law[1]:

$$\frac{da}{dN} = C\left[\Delta G\right]^m = C\left[G_{\max} - G_{\min}\right]^m \tag{1}$$

where a is the crack length, N is the number of cycles, ΔG is the variation of the Energy Release Rate between the maximum and minimum load configurations during the load cycle, and C and m are experimentally determined parameters which depend on the material and on the load conditions. The Paris' law, initially developed for fatigue crack evolution in metallic materials, relates the crack growth rate (da/dN) to the Energy Release Rate (G).

The numerical approaches for the analysis of delamination under fatigue load can be divided in two categories: Fracture Mechanics (FM) and Damage Mechanics (DM)[2]. The FM methods are based on the direct application of the Paris' law in combination with a procedure for the evaluation of the Energy Release Rate, as the Virtual Crack Closure Technique (VCCT)[3-7]. In Damage Mechanics approach the degradation of the material interface is described by the evolution of one or more damage variables, and the Cohesive Zone Model (CZM) is adopted to represent the fracture[8-11]. Usually, for fatigue problems, the CZM approach is implemented together with an approach called "envelope load method"[2]. By using this method, only the maximum load is modeled, and the load ratio is taking into account inside the constitutive model which relates the cohesive properties degradation with the number of cycles.

One of the main limitations in adopting the envelope load method is that, by simulating only the maximum load configuration, the load ratio used in the constitutive model for the calculation of the crack growth rate is considered constant during the duration of the analysis and equal to the applied (external) load ratio ($R = P_{\min}/P_{\max}$). However, the local stress ratio ($R_{Local} = \sqrt{G_{\min}/G_{\max}}$) may or may not be equal to the applied (external) load ratio[12], and can change during the damage evolution and along the delamination front. This can happen, for example, when two or more non-synchronized loads act simultaneously on the structure, or when stiffened structures are tested in post-buckling load fatigue conditions, where the buckling mode shape may change between the maximum and the minimum load of the fatigue cycle[13-14].

The objective of this work is to develop a methodology based on the Finite Element (FE) method and on the CZM able to dynamically acquire the local stress ratio along the delamination front and during the evolution of the damage.

The paper is organized as follows. In Section III, the fatigue damage model based on cohesive element adopted in this work is briefly described, while in Section IV the theory behind the proposed numerical approach and its implementation in the FE code ABAQUS[15] are presented. In Section V the methodology is, at first, validated with results of Double Cantilever Beam (DCB) and Mixed-Mode Bending (MMB) tests taken from literature[5-7], and then applied to a specimen with loading and boundary conditions designed to produce a local stress ratio different from the applied load ratio and which can change during the propagation of the damage. Finally, conclusions are reported.

III. Quasi-Static and Fatigue Cohesive Model

Cohesive elements have been developed in the last decades to simulate the initiation and propagation of delamination under quasi-static loading conditions using FE. The cohesive law relates tractions to the separation at the interface where the crack propagation occurs. After an initial elastic part defined by the penalty stiffness, K, the damage initiation displacement (Δ^0) is related to the interfacial strength of the material (τ^0), while the final displacement (Δ^f) is defined by the Critical Energy Release Rate (G_C), which represents the area under the softening curve (Fig.1).

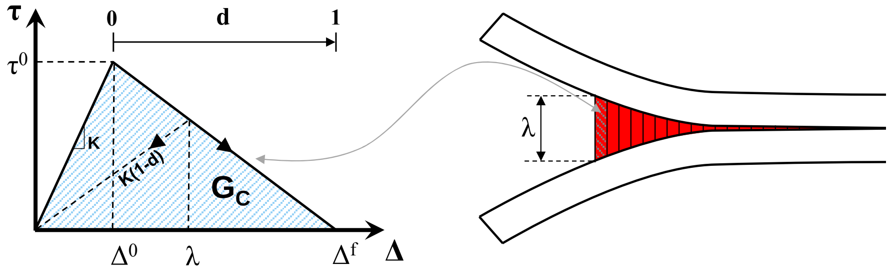

Fig. 1 Constitutive response for traction-separation cohesive elements.

The loss of stiffness of the cohesive element is directly related to the evolution of a damage variable (*d*), which can be calculated from Eq. (2).

$$d = \frac{\Delta^f(\lambda - \Delta^0)}{\lambda(\Delta^f - \Delta^0)} \tag{2}$$

CZM formulation has been extended to simulate also fatigue damage propagation. In this work, the fatigue constitutive model developed by Turon et al.[9] has been adopted. The model of Turon is based on the approach usually called "envelope load method", which takes into account, during the numerical simulation, only the maximum load of a single load cycle, while the load oscillation is included within the cohesive constitutive model. The evolution of the cohesive damage can be expressed as sum of a component related to the quasi-static damage (d_s) and one related to the fatigue damage (d_f), as shown in Eq. (3):

$$\frac{\partial d}{\partial N} = \frac{\partial d_s}{\partial N} + \frac{\partial d_f}{\partial N} \tag{3}$$

The fatigue damage rate ($\partial d_f / \partial N$) defines the evolution of the damage variable as a function of the number of cycles, and, referring to Fig. 1, is formulated as follow:

$$\frac{\partial d_f}{\partial N} = \frac{1}{l_{CZ}} \frac{\left[\Delta^f(1-d) + d\Delta^0\right]^2}{\Delta^f \Delta^0} \frac{\partial a}{\partial N} \tag{4}$$

where l_{CZ} is the length of the cohesive zone and $\partial a / \partial N$ is the Crack Growth Rate, defined as a piecewise function using the Power law:

$$\frac{\partial a}{\partial N} = \begin{cases} C\left(\dfrac{\Delta G}{G_C}\right)^m, & G_{th} < G < G_C \\ 0 & otherwise \end{cases} \tag{5}$$

The variation of the energy release rate (ΔG) is calculated using the constitutive law:

$$\Delta G = G_{\max} - G_{\min} = \frac{\tau^0}{2}\left[\Delta^f - \frac{\left(\Delta^f - \lambda^{\max}\right)^2}{\Delta^f - \Delta^0}\right]\left(1 - R^2\right) \tag{6}$$

In order to avoid a cycle-by-cycle analysis, a cycle-jump strategy is implemented, considering a number of cycles that can be jumped without expecting any relevant change in the damage state. The damage at each cycle jump is calculated as following:

$$d^J_{i+\Delta N_i} = d^J_i + \frac{\partial d^J_i}{\partial N}\Delta N_i \quad with \quad \Delta N_i = \frac{\Delta d_{\max}}{\max\left\{\frac{\partial d^J_i}{\partial N}\right\}} \tag{7}$$

where $d^J_{i+\Delta N_i}$ is the damage variable at integration point J at cycle $N_{i+\Delta N_i}$, d^J_i is the damage variable at integration point J at cycle N_i, $\partial d^J_i / \partial N$ is the damage rate at integration point J at cycle N_i evaluated by using Eq. (4), and ΔN_i is the number of cycle that can be jumped. The cycle jump is evaluated in order to maintain a fixed level of accuracy during the analysis. In particular it corresponds to the number of cycles that ensure a maximum variation of the damage variable inside the cohesive element layer equal to $\Delta d_{\max}$, which is a fixed number. It represents the sensitivity of the analysis, and the smaller the value of $\Delta d_{\max}$ is, the higher is the accuracy of the simulation.

IV. Min-Max Load Approach

In order to dynamically capture the local stress ratio is necessary to have information about the deformed shape of the structure when it is at the minimum and at the maximum load of the fatigue cycle. The idea of the technique developed in this work, called "Min-Max Load Approach", is to perform a single simulation with two models representing the same structure but with different applied loads. Fig. 2 shows the example of a Double Cantilever Beam (DCB) specimen subjected to a sinusoidal load. Instead of analyzing all the fatigue cycles, two identical models are created and analyzed at constant load. One model simulates the deformed shape of the structure when the applied load is equal to the minimum value of the fatigue cycle, and the other one represents the deformed configuration of the specimen at the maximum load. The fatigue calculations, based on the constitutive model described in the previous section, are performed on the model representative of the maximum load configuration, which needs the value of the energy release rate from the minimum configuration in order to calculate the local stress ratio and the crack growth rate according to the Paris' law equation. On the other hands, the minimum load configuration requires information regarding the damage state in the cohesive layer, in order to update the crack front. These data are exchanged at the beginning and at the end of each cycle-jump, as illustrated in Fig. 2.

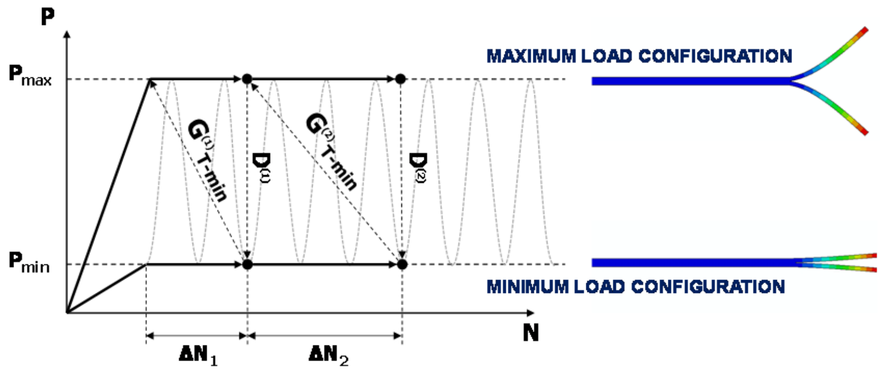

Fig. 2 Min-Max Load Approach.

The proposed approach has been implemented in the finite element code ABAQUS. After discretizing the structure, a mirror copy is performed. This results in two identical finite element models subjected to the minimum and the maximum loads, respectively.

In order to identify the two configurations during the analysis, each element belonging to one configuration has the same number of the corresponding element in the other configuration but with a constant offset (Fig. 3).

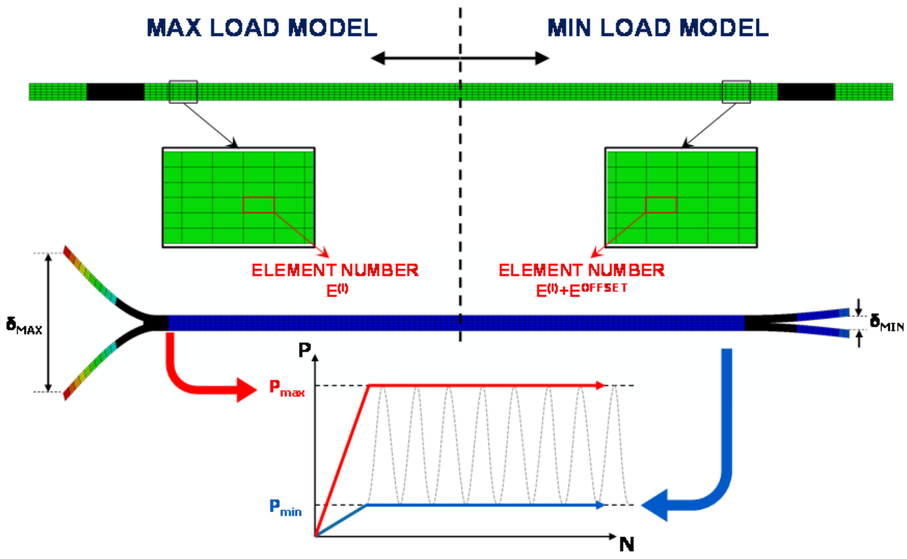

Fig. 3 FE application of Min-Max Load Approach.

The fatigue damage model has been implemented by means of a User Material Subroutine[15] (UMAT). The subroutine, written in Fortran language, is called at each integration point during each load increment of the non-linear analysis, allowing to define a completely user-defined material behavior. The operations performed inside the developed subroutine are schematically summarized in Fig. 4.

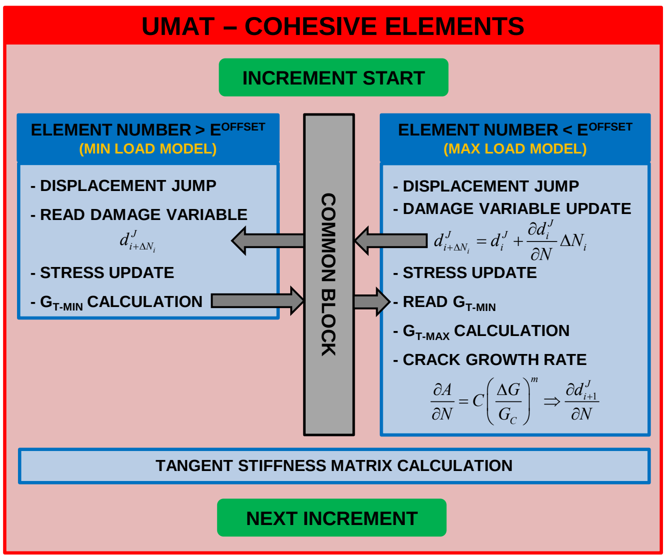

Fig. 4 Fatigue damage model implemented in ABAQUS UMAT subroutine.

At first, a check is performed to verify if the integration point under consideration belongs to the maximum or minimum load model. If the element is part of the structure subjected to the minimum load, then the value of the damage variable is read from the corresponding element of the maximum load configuration, the stresses are update and the calculations of the energy release rate is performed. On the other hand, if the integration point under investigation belongs to the maximum load model, then, the damage variable is update together with the stresses, and the fatigue damage calculations are performed using the value of G taken from the corresponding element in the minimum load configuration. All information between the two models is exchanged by using a COMMON BLOCK, which allows sharing variables between subroutines in the Fortran environment.

V. Results and Discussion

Several numerical simulations have been performed to analyze and validate the response of the developed fatigue approach for crack propagation under pure mode I and mixed-mode condition at different range of the energy release rate. Then, to demonstrate its effectiveness, a numerical investigation has been carried out on a specimen whose boundary and loading conditions are designed to produce a local stress ratio which can change during the propagation of the damage and which is different from the applied load ratio.

A. Simulation of DCB Specimen

Numerical simulations have been conducted on a DCB specimen to investigate fatigue crack growth under pure mode I loading[5]. The geometrical characteristics and the material properties are shown in Fig. 5 and Table 1, respectively.

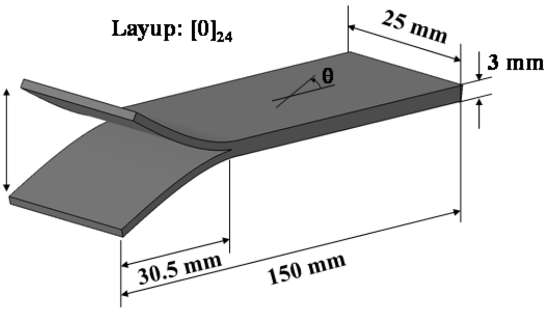

Fig. 5 DCB specimen.

Table 1. Material properties T300/1076.

Lamina properties		Interface properties		Fatigue properties	
E_1	139400 MPa	G_{1C}	0.17 kJ/m^2	C_I	2.44E6 mm/cycles
$E_2 = E_3$	10160 MPa	G_{2C}	0.49 kJ/m^2	m_I	10.61
$G_{12} = G_{13}$	4600 MPa	η	1.62		
G_{23}	3540 MPa	τ_3^0	60 MPa		
$\nu_{12} = \nu_{13}$	0.25	K	10^6 N/mm^3		
ν_{23}	0.45				

The two FE models, representing the DCB subjected to the minimum and the maximum loads, have been discretized by using 2D plane strain elements and 4-node cohesive elements with zero-thickness. To guarantee an accurate representation of the cohesive process zone, an element length of 0.05 mm has been adopted in the propagation region. The analysis has been divided in two steps. In the first quasi-static step, the displacement at each arm tip is increased up to the maximum and minimum values of the fatigue cycle, and only the static damage is taking into account. In the second step, the displacement is kept constant and the fatigue calculation are performed, according to Eq. (3) and Eq. (4).

The results in terms of deformed shape and damage propagation of the analysis performed at applied load ratio R = 0.1 are shown in Fig. 6.

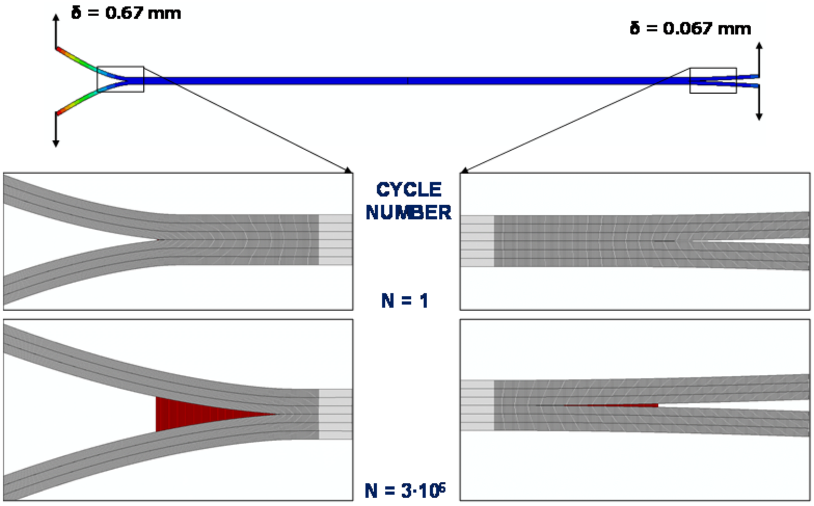

Fig. 6 DCB deformed shape and crack propagation at R=0.1.

In Fig. 7 the crack length, as a function of the number of cycles obtained from the numerical analyses at different values of the applied load ratio, is compared to the analytical solution derived from the Corrected Beam Theory[16] and to the simulation performed adopting the VCCT approach implemented in ABAQUS. The results of the different numerical approaches are in excellent agreement with each other, while the small deviations from the analytical predictions are due to the linear formulation of the analytical model.

For an applied load ratio equal to 0.1, the delamination length changes from an initial value of 30.5 mm to a final length of 37.5 mm at about 3,000,000 cycles, when the energy release rate at the crack tip is below the threshold value and the crack stops propagating.

The effect of changing the applied load ratio has been investigated by simply increasing the displacement of the minimum load model to half the maximum displacement, resulting in an applied load ratio equal to 0.5. When the load ratio is increased, a reduction of the propagation velocity is obtained, as it can be observed in Fig. 7.

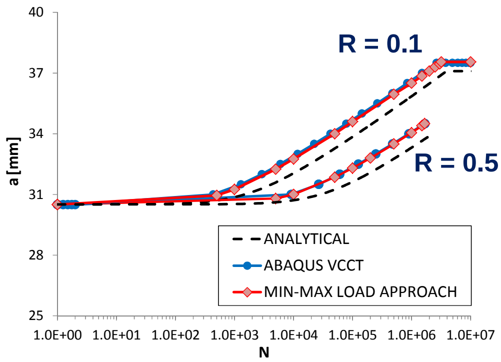

Fig. 7 DCB fatigue crack propagation at different load ratio.

B. Simulation of MMB Specimen

In order to investigate delamination propagation in mixed-mode conditions, the Mixed-Mode Bending (MMB) test has been simulated[7]. The geometrical characteristics and the material properties of the specimen with a 20% mixed-mode ratio are shown, respectively, in Fig. 8 and Table 2.

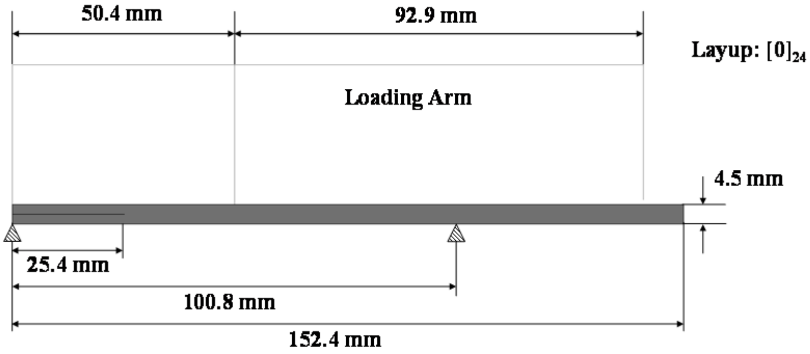

Fig. 8 MMB specimen.

Table 2. Material properties IM7/8552.

Lamina properties		Interface properties		Fatigue properties	
E_1	161000 MPa	G_{1C}	0.212 kJ/m^2	G_{II}/G_T	0.2
$E_2 = E_3$	11373 MPa	G_{2C}	0.774 kJ/m^2	$C_{20\%}$	2412 mm/cycles
$G_{12} = G_{13}$	5200 MPa	η	2.21	$m_{20\%}$	8.4
G_{23}	3900 MPa	τ_3^0	60 MPa	δ_{max}	1.27 mm
$\nu_{12} = \nu_{13}$	0.32	$\tau_1^0 = \tau_2^0$	90 MPa	G_{II}/G_T	0.2
ν_{23}	0.45	K	10^6 N/mm^3		

The results obtained from the numerical simulation in terms of deformed shape and damage propagation are shown in Fig. 9.

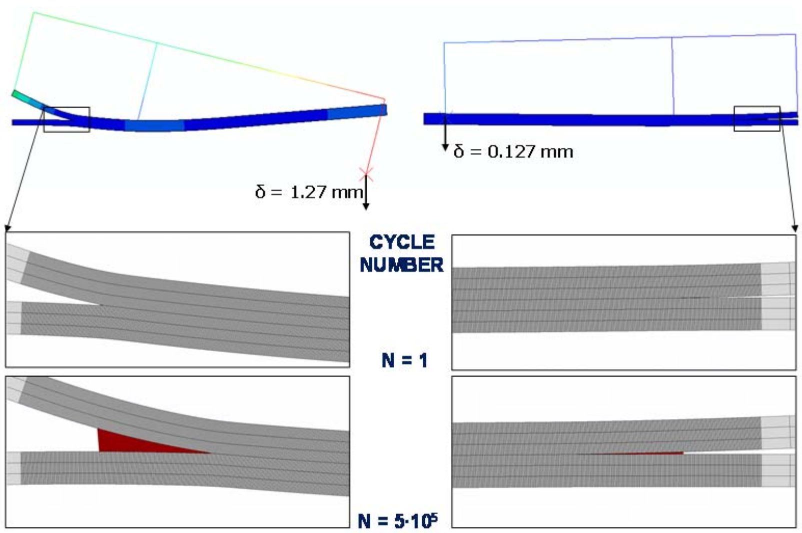

Fig. 9 MMB 20% deformed shape and crack propagation at R=0.1.

In Fig. 10, the crack length variation as function of the number of cycles is compared with the results obtained from the VCCT approach implemented in ABAQUS and with analytical solution calculated by using the corrected beam theory with a superposition of pure mode I and mode II loadings[17], showing also in this case good agreement.

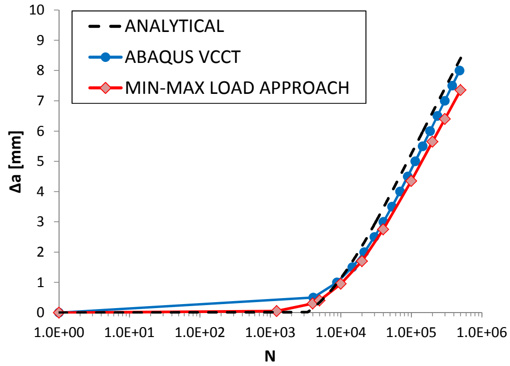

Fig. 10 MMB 20% fatigue crack propagation.

C. Modified MMB Test

The results of the simulations performed in the previous subsections have shown very good agreement with the analyses performed using the VCCT approach implemented in ABAQUS. Apparently, no advantages are obtained by using the "Min-Max Load Approach" because, for the specimens previously considered, the applied load ratio is always equal to the local stress ratio. However, they have been analyzed to prove that the approach is able to simulate the propagation of the delamination and to correctly predict the load ratio without giving this value as input in the constitutive model.

The developed approach has been then applied to a specimen in which the local stress ratio is not equal to the applied load ratio, cannot be predicted in advance, and can change during the propagation of the delamination. In order to meet these requirements, a modified MMB test has been considered, with the same geometrical characteristics and boundary conditions reported in the previous subsection in Fig. 8 and Table 2. In the classic MMB test, the load or displacement is applied by means of a lever which distributes the load into a mode I and a mode II bending component in a ratio which depends on the length of the lever. In the modified version of the MMB test investigated in this work, the mode I and mode II loads are decoupled from each other and applied separately without using a lever. In particular, a constant mode II displacement (δ^{II}) and a sinusoidal mode I opening displacement, oscillating between the minimum value δ^{I}_{MIN} and the maximum value δ^{I}_{MAX}, have been considered, as shown in Fig. 11.

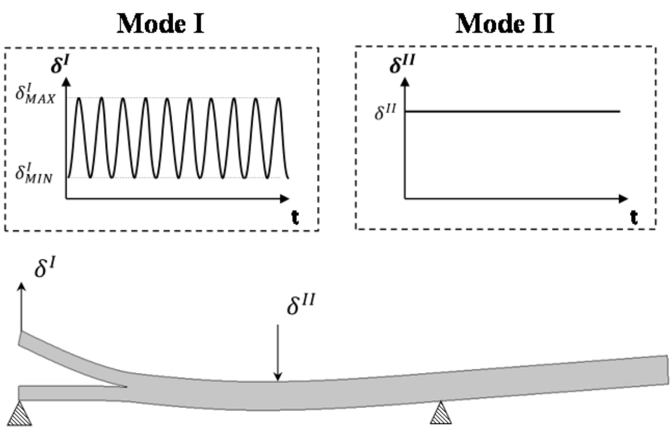

Fig. 11 Modified MMB specimen.

Two identical models have been realized and analyzed in the same analysis at constant load. In particular, one model simulates the behavior of the structure when subjected to the maximum load, which means that a mode I displacement, equal to the maximum value of the displacement during the fatigue cycle (δ^{I}_{MAX}), has been applied together with a mode II displacement (δ^{II}). On the other hand, the model representing the minimum load configuration is characterized by a mode I displacement equal to δ^{I}_{MIN} and the same constant mode II displacement (δ^{II}). The two configurations with the boundary and loading conditions are summarized in Fig. 12.

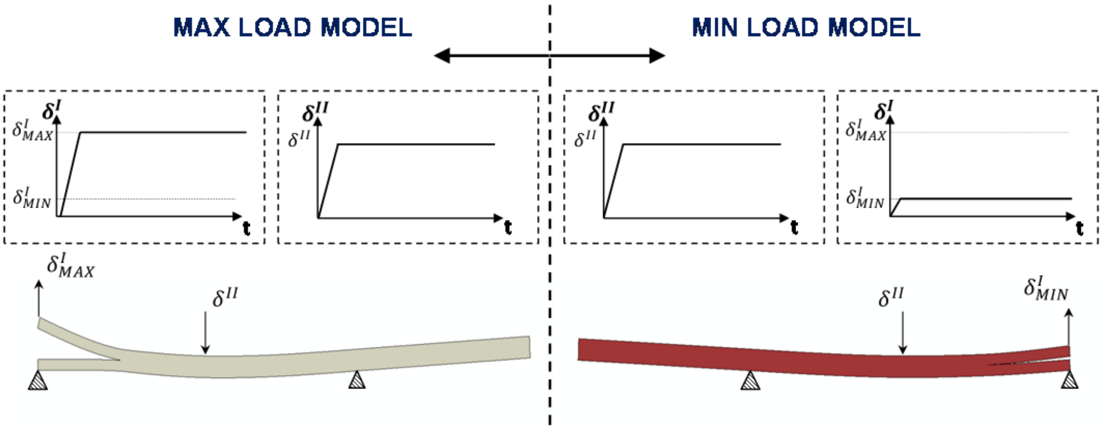

Fig. 12 Min-Max Load Approach for modified MMB specimen.

To take into account the variation of the local stress ratio into the constitutive damage model, the semi-empirical fatigue delamination growth law developed by Allegri et al.[18] has been adopted in this work. The law describes the effect of mode-mixity and load ratio on the delamination growth rate by using a single formula, as shown in Eq. (8).

$$\frac{\partial a}{\partial N} = C\left[\frac{G_{\max}}{G_C(\phi)}\right]^{\frac{b_{0I}}{(1-R_{Local})^{1+\alpha(\phi)}}e^{-h\phi}} \tag{8}$$

where ϕ is the mode-mixity ($\phi = G_{II\max}/G_{\max}$) and $G_{\max}$ is the peak value of the energy release rate, defined as the sum of the maximum values of mode I and mode II energy release rate components:

$$G_{\max} = G_{I\max} + G_{II\max} \tag{9}$$

The value $\alpha(\phi)$ is a function of the mode-mixity and of the fracture toughness $G_C(\phi)$:

$$\alpha(\phi) = \frac{G_C(\phi) - G_{IC}}{G_{IIC} - G_{IC}} \tag{10}$$

The mixed-mode fracture toughness can be expressed by using the formula proposed by Benzeggangh and Kenane[19]:

$$G_C(\phi) = G_{IC} + (G_{IIC} - G_{IC})\phi^{\eta} \tag{11}$$

The factor C, the exponent b_{0I} and the coefficient h are material dependent parameters, evaluated by using several set of experimental data on the material IM7/8552 and are reported in Table 3[18].

Table 3. Fatigue coefficients.

Material	*C [mm]*	b_{0I}	h
IM7/8552	3.51E-2	14.05	1.47

Three different simulations have been carried out at different values of mode II displacement (δ^{II}), while the fatigue load on mode I component is kept constant with an applied load ratio equal to 0.1 (R = 0.1). The aim is to investigate how the addition of a constant load (δ^{II}) during the entire fatigue cycle, from the minimum to the maximum load, affects the local stress ratio and therefore the propagation of the crack. In Table 4 the loading conditions adopted for the three performed analyses are summarized.

Table 4. Applied displacements in modified MMB specimen analyses.

δ^{I}_{MAX} [mm]	δ^{I}_{MIN} [mm]	δ^{II} [mm]
0.65	0.065	0.15
0.65	0.065	0.10
0.65	0.065	0.05

The deformed shape obtained by the first analysis is reported in Fig. 13. It can be observed that, even when the fatigue displacement reaches the minimum value, the structure is still loaded due to the presence of the constant displacement δ^{II}.

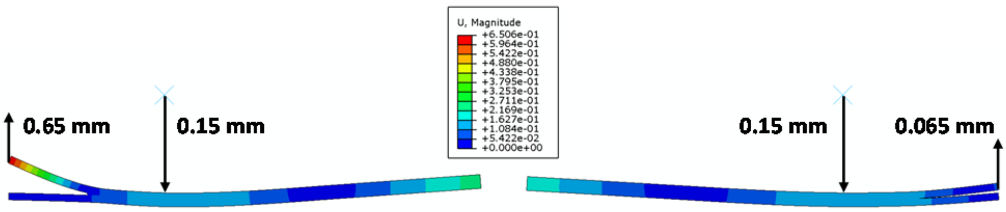

Fig. 13 Modified MMB deformed shape.

In Fig. 14 the values of the crack length variation are reported in function of the number of cycles for all three performed analyses. As expected, increasing the mode II displacement (δ^{II}) leads to higher delamination length due to the increase of the total applied energy release rate.

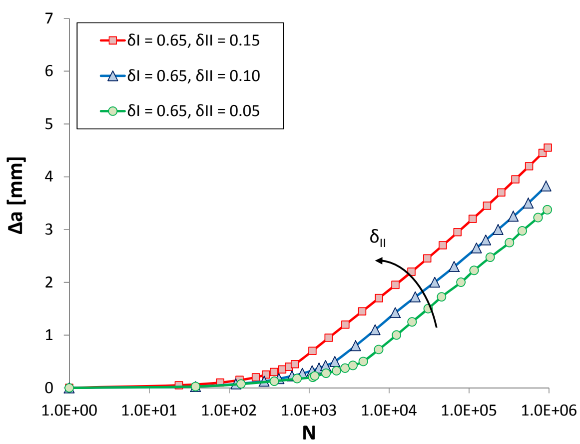

Fig. 14 Fatigue crack growth at different value of applied mode II displacement: modified MMB specimen.

The effectiveness of the approach proposed in this paper can be appreciated by observing Fig. 15, where the crack length variation as a function of the number of cycles is compared with the results obtained by using the same approach but without performing the calculation of the local stress ratio and assuming that it is constant and equal to the applied load ratio (R_{Local} = 0.1).

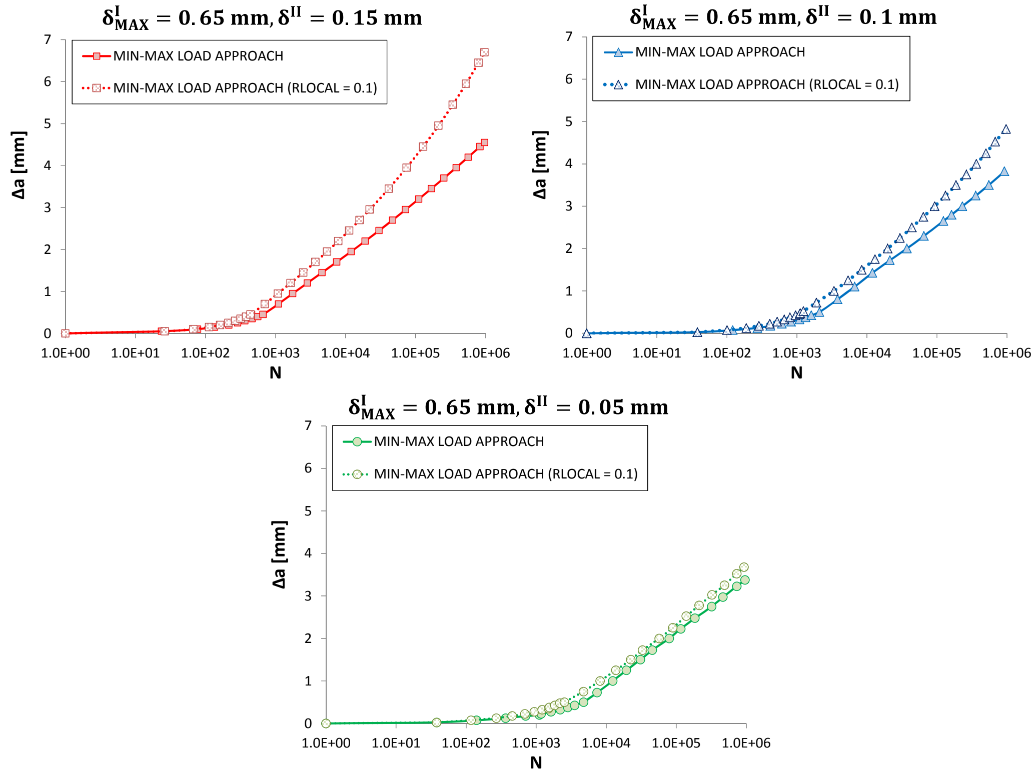

Fig. 15 Comparison with and without application of Min-Max Load Approach: modified MMB specimen.

It can be observed from Fig. 15 that, for all the performed analyses, the crack length predicted by using the actual value of the local stress ratio is always smaller than the one obtained by considering its value equal to the applied load ratio. Indeed, by adding a constant load component to both the minimum and maximum load configurations, the resulting local stress ratio will be always greater than the applied load ratio, leading to a reduction of the crack growth rate as predicted by the adopted delamination growth law presented in Eq. (8). Furthermore, from Fig. 15 it is evident that the difference between the results decreases when the mode II displacement is reduced because the actual value of the local stress ratio decreases and approaches to the applied load ratio.

VI. Conclusions

A new strategy for the simulation of fatigue delamination propagation, called "Min-Max Load Approach", is proposed. The new methodology, based on the CZM technique, is able to dynamically capture the local stress ratio during the evolution of the damage. A single simulation is performed with two models representing the same structure but with different applied loads. One model represents the deformed shape of the structure when the applied load is equal to the minimum value of the fatigue cycle, and the other one represents the deformed configuration of the structure at the maximum load. By using the minimum and the maximum values of the energy release rate taken from the two models, it is possible to evaluate the local stress ratio. The subroutine UMAT is adopted in ABAQUS to implement the cohesive damage, allowing the two models to communicate with each other exchanging information in terms of energy release rate and damage propagation. The methodology has been validated by performing analyses on DCB and MMB specimens. Then, the "Min-Max Load Approach" has been adopted to numerically investigate a specimen equal to the MMB but with modified loading conditions such as to produce a variable local stress ratio different from the applied load ratio. The results of the analyses indicate that the approach is able to correctly predict the fatigue damage propagation without introducing any information regarding the applied load ratio in the damage constitutive model. Indeed, the local stress ratio is calculated during the analysis allowing capturing any possible changes of its value along the delamination front and during the damage evolution.

The work will be extended to other situations where the local stress ratio changes during the damage evolution, such as the case of a structure subjected to fatigue compressive load in post-buckling regime.

Acknowledgments

This work was sponsored by the Office of Naval Research (ONR), under grant award number N62909-17-1-2129. The views and conclusions contained herein are those of the authors only and should not be interpreted as representing those of ONR, the U.S. Navy or the U.S. Government.

References

[1]Paris P.C., Gomez M.P., and Anderson W.E., "A Rational Analytic Theory of Fatigue," *The Trend in Engineering*, Vol. 13, No. 1, 1961, pp. 9-14.

[2]Bak B., Sarrado C., Turon A., and Costa J., "Delamination under Fatigue Loads in Composite Laminates: A Review on the Observed Phenomenology and Computational Methods," *Applied Mechanics Reviews*, Vol. 66, No. 6, 2014, pp. 1-24.

[3]Di Memmo I., and Bisagni C., "Fatigue Simulation for Damage Propagation in Composite Structures," Proceedings of *32nd Annual American Society for Composites Technical Conference*, October 22 - 25, 2017, Purdue University, West Lafayette (IN), USA, pp. 1009-1019.

[4]Krueger R., "Virtual Crack Closure Technique: History, Approach, and Applications," *Applied Mechanics Reviews*, Vol. 57, No. 2, 2004, pp. 109-143.

[5]Krueger R., "Development of a Benchmark Example for Delamination Fatigue Growth Prediction," NASA/CR-2010-216723, 2010.

[6]Krueger R., "Development and Application of Benchmark Examples for Mixed-Mode I/II Quasi-Static Delamination Propagation Predictions," Technical Report NASA/CR-2012-217562, 2012.

[7]Krueger R. and Carvalho N., "In Search of a Time Efficient Approach to Crack and Delamination Growth Predictions in Composites," Proceedings of *31st ASC Technical Conference*, Vol. 1, September 19 – 21, 2016, Williamsburg (VA), USA, pp. 368-383.

[8]Bak B.L.V., Turon A., Lindgaard E., and Lund E., "A Simulation Method for High-Cycle Fatigue-Driven Delamination using a Cohesive Zone Model," *International Journal of Numerical Methods in Engineering*, Vol. 106, 2016, pp. 163-191.

[9]Turon A., Costa J., Camanho P.P., and Dávila C.G., "Simulation of Delamination in Composites under High-Cycle Fatigue," *Composites Part A: Applied Science and Manufacturing*, Vol. 38, No. 11, 2007, pp. 2270-2282.

[10]Pirondi A., and Moroni F., "Simulation of Mixed-Mode I/II Fatigue Crack Propagation in Adhesive Joints with a Modified Cohesive Zone Model," *Journal of Adhesion Science and Technology*, Vol. 25, No. 18, 2011, pp. 2483-2499.

[11]Harper P.W., and Hallett S.R., "A Fatigue Degradation Law for Cohesive Interface Elements - Development and Application to Composite Materials," *International Journal of Fatigue*, Vol. 32, No. 11, 2010, pp. 1774-1787.

[12]Krueger R., Deobald L., Mabson G.E., Engelstad S., Rao M.P., Gurvich M., Seneviratne W., Perera S., O'Brien T.K., Murri G., Ratcliffe J., Dávila C.G., and De Carvalho N., "Guidelines for VCCT-Based Interlaminar Fatigue and Progressive Failure Finite Element Analysis," NASA/TM–2017-219663, 2017.

[13]Dávila C.G., and Bisagni C., "Fatigue Life and Damage Tolerance of Postbuckled Composite Stiffened Structures with Initial Delamination," *Composite Structures*, Vol. 161, 2017, pp. 73-84.

[14]Dávila C.G., and Bisagni C., "Fatigue Life and Damage Tolerance of Postbuckled Composite Stiffened Structures with Indentation Damage," *Journal of Composite Materials*, Vol. 52, No. 7, 2018, pp. 931-943.

[15]Simulia, 2017. Abaqus Analysis User's Guide.

[16]Williams J.G., "The Fracture Mechanics of Delamination Tests," *The Journal of Strain Analysis for Engineering Design*, Vol. 24, No. 4, 1989, pp. 207-214.

[17]Reeder, J.R., "Refinements to the Mixed-Mode Bending Test for Delamination Toughness," *Journal of Composites, Technology and Research*, Vol. 25, No. 4, 2003, pp. 1-5.

[18]Allegri G., Wisnom M.R., and Hallett S.R., "A New Semi-Empirical Law for Variable Stress-Ratio and Mixed-Mode Fatigue Delamination Growth," *Composites Part A: Applied Science and Manufacturing*, Vol. 48, 2013, pp. 192-200.

[19]Benzeggagh M.L. and Kenane M., "Measurement of Mixed-Mode Delamination Fracture Toughness of Unidirectional Glass/Epoxy Composites with Mixed-Mode Bending Apparatus," *Composites Science and Technology*, Vol. 56, 1996, pp. 439-449.