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On Centrality-Related Disaster Vulnerability of Network Regions

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Abstract—Networks are typically embedded in non-homogeneous areas and different parts/regions of the network may therefore be at risk from different types of disasters. This non-homogeneity leads to difficulties in protecting the network against (the risk of) disasters. Network operators need to be able to integrate predictions on possible future disaster events in the planning of their network operation. Especially the (future) availability of network links is crucial in configuring network connections, since the requested availability of network connections is stipulated in Service Level Agreements and must be satisfied, even under the threat of disasters. In this paper, we propose (1) a novel model to characterize disaster areas, with occurrences of each type of disaster represented by a temporal distribution (e.g., Poisson process), and (2) two metrics, namely a betweenness-centrality metric for network regions and an impact metric that indicates the magnitude of the threat posed by disasters within a network region during a given time period.

I. INTRODUCTION

Modern telecommunication networks deliver a multitude of high-speed network services via connections that are configured over optical networks. The availability of a connection depends on the availability of the links constituting its assigned end-to-end path. Links can fail in the event of natural disasters [1] (e.g., earthquakes, hurricanes or tsunamis) or anthropogenic disasters [2] (e.g., anchor drags/drops, sabotage or terrorist attacks). Such disasters often affect different network regions differently, be it in terms of impact level, probability of occurrence, or the size of its area-of-effect. In the absence of proper service compensating measures, link failures degrade and interrupt connections, possibly leading to monetary penalties to network operators due to breached Service Level Agreements (SLAs), where an SLA is a contract between the network operator and a client. Even a single link failure can be disastrous, since the failure may trigger the failure of multiple connections that each carry a huge amount of data. It is estimated that losses due to service downtime can range between \$25,000 to \$150,000 per hour [3].

Different network regions tend to be at risk from different types of disasters, as illustrated in Fig. 1, since disaster events often occur in specific geographic areas. For instance, the West Coast of the United States is more vulnerable to earthquakes, while the East Coast of the United States is more vulnerable to hurricanes [4]. Similarly, ninety percent of the world’s earthquakes occur along the Pacific Ring of Fire [5]. Hence, network operators must utilize any spatial information on

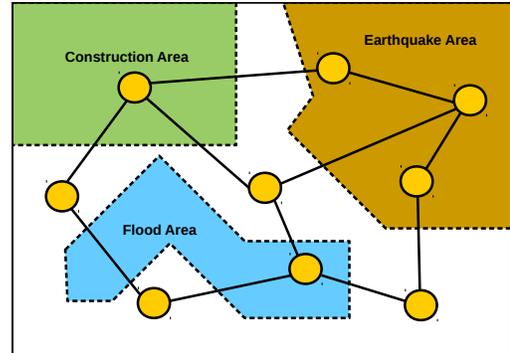


Fig. 1. Example of different disaster vulnerabilities.

their network regions in ensuring that their connections are as reliable as possible. A higher level of disaster resiliency should be provided to network regions with higher disaster risks, e.g., a higher level of duct armoring for submarine cables that cross areas where sharks bites are prominent. Different network regions also tend to have different importance to the network operation, based on the centrality of the network region. The concept of centrality [6] is often used in the field of *Network Science* to typify important nodes or links in the network, but the concept has not been extended to network regions. In the context of this paper, the betweenness centrality of a network region is defined and considered in terms of the number of shortest paths (between all nodes) that pass through the network region, via any links that cross the network region. Network regions with higher centrality value should be provided with more disaster resiliency measures compared to network regions with lower centrality value.

Our main contributions in this paper are organized as follows. In Section II, we develop a model for representing disaster areas, with the occurrences of each disaster type characterized by a probability distribution (we use the Poisson distribution, but the principle extends to other distributions as well), and we formulate the computation of link availability within a time period, based on the temporal distribution of possible disaster events. In Section III, we propose two metrics, namely (1) a regional betweenness centrality metric to represent the importance of a network region in terms of the number of shortest paths (between all nodes) that pass through the network region, and (2) a disaster-risk impact metric to

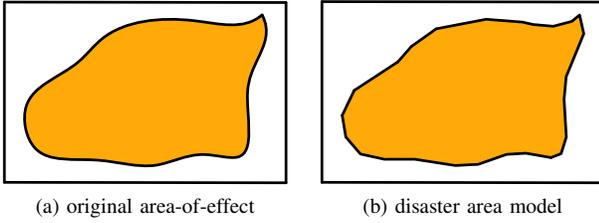


Fig. 2. Approximating the boundary of a disaster area.

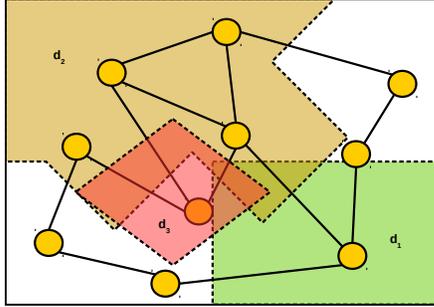


Fig. 3. Overlapping disaster areas.

reflect the magnitude of the threat posed by disasters within a network region. We analyze the effect of link density and disaster area size on the betweenness centrality of network regions in Section IV, and conclude the paper in Section V.

II. MODELING OF DISASTER AREAS

A disaster area implies a subset of the network area, in which all of the links (and consequently nodes) that cross the area will likely fail in the event of the disaster. Note that we model the area that is prone to a certain type of disasters (e.g., the Pacific Ring of Fire) and not the exact shape of the area that will be destroyed by a single pending disaster, which is nearly impossible to predict. Unlike in [7], [8], where the disaster area is considered bounded by a confined shape, we model the boundary of the disaster area using a non-unilateral simple closed polygon within the two-dimensional Cartesian plane. In the case of an irregularly shaped area, the accuracy of the polygon in representing the area often increases with more line segments defining the polygon. Fig. 2 shows an example of a disaster area that is represented by our model.

Predicting disaster events is happening frequently already, e.g. think of tsunami early warning systems, extreme weather predictions, or intelligence from national security agencies. Utilizing such disaster prediction data is possible via an integrated network-warning system, as proposed in [9]. Some future disaster events can also be estimated based on the studies of earlier disaster events. We consider that the occurrence of a disaster within a disaster area d_i can be characterized by a Poisson process $Poisson(\lambda_{d_i})$, where the rate $\lambda_{d_i} > 0$ of disaster occurrences depends on the nature of the disaster (other probability distributions could be used as well). For instance, a flood may occur once every 100 years on average for a given area d_i . While different manifestations of a type of

a disaster in a certain disaster area (e.g., floods in a flood-prone area) may be different, in the planning process a worst-case scenario is assumed, where any manifestation of a type of disaster would take out the complete corresponding disaster area. The Poisson process has long been used to characterize the occurrence of various types of disasters, such as hurricanes [10] and earthquakes [11], [12]. We assume that in the event of a disaster, all the links that cross the disaster area will fail.

We consider that disaster events occur independently of one another and that a disaster area corresponds to a single type of disaster. d_i can therefore refer to both the (possibly scattered [13]) disaster area as well as the type of disaster. A specific network region may be vulnerable to multiple types of disasters, which means that disaster areas may overlap (as shown in Fig. 3). The Poisson process $X(t)$ representing the failure event of a link $(u, v) \in L$, due to the occurrence of at least one type of disaster $d_i \in D'$ (where $D' \subseteq D$ is the set of disasters that overlap with link (u, v)), is

$$X(t) = Poisson\left(\sum_{d_i \in D'} (\lambda_{d_i})\right) \quad (1)$$

Disaster-Aware Link Availability (DALA) problem: Given are a network G comprising a set N of $|N|$ nodes and a set L of $|L|$ links, a set D of $|D|$ types of disasters that can occur within the geographic plane in which G is embedded, and a time period τ . Each type of disaster $d_i \in D$ is characterized by a Poisson process $Poisson(\lambda_{d_i})$. Each link $(u, v) \in L$ connects nodes u and v , and crosses the disaster areas within the set $D'_{uv} \subseteq D$. Compute the availability of each link $(u, v) \in L$ within the time period τ , with respect to $D'_{uv} \subseteq D$.

We propose Algorithm 1 for finding the set of disaster areas $D'_{uv} \subseteq D$ that overlap with link (u, v) . Link (u, v) fails in the event of any disaster in $D'_{uv} \subseteq D$. The algorithm utilizes the R-tree (a depth-balanced data structure for organizing objects using bounded rectangles) [14] and minimum bounding rectangles (the smallest rectangle that encloses an area) for eliminating the need of naively checking whether each link and each disaster area are overlapping (thus leading to significant savings of running time, as in [15]). The probability that link (u, v) fails ($Pr_{uv}(\tau)$) within a time period τ is equal to the probability that a least one type of disaster $d_i \in D'_{uv}$ occurs within the time period τ . The selection of τ is particularly important in planning the network ahead, e.g., a 5-10-years plan. The probability that at least one type of disaster $d_i \in D'_{uv}$ will be observed during the specific time period τ can be described as $Pr[X(\tau) \geq 1]$. Hence, the probability that link (u, v) will fail at least once during time period τ is

$$\begin{aligned} Pr_{uv}(\tau) &= Pr[X(\tau) \geq 1] \\ &= 1 - e^{-\left(\sum_{d_i \in D'_{uv}} (\lambda_{d_i})\right)\tau} \end{aligned} \quad (2)$$

$$A_{uv}(\tau) = 1 - Pr_{uv}(\tau) \quad (3)$$

By computing the corresponding $Pr_{uv}(\tau)$ for each link $(u, v) \in L$, we can assign each link with an availability value

$A_{uv}(\tau)$ (a value between zero to one), which represents the probability that link (u, v) is free from the impact of disasters during the time period (τ) . The information of link availability is useful for computing availability-based shortest paths [16].

III. RISK ASSESSMENT OF NETWORK REGIONS

In this section, we generalize the concept of betweenness centrality [6] as a metric for the betweenness centrality of a network region, and also propose another metric referred to as the *risk impact* (R) to assess the probabilistic impact of disaster events within the network region to the network operation as a whole, with respect to betweenness centrality of the network region and the temporal distribution of the disasters overlapping with the network region.

The betweenness centrality of a network region will represent the importance of the region in the network operation. We consider the betweenness centrality of a network region in terms of the number of shortest paths (between all nodes) that pass through the region, via any links that cross the region. The failure of a network region with higher betweenness centrality can thus have a more adverse effect on the network operation, compared to network regions with lower betweenness centrality. By being able to compare network regions of different shapes and sizes in terms of their betweenness centrality, network regions with higher centrality value can be provided with higher disaster resiliency measures, compared to network regions with lower centrality value. Considering centrality of network regions is more useful when vulnerability is combated in terms of regions, instead of based on the node/link centrality as conducted in [17]. Since protection costs may vary (e.g., due to municipality costs, or proximity from land for undersea links), it might be important to focus on protecting higher-valued network regions.

Betweenness Centrality of Network Region (BCNR) problem: Given are a network G of a set N of $|N|$ nodes and a set L of $|L|$ links, and a network region r that is embedded within the geographic plane of G . Each link $(u, v) \in L$ connects nodes u and v , and a set of links $L' \subseteq L$ crosses network region r . Find the betweenness centrality of the network region r .

We use the notion of shortest path loosely in this paper, i.e., the shortest path can be used to represent the least-distance path, the path with the minimal energy consumption, the most reliable path, and many more depending on the defined link weight. Hence, the interpretation of the network region betweenness will vary depending on the defined link weight. Nevertheless, different network regions will have different network importance based on their betweenness centrality values. The failure of links within a network region with high betweenness centrality will significantly and negatively affect the network operation, since a high number of shortest paths will be unavailable, thus increasing the lower bound of the best operational condition available to the network operators (since the replacement/backup path [16] is often suboptimal compared to the assigned initial/primary path).

Algorithm 1 Finding Disaster Areas Overlapping With Links

- 1: populate an R-tree Y with all the minimum bounding rectangles MBR_{d_i} of each disaster area $d_i \in D$
 - 2: **for** each link $(u, v) \in L$
 - 3: find the set $D'_{uv} \in Y$ that overlaps the minimum bounding rectangle MBR_{uv} of link (u, v)
 - 4: **for** each disaster area $d_i \in D'_{uv}$
 - 5: **if** link (u, v) does not overlap with disaster area d_i
 - 6: remove d_i from D'_{uv}
-

Algorithm 2 Finding Links Overlapping a Network Region

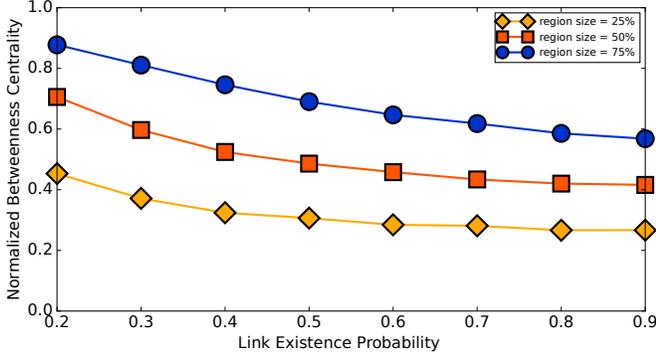
- 1: populate an R-tree Y with all the minimum bounding rectangles MBR_{uv} of each link $(u, v) \in L$
 - 2: find the set $O_r \in Y$ that overlaps with the minimum bounding rectangle MBR_r of network region r
 - 3: **for** each link $(u, v) \in O_r$
 - 4: **if** link (u, v) does not overlap network region r
 - 5: remove (u, v) from O_r
-

$$B_r = \sum_{s \in N, t \in N, s \neq t, s \notin O_r^n, t \notin O_r^n} \frac{\sigma_{st}(O_r)}{\sigma_{st}} \quad (4)$$

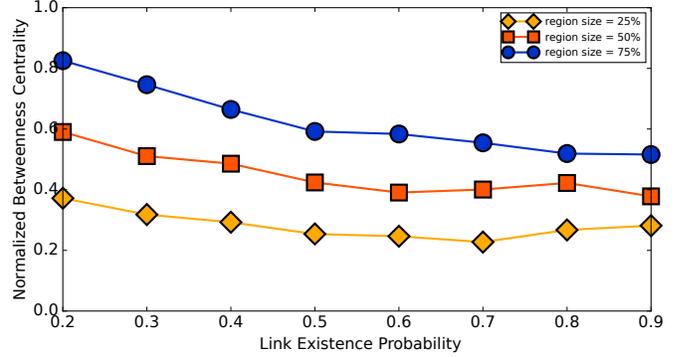
$$B'_r = \frac{2B_r}{(|N| - |O_r^n|)(|N| - |O_r^n| - 1)} \quad (5)$$

We compute the betweenness centrality B_r of a network region r using Equation 4, where the computation of B_r is derived analogously to the computation of group betweenness centrality [18], σ_{st} is the total number of shortest paths from node s to node t , O_r is the set of links that cross the network region r , $\sigma_{st}(O_r)$ is the total number of shortest paths from node s to node t that traverse any links that cross the network region r , and O_r^n is the set of nodes that reside within the network region r . We also ensure that neither node s nor node t is within the inspected network region during the computation, and that each undirected shortest path is only counted once in the computation. We find O_r using Algorithm 2. We can also compute the normalized betweenness centrality (B'_r) using Equation 5, such that its value lies between zero and one, by dividing the value by the total number of node pairs from the set of nodes that are not within the network region r .

While [8] finds the most vulnerable network region of predefined size, they assume that the risk profiles of all possible network regions are similar, while we argue that a small network region with a high centrality and high disaster occurrence rate can lead to worse impact on network operation than a larger region with a lower centrality and lower disaster occurrence rate. Different network regions have different risk impact, since beside the network region size, the risk also depends on the network region unavailability (due to disasters) and the number of shortest paths that traverse any links crossing the network region. [19] used the loss per unit time of the network operator as an indicator of the losses due in the event of a disaster, for a predefined set of network connections.

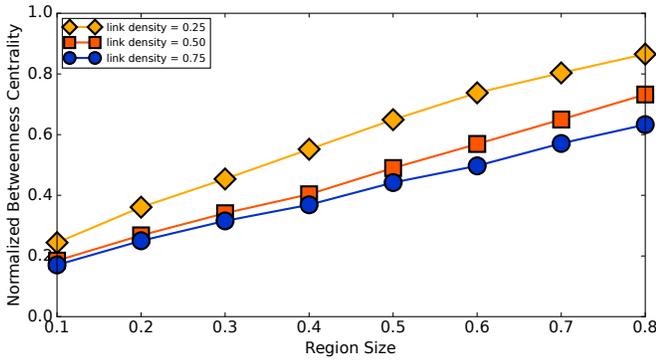


(a) Waxman topologies

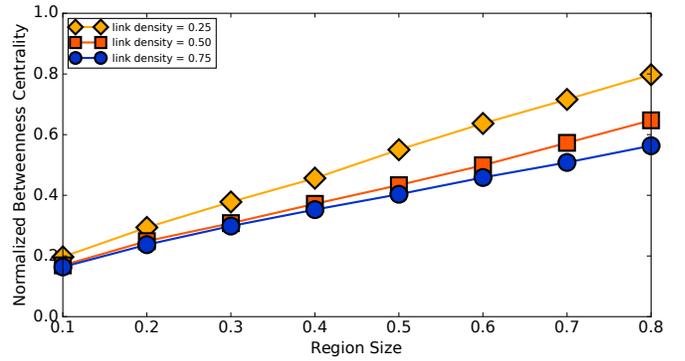


(b) Erdős-Rényi topologies

Fig. 4. Effect of the network link density on the betweenness centrality of network regions.



(a) Waxman topologies



(b) Erdős-Rényi topologies

Fig. 5. Effect of the region size on the betweenness centrality of network regions.

Risk Assessment of Network Region (RANR) problem: Given are a network G of a set N of $|N|$ nodes and a set L of $|L|$ links, a set D of $|D|$ types of disasters that can occur within the geographic plane into which G is embedded, a network region r that fully overlaps with a set of disasters $D' \subseteq D$, and a time period τ . Each type of disaster $d_i \in D$ is characterized by a Poisson process $Poisson(\lambda_{d_i})$. Each link $(u, v) \in L$ connects nodes u and v , and may cross the disaster areas within the set $D'_{uv} \subseteq D$. Compute the risk that the disasters D pose to the network region r within the time period τ .

$$R_r(\tau) = B_r P r_r(\tau) = B_r \left(1 - e^{-\left(\sum_{d_i \in D'} \lambda_{d_i} \right) \tau} \right) \quad (6)$$

We consider the disaster impact in terms of the number of affected shortest paths (via the regional betweenness centrality), such that our metric does not depend on the current/predicted traffic matrix utilized in the network, since these traffic matrices do change in time, while the links that compose the shortest paths between the network nodes remain the same as long as the network topology and link weights remain unchanged. The risk impact (R_r) of a network region r for a time period (τ) can be computed using Equation 6, where

$P r_r(\tau)$ is the probability that at least one type of disaster occurs during the time period τ within the network region r .

IV. ANALYSIS

We analyze the effect of the network link density and region size on the regional betweenness centrality for randomly generated Waxman [20] and Erdős-Rényi [21] topologies. The Waxman graph is frequently used for representing spatial networks, e.g. optical networks [22], due to its unique property of decaying link existence over distance. In both topologies, $|N| = 50$ nodes are placed uniformly at random coordinates in the network area. In our scenario, the link weight corresponds to the Euclidean distance between each of its adjacent nodes (other weights may also be used, as discussed earlier in Section III). In the Waxman topology, the link existence is reflected by $i e^{-\frac{\ell_{uv}}{j a}}$, where ℓ_{uv} is the Euclidean distance between nodes u and v , and a is the maximum Euclidean distance between any nodes. Higher i leads to higher link densities, and lower j leads to shorter links. We set j to 0.5. We consider only connected graphs, such that there is at least one path between each node. No self-loops or parallel links are allowed. We generate rectangular regions of predefined size for our simulation (other area types may also be used, as discussed earlier in Section II). Simulations were conducted on an Intel(R) Core i7-4600U

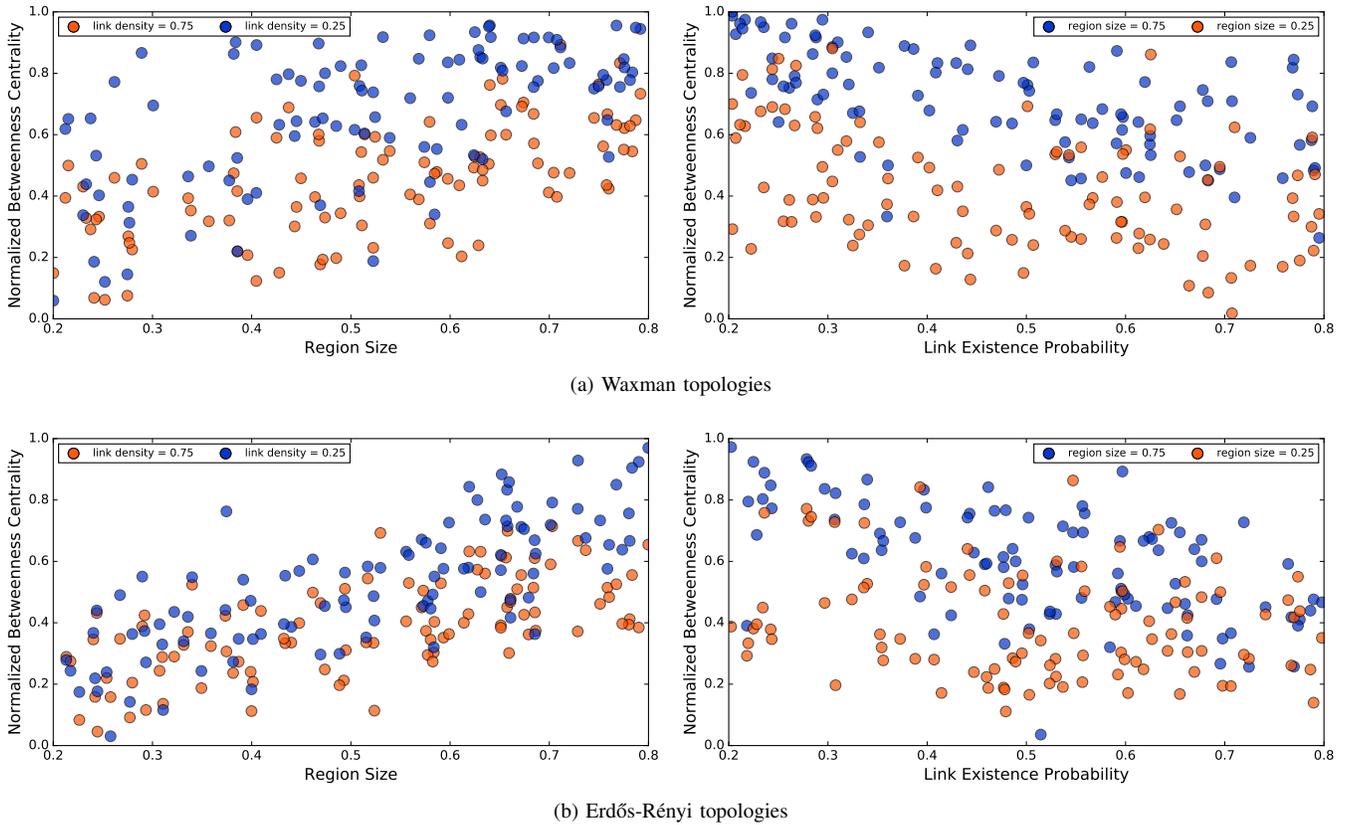


Fig. 6. Case-by-case observations of the betweenness centrality of network regions.

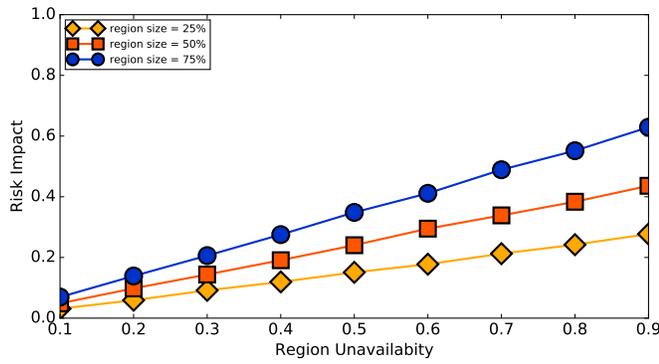
2.1GHz machine of 16GB RAM memory. Simulation results were averaged over a thousand runs.

As illustrated in Fig. 4, the normalized betweenness centrality of network regions with fixed size decreases, for both Waxman and Erdős-Rényi topologies, as the network link density increases. In this instance, we vary parameter i of the Waxman topologies to get the corresponding link existence probability. When the link density is small, the chance that a network region will encompass links that function as part of the shortest paths is higher. However, as the link density increases the decrease does not become so apparent anymore, since the number of links is high. The same trend is observed under different network region sizes, where Fig. 5 shows that the normalized betweenness centrality of the network regions increases, for both Waxman and Erdős-Rényi topologies, with the increase of the network region size. When the size of the network region is larger, more links and shortest paths will cross the network region. The occurrence of a disaster event within the network region will then affect all the links within the network region. The same trend is observed under different network link densities, although, as shown earlier in Fig. 4, an increase in link density decreases the normalized betweenness centrality of the network regions.

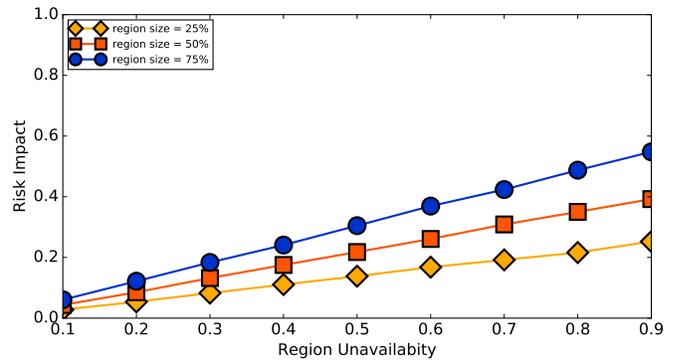
Based on these observations, unlike conventional node/link betweenness centrality that varies based on the network link density, both network link density and the region size play a role in determining the regional betweenness centrality. Fig. 6

shows the normalized betweenness centrality of a hundred randomly positioned network regions of various size for network link density of 0.25 and 0.75. In this sense, on a case-by-case basis, a larger network region may not necessarily have larger betweenness centrality value compared to a smaller network region, nor do network regions in a network with smaller link density necessarily have higher centrality value compared to network regions in a network with higher link density. The network link density, region size and position of the network region are important in determining the regional betweenness centrality. Hence, in some cases, it might be more beneficial for the network operator to increase the robustness of a smaller network region against the threat of disaster occurrences, since the region carries a more vital role to the network operations compared to other larger, but less important, network regions.

From Fig. 7, the risk impact of network regions increases, for both Waxman and Erdős-Rényi topologies, as the region unavailability increases (where the unavailability of network regions can be derived from the occurrence distribution of disasters that overlap the network region). The unavailability of a network region implies that at least a single disaster occurs within the time period in the network region. The same trend is observed under different network region sizes. However, as mentioned earlier, on a case-to-case basis, the network density and position of the network region also play a role in determining the risk impact of a network region.



(a) Waxman topologies



(b) Erdős-Rényi topologies

Fig. 7. Effect of the region unavailability on the risk impact of network regions.

V. CONCLUSION

We have proposed a model for representing the area-of-effect of disasters with different shapes and sizes, with each type of disaster characterized by a probability distribution over a certain disaster area. We subsequently computed the link availability within a time period with respect to the frequency distribution of the possible disasters. We have also proposed metrics referred to as region betweenness centrality and the region risk impact. The latter indicates the probabilistic failure impact of a network region on the whole network operation, due to possible disaster events within the network region, with respect to the number of expected affected shortest paths and the temporal distribution of the disasters. We also show that, on a case-by-case basis, a larger network region may not necessarily have larger betweenness centrality value compared to a smaller network region. Nor do network regions in a network with smaller link density necessarily have higher betweenness centrality value compared to network regions in a network with higher link density. For future work, our framework for modeling network regions can also be extended to cover different temporal distributions. Instead of considering disasters as independent to each other, future work could also consider dependent disasters.

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