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DOI
10.1109/PTC.2019.8810916

Publication date
2019

Document Version
Final published version

Published in
2019 IEEE Milan PowerTech

Citation (APA)
Markensteijn, A., & Vuik, K. (2019). On the Solvability of Steady-State Load Flow Problems for Multi-Carrier Energy Systems. In *2019 IEEE Milan PowerTech* (pp. 1-6). Article 8810916 IEEE. https://doi.org/10.1109/PTC.2019.8810916

On the Solvability of Steady-State Load Flow Problems for Multi-Carrier Energy Systems

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***Abstract*—The coupling of single-carrier network into multi-carrier energy systems (MES) has recently become more important. Conventional single-carrier steady-state load flow models are not able to capture the full extent of the coupling. Different models for multi-carrier networks have been proposed, either based on the energy hub concept or using a case specific approach. However, the effect of the coupling on solvability and well-posedness of the integrated system of non-linear equations has not been discussed. Using a general load flow model on a small example MES, this paper discusses the problems arising due the coupling of single-carrier networks, and provides guidelines to obtain a solvable steady-state load flow model for MES.**

***Index Terms*—Integrated energy systems, Load flow analysis, Multi-carrier energy networks, Natural gas, Power flow analysis**

I. Introduction

In recent years, multi-carrier energy systems (MES) have become more important, as they are considered to have better performance compared with the classical energy systems [1]. In MES, different energy carriers, such as electricity, heat, and natural gas, interact with each other to form one combined energy system. An important tool for designing and operating energy systems is steady-state load flow analysis of the energy transmission or distribution networks. Conventional load flow models developed for single-carrier (SC) networks are not able to fully capture the effect of integrating different networks into one multi-carrier network. Recently, different load flow models have been proposed for MES, either based on the energy hub concept, or using a more case specific approach.

The energy hub concept was first introduced in [2], and models the relation between input and output energies of different carriers. Unidirectional flow from input to output is assumed, and within the energy hub, transmission of the energy carriers is not taken into account. The extension of the energy hub concept to allow for bidirectional flow, and a detailed representation of energy flow within the energy hub, is provided in [3], [4]. However, the connection of the energy hub to the SC networks is not discussed. Explicit modeling of both the energy hub and the SC network is studied in [5].

This research received funding from the Netherlands Organisation for Scientific Research (NWO), Alliander N.V., and Amsterdam Institute for Advanced Metropolitan Solutions (AMS).

However, they do not describe the representation of the energy hub as a network element.

The second type of multi-carrier load flow models combines the load flow equations of the SC networks into one integrated system of equations. Such a model is given for a combined gas and electricity network in [6], [7], for electricity and heat in [8], [9], and for electricity, gas, and heat in [10]–[12]. However, since these models are case specific, they are difficult to apply to a general MES.

To the best of the authors knowledge, the currently available models for MES do not discuss the effect of the coupling on the solvability and well-posedness of the resulting integrated system of non-linear steady-state load flow equations. Usually, the coupling models introduce more unknowns than equations, such that additional equations or boundary conditions are needed for the system to be solvable. We analyze the effect of coupling on the integrated system of equations, using a general graph-based load flow model on a small example MES. The effect of the additional boundary conditions on the well-posedness of the load flow problem is analyzed by solving the system of non-linear equations using the Newton-Raphson method.

II. Network Representation

A. Terms and Definitions

Energy systems are mathematically represented by a network or graph. A graph $\mathcal{G}$ is a pair $((V),(E))$, where $\mathcal{V}$ is a set of nodes v_i and $\mathcal{E}$ is a set of links e_k. A link is a set of two nodes, such that $e_k = \{v_i, v_j\}$, or an ordered pair of nodes $e_k = (v_i, v_j)$. If all links in $\mathcal{E}$ are ordered, the graph is directed, if none of the links are ordered, the graph is undirected. Flow can enter the network through sources, and leave the network through sinks or loads. Both are represented by terminal nodes and terminal links. A terminal link is a link that is only connected to one node, denoted by $t_l = \{v_i\}$, and is also called a half link. It is a representation of flow entering or leaving the network. By definition, a terminal link can only be connected to a terminal node, and, conversely, a node with a terminal link connected to it is called a terminal node. One node can have more terminal links connected to it.

Balanced ac power grids are represented by an undirected graph, while gas pipe networks and heat pipe networks are represented by directed graphs. The physical pipeline system

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TABLE I
VARIABLES FOR A GAS, HEAT, AND ELECTRICAL NETWORK.

Network	Node	Link	Terminal node
Gas	pressure p	flow q	injected flow q
Heat	head h supply temperature T^s return temperature T^r	flow m	injected flow m outflow temperature T^o heat power φ
Electricity	voltage V	current I	injected current I injected complex power S

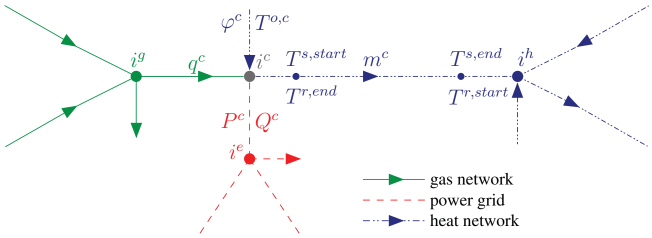


Fig. 1. Coupling node i^c, connected by dummy links to a gas node i^g, an electrical node i^e and a heat node i^h. The coupling variables are shown next to the (terminal) links they are associated with.

of a heat network consists of a supply line and a return line, connected to each other through heat sources and loads. We assume that the water flow in the return lines is opposite in direction, but equal in size, to the water flow in the return lines. The heat pipeline system is then represented as a directed graph, where the links represent pipelines in the supply line, outgoing terminal links represent heat loads, and incoming terminal links represent heat sources.

Variables are associated with the links and nodes. For basic steady-state load flow analysis, these variables, and the network element they are associated with, are given in Tab. I. Variables associated with terminal links are seen as nodal variables. To distinguish between (terminal) link and nodal variables, the nodal variables are called injected. If a node has more than one terminal link connected to it, the injected flow is the sum of all the flows of the terminal links.

B. Coupling of Single-Carrier Networks

We introduce the coupling node to combine SC networks into one multi-carrier network. The coupling node does not belong to any of the SC networks. If the coupling node is used to couple networks with the same energy carrier, it is called homogeneous. Similarly, it is called heterogeneous if it couples networks of different energy carriers. Nodes and links of a SC network are called homogeneous. A network is called homogeneous if it consists of only homogeneous nodes and links, and heterogeneous if at least one of the nodes in the network is heterogeneous. A heterogeneous coupling node can be connected to (terminal) links of any type. However, no variables are associated with a coupling node, so that some links cannot be connected to a coupling node. For instance, a link representing an electrical transmission line cannot be connected to a coupling node, since the coupling node does not have a voltage associated with it. Therefore, a coupling node is connected to any other node by a dummy link. They do not represent any physical component, they merely show a connection between nodes. If the dummy link connects a coupling node and a SC node, the dummy link is considered to be homogeneous and of the same carrier type as the SC node. As such, it has the same variables associated with it as any link of that carrier type. Fig. 1 shows the graph representation of a heterogeneous coupling node connected to a gas network, a power grid, and a heat network. The arrows show the direction of the heat and gas (terminal) links, not the actual direction of flow. Hence, the coupling node allows for bidirectional flow.

III. LOAD FLOW EQUATIONS

Conservation of energy holds for all homogeneous nodes in the SC networks. All SC links representing a physical element have a link equation that relates the link variables to the nodal variables. In this paper, we use the following models.

A. Electricity

The (non-dummy) links in the power grid represent short transmission lines. The active and reactive power for a short transmission line k from node i to node j are given by

$$\begin{aligned} P_{ij} &= g_{ij}|V_i|^2 - |V_i||V_j| \left(g_{ij}\cos\delta_{ij} + b_{ij}\sin\delta_{ij}\right) \\ Q_{ij} &= -b_{ij}|V_i|^2 - |V_i||V_j| \left(g_{ij}\sin\delta_{ij} - b_{ij}\cos\delta_{ij}\right) \end{aligned} \tag{1}$$

with P the active power, Q the reactive power, δ the voltage angle, $|V|$ the voltage amplitude, g_{ij} and b_{ij} the conductance and susceptance of the line, and $\delta_{ij} := \delta_i - \delta_j$. At every power node i, conservation of energy holds:

$$P_i = \sum_{j,\, j\neq i} P_{ij}, \quad Q_i = \sum_{j,\, j\neq i} Q_{ij} \tag{2}$$

with P_i and Q_i the injected active and reactive power.

B. Gas

At every gas node i, conservation of flow holds:

$$q_i = \sum_{j,\, j\neq i} q_{ij} \tag{3}$$

with q_i the injected gas flow, and q_{ij} the link gas flow. The non-dummy links k, from node i to node j, represent gas pipes with a steady-state pipe flow equation. Different models exists (see e.g. [13]), we use:

$$q_k = C_k \operatorname{sign}\left(p_i - p_j\right) \sqrt{|p_i - p_j|} \tag{4}$$

Here, q_k is the gas flow, p_i is the nodal pressure, and C_k is the pipe constant of pipe k.

C. Heat

At every heat node i, conservation of mass holds:

$$m_i = \sum_{j,\, j\neq i} m_{ij} \tag{5}$$

with m_i the injected water flow, and m_{ij} the link water flow in the supply line. The non-dummy links k, from node i to node j, represent pipes with a steady-state head loss equation [8]:

$$f_k^{\Delta h} = h_i - h_j - K_k |m_k| m_k = 0 \tag{6}$$

with h the nodal head, m the mass flow, and K_k the pipe constant. The head loss equation for every link in the supply line, combined with conservation of mass in every node, give the hydraulic model of the heat network. For the thermal model we use an exponential temperature drop for both supply and return pipelines:

$$T_k^{\text{end}} - \psi(m_k)\, T_k^{\text{start}} = 0, \quad \psi(m_k) := \exp\left(\frac{-\lambda_k L_k^h}{C_p |m_k|}\right) \tag{7}$$

where T_k^{end} and T_k^{start} are the temperatures at the end and the start of the pipe, λ_k is the heat transfer coefficient of the pipe, L_k^h the length of the pipe, and C_p is the specific heat of water. The start and end of a pipeline are defined for the actual direction of flow. Assuming only heat sinks or only heat sources can be connected to a single heat node, the heat power equation holds for every terminal link l connected to node i:

$$\varphi_{i,l} = \begin{cases} C_p m_{i,l}\left(T_i^s - T_{i,l}^o\right), & \text{if node } i \text{ is a sink} \\ C_p m_{i,l}\left(T_{i,l}^o - T_i^r\right), & \text{if node } i \text{ is a source} \end{cases} \tag{8}$$

T^s and T^r are the supply and return temperatures, and T^o the outflow temperature directly after the component (see e.g. [8]). At every node i, the supply and return temperatures are determined by the mixing rule, which is the weighted average of the inflow temperatures:

$$\begin{aligned} f_i^{T^s} &= \sum \left(m_{\text{out}}^s\right) T_i^s - \sum \left(m_{\text{in}}^s T_{\text{in}}^s\right) = 0 \\ f_i^{T^r} &= \sum \left(m_{\text{out}}^r\right) T_i^r - \sum \left(m_{\text{in}}^r T_{\text{in}}^r\right) = 0 \end{aligned} \tag{9}$$

$\sum m_{\text{out}}^s$ denotes the sum of all outgoing flows of node i in the supply line. Similarly, $\sum m_{\text{in}}^s$ denotes the sum of all ingoing flows of node i in the supply line, $\sum m_{\text{out}}^r$ the sum of all outgoing flows of node i in the return line, and $\sum m_{\text{in}}^r$ the sum of all ingoing flows of node i in the return line. It holds that $\sum m_{\text{in}}^r = \sum m_{\text{out}}^s$ and $\sum m_{\text{out}}^r = \sum m_{\text{in}}^s$.

D. Coupling

There are two main ways of modeling a MES as an integrated energy network. The first models all coupling components explicitly as a network involving heterogeneous coupling nodes, such that the SC networks are connected with each other through a coupling network. The second is based on the energy hub (EH) concept [2]. The SC networks are connected through the energy hubs, which are represented as single heterogeneous coupling nodes.

In this paper, we consider a combined heat and power plant (CHP) and a gas boiler (GB) as coupling components. For both we use a linear model, although the concept of the coupling node allows for more complex (non-linear) models:

$$\begin{aligned} \varphi_{\text{GB}} &= \eta_{\text{GB}} \text{GHV} q_{\text{GB}} \\ \varphi_{\text{CHP}} + P_{\text{CHP}} &= \eta_{\text{CHP}} \text{GHV} q_{\text{CHP}} \end{aligned} \tag{10}$$

Here, GHV is the gross-heating value of the gas, q_{GB} and q_{CHP} are the gas flows consumed by the gas boiler and CHP,

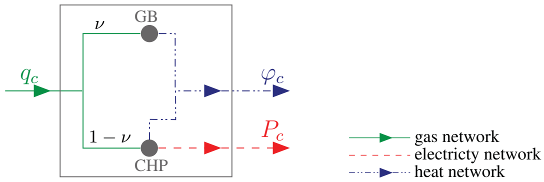


Fig. 2. Schematic representation of an energy hub, consisting of a gas boiler (GB) and a CHP.

η_{GB} and η_{CHP} are their efficiencies, and φ_{GB}, φ_{CHP}, and P_{CHP} the produced powers. Based on the schematic representation shown in Fig. 2, these two components can be modeled as a single EH, for which the coupling equations are

$$\begin{aligned} P_{\text{EH}} &= \mu(1-\nu)\eta_{\text{CHP}} \text{GHV} q_{\text{EH}} \\ \varphi_{\text{EH}} &= \nu \eta_{\text{GB}} + (1-\mu)(1-\nu)\eta_{\text{CHP}} \text{GHV} q_{\text{EH}} \end{aligned} \tag{11}$$

with q_{EH} the gas consumed by the energy hub, P_{EH} and φ_{EH} the produced powers, ν the factor of gas dispatched to the gas boiler, and μ the factor of gas converted to active power by the CHP. The heat produced by the gas boiler and the CHP are modeled as (terminal) half links. Since both are heat sources, the heat power equation (8) becomes

$$\begin{aligned} \varphi_{\text{GB}} &= C_p m_{\text{GB}} \left(T_{GB}^o - T_i^r\right) \\ \varphi_{\text{CHP}} &= C_p m_{\text{CHP}} \left(T_{CHP}^o - T_i^r\right) \end{aligned} \tag{12}$$

for a gas boiler and CHP connected to heat node i, and

$$\varphi_{\text{EH}} = C_p m_{\text{EH}} \left(T_{EH}^o - T_i^r\right) \tag{13}$$

for an energy hub connected to heat node i.

In total, the gas-boiler and the CHP introduce 4 equations ((10) and (12)) and 10 unknowns (q_{GB}, q_{CHP}, P_{CHP}, Q_{CHP}, m_{GB}, m_{CHP}, φ_{GB}, φ_{CHP}, T_{GB}^o, and T_{CHP}^o). The EH introduces 3 equations ((11) and (13)) and 6 unknowns (q_{EH}, P_{EH}, Q_{EH}, m_{EH}, φ_{EH}, and T_{EH}^o). The energy hub concept assumes the ratios ν and μ to be specified. Then, if one of the coupling powers q_{EH}, P_{EH}, or φ_{EH} is known, the other two can be determined from the coupling equations (11). This is not the case for the coupling network. For instance, if P_{CHP} is known, non of the other coupling energies can be determined solely from the coupling equations (10). Depending on the application, either the energy hub concept or a coupling network is preferable.

IV. Node Types

Typically, the load flow equations of the SC networks have more variables than equations. In that case, boundary conditions are imposed to reduce the number of variables by prescribing values for some (nodal) variables. Nodes are then classified according to which variables are specified. The standard node types for SC network are shown in Tab. II. When the SC networks are combined using coupling nodes, the coupling equations introduce more variables than equations to the total system. Thus, additional variables are then prescribed. One commonly used option is to prescribe one or more of the coupling energies (e.g. [2], [5], [10], [12]). However, this

TABLE II
STANDARD NODE TYPES FOR SINGLE-CARRIER NETWORKS.

Network	Node type	Specified	Unknown
Gas	reference	p	q
	load	q	p
Electricity	slack	$\lvert V\rvert$, δ	P, Q
	generator (PV)	P, $\lvert V\rvert$	Q, δ
	load (PQ)	P, Q	$\lvert V\rvert$, δ
Heat	source reference slack	T^s, h	T^r, T^o, φ, m
	source	T^o, φ	T^r, T^s, h, m
	load	T^o, φ	T^r, T^s, h, m
	junction	$m = 0$	T^r, T^s, h

effectively decouples the integrated network. If one or more of the coupling energies are known, the coupling equations can be used to directly determine (some of) the other energies. These energies, and the already prescribes ones, can then be used as boundary conditions for the SC networks. Therefore, we will assume all coupling energies unknown, and impose additional boundary conditions elsewhere in the SC networks.

The coupling energies can be seen as unknown injected flows or energies from the perspective of the SC networks. Imposing additional boundary conditions in the SC networks may lead to new node types [7], [14]. Consider for instance a power grid connected to a gas network through some generator. If the power node that is coupled was originally a load node, it can be seen as a generator with unknown active power from a technical perspective. Since the coupling power flows into the SC network through a dummy link, the node could be modeled as a generator node with known injected active and reactive power, called a PQV-node, which adds one additional boundary condition. The coupling equations (10) or (11) can be seen as boundary conditions for SC networks, for instance for the gas network. The additionally required boundary conditions then need to be imposed such that the other SC networks, for instance heat and power, are able to determine the remaining coupling energies. Imposing the boundary conditions in such a way leads to a solvable combined non-linear system of equations. For optimization purposes, the coupling parameters can be kept unknown, without imposing additional boundary conditions.

A. System of Equations

The load flow models of the SC networks can be combined with the coupling equations to form one integrated non-linear system of equations for the multi-carrier energy network. Since the coupling components are connected to the SC networks by dummy links, the coupling active and reactive powers, P_c and Q_c, are included in the nodal conservation of energy (2), the coupling gas flow q_c is included in conservation of flow (3), and the coupling water flow m_c in conservation of mass (5) and in the mixing rules (9).

Different formulations of the SC systems of equations exist. For power, we use the standard complex power formulation in polar coordinates (e.g. [15]). For every node with specified injected active or reactive power, conservation of energy is used to form the non-linear system of equations:

$$\mathbf{F^e} = \begin{pmatrix} \sum\limits_{\mathbf{j},\,\mathbf{j}\neq\mathbf{i}} \mathbf{P_{ij}} - \mathbf{P}^{\text{inj}} \\ \sum\limits_{\mathbf{j},\,\mathbf{j}\neq\mathbf{i}} \mathbf{Q_{ij}} - \mathbf{Q}^{\text{inj}} \end{pmatrix} = \mathbf{0}, \quad \mathbf{x^e} = \begin{pmatrix} \delta \\ |\mathbf{V}| \end{pmatrix} \tag{14}$$

$\mathbf{P}^{\text{inj}}$ and $\mathbf{Q}^{\text{inj}}$ are the vectors of known injected active and reactive power, and δ and $|\mathbf{V}|$ are the vectors of unknown nodal voltage angle and voltage amplitude.

For the gas network, we adopt the nodal formulation based on [13]. Collecting all equations for which the injected gas flow is known, the non-linear system of equations is given by

$$\mathbf{F^g} = A^{g\prime}\mathbf{q} - \mathbf{q}^{\text{inj}\prime} = \mathbf{0}, \quad \mathbf{x^g} = \mathbf{p} \tag{15}$$

with $\mathbf{p}$ the vector of unknown nodal pressures, $\mathbf{q}^{\text{inj}\prime}$ the reduced vector of known injected flows, $\mathbf{q}$ the vector of gas link flows, $A^{g\prime}$ the reduced incidence matrix, which entries are given by

$$A_{ik} = \begin{cases} 1, \text{ if } e_k = (v_j,\, v_i) \\ -1, \text{ if } e_k = (v_i,\, v_j) \\ 0, \text{ otherwise} \end{cases} \tag{16}$$

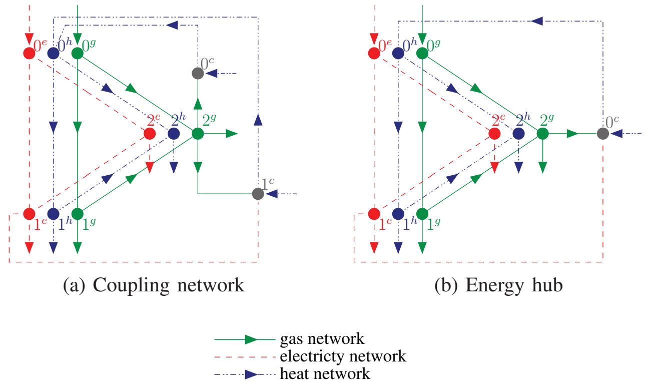
For the heat network, the hydraulic and thermal model can be combined into one hydraulic-thermal model by substituting the heat equations (8) in the head loss equation (6) and in nodal conservation of mass (5). The conservation of mass and the supply mixing rule are not taken into the system of equations for nodes that have a terminal link with unknown injected heat power. Furthermore, the outflow temperature T^o of every component is assumed known, such that T^o is not a part of $\mathbf{x^h}$. The non-linear system of equations for heat is then given by

$$\mathbf{F^h} = \begin{pmatrix} A^{h\prime}\mathbf{m} - \mathbf{m}^{\text{inj}\prime} \\ \mathbf{F^{\Delta h}} \\ \mathbf{F^{T^s}} \\ \mathbf{F^{T^r}} \end{pmatrix} = \mathbf{0}, \quad \mathbf{x^h} = \begin{pmatrix} \mathbf{m} \\ \mathbf{h} \\ \mathbf{T^s} \\ \mathbf{T^r} \end{pmatrix} \tag{17}$$

with $\mathbf{m}$ the vector of link mass flows, $\mathbf{h}$ the vector of unknown nodal heads, $\mathbf{T^s}$ and $\mathbf{T^r}$ the vectors of unknown supply and return temperatures, $\mathbf{F^{\Delta h}}$ the vector of head loss equations, $\mathbf{F^{T^s}}$ the vector of supply line mixing rules, $\mathbf{F^{T^r}}$ the vector of return line mixing rules, $A^{h\prime}$ the reduced incidence matrix, which entries are given by (16), and $m_i^{\text{inj}} = \sum\limits_{l} m_{i,l}$ where $m_{i,l}$ can be found from the heat power equation (8).

For the coupling part, if T^o of a (coupling) component is known, it is added as an equation $T^o - T^{o,\text{known}}$. Combining this with all the coupling equations, such as (10) or (11), and all heat power equations for the coupling components, such as (12) or (13) gives

$$\mathbf{F^c} = \begin{pmatrix} \mathbf{f^c} \\ \mathbf{F}^{\varphi} \\ \mathbf{T^o} - \mathbf{T}^{\mathbf{o},\text{known}} \end{pmatrix} = \mathbf{0}, \quad \mathbf{x^c} = \begin{pmatrix} \mathbf{q} \\ \mathbf{P} \\ \mathbf{Q} \\ \mathbf{m} \\ \varphi \\ \mathbf{T^o} \end{pmatrix} \tag{18}$$



(a) Coupling network (b) Energy hub

Fig. 3. Network topologies for the example MES, (a) connected by a gas-boiler 0^c and a CHP 1^c, (a) connected by an energy hub 0^c.

Here, $\mathbf{f^c}$ is the vector of coupling equations, $\mathbf{F}^\varphi$ is the vector of heat power equations, $\mathbf{T^o}$ and $\mathbf{T}^{\mathbf{o},\text{known}}$ are the vectors of unknown and known outflow temperatures, and $\mathbf{q}$, $\mathbf{P}$, $\mathbf{Q}$, $\mathbf{m}$, and φ are the vectors of the coupling gas flow, active power, reactive power, water flow, and heat power.

Combining the SC systems of equations (14), (15), and (17) with the coupling part (18) leads to an integrated system of equations:

$$\mathbf{F} = \begin{pmatrix} \mathbf{F^g} \\ \mathbf{F^e} \\ \mathbf{F^h} \\ \mathbf{F^c} \end{pmatrix} = \mathbf{0}, \quad \mathbf{x} = \begin{pmatrix} \mathbf{x^g} \\ \mathbf{x^e} \\ \mathbf{x^h} \\ \mathbf{x^c} \end{pmatrix} \tag{19}$$

B. Newton-Raphson

We use Newton-Raphson iteration (NR) to solve the non-linear system of equations (19). The iteration scheme for multiple dimensions is given by

$$\mathbf{x}^{k+1} = \mathbf{x}^k - J\left(\mathbf{x}^k\right)\mathbf{F}\left(\mathbf{x}^k\right) \tag{20}$$

with k the iteration number and $J\left(\mathbf{x}^k\right)$ the Jacobian matrix. Due to the choice for a (heterogeneous) coupling node connected to the SC networks by (homogeneous) dummy links, the Jacobian matrix of the integrated system of equation (19) has the following form:

$$J = \begin{pmatrix} J_{gg} & J_{ge} & J_{gh} & J_{gc} \\ J_{eg} & J_{ee} & J_{eh} & J_{ec} \\ J_{hg} & J_{he} & J_{hh} & J_{hc} \\ J_{cg} & J_{ce} & J_{ch} & J_{cc} \end{pmatrix} = \begin{pmatrix} J_{gg} & 0 & 0 & J_{gc} \\ 0 & J_{ee} & 0 & J_{ec} \\ 0 & 0 & J_{hh} & J_{hc} \\ 0 & 0 & J_{ch} & J_{cc} \end{pmatrix} \tag{21}$$

where the submatrices are defined as

$$J_{\alpha\beta} = \frac{\partial \mathbf{F}^\alpha}{\partial \mathbf{x}^\beta}, \quad \alpha, \beta \in \{g, e, h, c\} \tag{22}$$

Since the required additional boundary conditions are not imposed in the coupling part, the submatrices will in general not be square. J_{cc} will have more columns than rows, whereas J_{gg}, J_{ee}, and J_{hh} will have more rows than columns.

TABLE III
NODE TYPE SETS FOR THE EXAMPLE MES

	set 1		set2		set 1 EH		set 2 EH	
Node	***Type***	***Specified***	***Type***	***Specified***	***Type***	***Specified***	***Type***	***Specified***
0^g	ref.	p	ref.	p	ref.	p	ref.	p
1^g	load	q	load	q	load	q	load	q
2^g	load	q	ref. load	p, q	load	q	ref. load	p, q
0^e	slack	$\lvert V\rvert$, δ	slack	$\lvert V\rvert$, δ	slack	$\lvert V\rvert$, δ	slack	$\lvert V\rvert$, δ
1^e	PQVδ	P, Q, $\lvert V\rvert$, δ	PQV	P, Q, $\lvert V\rvert$	PQV	P, Q, $\lvert V\rvert$	PQV	P, Q, $\lvert V\rvert$
2^e	load	P, Q	load	P, Q	load	P, Q	load	P, Q
0^h	ref. temp.	T^s, h	ref. temp.	T^s, h	ref.	h	junction	$m = 0$
1^h	load	T^o, φ	load	T^o, φ	load	T^o, φ	load ref. slack	T^r, h
2^h	load	T^o, φ	load	T^o, φ	load	T^o, φ	load	T^o, φ
0^c	temp	T^o	temp	T^o	temp	T^o	temp	T^o
1^c	temp	T^o	temp	T^o	-	-	-	-

V. EXAMPLE NETWORKS

To illustrate the effect of coupling on the total system, we consider the small MES as shown in Fig. 3. To show the effect of different coupling equations, we couple the SC networks by using a gas-boiler and a CHP (Fig. 3a), or by an energy hub (Fig. 3b). Using the energy hub as shown in Fig. 2 means that these two networks model the same MES. The SC networks consists of three nodes, all connected to each other. The gas network and power grid have an external source, connected at node 0^g and 0^e respectively. The heat network has no external source; all heat is provided by the gas network. The networks are coupled at node 2^g of the gas network, 1^e of the power grid, and 0^h of the heat network.

The loop created between nodes 2^g and 0^h in the first network (Fig. 3a) causes some difficulties. If only the total amount of gas consumed and total amount of heat provided by the coupling components is known, it leaves infinitely many options to distribute those energy flows over the gas boiler and the CHP. For the first network, the node types must be chosen such that either both gas flows, or both heat flows, can be determined uniquely. Since none of the coupling flows are specified, this is impossible in the gas network. In the heat network, it is possible if both outflow temperatures are specified such that $T_{GB}^{o,\text{known}} \neq T_{CHP}^{o,\text{known}}$, and if the supply temperature in node 0^h is specified. If a heat power equation without outflow temperature is used (e.g. [5]), this would not be possible.

This problem does not arise for the network with the energy hub (Fig. 3b), because the energy hub concept specifies both ratios ν and μ, and because there is no loop in the network. However, if for this network one of the coupling energies q_{EH}, P_{EH}, or φ_{EH} is determined by one of the SC networks the other two energies are known through the coupling equations (11). This effectively fixes those two energies as boundary conditions in the other two SC networks, limiting the allowable node types in those two SC networks.

Due to the differences in network topology between Fig. 3a and Fig. 3b, and in coupling equations (10) and (11), different node types are needed for the total system (19) to be solvable. Tab. III gives 2 sets for both networks for which the system is well posed. The first set has no additional boundary conditions in gas, while the second set has.

In the first example, with the coupling network, node types are chosen such that the heat network can determine the heat

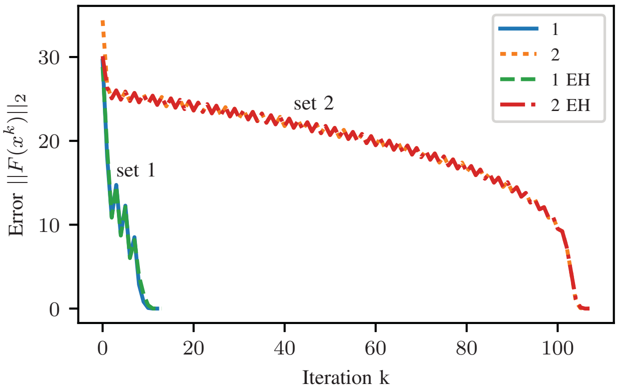


Fig. 4. Convergence plot of NR for both node sets of both example networks. The curves for node set 1 of both examples are indistinguishable at this scale, as are the curves for node set 2.

power flows. For the first node set, the nodes are chosen such that the power grid determines the active power required from the CHP. The coupling equations then determine the coupling gas flows. For the second node set, the nodes are chosen such that, given the heat flow produced by the gas boiler (node 0^c), the gas network can determined the gas flow supplied to the CHP. The coupling equations then determine the active power produced by the CHP.

For the example with the EH, the first node set is chosen such that the heat network determines the coupling heat power. The coupling equations then determine the coupling gas flow and active power. The second node set is chosen such that the gas network determines the coupling gas flow. The coupling equations then determine the coupling heat power and active power. Taking nodes 1^h and 2^h as load nodes, and assuming there is no external heat source, this leaves the heat network without a slack for the heat power, which could lead to an ill-posed problem. If $\varphi_{\text{EH}} \gg \varphi_{1^h} + \varphi_{2^h}$, the water mass flow in the pipes will become very small, that is $m_{ij} \approx 0$. Since all outflow temperatures and heat powers are specified, it follows from the heat power equations (8) and (13) that $T^r_{0^h} \ll T^o_{\text{EH}}$, $T^s_{1^h} \gg T^o_{1^h}$ and $T^s_{2^h} \gg T^o_{2^h}$. In this example, this leads to a numerically singular Jacobian matrix. To avoid this, a slack for the heat power must be introduced. One option is to make node 0^h a slack node. However, this would model a situation with an external heat source connected to node 0^h. Another option to take one of the load nodes 1^h or 2^h as slack nodes. Although this is not realistic, we choose the second option to show the effect of node types on convergence behavior. We use NR to solve the combined non-linear system of equations (19) for both node sets for both networks. Fig. 4 shows the convergence behavior of NR for all four examples. For all examples NR converges, that is $||\mathbf{F}\left(\mathbf{x}^k\right)||_2 \leq 10^{-6}$ for some iteration k. Both coupling methods show similar behavior; node set 1 converges faster than node set 2. This difference is due to the additional boundary condition in gas for the second node set. These examples show that the choice of node types can influence the convergence behavior of NR.

VI. Conclusion

The heterogeneous coupling node can be used as representation of a physical coupling between different energy carriers. It allows bidirectional flow, it can represent different physical coupling components, and it can be combined with different coupling models. Therefore, it extends and generalizes the currently available steady-state load flow models of MES.

Using the coupling node, we modeled a small example MES in two different ways, with a coupling network or with an energy hub. The chosen coupling model determines the topology of the multi-carrier network and the used coupling equations. This influences the possibilities for imposing the additionally required boundary conditions in the SC networks, and subsequently influences the integrated system of load flow equations. Choosing the wrong node types could lead to bad convergence behavior, or even to an ill-posed or unsolvable load flow problem.

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