



Topology optimisation of a concrete floor slab guided by manufacturability constraints from a vacuumatic formwork

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Abstract

This paper presents a design for a topology-optimised concrete floor slab, of which the structural optimisation process is guided by manufacturability constraints from a vacuumatic formwork. The design has been obtained using an open-source, three-dimensional topology optimisation algorithm. Traditional floor systems are cost-optimised but can have the disadvantage of being structurally inefficient. Topology optimisation allows for efficient material distribution, and thus a reduction in weight. Topology-optimised floors are typically regarded as being difficult to produce, however, and cost too much to be considered in building designs. In order to reach a compromise between a low self-weight and low production costs, two features are included in the optimisation process. First, manufacturability is directly incorporated in the optimisation, rather than afterwards. Secondly, the highly malleable vacuumatic formwork system by Huijben [7] has been used as a premise. Its advantages may cause the formwork costs to be reduced considerably when producing floor slabs.

Keywords: structural optimisation, topology optimisation, SIMP, vacuumatic formwork, concrete, floor slab

1. Introduction

Concrete floors are a common component of buildings and over the years, these building elements have been greatly optimised for cost instead of weight. For almost every span or support type, there is a floor system that can satisfy those requirements for a reasonable price. The majority of standard floor systems are flat plates because formwork takes up roughly sixty percent of the production cost in the US (Fanella and Alsamsam [6]). Other Western countries will show similarities in this cost distribution. The superfluous concrete that is used consequently is not enough to justify complex surfaces. However, with the recent increase in interest in sustainability and architectural value, there is potential in a topology-optimised floor system, provided there is additional funding.

Up to now, the focus of sustainable building has been on reducing energy consumption in heating, cooling, et cetera. While this has resulted in considerable improvements in energy consumption, a truly sustainable building should also take structural material usage into account. Material usage is becoming relatively more important as energy consumption is being reduced more and more. It can be expected that in the near future there will be stricter regulations on the environmental impact of material use, so a lighter topology-optimised floor can have a positive effect on the total life-cycle costs. Together with the additional aesthetic enhancements, this gives a building more value for its users.

Usually, the final design of a concrete floor slab in the construction stage differs from the topology-optimised structure because of its manufacturability. In theory, a structural topology-optimised floor is very efficient, but in practice, it can be too expensive to produce. Complex connections and irregular members cause the price to increase considerably. Therefore, what usually happens is that, after the structural topology optimisation of an element, the design is manually modified in order to be suitable for manufacturing. This implies that the element is designed in terms of manufacturability and not entirely optimised in terms of its structural performance anymore.

In order to reach a compromise between a low self-weight and low production costs, two features are included in the optimisation process in this paper. First, manufacturability is directly incorporated in the optimisation loop, rather than being applied as a modification afterwards. Secondly, the highly malleable vacuumatic formwork system by Huijben [7] has been used as a premise. Its advantages may cause the formwork costs to be reduced considerably when producing floor slabs. Figure 1 displays the three main components that make up this paper. A concrete floor has been topology-optimised with manufacturability constraints from a vacuumatic formwork.

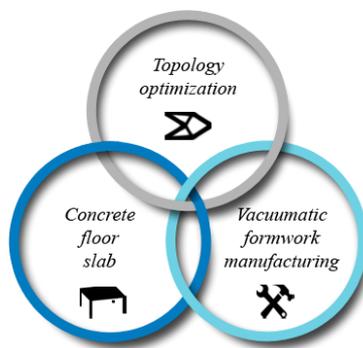


Figure 1: Three main components of this paper

2. Vacuumatic formwork

The field of digital design and structural optimisation is progressing quickly. However, making the design is only one part of the building process since manufacturability of said design is just as influential. For complex structures, a manufacturing method referred to as sand casting is often applied in the metal industry. It is characterised by using a mould made of sand which is stabilised with either clay, another bonding agent, or a vacuum. The latter is used as the chosen manufacturing method of the topology-optimised floor.

2.1 The V-process

Sand casting is the most accepted production method for both ferrous and nonferrous metals, accounting for roughly ninety percent of all castings (Ravi [12]). It is used to manufacture complex, small-scale elements, with a high degree of precision. Generally, the steps involved in producing a casting using sand include pre-casting, where the sand is prepared and compacted. The production of the master pattern can be done by any other manufacturing technique, and is often made of wood, metal, plastic, or expanded polystyrene foam. The pre-casting is followed by casting, where the metal is melted and poured into the mould. After cooling, the process finishes with post-casting. The stabilised sand is removed and the cast element is ready for inspection (Wang et al. [14]).

A casting method with which the sand is enclosed by a flexible membrane and stabilised by means of an internal under-pressure is referred to as the V-process (Nakata and Kubo [11]). The method was developed in Japan in 1969 and compared to the other sand casting techniques for metal, there are many advantages, according to Clark [5]. They include a zero degree draught, which reduces weight and machining, while still allowing the cast to be removed very easily from the mould. Thin walls, a

high surface finish, and tight tolerances reduce weight even further and create a product twice as accurate as typical sand castings. The unlimited pattern life and a very high return of sand are sustainable benefits.

2.2 Vacuumatic formwork for the building industry

The technique of applying vacuum stabilised sand for producing free-form structures in concrete is referred to as 'vacuumatic formwork' (Huijben [7]). It is the proposed construction method for the topology-optimised floor system. It combines the basic principles of the V-process method, with an application in the building industry. There are no practical examples yet of the vacuumatic formwork being used to cast complex reinforced concrete elements. It is assumed, however, that the advantages of the V-process translate well to a vacuumatic formwork. One major difference with the V-process is that for a floor, a one-sided mould is sufficient when the concrete is cast from the top.

The feasibility of using vacuumatic formwork for producing (potentially topology-optimised) ribbed structures in concrete with low-tech equipment has been illustrated by Huijben [8] during a workshop at last year's IASS symposium in Amsterdam. As feasibility was one of the goals of the workshop, accessible and simple tools were used to test the method. Timber planks for the outer mould, an EPS pattern, and a vacuum cleaner were all easy to come by. Highly elastic film, in this case, made from polyolefin copolymer and a fine-grained sand complemented the applied equipment. While using identical tools, all four panels in Figure 2 have a very different topology, showcasing the reusability, adaptability, and potential of the method.



Figure 2: Result of the vacuumatic formwork workshop

2.3 SWOT Analysis

By determining the strengths, weaknesses, opportunities, and threats (SWOT analysis) of the vacuumatic formwork in Figure 3, strategies can be formed by identifying relations between these four aspects. The required research that needs to be done might benefit from the increase in popularity of topology optimisation. This would then also help close the development gap with comparable digital production methods. The current high costs of labour might be intimidating, but the trend of the automation of the industry can be considered an opportunity. For example, in the future, robots might be able to cut, bend, and binding the reinforcement for a competitive price. The floor would be made in prefabricated parts off-site, after which they are shipped to the building site to be combined. From the restrictions on the size of a lorry, it makes sense to divide a floor into parts of less than 12.0×2.5 m². At this size, the vacuumatic formwork system should still be able to operate while allowing the workers in the factory to still place and bind the reinforcement.

2.4 Combining vacuumatics and topology optimisation

The strength of topology optimisation lies in placing the material where it is the most efficient, resulting in relatively lightweight structures with organic shapes. For traditional manufacturing methods, this means that many man-hours are required to produce the formwork. In Western countries, man-hours are the major component of production costs, meaning that topology-optimised designs are extremely expensive.

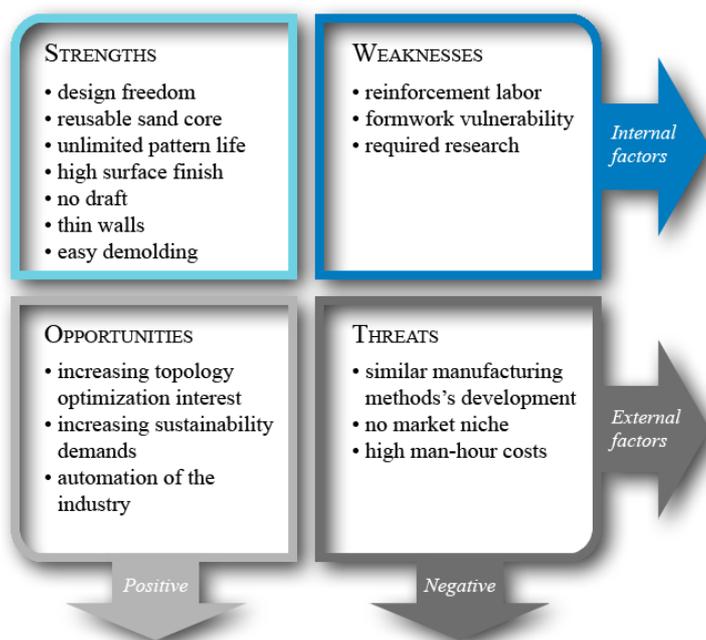


Figure 3: SWOT analysis

The costs of these highly organic shapes that follow from topology optimisation can be scaled down considerably by including manufacturability constraints inside the design process. The second solution to potentially decreasing production costs is the application of a vacuumatic formwork. The extent to which that is possible is not yet clear for this new production method, however. Further research is needed to reveal how a vacuumatic formwork influences the production costs. As displayed in Table 1, vacuumatic formwork strengthens the advantages of topology optimisation, while counteracting the disadvantages.

Table 1: Advantages and disadvantages of topology optimisation and vacuumatic formwork

Topology optimisation	Vacuumatic formwork
+ Sustainability increase	+ Sustainable manufacturing technique
+ Weight reduction	
+ Aesthetics increase	
– Complex shapes are difficult to manufacture	+ Capable of producing complex shapes that are easily demoulded
– Complex shapes are expensive to manufacture	+ Low-tech procedure that has the potential to be inexpensive

3. Methodology

For this paper, the combination of several requirements made the production of a new algorithm necessary, as their combination is not found in other software packages. The first requirement is the option of three-dimensional analysis. A simplification of the real scenario towards a two-dimensional analysis would not include load dispersion in two directions and thus, give a very unfavourable result. The second requirement is that the algorithm is available as open-source software. Closed-source software, such as Matlab, Rhinoceros, Altair, or Abaqus, is closed in a sense that they are closed to

anyone who doesn't pay the steep price, but also, they are often not transparent. Transparency is essential for adding manufacturability constraints, for example.

The Solid Isotropic Material with Penalization (SIMP) method by Bendsøe [2] is applied in the topology optimisation algorithm. The method does not remove elements that have a relatively low stress, but it changes their stiffness to such a small number the element practically has no use anymore. It is regarded as an efficient and effective method and has proven itself in a wide range of models. The input of the algorithm consists of all the factors that may affect the end result. In topology optimisation software this generally consists of the design space, volume fraction, load definition, boundary conditions, relative material properties, and, in this case, manufacturability constraints. The names of the input variables are very similar to the ones in the notable Matlab topology optimisation algorithms with the SIMP method by Sigmund [13], Andreassen et al. [1], and Liu and Tovar [10]. This makes comprehension of the code easier for users that are already familiar with those algorithms.

There are several assumptions made in the algorithm that need to be borne in mind. First, the design space that makes up the topology optimisation problem is a rectangular cuboid or box. Secondly, the elements that make up the design space are cubes and, as is common in topology optimisation, they have linear and isotropic properties. Both the shape of the design space and the elements simplify the numbering of the elements/nodes significantly in the Python script.

3.1 Optimisation objective

The Eurocode prescribes a maximum deflection of $0.004L$ in the Serviceability Limit State (SLS), where L represents the span of the floor. One common method of defining the total stiffness is by expressing it in a total strain energy, which is the external work done by the applied load. The total strain energy of the structure can be used to define the compliance. The compliance is the inverse of the total stiffness, so maximising the overall stiffness can be formulated as minimising the compliance in force-length. Additionally, compliance minimization is a common optimisation objective in topology optimisation literature. A minimum compliance problem with the SIMP method is defined by Equation (1) and (2). In Equation (1), the compliance is given by c , the force vector by f , and the displacement vector by u . This equation is subject to a volume fraction V^* , that is composed of the volumes of individual elements V_i multiplied with their relative densities x_i . The relative densities are anywhere between a minimum density x_{min} and 1.

$$\text{Minimise } c = f^T u \quad (1)$$

$$\text{Subject to } V^* - \sum_{i=1}^N V_i x_i = 0 \quad (2)$$

$$0 < x_{min} < x_i < 1$$

3.2 Design space and boundary conditions

The design space represents the space in which the optimal solution must be found. For the floor design, it will be shaped as a rectangular cuboid, because buildings and floors usually consist of rectilinear polygons. The span of the floor slab is assumed to be seven meters. The width of 2.5 m follows from the width of the comparable, traditional composite plank floor. The width of these prefabricated elements is adopted because of the width of a truck, so the floor slabs can easily be moved from the factory to the building site. The height of the design space is assumed to be 312 mm. For the boundary conditions, a slab supported by a beam or wall will be assumed. This translates to a hinged line support over two opposing edges (Figure 4). One line support is not allowed to translate in any direction, the other is allowed one degree of freedom in the z-direction to simulate a roll.

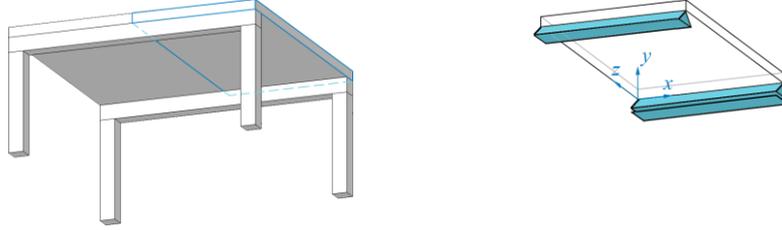


Figure 4: Context and model of the design space

3.3 Live load

The potential for development of a topology-optimised floor is the highest when its advantages are adequately exploited. A floor that can be produced with a complex shape, can withstand high loads, and has additional value from its aesthetics, benefits the most from a location where many people come together. Meeting areas in nonresidential building construction appear to be suitable for fulfilling this function. In that case, the load class will be based on class C3, according to Table 6.1 of NEN-EN 1991-1-1. This corresponds to an equally distributed load over the top surface, with a value of 5.0 kN/m² in the Serviceability Limit State.

3.4 Self-weight

Equation (3) displays that the self-weight $F_{i,sw}$ of a single element in the model is directly related to the relative stiffness x_i and specific weight ρ_0 of that corresponding unit. As Equation (4) implies, this means it differs slightly from the definition of the elemental stiffness E_i in which a penalization p is applied. Because of the penalty factor in the stiffness function, the solution of the topology optimisation already approaches a black-and-white structure. It is, therefore, unnecessary to also penalise the self-weight. Additionally, it prevents the algorithm from not achieving convergence.

$$F_{e,sw} = x_i \rho_0 \quad (3)$$

$$E_i = x_i^p E_0 \quad (4)$$

3.5 Material properties

In topology optimisation, the models generally consist of linear, isotropic elements, with equal compressive and tensile strength. In reality, concrete performs very well under compression but fails relatively quickly under tension. The addition of reinforcement bars results in a composite, inhomogeneous material that needs to be described differently from the elements traditionally used in topology optimisation. Therefore, the material definition has been changed to model concrete more accurately than regular topology optimisation algorithms.

In this paper, it is assumed that the floor slabs are prefabricated in a factory off-site. Prefab elements usually utilise a higher strength grade concrete than cast in-situ elements. The higher strength concrete hardens quicker, which means that they can be demoulded earlier and that more elements can be produced per day. Therefore, the relatively high concrete strength class C50/60 is considered for the floor slab design in this paper. It is advised to use a concrete mixture with self-compacting abilities, in order to prevent technicalities during casting with the vulnerable vacuumatic formwork.

The biggest difference with regular topology optimisation can be found in the addition of the cracked Young's modulus. When an element is under tension, it is considered to be cracked. As a result, its maximum Young's modulus is reduced to a third.

Table 2: Assumed properties for C50/60 in the algorithm

Material property			Unit
Uncracked Young's modulus	E_{cm}	37,000.00	MPa
Cracked Young's modulus	E_{ctm}	12,000.00	MPa
Compressive strength	f_{ck}	N/A	MPa
Tensile strength	$f_{ctk;0.05}$	N/A	MPa
Specific weight	γ_c	24.00	kN/m ³
Poisson's ratio	ν	0.20	–

3.6 Minimum concrete thickness

Having a minimum thickness of the concrete is essential because the result of a topology optimisation can have very thin members. These thin members may perform very well in creating a stiff structure, which the algorithm optimises for, but they can also be prone to breakage in practice. A minimum thickness constraint provided by a filter prevents this problem. A filter has two functions: it prevents checker boarding and it assigns a minimum thickness to the model. The density filter is often used in topology optimisation and was proposed by Bourdin [3] and Bruns and Tortorelli [4].

3.7 Flat top surface

This constraint follows from functional requirements in the utilisation stage. A floor without a flat top surface would not fulfill its function very well. As a consequence, the top of the design space needs to be flat. This can be accomplished by prohibiting the top layer(s) of elements from taking an 'empty' value. These are then called 'active elements'. That way, they are always present in each iteration.

3.8 Casting constraint

The casting constraint places limits on how material is distributed in the design space. During construction, it makes sense for a floor to have the vacuumatic formwork on the bottom and to pour the concrete from the top. It would then be less than ideal to either have internal voids in the structure, around which the concrete has trouble flowing or to have undercuts, which trouble demoulding. These problems result in a so-called casting constraint, defined in Equation 5 and visualised in Figure 5. In this definition, an element has its density updated whenever there is an element below it with a larger density.

$$x_i \geq x_{i+1} \geq x_{i+2} \geq \dots \geq x_n \quad (5)$$

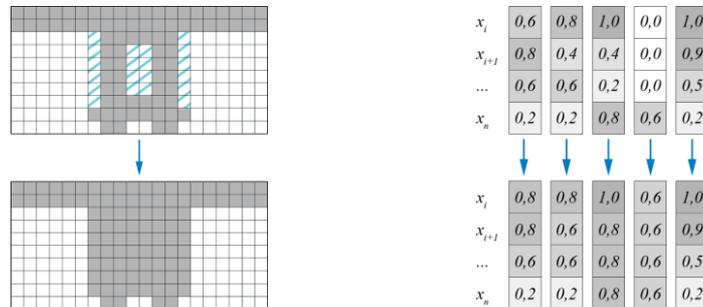


Figure 5: left: problem and solution, showing one void and two undercuts; right: examples of the applied logic

4. Analysis

4.1 Traditional topology optimisation

Without any additional constraints, the problem of designing an efficient floor slab reduces to a basic topology optimisation of a design space with a volume constraint. In theory, this results in the most optimal structure according to the objective function. However, the considered material is not appropriate for concrete, and manufacturability and functionality are neglected. For the sake of comparison, the traditional optimal design is displayed in Figure 7. The design has a maximum deflection of 2.1 mm and an objective of 0.0071 kNm. What follows from the optimisation process, is a structure with a big internal arm at midspan, and very slender webs near the supports. Both the internal openings and very thin members are not desirable when utilising a vacuumatic formwork.

4.2 Topology-optimised floor slab design for a vacuumatic formwork

When manufacturability constraints, a concrete material, and functional constraints are all activated in the optimisation process, then their functioning operates as displayed in Figure 6. Each addition in the process changes the outcome of the iterations to something that is more ‘manufacturable’ in combination with the vacuumatic formwork and that is more functional as a concrete floor. The result of the topology optimisation with the aforementioned input is displayed in Figure 7. The design has a maximum deflection of 14.1 mm and an objective of 0.0613 kNm.

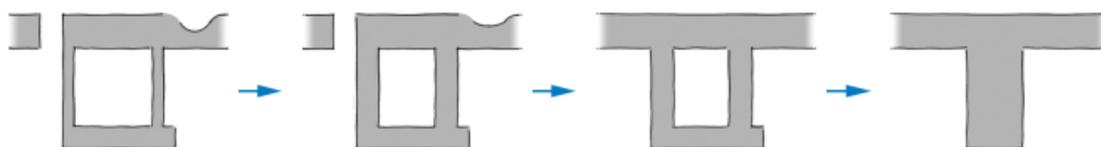


Figure 6: Functioning of the constraints for a hypothetical cross-section. a: topology optimisation result; b: inclusion of minimum thickness; c: inclusion of flat top; d: inclusion of casting constraint

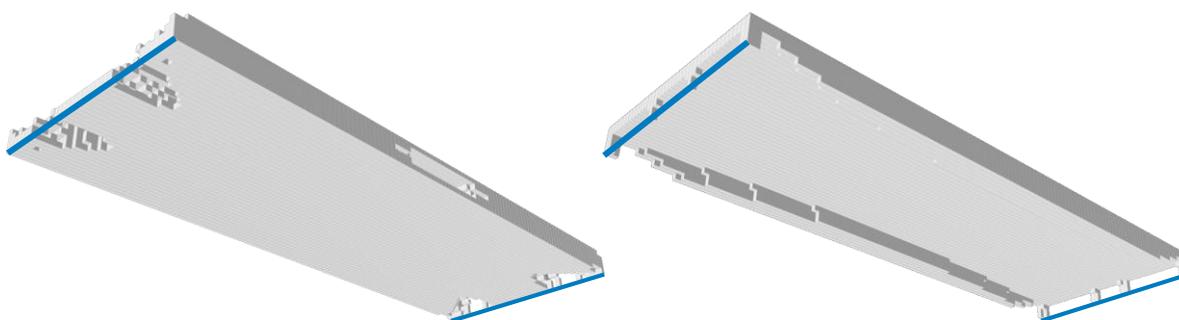


Figure 7: Left: traditional topology-optimised floor slab; right: topology-optimised floor slab to be produced with a vacuumatic formwork (supports in blue)

4.3 Advantages in weight, costs, sustainability, and aesthetics

The Eurocode prescribes a maximum deflection of $0.004L$ in the Serviceability Limit State. For the given span that means that the deflection of the floor under the design load is allowed to be anything up to 28 mm. The topology-optimised floor slab reaches a maximum deflection of 14.1 mm for a space that is 375 mm high and has a volume fraction of 50%. For a traditional monolithic slab, we arrive at a floor with a maximum deflection of 27 mm for a space that is 188 mm high. The two floors can be compared in accordance with Figure 8, now. The traditional floor slab has a total concrete volume of 3.28 m^3 , while the topology-optimised floor has a total volume of 2.73 m^3 . It can be concluded that the topology-optimised floor slab reduces the weight of the floor by approximately 17%.

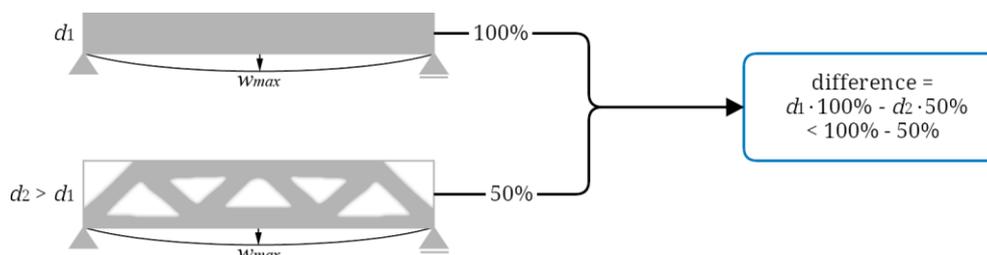


Figure 8: Applied method for comparing the traditional floor slab with the topology-optimised floor slab

The damage a product does to the environment can be expressed in the amount of carbon dioxide it produces in its creation and transport to the building site. Aspects such as sound insulation/reflection, waste, and reusability and adaptability of the vacuumatic formwork are not examined. To calculate the carbon emission of a floor slab, the amount of concrete and reinforcement need to be determined. The volume of the floors was established to be 3.28 m³ for a traditional floor and 2.73 m³ for the topology optimised floor. A distance of 30 km is adopted for the transportation. The amount of reinforcement for the traditional slab is assumed to be 100 kg/m³. For the topology-optimised floor slab, 120 kg/m³ is adopted. It can be concluded that the topology-optimised floor slab design is more sustainable than its traditional counterpart, reducing CO₂ emission by 10%. When taking the reusability and adaptability advantages of a vacuumatic formwork into account, this improvement will be even greater.

Compared to traditional floor slabs, it is reasonable to assume that the topology-optimised floor slab is more expensive since the floor slab industry is greatly optimised for costs. However, in the near future, this difference might become smaller or it might even invert completely if workers become more familiar with the technology or with the automation of the industry. Additionally, the formwork efficiency plays a significant role in the costs. A vacuumatic formwork can be used over and over, whereas a timber or steel formwork has a limited number of repetitions. Table 3 displays how a structure produced with a vacuumatic formwork compares to a traditional timber or steel formwork, now and in the near future.

Table 3: Comparing reinforced concrete costs for a topology-optimized slab produced with vacuumatics and a traditional slab with timber/steel formwork

Cost component		Presently	Prospective
Concrete	Material	+	+
	Transportation	0	0
	Labour	0	0
Reinforcement	Material	-	-
	Labour	-	0
Formwork	Labour	-	0
	Form efficiency	++	++

The aesthetics of the topology-optimised floor slab cannot be measured quantitatively. It is therefore left up to the reader to conclude for him or herself whether the design as displayed in Figure 7 on the right has more architectural value than a traditional floor slab.

5. Discussion

The result that follows from the topology optimisation algorithm can help with making a design. It is strictly a tool to come up with designs and the result cannot be viewed as definitive. There is still a step to be made in post-processing in order to arrive at a structural design. Among other things, part of this post-processing would include modelling of reinforcement and smoothing of the display. It can be

concluded that the bottleneck in the process is not caused by the manufacturing method, but by the engineering, which is the opposite of what usually occurs in the building industry.

There are also limitations on the floor slab presented in this paper. First of all, the reduction in weight also brings a reduction in sound insulation. Secondly, the higher structural efficiency of the present material can result in a lower fire resistance. Lastly, the costs are probably still higher than that of a traditional floor slab.

6. Conclusion

The problem of diminishing benefits of topology optimisation due to manufacturability constraints can be solved by reaching a compromise between a topology-optimised floor and a cost-optimised monolithic floor. To achieve this, first, manufacturability is incorporated in the optimisation loop, rather than afterwards as a manual modification. Secondly, the highly malleable vacuumatic formwork system allows sand to take up more complex shapes than its own angle of repose will support. Major advantages of this technique are reusability, demouldability, and adaptability of the formwork. These advantages mean that the formwork costs may be reduced severely when producing floors, especially since floors have a high rate of repetition in most multi-story buildings. The final topology-optimised floor slab design has advantages in weight reduction, sustainability, and aesthetics.

References

- [1] Andreassen, E., Clausen, A., Schevenels, M., Lazarov, B., and Sigmund, O., Efficient topology optimisation in Matlab using 88 lines of code. *Structural and Multidisciplinary Optimisation*, 2011, **43**(1):1–16.
- [2] Bendsoe, M., Optimal shape design as a material distribution problem. *Structural Optimization*, 1989, **1**(4):193–202.
- [3] Bourdin, B., Filters in topology optimisation. *International journal for numerical methods in engineering*, 2001, **50**:2143–2158.
- [4] Bruns, T. E. and Tortorelli, D. A., Topology optimisation of non-linear elastic structures and compliant mechanisms. *Computer Methods in Applied Mechanics and Engineering*, 2001, **190**.
- [5] Clark, T., *McCann's casting process and information guide*, N.D., <http://www.mccannsales.com/book/vprocess.pdf>. [Online; accessed 25-January-2016].
- [6] Fanella, D. and Alsamsam, I., Design of concrete floor systems. *Cement*, 2007.
- [7] Huijben, F.A.A., Vacuumatics: 3D Formwork Systems – Investigations of the structural and morphological nature of vacuumatic structures so as to be used as semi-rigid formwork systems for producing ‘free forms’ and customised surface textures in concrete for architectural applications. *PhD research, Eindhoven University of Technology*, 2014.
- [8] Huijben, F.A.A., Vacuumatic formwork: a novel granular manufacturing technique for producing topology-optimised structures in concrete. *Granular Matter*, 2016, **18**(2):1–8.
- [9] Kumar, P. and Gaindhar, J. L., Off-line quality control for v-process castings. *Quality And Reliability Engineering International*, 1995, **11**(3):175–181.
- [10] Liu, K. and Tovar, A., An efficient 3D topology optimisation code written in Matlab. *Structural and Multidisciplinary Optimisation*, 2014, **50**(6):1175–1196.
- [11] Nakata, K. and Kubo, Y., *Molding method with shielding member drawn against particulate material*. Patent, 1974.
- [12] Ravi, B., *Metal casting: Computer-Aided Design and Analysis*. Prentice-Hall of India, 2005.
- [13] Sigmund, O., A 99 line topology optimisation code written in Matlab. *Structural and Multidisciplinary Optimisation*, 2001, **21**(2):120–127.
- [14] Wang, W., Stoll, H., and Conley, J., *Rapid tooling guidelines for sand casting*. Springer, 2010