An approach on form-diversity of free-form shells generated from numerical hanging models

Qingpeng LI *, Andrew BORGART 1, Yue WU 2

* Faculty of Architecture and Built Environment, Delft University of Technology, Julianalaan 134, 2628 BL Delft, The Netherlands
Key Lab of Structures Dynamic Behavior and Control of China Ministry of Education, Harbin Institute of Technology, Harbin 150090, PR China
Email: hitssrclqp@gmail.com

1 Faculty of Architecture and Built Environment, Delft University of Technology
2 Key Lab of Structures Dynamic Behavior and Control of China Ministry of Education, Harbin Institute of Technology

Abstract
Based on its structural principle, the form of a hanging model is self-forming and capable of transferring its self-weight and area load solely by means of tension, and when it is turned upside down a pure compression model arises. Isler H. designed many shells by using this kind of models but with relatively limited structural forms. Recently, these models can be simulated by many numerical methods, and as a result, this brings a more convenient and efficient approach to generate diverse hanging structural forms in a very short time. The structural form and its mechanical behavior can be influenced by five categories of parameters which are used to describe geometry, cross-sectional properties, material properties, boundary conditions and loads in the initial structural system. Therefore, for free-form shells generated from hanging models, considering the requirements from both architecture and structure, an approach on form-diversity can be presented by varying the five categories of parameters of the initial structural system in the form-finding process. This paper uses the Non-linear Finite Element Method as the form-finding method to generate the equilibrium shape of the hanging model, and thus several numerical examples are shown to illustrate the specific procedures and results of the form-diversity approach.

Keywords: form-finding; free-form shells; form-diversity; numerical hanging models

1. Introduction
Based on its structural principle, the form of a hanging model is self-forming and capable of transferring its self-weight and area load solely by means of tension, and when it is turned upside down a pure compression model arises. This experiment is based on the mechanical balance principle and represents structural behavior independent of scale. Because of its appealing characteristics, such as its clear mechanics concept and direct visual image, it was appreciated by Gaudi A., Isler H., Otto F., and other design masters. They designed many projects with attractive shapes and sensible structures using this method. And Isler H. designed many shells by using this kind of models but with relatively limited structural forms.

And one question arises, how to obtain more diverse structural forms in a very short time, in order to overcome the relative shortcomings of the physical experiment with relatively homogeneous forms.

With the development of numerical analysis techniques and because of the disadvantages of the hanging model experiment, such as its complex model making process, this physical method has been
gradually replaced by numerical analysis methods. Therefore, these models can be simulated by these numerical methods, and as a result, this brings a more convenient and efficient approach to generate diverse hanging structural forms with high efficiency.

The structural form and its mechanical behavior can be influenced by five categories of parameters which are used to describe geometry, cross-sectional properties, material properties, boundary conditions and loads in the initial structural system. Therefore, for free-form shells generated from hanging models, considering the requirements from both architecture and structure, an approach on form-diversity can be presented by varying the five categories of parameters of the initial structural system in the form-finding process.

2. Numerical hanging-experiment method

With a combination of the basic rationale of this experiment and the computer numerical simulation technique, this paper develops a numerical form-finding method for free-form shells which is based on the inverse hanging experiment by using non-linear finite element technique, both in order to exploit the advantages of the inverse hanging method, and to overcome the innate disadvantages of it, what's more, to realize the harmony and unity between the delicate forms of the structure and its reasonable structural behaviors.

Shown in Figure 1, the basic procedure of the computational generation method is as follows:

1. It establishes an initial plane numerical model with cable elements in the boundary and membrane elements in the interior part, and applies loads perpendicular to the plane;
2. It obtains the hanging balanced configuration with a pure tension state by using the non-linear finite element method with the help of the ANSYS software;
3. It eventually obtains the pure compression shell structure after inversing the structure's coordinates and altering the element type of the finite element model.

And then it obtains the shell structure with a pure compression state.

![Figure 1: The basic procedure of the numerical method](image)

It should be noted that the primary purpose of this numerical method is to generate a balanced form of the structure, and it is obvious that the numerical balanced form-finding of the hanging flexible cable-net structure, under its self-weight or other specific loads, is the most difficult and key point in the procedure. More specifically, the numerical equilibrium form-finding of the hanging flexible cable-net structure has many characteristics, which are:

1. The initial structural form of the model is a geometric plane shape, therefore it requires pre-stress to provide stiffness to bear the out-of-plane loads. And the initial pre-stress of the cable elements is dependent on their elastic modulus and initial strain.
2. The numerical process of the form-finding, which is from a plane non-balanced state to a balanced state, is a process with a large deformation, therefore, the process will have a strong geometric nonlinearity. It can be solved using the non-linear finite element method relatively accurately.

3. The result of the form-finding is a balanced free-form structure. It can have numerous balanced forms with the same boundary conditions and certain loads. Every specific balanced form is related to the load distribution and the pre-stress distribution in the structural model. Moreover, it is dependent on relative values of the former two parameters but not their absolute values.

With the analysis of the above characteristics, it can be concluded that: the hypothetical numerical value of the structural parameters can be used to simulate the process of form-finding, and it will limit the lateral nodal displacement at the boundary to ensure the load distribution will not change significantly in the process.

However, one more question arises, how to obtain more diverse structural forms in a very short time, in order to overcome the relative shortcomings of the physical experiment with relatively homogeneous forms. For free-form shells generated from hanging models, considering the requirements from both architecture and structure, an approach on form-diversity can be presented by varying the five categories of parameters of the initial structural system in the form-finding process. It will present many form diversity measures to obtain more diverse structural forms after appropriate analysis of the structural system and a partition of the influence parameters for the structural form.

3. Form-diversity of free-form shells

3.1. Methodology of numerical form-finding

Form-finding is a forward process in which parameters are explicitly/directly controlled to find an 'optimal' geometry of a structure which is in static equilibrium with a design loading. It is always used to express the process of determining the equilibrium shape of lightweight structures. In the pre-computer-age, architects and engineers used physical model experiments to conduct the form-finding process, among which the hanging model experiment and the soap film experiment are very famous. With the development of computational techniques, most of the form-finding processes are carried out by numerical methods. In the last decades, the term 'form-finding' has been used in a more special context, It has been related to mechanical methods which are applied to find the shape of structures subjected to the action of forces in equilibrium.

For this kind of problems, the final equilibrium structural form always has a very close relationship with the mechanical behavior. For instance, in tension structures, the equilibrium structural form provides the structure with geometric stiffness to withstand loads. It is acceptable for most people that the mechanical behavior is reasonable after the process of form-finding, and what they concern most is to find the structural form under certain constraint conditions.

![Figure 2: The conceptual scheme of numerical form-finding](image-url)
Figure 2 shows the conceptual scheme of the numerical form-finding: for most of these problems, after giving the five categories the parameters of the initial system, the equilibrium structural form can be obtained by the analysis at one time. While in order to get the form with some other constraint conditions, it also needs some adjustment of the initial system. However, the adjusting process does not have systematic methods, and it depends mostly on designers’ subjective approach. Moreover, it should be noted that the initial structural system always performs as a geometrically unstable system, and the analysis for the initial structural system has strong geometric nonlinearity.

It shows five influence parameters for the structural system, which are geometry, distribution of the material, mechanical properties of the material, the boundary conditions and loads or other actions. To be specific, it explains the five categories of parameters as follows:

1. Geometric parameters, which can define the initial shape and space of the structure.

2. Distribution parameters of the material, which can define the sections for beam elements or thickness for shell elements, etc that are in the structure.

3. Mechanical parameters of the material, which describe the constitutive models of all building materials in the structure.

4. Parameters of the boundary conditions, which describe the mechanical features for both the supports and the joints in the structure.

5. Parameters of the loads or other actions, which refers to all the loads, temperature actions, earthquake actions, etc that apply to the structure.

These parameters can describe a real structure and be used to establish a computing model of it. Then using a numerical analysis method (the non-linear finite element method in this paper), the final shape and structural behavior of the structure can be obtained. Moreover, the analysis of the structural system can also provide a new understanding of the basic concepts of Structural Morphology from a perspective of numerical analysis.

Clearly, the different sets of these parameters will have an effect on the eventual structural form and behavior, as a result, a diversity of structural forms can be realized by adjusting the parameters for the initial structures. However, in practical projects, some parameters are not allowed to be changed in the process of structural design or analysis, such as the material parameters, the boundary conditions and the load distribution parameters. However, in the process of form-finding for the hanging cable-net structures, the parameters are more optionally to change in order to get diverse balanced structural forms.

3.2. Form diversity measures for the inverted structural forms

Hanging models represent a kind of equilibrium state of flexible materials under their self-weight and certain constraint conditions, whose stress states are pure tension. When these equilibrium shapes are used as the geometry of rigid structures after inverting them, they will perform with an effective structural behavior under their self-weight or loads with a same distribution. For form-finding of shell structures generated from this kind of models, a piece of fabric or rubber applied with gypsum may be used, and when the gypsum is hardened, the models can be turned upside down and then be observed and scaled to real design process.

In order to expound those measures in detail, consider adjusting a basic the following plane structure as an example. Figure 3 shows the initial conditions of this example. The initial shape of this example is a hexagon in the XY plane with supports at the six corners, and the corners are beveled by lines AA’, BB’, CC’, DD’, EE’ and FF’. The plane area is 64.30 m² (hexagon with sides of 5.0 m). The initial numerical structural model is composed of particles and triangular membrane elements. The elastic modulus of the membrane material is 5.0E05 N/m², the Poisson's Ratio of it is 0.3, and the thickness of the membrane element is 0.001 m.
After the form-finding, it gets the equilibrium shape of the hanging membrane, and Figure 4 shows the inverted shape and its coordinate system.

Then undertake the basic adjusting measures to obtain the related structural forms as follows:

1. Adjusting the boundary condition, it mainly refers to adjust the support type and the support position of the initial numerical structural model. For instance, shown in Figure 5, it can get reasonable structural form with different appearance by altering the support type from six short-line corners to line supports.
2. Adjusting the material property, it mainly refers to adjust the elastic modulus of different types of elements or different parts of elements in the initial numerical structural model. For instance, shown in Figure 6, it can get reasonable structural form with different appearance by different ratios of cable elements' modulus of elasticity and membrane elements'.

![Figure 6: The structural form by changing material property](image)

3. Adjusting the distributions of the structural material, it refers to adjust the distributions of different elements in the initial numerical structural model, such as to apply cable elements in it, etc. For instance, showing in Figure 7, it can get reasonable structural form with different appearance by applying cable elements in the diagonals, and the cable elements would correspond to the arch ribs in the inversed structures.

![Figure 7: The structural form by changing the distributions of the structural material](image)

4. Adjusting the distributions of the loads in the initial numerical structural model, in consideration of that, the final form is relevant to the distribution of the loads but the exactly numerical magnitudes of the loads. For instance, shown in Figure 8, it can get reasonable structural form with different appearance by altering self-weight to loads of pressure.

5. Adjusting the mass distribution of structural material, it mainly refers to cut holes in the initial numerical structural models. For instance, shown in Figure 9, it can get reasonable structural form with different appearance by cutting one hexagonal hole in the central part, and by cutting seven hexagonal holes symmetrically in the central part.

However, it still must be noticed that the diversities of structural form should combine with the specifications of engineering practices, and it cannot adjust the parameters optionally in the process of form finding. And only in this way can the method have its feasibility in practical projects.
4. Conclusions
With a combination of the basic rationale of this experiment and the computer numerical simulation technique, this paper develops a numerical form-finding method for free-form shells which is based on the inverse hanging experiment by using non-linear finite element technique. Then it presents the methodology of numerical form-finding methods, and clearly, the different sets of these parameters will have an effect on the eventual structural form and behavior, as a result, a diversity of structural forms can be realized by adjusting the parameters for the initial structures. By exploring the influence of the five kinds of parameters of the initial structural model on the final structural form, this paper presents an approach on form-diversity of free-form shells generated from numerical hanging models.

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