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# Time-Domain Analysis of Thin-Wire Structures Based on the Cagniard-DeHoop Method of Moments

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**Abstract**—Thin-wire structures in the presence or absence of a ground plane are analyzed numerically in the time domain (TD) with the aid of the Cagniard-DeHoop method of moments (CdH-MoM). It is demonstrated that the TD solution of such problems can be cast into the form of discrete time-convolution equations. Under the assumption of piecewise linear space-time axial current distribution, the elements of the TD impedance array are derived analytically in terms of elementary functions. Their approximations applying to multi-conductor transmission lines are discussed. Illustrative numerical examples validating the TD solution are presented.

**Index Terms**—transient electromagnetic (EM) scattering, wire antenna, transmission line (TL), Cagniard-DeHoop method of moments, marching-on-in-time technique, time-domain (TD) analysis.

## I. INTRODUCTION

THE performance of a linear-antenna array [1, Sec. 14.11.1] consisting of a set of conducting wires, possibly in the presence of a metal plane [1, Sec. 6.6], can be analyzed through electric network representations [1, Sec. 8.7.1]. The pertaining input and mutual impedances, as well as its electromagnetic (EM) radiation and scattering characteristics, are functions of the electric current induced in the antenna-array's wire elements (see [2, Sec. 8.6] and [3, Sec. 1.1]). A versatile numerical tool suitable for achieving this distribution is widely known as the method of moments [4, Chapter 4]. Under the assumption of sinusoidally in time varying EM fields, this solution procedure leads to a system of equations, say

$$\hat{\mathbf{Z}} \cdot \hat{\mathbf{I}} = \hat{\mathbf{V}}, \quad (1)$$

thus interrelating the (frequency-domain) axial electric currents,  $\hat{\mathbf{I}}$ , with the excitation voltage,  $\hat{\mathbf{V}}$ , via the impedance array,  $\hat{\mathbf{Z}}$ , at a fixed frequency of analysis. In this manner, the numerical solution offers an enlightening representation

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of a wire antenna in the form of an  $N$ -port network (see [4, Chapters 4 and 5]). Since the product of two frequency-domain functions corresponds to the time convolution in the original domain, the time-domain (TD) generalized representation of a wire antenna can be cast into the TD discrete-convolution form

$$\sum_k \mathbf{Z}_{m-k} \cdot \mathbf{I}_k = \mathbf{V}_m, \quad (2)$$

where indices  $m$  and  $k$  refer to instants in a time window of observation. The development of such TD network representations concerning thin wire structures (in the presence or absence of a ground plane) is exactly the main objective of this article.

The thin-wire problems under consideration are solved here with the help of the Cagniard-DeHoop method of moments (CdH-MoM) [5], recent applications of which can be found in [6]–[8], for instance. Regarding its novelty, the outcomes of this work are not limited to a thin conductor located just above the reference plane. In this sense, the present work can be understood as an extension of Ref. [7], where the CdH-MoM is applied to a single transmission line (TL). In contrast to the widespread approach relying on finite-difference approximations of the differential operator in the starting integro-differential equation (see [9]–[12] [13, Sec. 2.3.3]), the proposed TD solution is free of such approximations. As a matter of fact, as the EM scattering problem analyzed in this article is formulated via the EM reciprocity theorem of the time-convolution type [14, Sec. 28.2], the introduced TD result can be viewed as an “exact weak” solution for the piecewise-linear space-time basis. Moreover, since the elements of the pertaining TD impedance arrays are expressed merely in terms of elementary functions, their filling is computationally effortless. For alternative numerical approaches allowing the extraction of the TD impedance in an (almost) explicit form we refer the reader to [15]–[19], for example. These numerical approaches are mostly capable of handling more general problem configurations, but at the expense of introducing some additional mathematical intricacies. The TD expressions for the TD impedance arrays introduced in the present article are of greater simplicity and are, in their present form, believed to be entirely new.

The EM scattering problem under consideration is first formulated in Sec. II using the TD EM reciprocity theorem of the time-convolution type [14, Sec. 28.2]. Subsequently, in

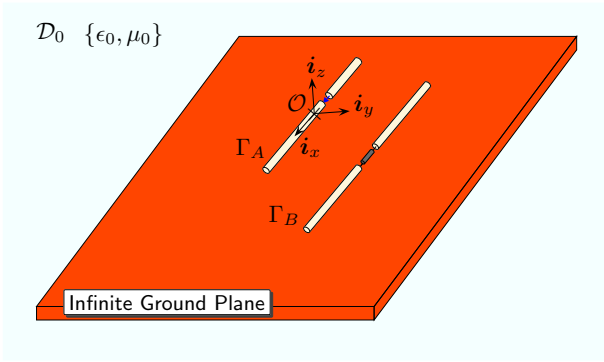


Fig. 1. Thin-wire antennas above a ground plane.

Sec. III, the pertaining TD interaction integrals are represented through complex slowness integrals. In Sec. IV, the space-time distribution of (unknown) axial currents is expanded in a piecewise-linear manner, which through the use of the Cagniard-DeHoop inversion technique [20] leads to closed-form expressions for the pertaining TD impedance arrays. For readers' convenience, analytical details of the joint inversion procedure are briefly summarized in the Appendix. Illustrative numerical results and their validation are presented in Sec. V. Finally, conclusions are drawn in Sec. VI.

## II. PROBLEM DEFINITION

The analyzed problem is shown in Fig. 1. Here, the position is specified by the coordinates  $\{x, y, z\}$  with respect to a Cartesian reference frame with the origin  $\mathcal{O}$  and the standard base  $\{\hat{i}_x, \hat{i}_y, \hat{i}_z\}$ . The time coordinate is denoted by  $t$ . The time-convolution operator is  $*_t$ . The Heaviside unit-step function is  $H(t)$  and the impulsive Dirac-delta distribution is denoted by  $\delta(t)$ .

The problem configuration consists of a set of mutually parallel wire antennas. Their radius,  $a > 0$ , is assumed to be relatively small such that the “reduced form” of Pocklington’s equation can be adopted [21]. In this thin-wire formulation, the electric currents at the end faces of the cylindrical wire antenna are neglected, thus reducing the original surface electric-field integral equation to the simplified 1-D one that can be solved for the electric current concentrated essentially along the wire’s axis. We shall analyze both the set of thin wires in the presence or absence of the infinite perfectly electrically conducting (PEC) plane. If present, the ground plane occupies  $\{-\infty < x < \infty, -\infty < y < \infty, -z_0 - \delta/2 < z < -z_0 + \delta/2\}$  with  $z_0 > 0$ . To describe the self- and mutual-interactions in a general system of parallel wires it is sufficient to analyze the transient EM response of two wires. Therefore, without any loss of generality, we shall further consider two wire antennas occupying  $\Gamma_A = \{-\ell_A/2 < x < \ell_A/2, y = 0, z = 0\}$  and  $\Gamma_B = \{-\ell_B/2 < x - x_0 < \ell_B/2, y = y_0, z = 0\}$  (see Fig. 1). Hence, the thin wires are located at the height  $z_0 > 0$  above the ground plane (if present) and at the distance  $|y_0| > 0$  far apart. The EM field coupling between two non-parallel wires is tractable via the CdH-MoM too, but this extension is outside the scope of the present work. The linear, homogeneous isotropic and lossless medium surrounding the

antennas is described by (real-valued and positive) scalars  $\epsilon_0$  and  $\mu_0$  implying the EM wave speed,  $c_0 = (\epsilon_0\mu_0)^{-1/2} > 0$ , and EM wave impedance  $Z_0 = 1/Y_0 = (\mu_0/\epsilon_0)^{1/2} > 0$ .

The presence of the wires is accounted for via the scattered EM field (denoted by  $^s$ ) that is hence defined as the difference between the total and excitation EM fields. The problem will be formulated through the TD Lorentz EM reciprocity theorem [14, Sec. 28.2]. Its application to the (actual) scattered and (computational) testing field states leads to (cf. [7, Eq. (2)])

$$\begin{aligned} & \int_{x=-\ell_A/2}^{\ell_A/2} E^{T;A}(x, a, t) *_t I^{S;A}(x, t) dx \\ & + \int_{x=x_0-\ell_B/2}^{x_0+\ell_B/2} E^{T;A}(x, |y_0|, t) *_t I^{S;B}(x, t) dx \\ & = \int_{x=-\ell_A/2}^{\ell_A/2} E^S(x, a, t) *_t I^{T;A}(x, t) dx, \quad (3) \end{aligned}$$

and

$$\begin{aligned} & \int_{x=x_0-\ell_B/2}^{x_0+\ell_B/2} E^{T;B}(x, a, t) *_t I^{S;B}(x, t) dx \\ & + \int_{x=-\ell_A/2}^{\ell_A/2} E^{T;B}(x, |y_0|, t) *_t I^{S;A}(x, t) dx \\ & = \int_{x=x_0-\ell_B/2}^{x_0+\ell_B/2} E^S(x, a, t) *_t I^{T;B}(x, t) dx, \quad (4) \end{aligned}$$

where  $I^{S;A}(x, t)$  and  $I^{S;B}(x, t)$  denote the (unknown) *axial* currents induced in  $\Gamma_A$  and  $\Gamma_B$ , respectively, and  $I^{T;A,B}(x, t)$  are the pertaining testing currents. Furthermore,  $E^{T;A,B}(x, \varrho, t)$  denotes the (*axial* component of) the testing electric-field strengths as generated by the testing currents,  $I^{T;A,B}(x, t)$ , respectively, and  $\varrho = \varrho(y, z) = (y^2 + z^2)^{1/2}$ . Symbolically, the TD reciprocity relations (3) and (4) can be cast into the matrix form

$$\begin{bmatrix} \mathbf{Z}_{AA} & \mathbf{Z}_{AB} \\ \mathbf{Z}_{BA} & \mathbf{Z}_{BB} \end{bmatrix} *_t \begin{bmatrix} \mathbf{I}_A \\ \mathbf{I}_B \end{bmatrix} = \begin{bmatrix} \mathbf{V}_A \\ \mathbf{V}_B \end{bmatrix}, \quad (5)$$

where the diagonal sub-arrays  $\mathbf{Z}_{AA}$  and  $\mathbf{Z}_{BB}$  represent the local interactions on wires  $\Gamma_A$  and  $\Gamma_B$ , respectively, while  $\mathbf{Z}_{AB}$  and  $\mathbf{Z}_{BA}$  describe the remote interactions from  $\Gamma_A$  to  $\Gamma_B$  and vice versa. A straightforward generalization of the matrix system allows the handling of a larger system of mutually parallel wire antennas. The pertaining TD interaction integrals are further represented by complex slowness integrals. Subsequently, these integrals will be transformed back to the TD using the CdH technique.

## III. COMPLEX SLOWNESS REPRESENTATIONS

The property of causality and the time invariance of the problem configuration is accounted for by the use of the one-sided Laplace transformation

$$\hat{E}(x, \varrho, s) = \int_{t=0}^{\infty} \exp(-st) E(x, \varrho, t) dt, \quad (6)$$

with  $\{s \in \mathbb{R}; s > 0\}$ . In the next step, the wave slowness representation is introduced

$$\hat{E}(x, \varrho, s) = (s/2\pi i)^2 \int_{\kappa=-i\infty}^{i\infty} \exp(-s\kappa x) d\kappa \times \int_{\sigma=-i\infty}^{i\infty} \exp(-s\sigma y) \hat{E}(\kappa, \sigma, z, s) d\sigma, \quad (7)$$

in which  $\kappa$  and  $\sigma$  are the wave-slowness parameters in the  $x$ - and  $y$ -direction, respectively. Note that this representation entails  $\partial_x \rightarrow -s\kappa$  and  $\partial_y \rightarrow -s\sigma$ . Using Eqs. (6) with (7), the interaction terms in the TD reciprocity relations (3) and (4) can be expressed through complex-slowness integrals. For instance, the local interaction integral on  $\Gamma_A$  in the presence of the PEC ground plane can be written as

$$\begin{aligned} & \int_{x=-\ell_A/2}^{\ell_A/2} \hat{E}^{\text{T};A}(x, a, s) \hat{I}^{\text{S};A}(x, s) dx \\ &= -\frac{sZ_0}{c_0} \frac{s}{2\pi i} \int_{\kappa=-i\infty}^{i\infty} c_0^2 \Omega_0^2(\kappa) \tilde{I}^{\text{T};A}(\kappa, s) \tilde{I}^{\text{S};A}(-\kappa, s) d\kappa \\ & \frac{s}{2\pi i} \int_{\sigma=-i\infty}^{i\infty} \frac{\exp[-s\Gamma_0(\kappa, \sigma)a] - \exp[-2s\Gamma_0(\kappa, \sigma)z_0]}{2s\Gamma_0(\kappa, \sigma)} d\sigma, \end{aligned} \quad (8)$$

where  $\Gamma_0(\kappa, \sigma) = [\Omega_0^2(\kappa) - \sigma^2]^{1/2}$ , with  $\text{Re}(\Gamma_0) \geq 0$ , and  $\Omega_0^2(\kappa) = 1/c_0^2 - \kappa^2$ . Furthermore, for the (transform-domain) axial testing field we used

$$\tilde{E}^{\text{T};A}(\kappa, \varrho, s) = -(sZ_0/c_0) c_0^2 \Omega_0^2(\kappa) \tilde{I}^{\text{T};A}(\kappa, s) \tilde{G}(\kappa, \varrho, s), \quad (9)$$

for  $\varrho = a$  and the modified Bessel functions in the pertaining Green's function, viz

$$\begin{aligned} \tilde{G}(\kappa, \varrho, s) &= \text{K}_0[s\Omega_0(\kappa)\varrho(y, z)]/2\pi \\ & - \text{K}_0[s\Omega_0(\kappa)\varrho(y, z + 2z_0)]/2\pi, \end{aligned} \quad (10)$$

were expressed through their integral representations (see [22, (9.6.23)]). In a similar fashion, the corresponding remote interaction from  $\Gamma_A$  to  $\Gamma_B$  can be described by

$$\begin{aligned} & \int_{x=x_0-\ell_B/2}^{x_0+\ell_B/2} \hat{E}^{\text{T};A}(x, a, s) \hat{I}^{\text{S};B}(x, s) dx \\ &= -\frac{sZ_0}{c_0} \frac{s}{2\pi i} \int_{\kappa=-i\infty}^{i\infty} c_0^2 \Omega_0^2(\kappa) \tilde{I}^{\text{T};A}(\kappa, s) \tilde{I}^{\text{S};B}(-\kappa, s) d\kappa \\ & \frac{s}{2\pi i} \int_{\sigma=-i\infty}^{i\infty} \exp(-s\sigma y_0) \frac{1 - \exp[-2s\Gamma_0(\kappa, \sigma)z_0]}{2s\Gamma_0(\kappa, \sigma)} d\sigma. \end{aligned} \quad (11)$$

The remaining interaction integrals can be represented in the same manner.

#### IV. TIME-DOMAIN SOLUTIONS

To solve the reciprocity relations numerically, the space-time solution is discretized. Hence, the thin-wire antennas  $\Gamma_A$  and  $\Gamma_B$  are divided into  $N_A + 1$  and  $N_B + 1$  segments, so that their partitions are  $\Delta\Gamma_A = \{x_n = -\ell_A/2 + n\Delta_A, y = 0, z = 0\}$  for  $n = \{1, \dots, N_A\}$  with  $\Delta_A = \ell_A/(N_A + 1)$ , and  $\Delta\Gamma_B = \{x_n - x_0 = -\ell_B/2 + (n - N_A)\Delta_B, y = y_0, z = 0\}$  for  $n = \{N_A + 1, \dots, N_A + N_B\}$ , where  $\Delta_B = \ell_B/(N_B + 1)$ . Likewise, the time axis is discretized uniformly via  $\{t_k =$

$k\Delta t; k = 1, 2, \dots, M\}$ , where  $\Delta t > 0$  denotes the time step. It is noted that the uniform discretization is not mandatory, but its use simplifies the resulting TD solution. Subsequently, the induced electric currents along the thin wires are expanded in terms of spatial and temporal basis (triangular) functions

$$I^{\text{S};A}(x, t) \simeq \sum_{n=1}^{N_A} \sum_{k=1}^M i_k^{[n]} \Lambda^{[n]}(x) \Lambda_k(t), \quad (12)$$

$$I^{\text{S};B}(x, t) \simeq \sum_{n=N_A+1}^{N_A+N_B} \sum_{k=1}^M i_k^{[n]} \Lambda^{[n]}(x) \Lambda_k(t), \quad (13)$$

where  $i_k^{[n]}$  denotes the (yet unknown) electric-current coefficients, and the basis functions can be specified by

$$\Lambda^{[n]}(x) = \begin{cases} 1 + (x - x_n)/\Delta & \text{for } x \in [x_{n-1}, x_n] \\ 1 - (x - x_n)/\Delta & \text{for } x \in [x_n, x_{n+1}], \end{cases} \quad (14)$$

$$\Lambda_k(t) = \begin{cases} 1 + (t - t_k)/\Delta t & \text{for } t \in [t_{k-1}, t_k] \\ 1 - (t - t_k)/\Delta t & \text{for } t \in [t_k, t_{k+1}]. \end{cases} \quad (15)$$

Next, the testing current is chosen to show the rectangular spatial distribution and the impulsive behavior in time. Accordingly, we write

$$I^{\text{T};A}(x, t) = \begin{cases} \delta(t) & \text{for } x \in [x_S - \Delta_A/2, x_S + \Delta_A/2] \\ 0 & \text{elsewhere,} \end{cases} \quad (16)$$

for all  $S = \{1, \dots, N_A\}$ , and

$$I^{\text{T};B}(x, t) = \begin{cases} \delta(t) & \text{for } x \in [x_S - \Delta_B/2, x_S + \Delta_B/2] \\ 0 & \text{elsewhere,} \end{cases} \quad (17)$$

for all  $S = \{N_A + 1, \dots, N_A + N_B\}$ . Now, making use of the transform-domain counterparts of Eqs. (12), (13) and (16), (17) in the complex-slowness integral representations of the interaction integrals (see Eqs. (8) and (11)), we end up with [7, Eq. (12)]

$$\sum_{k=1}^m (\underline{\mathbf{Z}}_{m-k+1} - 2\underline{\mathbf{Z}}_{m-k} + \underline{\mathbf{Z}}_{m-k-1}) \cdot \mathbf{I}_k = \mathbf{V}_m, \quad (18)$$

where  $\underline{\mathbf{Z}}_k = \underline{\mathbf{Z}}(t_k)$  represents a 2-D  $[(N_A + N_B) \times (N_A + N_B)]$  TD impedance array at  $t = t_k$ ,  $\mathbf{I}_k$  denotes a 1-D  $[(N_A + N_B) \times 1]$  array of the (unknown) electric-current coefficients,  $i_k^{[n]}$ , and,  $\mathbf{V}_m$  is a 1-D  $[(N_A + N_B) \times 1]$  excitation-voltage array at  $t = t_m$ . The elements of the excitation array pertaining to a given excitation-field distribution, say  $E^e(x, a, t)$ , follow upon evaluating the TD interaction integrals on the right-hand side of the TD reciprocity relations (3) and (4). This can be done by enforcing the explicit-type boundary condition on the wire's surface,  $E^s(x, a, t) = -E^e(x, a, t)$ , and using the chosen testing-current space-time distributions. Thanks to their simple form (see Eqs. (16) and (17)), the excitation elements can be for the majority of standard excitation mechanisms expressed in closed form. In order to cast the resulting time-convolution system of equations (18) into the form of Eq. (2), one may substitute

$$\underline{\mathbf{Z}}_n = \underline{\mathbf{Z}}_{n+1} - 2\underline{\mathbf{Z}}_n + \underline{\mathbf{Z}}_{n-1}, \quad (19)$$

which, as a matter of fact, represents the central second-order difference [22, (25.1.2)]. Solving then the system of equations with the aid of the marching-on-in-time technique, we get the following step-by-step updating scheme

$$\mathbf{I}_m = \underline{\mathbf{Z}}_1^{-1} \cdot \left[ \mathbf{V}_m - \sum_{k=1}^{m-1} \underline{\mathbf{Z}}_{m-k} \cdot \mathbf{I}_k \right], \quad (20)$$

for all  $m = \{1, \dots, M\}$ . The closed-form expressions for the (elements of the) TD impedance array, expressed in terms of the TD generic function (32), will be next given separately for the cases with and without the ground plane. Thanks to the exact evaluation of the TD interaction integrals, the filling of the TD impedance array takes typically a few seconds on a standard PC and the resulting marching-on-in-time scheme (20) yields stable and accurate TD electric-current responses.

#### A. Antennas in the Presence of the Ground Plane

In the presence of the ground plane, the local interactions on  $\Gamma_A$  can be described by the TD impedance sub-array  $\mathbf{Z}_{AA}$  (see Eq. (5)). Its elements can be expressed via

$$Z_{A;A}^{[S,n]}(t) = \frac{Z_0}{c_0 \Delta t \Delta_A} \left[ \Xi^{[S,n]}(3\Delta_A/2, 0, t) - 3 \Xi^{[S,n]}(\Delta_A/2, 0, t) \right], \quad (21)$$

for all  $S = \{1, \dots, N_A\}$  with  $n = \{1, \dots, N_A\}$ , where

$$\begin{aligned} \Xi^{[S,n]}(\Delta, y, t) &= \Upsilon(x^{[S,n]} + \Delta, y, a, t) - \Upsilon(x^{[S,n]} + \Delta, y, 2z_0, t) \\ &- \left[ \Upsilon(x^{[S,n]} - \Delta, y, a, t) - \Upsilon(x^{[S,n]} - \Delta, y, 2z_0, t) \right], \quad (22) \end{aligned}$$

In Eq. (22) we used  $x^{[S,n]} = x_S - x_n$  and recall that  $\Upsilon(x, y, z, t)$  is given by Eq. (32). A similar expression applies to the local interactions on  $\Gamma_B$ , viz

$$Z_{B;B}^{[S,n]}(t) = \frac{Z_0}{c_0 \Delta t \Delta_B} \left[ \Xi^{[S,n]}(3\Delta_B/2, 0, t) - 3 \Xi^{[S,n]}(\Delta_B/2, 0, t) \right], \quad (23)$$

for all  $S = \{N_A + 1, \dots, N_A + N_B\}$  with  $n = \{N_A + 1, \dots, N_A + N_B\}$ . Furthermore, the remote interactions from  $\Gamma_A$  to  $\Gamma_B$  is described by

$$Z_{AB}^{[S,n]}(t) = \frac{Z_0}{c_0 \Delta t \Delta_B} \left[ \Xi^{[S,n]}(\Delta_A/2 + \Delta_B, y_0, t) - 2 \Xi^{[S,n]}(\Delta_A/2, y_0, t) + \Xi^{[S,n]}(\Delta_A/2 - \Delta_B, y_0, t) \right], \quad (24)$$

for all  $S = \{1, \dots, N_A\}$  with  $n = \{N_A + 1, \dots, N_A + N_B\}$ . Finally, sub-array describing the remote interactions from  $\Gamma_B$  to  $\Gamma_A$  immediately follows as

$$Z_{BA}^{[S,n]}(t) = \frac{Z_0}{c_0 \Delta t \Delta_A} \left[ \Xi^{[S,n]}(\Delta_B/2 + \Delta_A, y_0, t) - 2 \Xi^{[S,n]}(\Delta_B/2, y_0, t) + \Xi^{[S,n]}(\Delta_B/2 - \Delta_A, y_0, t) \right], \quad (25)$$

for all  $S = \{N_A + 1, \dots, N_A + N_B\}$  and  $n = \{1, \dots, N_A\}$ . Alternatively, owing to the self-adjointness of the surrounding medium [23, Sec. 1.4.1], one may invoke the property

of reciprocity represented by  $Z_{BA}^{[S,n]}(t) = Z_{AB}^{[n,S]}(t)$  for all  $S = \{N_A + 1, \dots, N_A + N_B\}$ ,  $n = \{1, \dots, N_A\}$  and  $t > 0$ .

If the thin-wire antennas are relatively close to the ground plane, which is an underlying assumption of the standard TL theory (e.g. [5, Sec. 11.1] and [24]), approximation (33) can be used to further simplify the TD impedance array. Indeed, in such a case, we arrive at

$$Z_{AA}^{[S,n]}(t) \stackrel{\text{TL}}{\simeq} \frac{Z_c}{c_0 \Delta t \Delta_A} \left[ \Psi(x^{[S,n]} + 3\Delta_A/2, t) - 3\Psi(x^{[S,n]} + \Delta_A/2, t) + 3\Psi(x^{[S,n]} - \Delta_A/2, t) - \Psi(x^{[S,n]} - 3\Delta_A/2, t) \right], \quad (26)$$

for all  $S = \{1, \dots, N_A\}$  with  $n = \{1, \dots, N_A\}$ , where  $Z_c = (Z_0/2\pi) \log(2z_0/a)$  is the characteristic impedance of the TL above the PEC ground and the TD function  $\Psi(x, t)$  is given in the Appendix by Eq. (34). As a matter of fact, this result fully complies with the CdH-MoM analysis of TLs reported in a previous work [7, Eq. (14)]. Upon replacing  $A$  with  $B$  in Eq. (26), a similar expression for the approximate  $Z_{BB}^{[S,n]}(t)$  can be readily obtained. If, in addition, the TLs are relatively close to each other, their mutual EM coupling can be characterized by

$$Z_{AB}^{[S,n]}(t) \stackrel{\text{TL}}{\simeq} \frac{Z_d}{c_0 \Delta t \Delta_B} \left[ \Psi(x^{[S,n]} + \Delta_B + \Delta_A/2, t) - \Psi(x^{[S,n]} + \Delta_B - \Delta_A/2, t) - 2\Psi(x^{[S,n]} + \Delta_A/2, t) + 2\Psi(x^{[S,n]} - \Delta_A/2, t) + \Psi(x^{[S,n]} - \Delta_B + \Delta_A/2, t) - \Psi(x^{[S,n]} - \Delta_B - \Delta_A/2, t) \right], \quad (27)$$

for all  $S = \{1, \dots, N_A\}$  with  $n = \{N_A + 1, \dots, N_A + N_B\}$ , where  $Z_d = (Z_0/2\pi) \log[(y_0^2 + 4z_0^2)/|y_0|]$ . Finally, the TL approximation of  $Z_{BA}^{[S,n]}(t)$  can be found from Eq. (27) using the property of reciprocity, again.

#### B. Antennas in the Absence of the Ground Plane

The presence of the ground plane can be accommodated by the method of images. Consequently, the TD expressions applying to the configuration without the ground plane can be derived from Eqs. (21)–(25) upon removing the image-source contributions. Pursuing this line of reasoning, we get (cf. Eq. (21))

$$Z_{AA}^{[S,n]}(t) = \frac{Z_0}{c_0 \Delta t \Delta_A} \left[ \Upsilon(x^{[S,n]} + 3\Delta_A/2, 0, a, t) - 3\Upsilon(x^{[S,n]} + \Delta_A/2, 0, a, t) + 3\Upsilon(x^{[S,n]} - \Delta_A/2, 0, a, t) - \Upsilon(x^{[S,n]} - 3\Delta_A/2, 0, a, t) \right], \quad (28)$$

for all  $S = \{1, \dots, N_A\}$  with  $n = \{1, \dots, N_A\}$ . Furthermore, the remote interaction between two thin-wire antennas located in  $\mathcal{D}_0$  is described by (cf. Eq. (24))

$$Z_{AB}^{[S,n]}(t) = \frac{Z_0}{c_0 \Delta t \Delta_B} \left[ \Upsilon(x^{[S,n]} + \Delta_B + \Delta_A/2, y_0, 0, t) - \Upsilon(x^{[S,n]} + \Delta_B - \Delta_A/2, y_0, 0, t) - 2\Upsilon(x^{[S,n]} + \Delta_A/2, y_0, 0, t) + 2\Upsilon(x^{[S,n]} - \Delta_A/2, y_0, 0, t) + \Upsilon(x^{[S,n]} - \Delta_B + \Delta_A/2, y_0, 0, t) - \Upsilon(x^{[S,n]} - \Delta_B - \Delta_A/2, y_0, 0, t) \right], \quad (29)$$

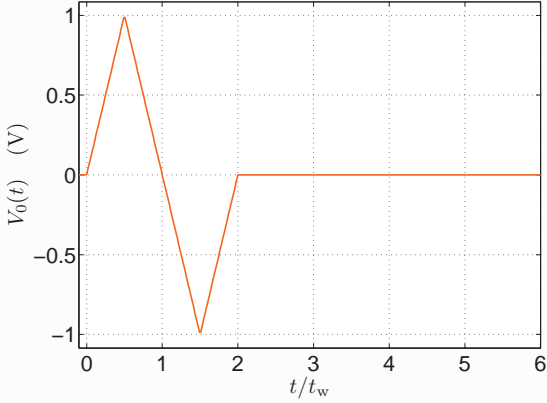


Fig. 2. Excitation voltage pulse shape.

for all  $S = \{1, \dots, N_A\}$  with  $n = \{N_A + 1, \dots, N_A + N_B\}$ . Using the replacements  $A \leftrightarrow B$ , similar expressions can be found at once for the local interactions on  $\Gamma_B$  and the remote interactions from  $\Gamma_B$  to  $\Gamma_A$ . For the sake of conciseness, these expressions are omitted.

## V. ILLUSTRATIVE NUMERICAL EXAMPLES

The validity of the presented CdH-MoM numerical solution will be demonstrated on numerical examples. Throughout the examples that follow, the transmitting thin-wire antenna,  $\Gamma_A$ , is activated via a voltage pulse applied to a relatively narrow gap located at its center at  $x = 0$ . As the excitation pulse shape, we take the bipolar triangular pulse [7, Eq. (28)]

$$V_0(t) = (2V_m/t_w) [tH(t) - 2(t - t_w/2)H(t - t_w/2) + 2(t - 3t_w/2)H(t - 3t_w/2) - (t - 2t_w)H(t - 2t_w)], \quad (30)$$

with  $c_0 t_w / \ell_A = 1/2$  and the unit amplitude  $V_m = 1.0$  V (see Fig. 2).

In the first example, we calculate the self-response of a single thin-wire antenna  $\Gamma_A$  located in the unbounded surrounding space in the absence of the ground plane. We take  $a/\ell_A = 1/500$ , implying  $a/c_0 t_w = 1/250$ , so that the thin-wire assumption is safely met. The electric-current pulse calculated using the TD impedance array from Sec. IV-B is shown in Fig. 3. For validation purposes, the problem has also been analyzed with the help of the finite-integration technique (FIT) as implemented in CST Microwave Studio<sup>®</sup>. It can be seen that the calculated pulse shapes correlate well. The *total* computational time of a demonstrational (non-optimized) MATLAB code with  $N_A = 49$  and  $M = 601$  grid points was about 6 s, with approximately 2/3 of the time being used for filling the TD impedance array and 1/3 for the marching-on-in-time scheme (20). On the other hand, the corresponding CST simulation took about 30 minutes. All simulations were conducted on a standard Intel(R) Core(TM) i7-2600 CPU 3.40 GHz platform with 16.0 GB RAM.

In the second example, the TD expressions presented in Sec. IV-A are employed to calculate the gap-excited, electric

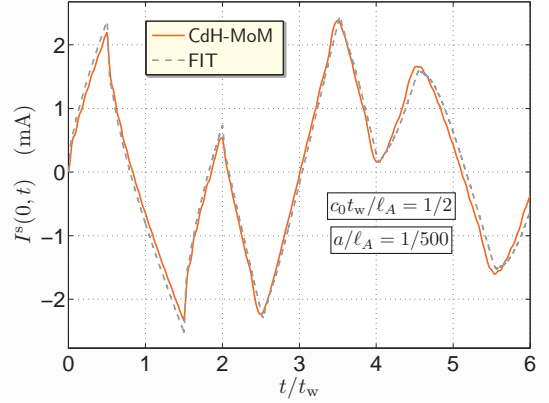


Fig. 3. Electric-current response of the thin-wire antenna in the absence of the ground plane.

current responses of the antenna from the previous example, but now in the presence of ground plane. The distance of the wire from the ground plane is first  $z_0/\ell_A = 1/5$ . Figure 4a shows the resulting pulse shapes as calculated using (21) (CdH-MoM) and its TL approximation (26) (CdH-MoM-TL) with  $N_A = 49$  and  $M = 601$ , again. Apparently, since the relative height of the thin-wire antenna above the ground plane,  $z_0/c_0 t_w = 2/5$ , is not sufficiently small, the TL approximation is not appropriate in this case. But, the full CdH-MoM solution agrees with the FIT based solution well, again. Owing to the presence of image-source contributions (cf. Eqs. (21) and (28)), the inclusion of the ground plane approximately doubles the time to fill the TD impedance array.

On the other hand, if the antenna is relatively close to the ground plane, the approximate expression (26) can replace Eqs. (21) to estimate the current distribution in a speedy manner. This observation is exemplified in Fig. 4b, where the current responses are calculated for  $z_0/\ell_A = 1/20$ . Since we keep  $c_0 t_w / \ell_A = 1/2$ , the height of the wire is a tenth of the excitation pulse's spatial support now, i.e.  $z_0/c_0 t_w = 1/10$ . While the CdH-MoM solution and its TL approximation then converge to each other, the use of the TL approximation is owing to its simplicity computationally more efficient. In the actual case with  $N_A = 49$  and  $M = 601$ , the TL approximation was about  $5 \times$  faster compared to the full CdH-MoM solution. Finally, it is noted that this TD result can be validated with the aid of the pertaining TD analytical solution based on the standard TL theory (see [5, Eq. (17.26)]). The discrepancies between the CdH-MoM-TL solution and the analytical one can be attributed to inevitable accumulated and space-time discretization errors. It has been verified that the correlation between the results can be improved by using a denser discretization of the solution domain.

In the third example, we shall analyze the pulsed EM field signal transfer between two thin-wire antennas,  $\Gamma_A$  and  $\Gamma_B$ , located at the height  $z_0/\ell_A = 1/20$  above the ground plane. The transmitting antenna,  $\Gamma_A$ , remains the same as in the previous scenarios. The receiving antenna,  $\Gamma_B$ , of length  $\ell_B = \ell_A/4$  is at its center loaded by a resistor of  $R^L = 100 \Omega$ . The

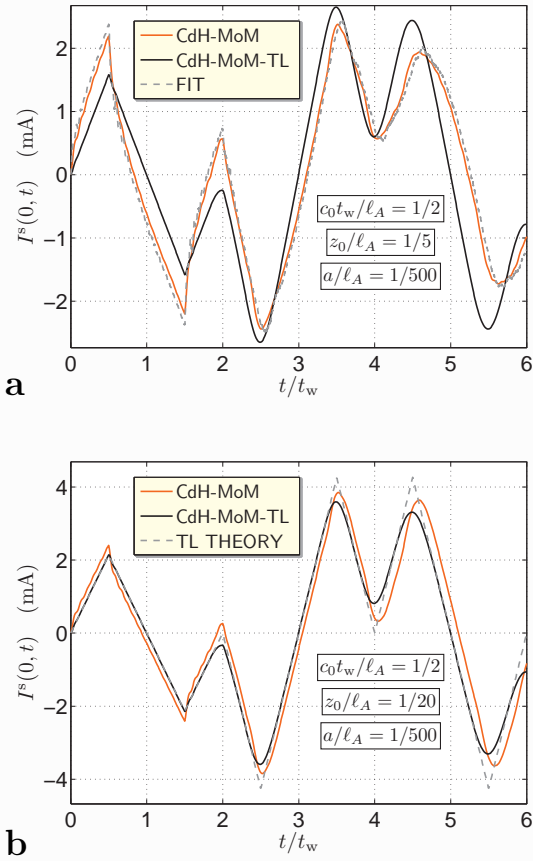


Fig. 4. Electric-current response of the thin-wire antenna in the presence of the ground plane. (a)  $z_0/\ell_A = 1/5$ ; (b)  $z_0/\ell_A = 1/20$ .

distance between the antennas is  $y_0/\ell_A = 1/5$  with  $x_0 = 0$ . As their radius we take  $a/\ell_A = 1/1000$ . The voltage induced across the load of the receiving antenna has been calculated via the impedance arrays (21)–(25) (CdH-MoM) as well as using their TL approximations (26) and (27) (CdH-MoM-TL) with  $N_A = 39$ ,  $N_B = 19$  and  $M = 1201$ . The calculated pulses are shown in Fig. 5a. Despite the relatively small distance from the ground plane,  $z_0/c_0 t_w = 1/10$ , their relative distance,  $y_0/c_0 t_w = 2/5$  is not sufficiently small to justify the approximation made in Eq. (27). Consequently, the CdH-MoM-TL approximation that predicts an instantaneous voltage response is not accurate. But, a good correspondence of the full CdH-MoM solution with the FIT based one is achieved, again. In this case, the total computational time of the CdH-MoM simulation amounted to about 26 s, out of which approximately 2/3 were spent to fill the TD impedance array and 1/3 to carry out the marching-on-in-time scheme, again. The TD impedance array can be filled roughly  $6 \times$  faster using the TL approximation.

Since the evaluation of the TD impedance array is very fast, the presented CdH-MoM approach lends itself to being used in design optimization procedures or/and parametric studies of ultra-wide-band wireless interconnects (e.g. [25], [26]). A simple parametric analysis is presented in Fig. 5b, where we demonstrate the influence of the receiving wire's

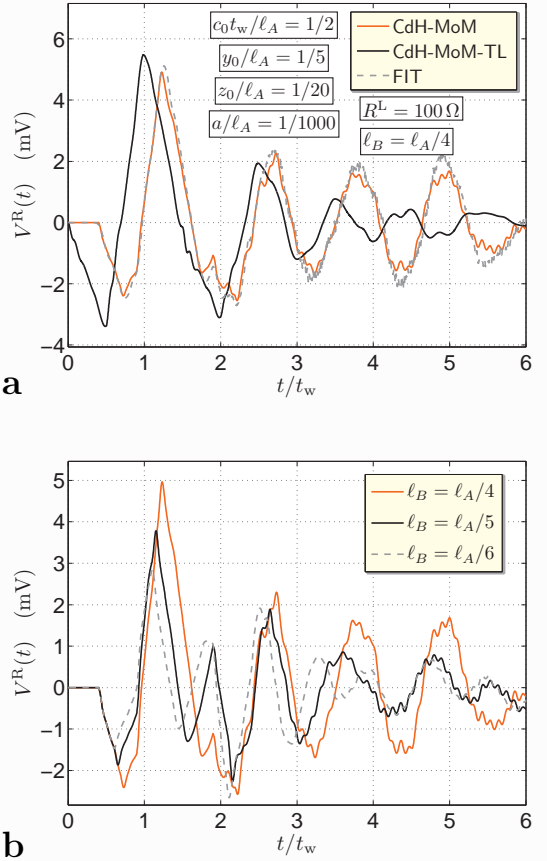


Fig. 5. Voltage pulses induced across the load of the receiving antenna. (a) Pulse shapes computed via the CdH-MoM, its TL approximation and FIT; (b) Influence of the receiving wire's length on the voltage response.

length on the induced-voltage pulse shape. The remaining configurational and excitation parameters were kept the same as in the previous example.

## VI. CONCLUSION

The CdH-MoM has been applied to analyze the EM transient response of (a system of) thin-wire antennas in the presence or absence of the ground plane. It has been demonstrated that such antenna configurations can be represented through a set of discrete time-convolution equations. Closed-form analytical expressions for the pertaining TD impedance arrays have been derived analytically with the aid of the CdH technique. The thus obtained elements of TD impedance arrays are expressed in terms of elementary functions only, which makes their implementation and evaluation in any computing environment such as MATLAB virtually effortless. Their approximations pertaining to multi-conductor transmission lines are discussed. Illustrative numerical examples were presented and validated using a commercial 3-D EM computational tool.

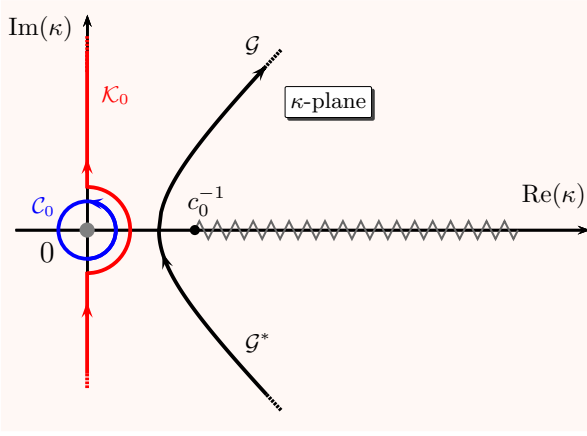


Fig. 6. Complex  $\kappa$ -plane.

## APPENDIX GENERIC FUNCTION

The generic integral representation has the following form

$$\hat{\Upsilon}(x, y, z, s) = -\frac{1}{2\pi i} \int_{\kappa \in \mathcal{K}_0} c_0^2 \Omega_0^2(\kappa) \frac{\exp(s\kappa x)}{s^3 \kappa^3} d\kappa \\ \times \frac{1}{2\pi i} \int_{\sigma=-i\infty}^{i\infty} \frac{\exp\{-s[-\sigma y + \Gamma_0(\kappa, \sigma)z]\}}{2\Gamma_0(\kappa, \sigma)} d\sigma, \quad (31)$$

for  $x, y \in \mathbb{R}$ ,  $\{z \in \mathbb{R}; z > 0\}$  and  $\{s \in \mathbb{R}; s > 0\}$ , and recall that  $\Gamma_0(\kappa, \sigma) = [\Omega_0^2(\kappa) - \sigma^2]^{1/2}$ , with  $\text{Re}(\Gamma_0) \geq 0$ , and  $\Omega_0^2(\kappa) = 1/c_0^2 - \kappa^2$ . Next,  $\mathcal{K}_0$  denotes the integration path that is indented to the right with a semi-circular arc with center at the origin,  $\kappa = 0$ , and a vanishingly small radius (see Fig. 6).

To transform (31) back to the original domain, we shall pursue the approach used in [5, Appendix G], for instance. The inversion procedure starts with the transformation of the inner integral with respect to  $\sigma$ . In the first step, its integrand is first continued into the complex  $\sigma$ -plane away from  $\text{Re}(\sigma) = 0$ . Consequently, using Cauchy's theorem and Jordan's lemma, the integration contour in the complex  $\sigma$ -plane is deformed into the so-called CdH path along which  $-\sigma y + \Gamma_0(\kappa, \sigma) = u \varrho \Omega_0(\kappa)$  is satisfied for all  $\{u \in \mathbb{R}; u \geq 1\}$  with  $\varrho = (y^2 + z^2)^{1/2}$ . Solving the latter equation for  $\sigma$ , we obtain a parametrization of the CdH hyperbolic path, say  $\mathcal{C} \cup \mathcal{C}^*$ , where  $*$  denotes the complex conjugate. It is noted that the CdH path intersects  $\text{Im}(\sigma) = 0$  at  $\sigma(u = 1) = -(y/\varrho)\Omega_0(\kappa)$ . Therefore, since we assume that  $\varrho > 0$ , the CdH path cannot not intersect the horizontal branch cuts extending along  $\{\Omega_0(\kappa) \leq |\text{Re}(\sigma)| < \infty, \text{Im}(\sigma) = 0\}$ . In the subsequent step, the integrations along  $\mathcal{C}$  and  $\mathcal{C}^*$  are combined and the parameter  $u$  is introduced as the new variable of integration. The thus obtained integral with respect to  $u$  is used in Eq. (31). Here, the order of the integrations with respect to  $\kappa$  and  $u$  are interchanged and in the resulting integrand with respect to  $\kappa$  is continued analytically away from  $\text{Re}(\kappa) = 0$ . It is noted that the integrand shows the triple pole singularity at  $\kappa = 0$  and the algebraic branch cuts along  $\{1/c_0 \leq |\text{Re}(\kappa)| < \infty, \text{Im}(\kappa) = 0\}$  due to  $\text{Re}(\Omega_0) \geq 0$  (see Fig. 6). Again, under the application of

Cauchy's theorem and Jordan's lemma, the indented path  $\mathbb{K}_0$  is deformed into the hyperbolic CdH path, say  $\mathcal{G}$  and  $\mathcal{G}^*$ , along which  $-\kappa x + u \varrho \Omega_0(\kappa) = \tau$  is met for  $\{\tau \in \mathbb{R}; \tau \geq R(u)/c_0\}$  with  $R(u) = (x^2 + u^2 \varrho^2)^{1/2}$ . In addition, if  $x > 0$ , the contribution of the pole singularity is taken into account by adding the integration around  $\mathcal{C}_0$ . Its evaluation via Cauchy's formula is straightforward. Upon combining the contributions from  $\mathcal{G}$  and  $\mathcal{G}^*$  and introducing the parameter  $\tau$  as the new variable of integration, we achieved the mapping of the inner integration from  $\kappa$  to  $\tau$ . In the obtained integral expression consisting of integrals with respect to  $u$  and  $\tau$ , we change the order of integrations. The thus obtained inner integral with respect to  $u$  is carried out analytically, which leads to the integral resembling the form of Laplace transformation (see Eq. (6)). Consequently, with the aid of Schouten-Van der Pol theorem [14, p. 1056] relying on Lerch's uniqueness theorem [23, Appendix], the inverse Laplace transform can be effectuated at once. This leads to a convolution-type integral with respect to  $\tau$  that can be evaluated in closed form, again. Pursuing this approach, we finally arrive at the TD original of (31) in the following form

$$\Upsilon(x, y, z, t) = \left\{ (c_0^2 t^2 + \varrho^2 - x^2) \log \left[ \frac{c_0 t + (c_0^2 t^2 - \varrho^2)^{1/2}}{\varrho} \right] \right. \\ \left. - 2c_0 t (c_0^2 t^2 - \varrho^2)^{1/2} \right\} \frac{\text{H}(x)\text{H}(c_0 t - \varrho)}{4\pi} \\ - \left\{ (c_0^2 t^2 + \varrho^2 - x^2) \log \left[ \frac{c_0 t + (c_0^2 t^2 - \varrho^2)^{1/2}}{R + |x|} \right] \right. \\ \left. - 2c_0 t (c_0^2 t^2 - \varrho^2)^{1/2} + 4|x|(c_0 t - R/2) \right\} \\ \times \frac{\text{sgn}(x)\text{H}(c_0 t - R)}{8\pi}, \quad (32)$$

where  $R = (x^2 + \varrho^2)^{1/2}$ ,  $\varrho = (y^2 + z^2)^{1/2}$  and  $\text{sgn}(x) = 2\text{H}(x) - 1$ . With reference to the standard TL theory assuming a relatively small conductor's height over the ground plane, it can be observed that

$$\Upsilon(x, 0, a, t) - \Upsilon(x, 0, 2z_0, t) \stackrel{\text{TL}}{\simeq} \frac{\log(2z_0/a)}{2\pi} \Psi(x, t), \quad (33)$$

as  $z_0 \downarrow 0$  and  $a \ll z_0$ , where [7, Eq. (17)]

$$\Psi(x, t) = \frac{1}{2} (c_0^2 t^2 - x^2) \text{H}(x)\text{H}(t), \quad (34)$$

for all  $x \in \mathbb{R}$  and  $t \in \mathbb{R}$ . These results are used in the main text to construct TD impedance arrays pertaining to various thin-wire antenna configurations.

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