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A Dynamic Stochastic Model for Evaluating Congestion and Crowding Effects in Transit Systems

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Abstract

One of the most common motivations for public transport investments is to reduce congestion and increase capacity. Public transport congestion leads to crowding discomfort, denied boardings and lower service reliability. However, transit assignment models and appraisal methodologies usually do not account for the dynamics of public transport congestion and crowding and thus potentially underestimate the related benefits.

This study develops a method to capture the benefits of increased capacity by using a dynamic and stochastic transit assignment model. Using an agent-based public transport simulation model, we dynamically model the evolution of network reliability and on-board crowding. The model is embedded in a comprehensive framework for project appraisal.

A case study of a metro extension that partially replaces an overloaded bus network in Stockholm demonstrates that congestion effects may account for a substantial share of the expected benefits. A cost-benefit analysis based on a conventional static model will miss more than a third of the benefits. This suggests that failure to represent dynamic congestion effects may substantially underestimate the benefits of projects, especially if they are primarily intended to increase capacity rather than to reduce travel times.

Keywords: Transit Assignment; Capacity; Dynamic Congestion; Agent-based Simulation; Cost-Benefit Analysis

1. Introduction

Transit assignment models (TAM) are used for predicting the distribution of passengers over a transit network. These models therefore play a critical role in evaluating the benefits of alternative network extensions as part of project appraisal. In many cities, insufficient capacity is perceived as the most serious problem in the public transport system, resulting in crowding, unreliability and long waiting times. However, appraisal methodologies for projects meant to increase capacity are relatively less well developed than methodologies for projects aiming to reduce travel times. Neglecting congestion-relief benefits results in an underestimation of the total benefit of an investment.

We present a method to capture the benefits of improved capacity more adequately by using a dynamic and stochastic TAM which accounts for dynamic congestion and crowding effects. The capabilities of the model are illustrated with a case study of a planned metro extension in Stockholm, Sweden, which exemplifies the magnitude of congestion-related effects when compared with nominal travel time savings as well as with the results of a conventional transit assignment model.

In the following, *congestion* in public transport refers to phenomena that are caused by high density of passengers and/or vehicles, which results in decreased service performance. A more saturated network element leads to an increase in the generalized travel cost. One of the effects of congestion is *crowding*, which refers to lower on-board comfort as the on-board load increases.

The main contributions of the study are:

- To the best of our knowledge, this is the first study that considers the dynamics of public transport congestion in appraisal of large public transport investments
- The development of a dynamic and stochastic TAM which represents the evolving interrelation between service reliability and passenger crowding and delay
- The model is implemented in an agent-based public transport simulation model
- The model is integrated in a project appraisal framework and the projected welfare benefits are compared with the results obtained from a state-of-the-practice static assignment model
- A simulation study of the congestion and crowding impacts of a large-scale real-world case study where an extensive but overloaded bus network in Stockholm is partially replaced by a metro line
- A demonstration of how the proposed model captures congestion and crowding relief benefits which are neglected by conventional static models
- Our simulation results suggest that the benefits that stem from including dynamic congestion and crowding effects in the analysis amount to at least one third of the total passenger benefits

Three distinct public transport congestion effects are considered: (1) *On-board discomfort* - crowding in the vehicles increases the value of time of passengers and hence their generalized travel cost; (2) *Denied boarding* - if the vehicle has no residual capacity, some passengers will be

denied boarding and have to wait for the next vehicle. (3) *Irregular vehicle arrivals* - boarding and alighting passenger flows as well as on-board passenger load are among the main determinants of dwell times at stops. The relation between passenger flows, dwell times and headways between successive vehicles results in a positive feedback loop that amplifies fluctuations in headways and gives rise to the bus bunching phenomenon. Insufficient capacity can therefore result in delays and reduced service reliability.

The developed model addresses the main challenges that were identified by Liu et al. (2010) in their review of TAM, dealing with supply uncertainties and adaptive user decisions. They identified the dynamic loading process and the agent-based simulation as two potential approaches, which are both utilized in this study.

The remainder of this paper comprises six sections. Section 2 presents a literature review on modelling congestion in TAM. Section 3 presents the dynamic model we propose for modelling congestion effects. Section 4 presents the application and its specifications. Section 5 presents the results from the case study, followed by a discussion of their implications on the cost-benefit analysis as compared with a schedule-based assignment model in Section 6. Section 7 presents the main conclusions and outlines directions for further research.

2. Literature review

There is a growing literature on modelling congestion in TAM with a remarkable increase in interest in the last decade, see a review by Fu et al. (2012). TAMs are conventionally classified into frequency-based and schedule-based models, differing in their network supply representation and its implications for the passenger loading procedure. Previous studies have developed a number of approaches to address congestion in transit networks. Most of the developments have focused either on accounting for on-board discomfort or on considering capacity effects on passengers' queuing. Flow-dependent in-vehicle times were introduced already in the seminal work by Spiess and Florian (1989). They suggested penalizing congested links by assigning travel times that were increasing functions of the *flow-capacity ratio multiplier*, inspired by the BPR function used in traffic assignment. This approach was then adopted by Lam et al. (1999) and Hamdouch et al. (2011). de Palma et al. (2015) formulated the user equilibrium and optimal equilibrium for a crowding definition inspired by the BPR function as well as for two alternative step functions. Alternatively, the congestion effect could be considered through assigning weights to waiting times by computing the *effective frequency* (de Cea and Fernández 1993, Cominetti and Correa 2011), hence shifting the travel impedance caused by congestion from links to nodes. This approach is based on queuing theory where waiting times becomes infinite at flow saturation. Szeto and Jiang (2014) formulated frequency-based TAM as a link-based variational inequality where the travel cost function includes a term for the additional waiting time due to in-vehicle crowding when headways are assumed to be perfectly irregular.

Both the flow-capacity ratio multiplier and the effective frequency methods discourage passengers from choosing saturated links. However, they do not guarantee that capacity will not be exceeded. Nuzzolo et al. (2001) and Cepeda et al. (2006) introduced an infinite

penalty for exceeding total capacity in schedule- and frequency-based assignment models, respectively. Similarly to static traffic assignment models, static TAMs do not guarantee that capacity is not exceeded, as all passenger demand is loaded onto the network even if it cannot be absorbed by the capacity available. Cepeda et al. (2006) applied a capacitated equilibrium static transit assignment model for the Stockholm transit network. The iterative process reduced the number of oversaturated links but retained flow/capacity ratios exceeding one without reaching a feasible flow distribution. This is especially important for highly-saturated networks where capacity constraints are binding for important network elements.

The static representation of service capacity can result in unrealistically long travel times when using the effective frequency method. Kurauchi et al. (2003) suggested addressing this limitation by considering the *failure-to-board probability*. Further developed in Schmöcker et al. (2008), a quasi-dynamic frequency-based model is used where the share of passengers that exceeds the residual capacity in the respective time period is transferred to the next period. Poon et al. (2004) and Hamdouch and Lawphongpanich (2008) adopted this approach in schedule-based models where capacity constraints are satisfied at the individual vehicle level by introducing new arcs between successive vehicle trips. The impact of denied boarding on prolonged waiting times depends on the queuing model assumed. Trozzi et al. (2013) modelled FIFO priority rules by introducing a bottleneck queue model for deriving the excess queuing time. Similarly to the discriminative effect of capacity constraints, the on-board discomfort effect does not affect all passengers uniformly as implied by the aforementioned flow-capacity ratio method. Sumalee et al. (2009) and Schmöcker et al. (2011) therefore introduced *fail-to-sit probability* to satisfy the set of priority rules and the seat capacity constraint. In-vehicle seat priorities were also incorporated in a static and deterministic macroscopic TAM developed by Leurent et al. (2014).

Most transit systems provide sufficient capacity when considering average flow to capacity ratios. Nevertheless, passengers experience recurrent crowding and delays due to the dynamic interaction between supply uncertainty and passengers' decisions. These interactions underlie the evolution of congestion in public transport networks – an uneven spacing of vehicles and as a result an uneven distribution of passengers, resulting in increases in average waiting times and average crowding.

Recent developments have considered the role of service uncertainty in TAM. Szeto et al. (2013) modelled both waiting time and in-vehicle times as random variables which depend on the effective frequency embedded in a reliability-based user equilibrium. In their frequency-based TAM, travel time variability is modelled at the line level, and passengers are assumed to have perfect information of service reliability. Similarly, Leurent et al. (2014) introduced a fixed vehicle delay as a function of line irregularity. The schedule-based TAM presented in Hamdouch and Lawphongpanich (2008) was extended by Hamdouch et al. (2014) through introducing the covariance of travel time between links in the space-time diagram, whereas dwell times were assumed negligible. While these advancements contribute to the consideration of reliability in the TAM domain, none of the abovementioned studies captures the dynamic congestion effects of the interaction between service uncertainty and passenger flows. Service reliability propagates dynamically in transit systems with the bunching

phenomenon being the most noticeable phenomenon (Cats et al. 2011), where congestion reinforces this process (Babaei et al. 2014).

Previous studies that evaluated the impact of congestion were limited to estimating crowding by considering the ratio between average demand and average supply (e.g. number of seats) (Prud'homme et al. 2012, Pel et al. 2014). This implies that increased capacity has a uniform impact on on-board crowding without considering load variations (Li and Hensher 2011). In order to address these limitations, our work adopts a dynamic modelling approach to account for the on-board congestion effects mentioned in Section 1.

3. Modelling the effects of congestion in public transport

3.1 Public transport simulation modelling approach

An agent-based simulation model enables the emulation of public transport dynamics by representing individual vehicles and passengers. BusMezzo, a dynamic public transport simulation model, is used in this study for modelling the effects of capacity and congestion in public transport. The progress of public transport vehicles between stops is modelled within a joint car and public transport mesoscopic simulation model, while time at stops is determined by their interaction with passengers at stops. Different public transport modes – such as metro, light rail, commuter trains and buses - have distinct vehicle types, operating speeds, travel time variability and dwell time functions, and are operated with different holding control strategies. These operational attributes yield different reliability and capacity levels depending on service design and right-of-way. Each vehicle is assigned with a chain of trips that is undertaken during the simulation period. The explicit modelling of vehicle scheduling enables capturing the dependency between successive vehicle trips and the potential propagation of delays from trip to trip. A detailed description of the supply representation as well as model validation, where the model's capability to replicate the bunching phenomenon is demonstrated, is available in Toledo et al. (2010).

The dynamic and disaggregate representation of both public transport supply and demand in BusMezzo explicitly models the underlying sources of congestion – supply uncertainty, load variations and vehicle capacity constraints. This enables the model to replicate how congestion evolves and determines system performance and ultimately influences passenger travel time components. In the following we formulate these relations and their representation in BusMezzo which enables their integration in the generalized travel cost function, which in turn is a key part of the project appraisal. Figure 1 illustrates the congestion-related interactions between passenger flows (denoted by rectangles), vehicle time components (ovals) and passengers' travel time components (highlighted cylinders), where the number of seats and the total on-board capacity are taken as parameters (parallelograms). Passenger flow relations are depicted by dashed arrows, while travel time relations are displayed using solid arrows. Vehicle dwelling and running times determine arrival times at subsequent stops. The difference between successive vehicle arrival times at the same stop, the headway, determines passenger arriving flows and their waiting times. The difference between vehicle arrival times at successive stops determines nominal passenger in-vehicle times while their perception depends on on-board crowding. It is evident that the

passenger travel time components which are included in the generalized travel cost are the outcome of complex relations between passenger flows and vehicle movements. These components are obtained from the simulation model based on a sequence of stochastic and dynamic interactions that are explained in the following sections along with the respective notations and mathematical definitions.

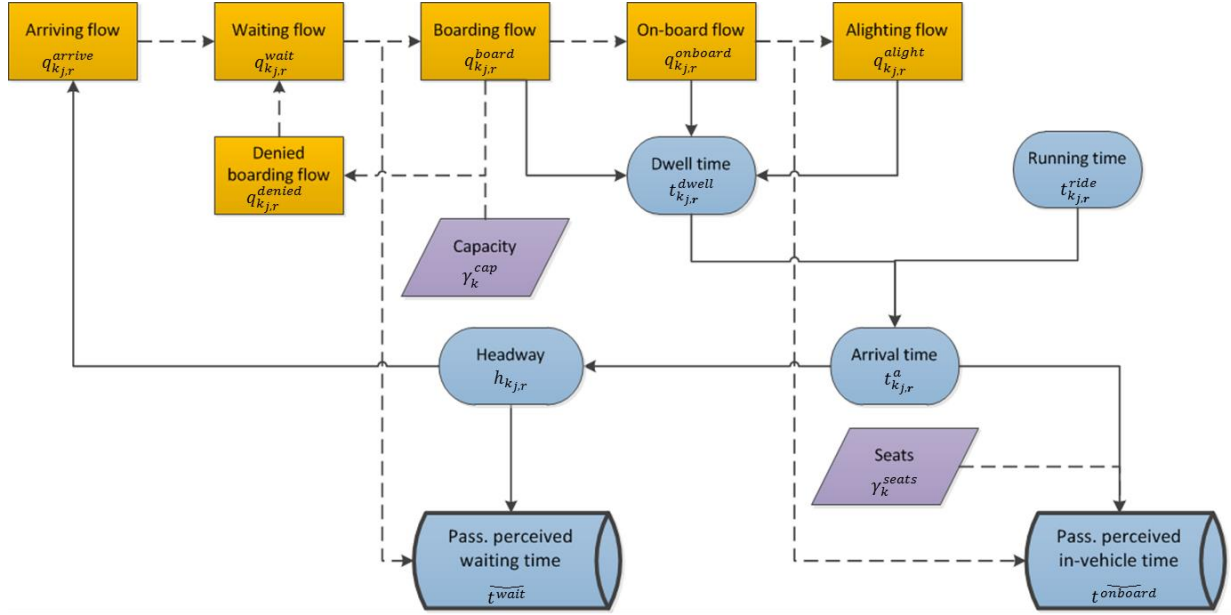


Figure 1: Relations between passenger flows, vehicle and passenger time components

The sets of public transport stops and lines are denoted by S and L , respectively. Each stop $j \in S$ may be operated by one or several lines. A line $l \in L$ is defined by a sequence of stops $l = (s_{l,1}, s_{l,2}, \dots, s_{l,|l|})$, and we let $j \in S_l$ mean that stop j is on line l . The set of vehicle trips that operate line l is denoted K_l . The set of vehicle trips that serve stop j is denoted by a sequence of trip arrivals $K_j = \{k_{j,1}, k_{j,2}, \dots, k_{j,\sum_{l \in L_s} |K_l|}\}$, and we let $l \in L_s$ mean that line l serves stop j . Travel demand is connected to the network through a subset of OD nodes, $S_{od} \subseteq S$. Without loss of generalization, let us consider a stop served by a single line where all incoming flows originate at stop j . Other incoming passenger flows – walking from nearby stops or alighting and interchanging at this stop – are secondary flows that are governed by the same processes discussed below.

The following sections describe how the three congestion effects – binding capacity constraints, deteriorating service reliability and on-board discomfort – are modelled in BusMezzo.

3.2 Dynamic passenger path choice

The flows of boarding and alighting passengers are obtained from numerous interdependent passenger decisions. The progress of individual passengers is modelled in BusMezzo as a sequence of travel decisions which are formulated as discrete random utility choices. An initial choice-set generation model results in a set of attractive path alternatives for each origin-destination pair. A path alternative $a \in A^{od}$ is a member of the path set for origin o to

destination d and is defined by an ordered set of stops, lines and connection links (Cats et al. 2011).

Each decision is triggered by a simulation event (e.g. vehicle arrival) and is defined by the need to choose the next path element (stop, vehicle or walking link) by evaluating the utility associated with each travel alternative. The utility that passenger n attaches to action g , $v_{n,g}$, is computed as the logsum over the path set $A^g \subseteq A^{od}$ associated with the action:

$$v_{n,g} = \ln \sum_{a \in A^g} e^{v_{n,a}} \quad \forall n \in N, g \in G \quad (1)$$

where N is the set of all passengers, G is the set of alternative actions that are available in the particular decision context and $v_{n,a}$ is the utility associated with path alternative a . The deterministic part of the utility function for a single path alternative takes the form

$$v_{n,a} = \boldsymbol{\beta}_{n,a} \widetilde{\mathbf{z}}_{n,a} \quad \forall n \in N, a \in A^{od} \quad (2)$$

where $\boldsymbol{\beta}_{n,a}$ is a vector of utility function coefficients and $\widetilde{\mathbf{z}}_{n,a}$ is the corresponding vector of expected values of path alternative attributes. Passengers take into consideration the anticipated travel attributes associated with each travel action based on passengers' preferences and expectations of downstream attributes. The utility that passenger n associates with path $a \in A^g$ is defined as

$$v_{n,a} = \beta_{n,a}^{walk} \widetilde{t}_{n,a}^{walk} + \beta_{n,a}^{wait} \widetilde{t}_{n,a}^{wait}(t) + \beta_{n,a}^{ivt} \widetilde{t}_{n,a}^{ivt}(t) + \beta_{n,a}^{trans} trans_a \quad \forall n \in N, a \in A^{od} \quad (3)$$

where $\widetilde{t}_{n,a}^{wait}(t)$ and $\widetilde{t}_{n,a}^{ivt}(t)$ are the time-dependent expected waiting time and in-vehicle time, respectively. $\widetilde{t}_{n,a}^{wait}(t)$ is the sum of waiting times along the respective path alternative where the waiting time for each leg is based on the joint frequency of the respective lines during the relevant time interval, assuming a random passenger arrival process. This expected value is substituted by real-time information if it is available en-route (see Cats et al. 2011 for details). Similarly, $\widetilde{t}_{n,a}^{ivt}(t)$ is the sum of in-vehicle times along the respective path alternative where the expected in-vehicle time on each leg is based on the timetable of the respective line between the relevant pair of stops. $\widetilde{t}_{n,a}^{walk}$ is the expected walking time and $trans_a$ is the number of transfers involved with the path alternative. $\boldsymbol{\beta}$'s are the corresponding parameters sampled from a normal distribution with a mean value based on the value-of-time associated with each attribute (see Section 4.4) and the standard deviation equal 25% of the respective value.

The logsum term in Eq. 1 expresses the utility of an action as the joint utility for a bundle of path alternatives. A multinomial logit (MNL) model is used for computing the probability of choosing a certain action, implying that the probability that passenger n will choose action g is

$$p_{n,g} = \frac{e^{v_{n,g}}}{\sum_{g \in G} e^{v_{n,g}}} \quad \forall n \in N, g \in G \quad (4)$$

The independence of irrelevant alternatives (IIA) property of the MNL model is partially counteracted by the action-path choice tree structure. To further counteract the IIA property, the most correlated paths (i.e., common stops and lines) are merged into hyper-paths. The dynamic path choice process involves three types of decisions: boarding, alighting and walking. Depending on the decision undertaken, $p_{n,g}$ is substituted by $p_{n,k_j,r}^{board}$, $p_{n,k_j,r}^{alight}$ and $p_{n,k_j,r}^{walk}$, as described in the following.

Passenger arrival. In the context of high-frequency services it is assumed that passengers do not consult timetables prior to their departures. Passenger arrival at the stop is hence regarded as a sum of Poisson arrival processes. The flow of passengers that arrived at stop j during the elapsed time between the arrival of trip $k_{j,r-1}$ and trip $k_{j,r}$ therefore also follows a Poisson distribution:

$$q_{k_{j,r}}^{arrive} \sim \text{Poisson} \left(\sum_{d \in S_{od} \cap S_i \setminus \{s_{i,1}, \dots, s_{i,j}\}} \lambda_{j,d} \cdot (t_{k_{j,r}}^a - t_{k_{j,r-1}}^a) \right) \quad \forall k_{j,r} \in K_j, j \in S \quad (5)$$

Where $\lambda_{j,d}$ is the arrival rate of passengers travelling from stop j to destination d , and $t_{k_{j,r}}^a$ is the time vehicle trip $k_{j,r}$ arrives.

Boarding decision. Let $x_{n,k_{j,r}} \in X_{k_{j,r}}$ denote the boarding decision of passenger n that upon the arrival of vehicle visit $k_{j,r}$, where $X_{k_{j,r}}$ is the set of waiting passengers. The number of passengers boarding $k_{j,r}$ is then

$$q_{k_{j,r}}^{board} = \min \left(\gamma_k^{cap} - q_{k_{j,r}}^{onboard} + q_{k_{j,r}}^{alight}, \sum_{x_{n,k_{j,r}} \in X_{k_{j,r}}} x_{n,k_{j,r}} \right) \quad \forall k_{j,r} \in K_j, j \in S \quad (6)$$

Where $x_{n,k_{j,r}} \in \{0,1\}$ is sampled based on the respective boarding probability, $x_{n,k_{j,r}} \sim \text{Bernoulli}(p_{n,k_{j,r}}^{board})$. Boarding decisions involve a binary decision where $G = \{board, stay\}$. γ_k^{cap} is the vehicle capacity and $q_{k_{j,r}}^{onboard}$ and $q_{k_{j,r}}^{alight}$ are the number of passengers on-board trip k prior to arrival at stop j and the number of passengers alighting at this stop, respectively. The number of boarding passengers depends thus on the waiting flow and the residual on-board capacity.

The number of waiting passengers upon the arrival of trip $k_{j,r}$, $q_{k_{j,r}}^{wait} = |X_{k_{j,r}}|$, is the sum of those that stayed in stop j after the departure of trip $k_{j,r-1}$ – either because they choose not to board the previous vehicle or were denied from boarding it – and the number of passengers that arrived during the elapsed time

$$q_{k_{j,r}}^{wait} = q_{k_{j,r-1}}^{wait} - q_{k_{j,r-1}}^{board} + q_{k_{j,r}}^{arrive} \quad \forall k_{j,r} \in K_j, j \in S \quad (7)$$

Alighting decision. Let $y_{n,k_{j,r}} \in Y_{k_{j,r}}$ denote the alighting decision of passenger n that rides trip $k_{j,r}$ which approaches stop j , where $Y_{k_{j,r}}$ is the set of passengers on-board this vehicle. The number of alighting passengers is then

$$q_{k_j,r}^{align} = \sum_{y_{n,k_j,r} \in Y_{k_j,r}} y_{n,k_j,r} \quad \forall k_j,r \in K_j, j \in S \quad (8)$$

Where $y_{n,k_j,r} \in \{0,1\}$ is sampled based on the respective alighting probability, $y_{n,k_j,r} \sim \text{Bernoulli}(p_{n,k_j,r}^{align})$. Alighting decisions includes a subset of the remaining stops along the line thus $G \subseteq S_l \setminus \{s_{l,1}, \dots, s_{l,j}\}$. Eq. 8 formulates the alighting flow as a sequence of Bernoulli trials which will be equivalent to a Binomial distribution in the special case of identical alighting probabilities.

Walking decision. Upon alighting, each passenger takes a walking decision and chooses whether to walk to the final destination, transfer at the current stop or walk to a nearby stop and transfer there. The choice-set composition consists of the current stop and walking alternatives for which the set of path alternatives with passengers' destination is not empty. Eq. 4 determines the probability that a passenger will choose a certain action. Passengers that choose to walk to other nearby stops will join the respective waiting queue.

The on-board occupancy, $q_{k_j,r}^{onboard} = |Y_{k_j,r}|$, is a state variable that is updated as a function of the boarding and alighting flows, where the latter is a function of the upstream on-board flow. Eq. 9 denotes the flow conservation update

$$q_{k_{j+1},r}^{onboard} = q_{k_j,r}^{onboard} - q_{k_j,r}^{align} + q_{k_j,r}^{board} \quad \forall k_j,r \in K_j, j \in S \quad (9)$$

3.3 Impacts of congestion on denied boarding

Eq. 1-9 formulates the relations between passenger flows and individual decisions. The simulation model keeps track of the passengers' occupancy of each individual vehicle along its trip. The number of seats and vehicle capacity (number of seats plus the maximal number of standing passengers) are specified for each vehicle type. The number of passengers that want to board trip k at stop j but are unable due to capacity constraints is calculated as

$$q_{k_j,r}^{denied} = \sum_{x_{n,k_j,r} \in X_{k_j,r}} x_{n,k_j,r} - q_{k_j,r}^{board} \quad \forall k_j,r \in K_j, j \in S \quad (10)$$

As shown in Figure 1, these passengers are retained in the flow of waiting passengers. It is assumed that passengers wishing to board an arriving vehicle form a boarding queue based on their arrival time at the stop (i.e. a FIFO queuing regime, as in Papola et al. (2009) and Trozzi et al. (2013)).

3.4 Impacts of congestion on irregular vehicle arrivals

Passenger boarding, on-board and alighting flows are all influenced by service reliability. Vehicle travel times consist of riding times between stops, where $t_{k_j,r}^{ride}$ denotes the vehicle riding time between stops j and $j+1$, and dwell times at stops, $t_{k_j,r}^{dwell}$. The arrival time of vehicle trip k at stop j can therefore be expressed as

$$t_{k_j,r}^a = \sum_{j=1}^{j-1} t_{k_j,r}^{ride} + \sum_{j=1}^{j-1} t_{k_j,r}^{dwell} \quad \forall k_j,r \in K_j, j \in S \quad (11)$$

Riding times between stops are composed of running times on links and delays at intersections which are computed based on speed-density functions and stochastic queuing models, respectively. The mesoscopic traffic simulation model detailed in Toledo et al. (2010) determines riding times between stops. The effect of passenger congestion on public transport operations is primarily manifested through the dwell time. The dwell time is a monotonically increasing function of the number of boarding, alighting and on-board passengers. The dwell time at stop j , $t_{k_{j,r}}^{dwell}$, is determined by the number of boarding and alighting passengers while taking into consideration the non-linear effect of on-board crowding based on Weidmann (1994)

$$t_{k_{j,r}}^{dwell} = \alpha_0 + \left[\alpha_1 \cdot q_{k_{j,r}}^{board} + \alpha_2 \cdot q_{k_{j,r}}^{alight} \right] \cdot \left[1 + \frac{3}{4} \left(\max \left\{ 0, \frac{q_{k_{j,r}}^{onboard} - \gamma_k^{seats}}{\gamma_k^{cap} - \gamma_k^{seats}} \right\} \right)^2 \right] \quad \forall k_{j,r} \in K_j, j \in S \quad (12)$$

Where the α 's are the dwell time function coefficients. The coefficients of flow-dependent dwell time functions are specified for different public transport services depending on the respective boarding regime and number of doors.

The relation between headway, passenger flows and dwell times creates a positive feedback loop in supply variations as longer headways are reinforced and escalate along the line. Hence, congestion and irregularity exercise a bi-directional positive correlation. This leads to the degradation of service reliability along the line which is known as bunching. Cats et al. (2012) demonstrated that the simulation model can replicate this phenomenon. Since a larger share of passengers experience the headways that are longer than average, average passenger waiting time increases with headway variation.

It is postulated that the waiting time imposed by denied boarding induces a greater disutility because it induces a delay. The total perceived waiting time consists therefore of the total waiting time for the first vehicle and waiting times for further vehicles in case of denied boarding as follows:

$$\overline{t^{wait}} = \sum_{j \in S} \sum_{k_{j,r} \in K_j} \left[\beta^{initial_wait} \cdot \sum_{x_{n,k_{j,r}} \in X_{k_{j,r}}} \left(t_{k_{j,r}}^a - t_{n,j}^a \right) + \beta^{denied} \cdot q_{k_{j,r}}^{denied} \cdot h_{k_{j,r}} \right] \quad (13)$$

Where $\beta^{initial_wait}$ and β^{denied} are the value-of-time weights assigned to waiting for first and the residual penalty for excess waiting time for further vehicles. $t_{k_{j,r}}^a$ and $t_{n,j}^a$ are the arrival times of trip $k_{j,r}$ and passenger n at stop j , respectively. $h_{k_{j,r}} = t_{k_{j,r+1}}^a - t_{k_{j,r}}^a$ is the headway upon departure between successive vehicle trips at stop j . The formulation accounts for repeated failures to board as the first term in Eq. 13 sums over passengers' waiting time whereas the second term sums passengers' waiting time beyond the initial waiting time. Note that the disaggregate supply and demand representation enables computing waiting times directly from vehicle's and passengers' arrival times, rather than estimating them based on aggregate theoretical distributions.

3.5 Impacts of congestion on on-board discomfort

In addition to uneven waiting times, service irregularity also results in uneven passenger loads. The dynamic operations and assignment model enables replicating the uneven distribution of passengers between vehicles that run on the same line and the corresponding discomfort factors. In contrast, models that estimate on-board discomfort based on average crowding levels (e.g. volume/capacity or load/seats ratios, e.g. Cepeda et al. 2006, Nuzzolo et al. 2012, Pel et al. 2014) will result in an underestimation of the congestion effect on comfort since more passengers experience overcrowded vehicles compared to less crowded vehicles.

The disutility of in-vehicle time depends on whether a passenger has a seat or has to stand as well as on the on-board crowding. Previous studies have considered various seating priority rules which determine the allocation of seats to passengers. The following hierarchical set of rules is implemented in BusMezzo:

- passengers prefer to sit rather than to stand;
- passengers on-board have priority over boarding passengers;
- passengers who intend to alight further downstream have priority over those that have a shorter remaining travel segment.

The second rule implies that sitting passengers remain seated unless they alight and that standing passengers who boarded at upstream stops $\{1, \dots, j-1\}$ have priority over passengers boarding at stop j . Previous studies calculated the probability of failing to get a seat based on the combination of rules (a) and (b) in frequency-based (Schmöcker et al. 2011) and schedule-based (Hamdouch et al. 2011) transit assignment models. The relation between service regularity and seat availability was formulated analytically by Babaei et al. (2014). In contrast, the seating priority rules are applied explicitly in BusMezzo based on the interaction between individual passengers and vehicle trips. Similarly to Hamdouch et al. (2011), the third priority rule is designed to reflect the inclination of a passenger to sit as a function of the remaining travel time instead of applying random or FIFO seat allocation.

While seat allocation rules are relevant for calculating utility at the individual level, they do not influence the calculation of total discomfort across the network. In other words, while determining *who* sits and who stands, they do not influence *how many* passengers sit or stand on a given trip segment. The on-board crowding effect is assumed to be proportional to travel time (e.g. Tirachini et al. 2013). Occupancy and the number of vehicle seats are sufficient to determine the number of sitting and standing passengers for each line segment. The in-vehicle discomfort factor is then embedded in the total perceived in-vehicle time as follows:

$$\begin{aligned}
 \overline{t_{onboard}} = & \\
 \sum_{l \in L} \sum_{k \in K_l} \sum_{j \in S_l \setminus \{s_{l,l}\}} & \left[t_{k_{j+1},r}^a - t_{k_{j,r}}^a \right] \cdot \left[\min(\gamma_k^{seats}, q_{k_{j,r}}^{onboard}) \cdot \beta^{ivt} \alpha^{sit} (q_{k_{j,r}}^{onboard}, \gamma_k^{seats}) + \right. \\
 & \left. \max(0, q_{k_{j,r}}^{onboard} - \gamma_k^{seats}) \cdot \beta^{ivt} \alpha^{stand} (q_{k_{j,r}}^{onboard}, \gamma_k^{seats}) \right] \quad (14)
 \end{aligned}$$

γ_k^{seats} is the number of seats available. β^{ivt} is the value of in-vehicle time and α^{sit} and α^{stand} are the crowding multipliers for sitting and standing passengers, respectively. Both crowding

factors are defined as a function of $q_{k,j,r}^{onboard}$ and γ_k^{seats} in order to reflect the extra discomfort that is induced by each extra passenger on all other passengers (see section 4.4).

3.6 Overall scenario evaluation

Sections 3.2-3.5 define the relations between the passenger flows (Eq. 5-10), vehicle (Eq. 11-12) and passenger travel time components (Eq. 13-14) illustrated in Figure 1. The impacts of alternative scenarios can be summarized in terms of changes in welfare, essentially the total utility of all passengers expressed in monetary terms when the total demand, N_{od} , is constant. With $W_n(\sigma)$ denoting the welfare of passenger n in scenario σ , the total welfare in scenario σ is

$$W(\sigma) = E[\sum_{o \in S_{od}} \sum_{d \in S_{od}} \sum_{n \in N_{od}} W_n(\sigma)] = \sum_{o \in S_{od}} \sum_{d \in S_{od}} \sum_{n \in N_{od}} \beta_n z_n(\sigma) + u_n \quad (15)$$

where $N_{od} = \sum_{j \in S} \sum_{k_{j,r} \in K_j} q_{k_{j,r}}^{arrive}$ is the population of passengers for scenario σ , β_n is a vector of monetary cost valuations of the corresponding $z_n(\sigma)$ vector of experienced travel attribute values under scenario σ and u_n is the utility from other sources than travel. When the auxiliary utility, u_n , is constant, the welfare change between scenarios is the change in total generalized travel cost. This simplifies calculations, since it is sufficient to calculate aggregate generalized costs to compute welfare changes. The specification of the generalized cost, in particular the weights of the value-of-time components, is presented in the following section.

4. Application

The modelling/appraisal framework presented above was applied for a case study of a metro line extension in Stockholm, Sweden. The line extension is partly motivated by the high congestion levels experienced by passengers using the existing bus corridor. During peak periods, congestion results in on-board crowding, poor service reliability and in some cases denied boardings. The case study analyses the congestion effects with and without the metro line extension in order to assess the benefits attributed to congestion relief. In the following, the case study and the scenarios, including the benchmark static transit assignment model are presented in Section 4.1, followed by details on network, demand and value-of-time specifications which are presented in Sections 4.2-4.4, respectively.

4.1 Case study description

The Stockholm's metro system consists of 100 stations and 105 km of tracks. The metro system was mainly built 1950-1980 and has not been substantially expanded since then. The most substantial of the newly proposed expansion plans of the metro network is the extension of the Blue Line (Figure 2). The proposal is to extend the Blue Line from its current end station ('Kungsträdgården') in the historical city centre to the southern island of the inner-city Södermalm and further south-east to the suburban area of Nacka. This new subway stretch would serve an urban agglomeration corridor south-east of the city centre and will replace most of the buses that currently serve this corridor. The Stockholm Public

Transport Authority estimates that the demand in this corridor will be one of the four highest in the region in 2030 (SLL, 2013). The metro extension is expected to relieve both on-board crowding and bus traffic congestion, and hence shorten the experienced travel time for both passengers in the new metro and for the remaining bus passengers. To this end, BusMezzo was used to simulate the effects of the metro line extension and to estimate the social benefits in terms of generalized travel time savings including congestion effects.

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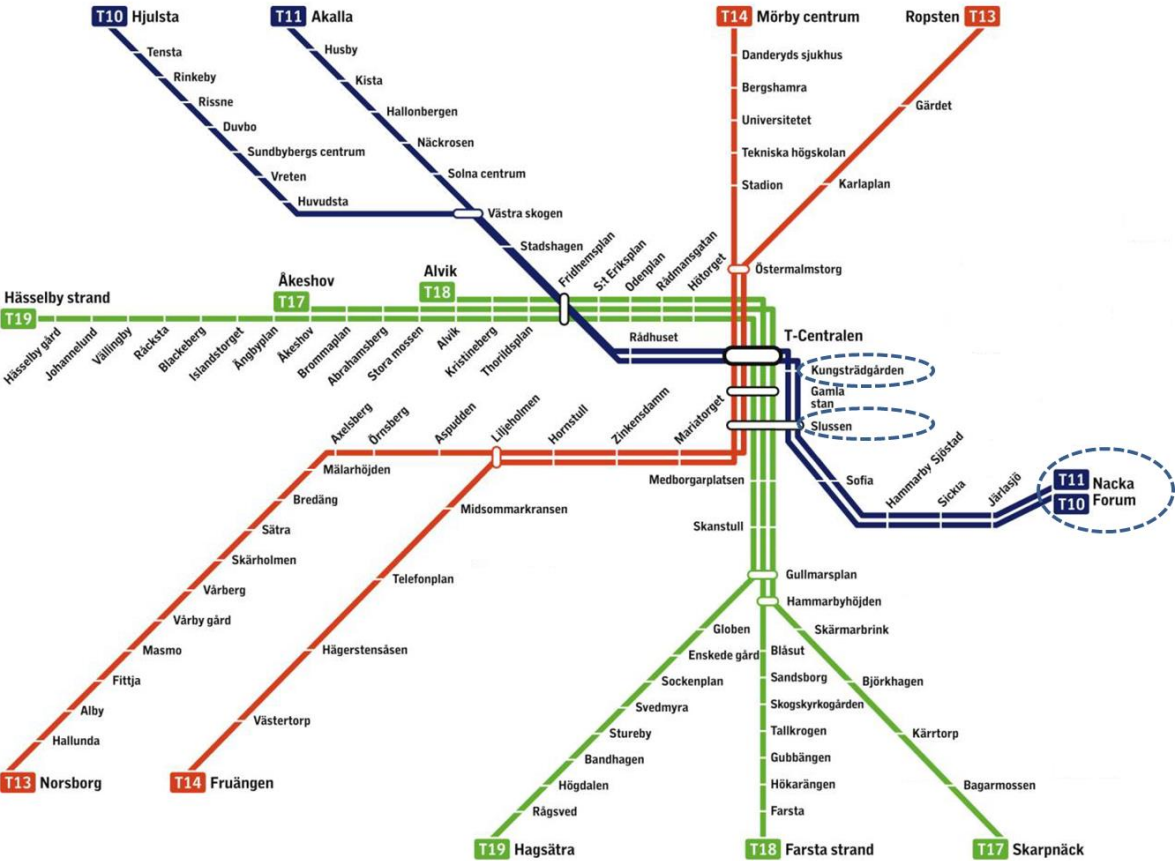


Figure 2: A schematic map of the extended metro network in Stockholm, with the suggested extension from *Kungsträdgården* to *Nacka Forum*. Currently, Nacka is connected to the metro network through bus and local train services to *Slussen*, a major interchange hub.

Two scenarios were simulated and analysed for the year 2030:

- I. *Do-nothing scenario (Base)* where the bus corridor is served with 200 buses in one direction during the rush hour
- II. *Extension scenario (TNacka)* where the Blue Line is extended to Nacka and the service on the bus corridor is adjusted so that some of the suburban bus lines terminate at the new end station of the Blue Line in Nacka Centrum.

Each scenario was simulated in BusMezzo, the dynamic transit assignment model presented in Section 3, as well as in PTV Visum (PTV AG 2012), a commercial software package for traffic analysis, forecasts and GIS-based data management which includes a conventional

static transit assignment model. The frequency-based public transport assignment model in Visum is based on the VIPS II algorithm (Jansson and Ridderstolpe 1992). This PTV Visum assignment model is used as the standard modelling tool of the Stockholm Public Transport Authority. In both the BusMezzo model and the Visum model, passengers take travel decisions based solely on service frequencies, scheduled travel times and network topology (i.e. disregarding crowding and risks of denied boarding). It is possible to include capacity constraints in the Visum route choice model, but in Visum (just as in other similar frequency-based static assignment models), the line capacity is simply the product of vehicle capacity and the number of vehicles, which in the studied cases exceeds demand by far. Visum without capacity constraints was therefore used as the benchmark model to demonstrate how the proposed model captures congestion-relief benefits.

4.2 Network representation

The transit network represented in BusMezzo in this study includes all lines with less than 15 minutes headways during the morning peak period (6:00-9:00). This gives a network of 70 lines consisting of all the metro lines, commuter trains, light rail trains, inner-city buses and suburban trunk buses. The network was coded in BusMezzo with detailed timetables, vehicle schedules and walking distances between stops. In total, 1050 stops are served by more than 2400 vehicle trips.

The bus lines connecting the southeast Stockholm region to the inner city were grouped into three distinct corridors, each of them consisting of multiple lines and all of them ending in the major interchange hub *Slussen* (Figure 2). Only these trunk corridors were considered in this study. Some of the lines stretched out beyond the study area limits. The headway distribution of these lines when entering the study area was estimated based on empirical automatic vehicle location (AVL) data and the respective expected occupancy was specified based on the outputs from the static transit assignment model, Visum. The initial occupancy of each vehicle was then assumed to be proportional to the respective headway.

BusMezzo enables the joint simulation of car traffic and public transport. However, in the absence of a calibrated car traffic OD-matrix for the case study network, interactions between cars and buses were not modelled directly, while traffic dynamics between buses were modelled endogenously. Bus travel time distributions between stops were estimated based on AVL data from 2013. Travel time between each pair of stops was therefore sampled from a shifted lognormal distribution with the minimum travel time equal to the free flow speed. Links with dedicated bus lanes obtained lower travel time variability while accounting for the dynamics between buses on links and at stops (e.g. queuing).

Given the importance of dwell times for congestion effects (Section 3.4), careful attention was given to the specification of the dwell time function for each public transport mode. The dwell time function reflects the corresponding vehicle type, number of doors, payment procedure and boarding regime. The impact of dwell time on congestion is especially important in the case of the bus lines to Nacka as boarding is allowed only from the front door and requires ticket validation next to the driver cabin.

4.3 Demand representation

Passenger demand was simulated only for the peak hour (7:00-8:00). The total number of public transport trips was assumed to be constant, i.e. induced demand due to modal shift towards public transport was disregarded. For the purposes of this paper, which is primarily to demonstrate how a dynamic simulation model can be used for evaluating congestion effects, this simplification is inessential. Approximately 125,000 passenger trips are initiated during this hour with 20,000 of them having either their origin or destination along the south-eastern corridor. The demand matrix was produced with the SIMS demand model (Algers et al., 1996), based on the travel time from models in Visum (for public transport) and Emme/2 (for other transport modes).

Congestion effects are directly included in the calculation of individual welfare outputs from BusMezzo, $W_n(\sigma)$. The mean value of the relative valuation of the utility function coefficients are described in the following section.

4.4 Monetary valuations of crowding, denied boarding and unreliability

Each of the consequences of capacity limitations – crowding, risk for denied boarding and unreliable waiting and travel times – increases the generalized travel cost. For appraisal, monetary valuations of each of these three phenomena are needed. The monetary values involved in this study are: $\beta_a = \{\beta^{walk}, \beta^{initial_waiting}, \beta^{denied}, \beta^{ivt}, \beta^{trans}\}$, the value of walking, initial waiting, waiting due to denied boarding, on-board times and transfer penalty, respectively.

The value¹ of in-vehicle time (before accounting for whether it was spent sitting or standing), β^{ivt} , is according to the Stockholm Public Transport Authority recommendations equal to the value for regional work trips in the Swedish Transport Administration guidelines, which is €6.9 per hour (Börjesson & Eliasson, 2014). This value forms the baseline for the subsequent valuations, since they are expressed as multipliers of the in-vehicle time. Waiting time for the first desired boarding – either at the first stop or when interchanging – and walking time – access, egress and when interchanging – are valued as $\beta^{initial_waiting} = \beta^{walk} = 2\beta^{ivt}$ (Wardman, 2004). Each interchange also has a fixed penalty equal to 5 minutes of in-vehicle time, $\beta^{trans} = 5\beta^{ivt}$ (Balcombe et al., 2004).

Waiting due to denied boarding presumably imposes a higher disutility per minute for passengers than normal waiting times since they are unpredictable, and moreover cannot be partly spent at home (or some equivalent) as normal waiting times can. The value of waiting time due to denied boarding can thus arguably be considered equivalent to the value of delay time. Börjesson et al. (2012) estimate a delay time multiplier of 3.5, meaning that the value of waiting time caused by denied boarding becomes $\beta^{denied} = 3.5 \cdot 2 \cdot \beta^{ivt}$.

Crowding is taken into account by multiplying $\beta^{in_vehicle}$ by factors that depend on the level of crowding. These multipliers are taken from the meta-study by Wardman and Whelan

¹ All values have been converted from SEK to € using 10 SEK = 1 €. Valuations are expressed in 2010 price levels.

(2011), and are found in Table 1. According to this meta-study, crowding affects the value of in-vehicle time for both seated and standing passengers. Seated passengers get multipliers from 0.95 to 1.71 when the occupancy, $q_{k,j,r}^{onboard}$, divided by the number of seats, γ_k^{seats} , increases from 50% to 200%. Standing passengers are only counted separately once all seats are taken (load factor > 100%); these multipliers range from 1.78 to 2.69. Note that the in-vehicle time multiplier increases as a non-linear function of the load factor.

Table 1: Crowding multipliers

Load factor ($q_{k,j,r}^{onboard}/\gamma_k^{seats}$)	Seated in-vehicle time multiplier α^{sit}	Standing in-vehicle time multiplier α^{stand}
50%	0.95	
75%	1.05	
100%	1.16	1.78
125%	1.28	1.97
150%	1.40	2.19
175%	1.55	2.42
200%	1.71	2.69

5. Results

To evaluate the benefits of the metro line extension, BusMezzo simulation runs and Visum assignments were conducted. Since BusMezzo is a stochastic simulation model, each scenario was analyzed based on the results of 10 simulation runs. This number of replications was determined to yield a maximum allowable error of less than 1% for the average passenger travel time. The execution time for a single run was less than 1 minute on a standard PC. The BusMezzo results are presented first, followed by a comparison with Visum results in terms of both passenger loads and travel times.

The inbound demand is approximately 13,000 passengers and the outbound demand is approximately 4,000 passengers in the morning rush hour. In the extension scenario, 35-45 % of the bus corridor passengers switch to the new metro extension in the inbound and outbound directions, respectively.

The BusMezzo results show a large variation in bus headways along the common corridor, leading to a large variation in on-board crowding in the do-nothing scenario. Figure 4 presents the distribution of vehicle occupancy in the most crowded bus corridor. More than 40% of the buses are predicted to become completely overloaded (load factor of 200 % resulting in crush capacity), while some buses are underutilized. This is a characteristic example of the bus bunching phenomenon. As expected, the metro extension makes this problem less prevalent. The number of overloaded buses decreases dramatically, yielding a more even passenger distribution. The number of underutilized buses with load factors below 40% decreases as well.

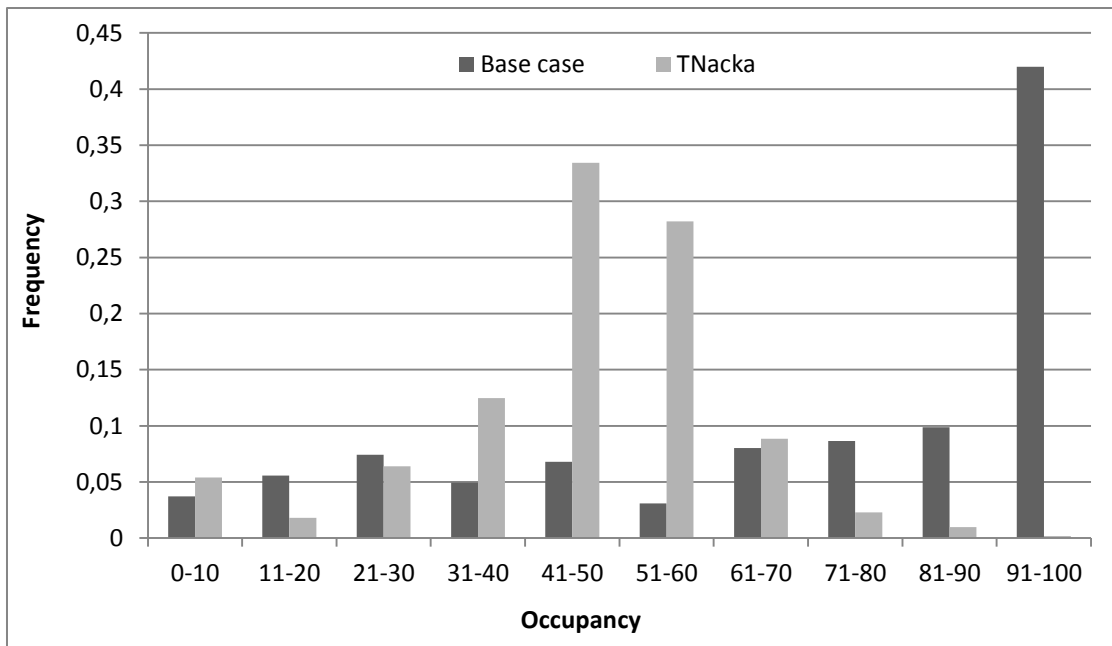


Figure 4: Occupancy distribution on line 474 (in the inbound direction, towards Slussen)

Due to the metro extension, passengers are able to reach their destinations with fewer interchanges, shorter walking distances and shorter in-vehicle times on average (Figure 5). The decrease in the number of overloaded buses leads both to shorter in-vehicle time (due to shorter dwell times) and to improved on-board comfort. The actual in-vehicle time is reduced by 9%, while the in-vehicle time weighted by crowding multipliers is reduced by 13%. The reduction of overloaded buses also leads to less denied boardings. In combination with a more regular service due to less crowding this leads to 25% shorter experienced waiting time, even though the bus service is actually less frequent in the scenario with metro extension.

The decrease in generalized travel time is 14% and 16% in the morning peak hour for the inbound and outbound directions, respectively. The congestion relief benefits caused by the shift from bus to metro are twofold: better comfort for passengers choosing the metro over the bus and better conditions for the remaining bus passengers due to less crowding, more even passenger loads and a more reliable service.

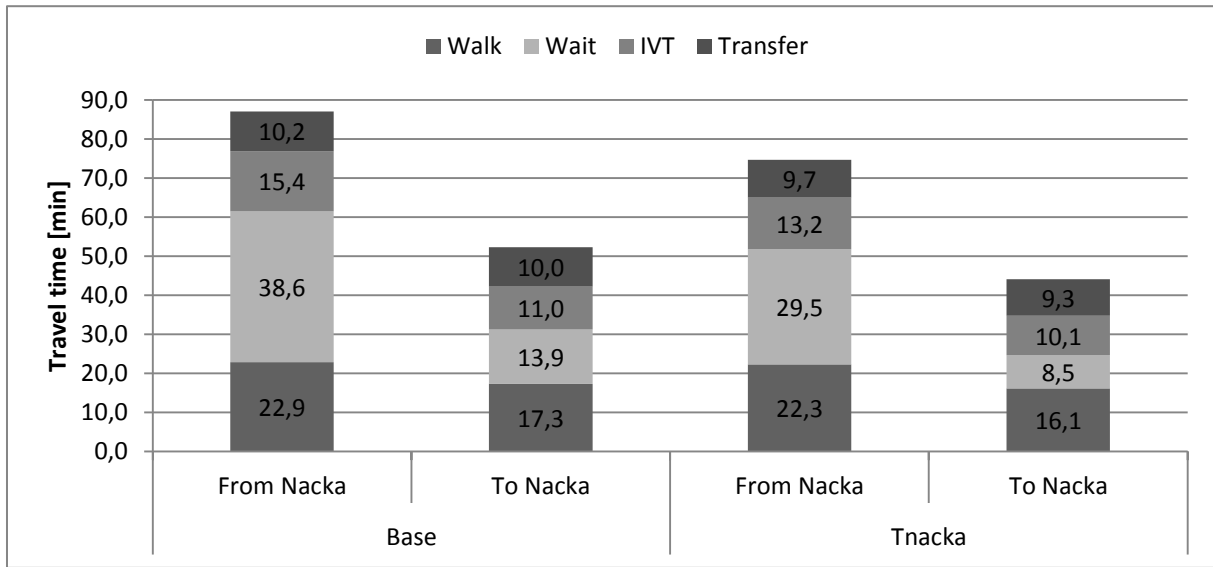


Figure 5: Average generalized travel time per passenger in BusMezzo

The difference between BusMezzo and Visum in terms of route volumes is shown in figures 6 – 9. For simplicity, flows that are irrelevant for the case study area are not generated in BusMezzo. In general, the relevant volumes are similar, but slightly lower in BusMezzo. The shift to the new metro is larger in Visum.

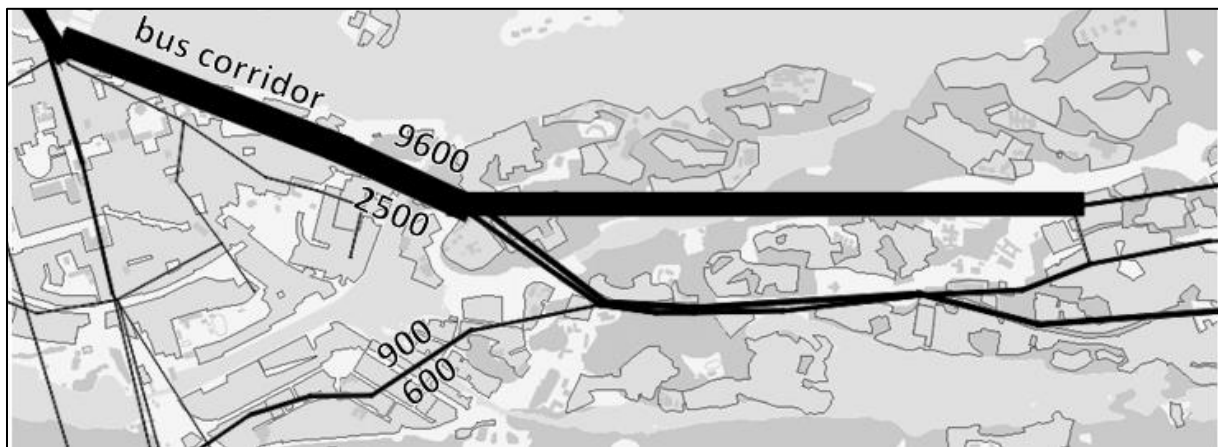


Figure 6: Link volumes during morning peak in base case in BusMezzo

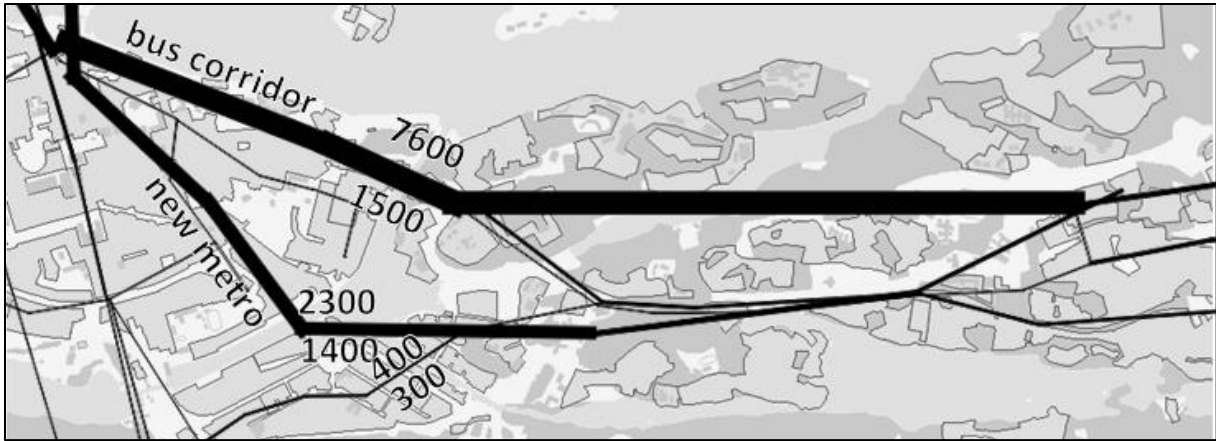


Figure 7: Link volumes during morning peak in TNacka scenario in BusMezzo

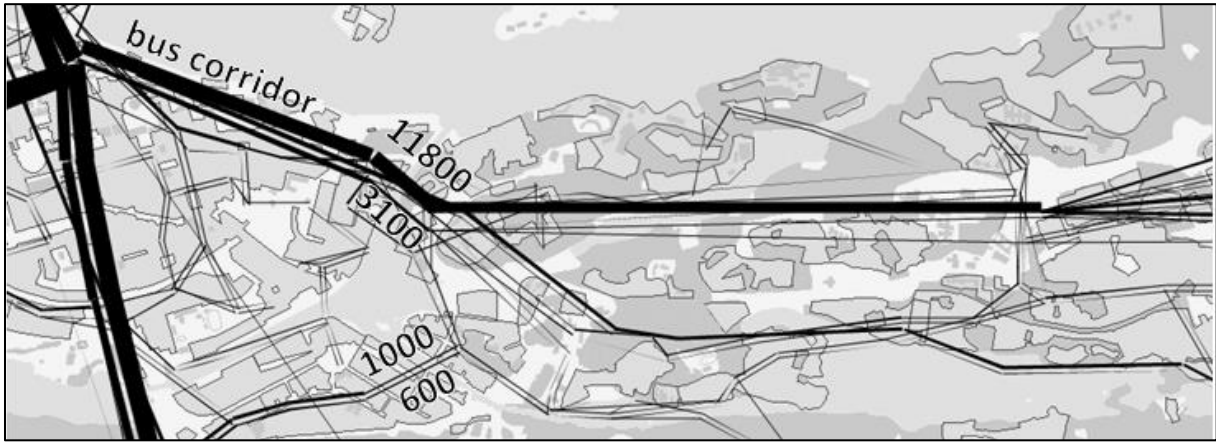


Figure 8: Link volumes during morning peak in base case in Visum

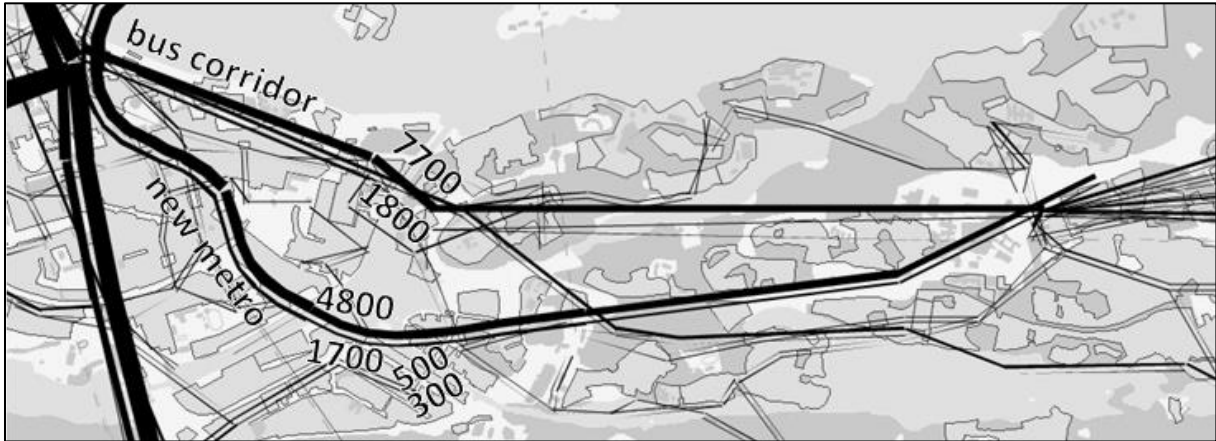


Figure 9: Link volumes during morning peak in TNacka scenario in Visum

Figure 10 shows the generalized travel time gains in the studied corridor, calculated for three different cases: (a) BusMezzo (as described in previous sections); (b) a deterministic and unconstrained instance of BusMezzo (where riding times and dwell times are constant, the in-vehicle multiplier is set to one and vehicle capacity is unlimited), and; (c) Visum. Due to differences in network coding and route choice representation in the two models, the proportion of the different travel time components is not identical, neither in the base case

nor in the TNacka scenario. As a result of this, the travellers in Visum walk longer to attain a shorter in-vehicle time and the total gain is larger in Visum than in the deterministic BusMezzo simulation. Link flows in BusMezzo are very similar for the deterministic and stochastic cases. However, as Figure 10 shows, differences in walk time and number of transfers do exist. In the stochastic simulation, a large part of the gains comes from waiting times, both in terms of denied boarding and extra waiting time due to irregular services. None of these effects exists in the deterministic or unconstrained simulation or in Visum.

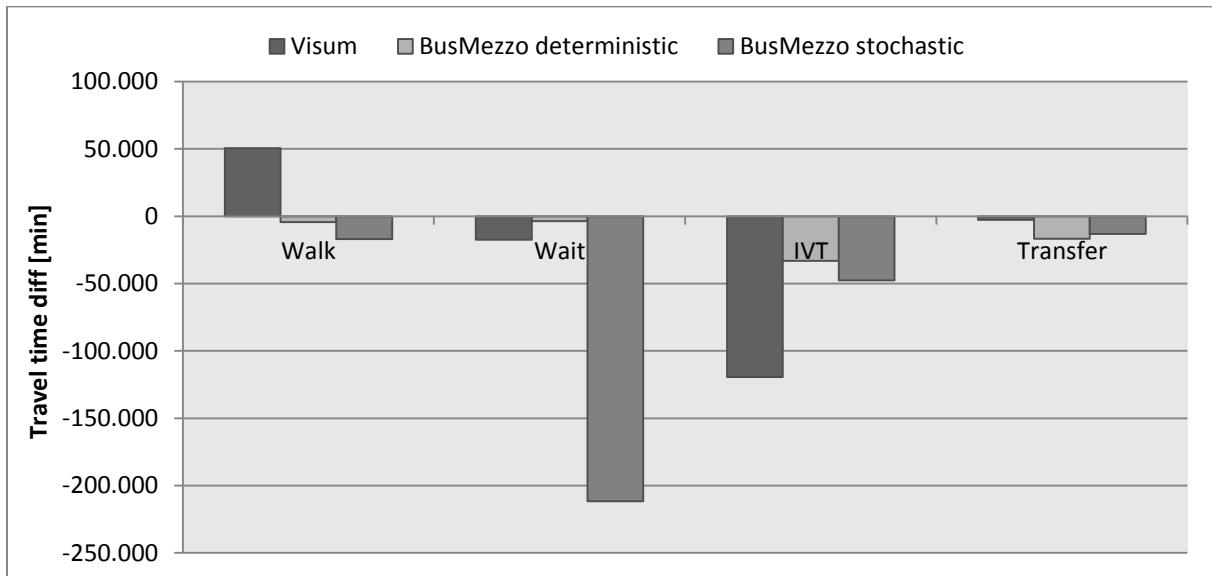


Figure 10: Comparison between generalized travel time gain components in BusMezzo and Visum

6. Implications for cost-benefit analysis

Since the alignment of the new metro line is not identical to the bus corridor, the extension of the Blue Line would also yield benefits that are not related to the studied bus corridor. The additional benefits coming from the areas not covered by the BusMezzo model were assessed by the static assignment in the Visum model covering the whole Stockholm area. In order to sum up benefits from the two models, a matrix of benefits per origin-destination pair was constructed.

The Visum model yields a total benefit of 1.1 million SEK per day for the entire O-D matrix, of which 0.6 million SEK are for trips that have their origin or destination along the south-eastern bus corridor. In the static model on-board crowding can only be measured as an average per line and on average the crowding level is rather low because of the large number of buses in operation. This makes the total benefit from the static assignment virtually unaffected of whether in-vehicle time is multiplied by the crowding factor or not as it only adds 3 % to the total generalized travel time.

The BusMezzo analysis was performed for the morning peak hour, neglecting any congestion effects in the morning rush outside the peak hour. The benefit is assumed to be equal in magnitude in the afternoon (the afternoon peak load is usually less pronounced but longer in duration), while the rest of the day is assumed to be unaffected by crowding and hence only

receive the travel time benefits from the Visum model. Hence, the inclusion of congestion effects as estimated by BusMezzo were based on the conservative assumptions that congestion-related benefits are limited to the morning and afternoon peak hours and the case study corridor.

The bus corridor benefit for one day calculated from the BusMezzo results equals 1.0 million SEK, which can be compared to the 0.6 million SEK resulting from the Visum model. The comparison with the deterministic and unconstrained settings in BusMezzo confirms that the travel time differences stem from the dynamic congestion effects rather than other differences between the models. Hence, the benefits due to congestion relief amount to 0.4 million SEK per day which corresponds to 37% of the benefits on the case study corridor and 25% of the total benefits of the Blue Line extension. Hence, accounting for the dynamics of public transport congestion on the case study corridor adds 60% to the travel time savings induced by the metro line extension. This implies that a cost-benefit analysis based on a conventional static model would miss more than a third of the total welfare benefits.

7. Conclusions

This paper presents a modelling framework that encompasses the essential elements that are necessary for quantifying the impacts of a public transport capacity increase and their inclusion in project appraisal. The modelling framework consists of a dynamic representation of public transport supply and demand which enables to capture passenger load variations. The model outputs are included in a cost-benefit analysis by assigning valuations of three travel time components related to congestion effects: increased waiting times due to denied boarding, discomfort caused by on-board crowding and longer waiting and in-vehicle times due to service irregularity.

A case study of a metro extension in Stockholm demonstrated that congestion effects constitute more than a third of the total benefits and that these effects are substantially underestimated by a conventional static model. In other words, accounting for the dynamic congestion effects added 60% to the benefits of a conventional static model which essentially only captures travel time savings.

(Generalized) travel time savings are often the dominant benefit of public transport projects. In practice, crowding effects are usually included by weighting the value of time savings with a crowding multiplier that depends on average load/capacity factors. However, using average load/capacity factors neglects variations in passenger loads across vehicles from which the three distinct public transport congestion effects stem. In our case study, crowding reduction calculated in this way added only 3% to the generalized travel time benefits. This is a typical magnitude; for example, Kroes et al. (2013) report that crowding reduction benefits amounted to only 8%² of generalized travel time benefits in a study of a line extension in Ile-de-France, calculating crowding benefits based on average load/capacity factors.

Our findings indicate that failure to represent dynamic congestion effects may substantially underestimate the benefits of projects primarily designed to increase capacity rather than

² Own calculation based on the information provided in Kroes et al. paper

reduce travel times such as the construction of high-capacity public transport, redesigning vehicle capacity or increased service frequency. The modelling framework developed in this paper therefore facilitates a more adequate appraisal method of increased public transport capacity to support policy makers in prioritizing investments. As part of the process of project assessment, alternative investments such as the deployment of operational and control measures to improve bus service reliability and mitigate bus bunching (Cats 2014) can be evaluated.

Future research should consider dynamic congestion effects in a TAM equilibrium framework. Cats and Gkioulou (2015) demonstrate how the agent-based public transport simulation model could be embedded in a day-to-day learning framework. Further research should extend the modelling framework by introducing an iterative network loading to obtain a congested equilibrium. This will allow considering the impact of congestion effects on individual route choice. The assumption used in this paper that passengers do not anticipate congestion effects when choosing routes presumably overestimates the welfare losses of congestion. Taking this into account through an iterative calculation is a topic for further development. Furthermore, an iterative day-to-day learning would facilitate the consideration of departure time adjustments due to the prevalence of congestion. Another direction for further research is a detailed representation of passenger distribution over platforms and vehicles. For example, Kim et al. (2014) demonstrated how metro car choice reproduces the uneven distribution of passengers over metro trains which results in higher crowding. Furthermore, more research on passengers' behaviour, perceived congestion and degree of adaptation is needed to model the interaction between congestion dynamics across network elements, beyond merely on-board congestion.

Acknowledgements

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