Min-max control of fuel-cell-car-based smart energy systems

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Farid Alavi[1], Nathan van de Wouw[1], and Bart De Schutter[1]

Abstract— **Recently, the idea of using fuel cell vehicles as the future way of producing electricity has emerged. A fuel cell car has all the necessary devices on board to convert the chemical energy of hydrogen into electricity. This paper considers a scenario where a parking lot for fuel cell cars acts as a virtual power plant. In order to describe the system behavior from the energy point of view, a hybrid (mixed logical dynamical) model is constructed. With this model, a control system is designed to determine the production profile for both the fuel cell and battery of each car in the parking lot subject to minimizing the operational cost. In order to deal with both the uncertainty in the demand profile and the power balance constraint, a robust min-max model predictive control algorithm is developed. The effectiveness of the proposed approach is illustrated in a numerical example.**

I. INTRODUCTION

The idea of using fuel cell cars for producing electricity is a relatively new concept in the distributed generation of power. In this paper, a parking lot for fuel cell cars is considered as a virtual power plant. We consider the scenario in which several fuel cell cars can exchange information with a central controller in the parking lot. In such a setting, a central controller can be employed to, on the one hand, manage the electricity production of fuel cell cars in order to balance the power production and consumption and, on the other hand, minimize the operational costs.

The control system that manages the demand and production of the electricity in a smart grid, of which the distributed power generation based on fuel-cell cars is an example, is called an energy management system. In recent years, several works have considered the controller design for such systems. In [1], the problem of charging and discharging of electric vehicles in a smart grid is considered in order to reach a power balance in the system and gain the maximum operational benefit. Two optimization methods based on centralized and decentralized schemes are proposed. However, the developed distributed solution requires the exchange of solutions for each agent during several iterations. Therefore, the network traffic is typically high in this approach.

Due to the intrinsic robust features and the ability to deal with the constraints in the system, Model Predictive Control (MPC) is a method that has gained much attention in design of an energy management system for smart grid applications. In [2], a distributed MPC method to operate a microgrid is developed. The problem of power scheduling in a microgrid is reviewed in [3] and [4].

Another example of using MPC in the operation of a microgrid system is [5], where it is assumed that the controlled microgrid has a renewable energy source, a storage battery system, and some electrical loads. The designed control algorithm is based on an optimization problem which takes into account the cost of system operation. However, this paper neglects binary variables that determine different operational modes of devices and, hence, some part of the operational cost is not considered.

In [6], control of a microgrid that contains several power plants, a photovoltaic system, and a fuel cell system is considered. The problem is formulated with mixed-logical dynamical models and the optimization is solved with the multiparametric programming techniques. Another example of using MPC in control of microgrids that contain fuel cells and solar photovoltaic systems is [7]. In this paper, the authors develop a method to control the system via the market trades. However, the developed method cannot guarantee the stability of the system.

The design of an MPC controller in order to operate several devices in a smart grid is discussed in [8]. In this paper, the optimization problem is expressed as a mixed integer linear programming problem. However, the authors have assumed that future load profile is predetermined and there is no uncertainty in the system.

In order to deal with the uncertainty in the energy management systems, robust control techniques are used in [9] and [10]. The authors in [9] use stochastic optimization in the energy management system, while in [10] a minimax formulation is used. In [11], a method is developed in order to model the uncertainty in power systems as a polytope, while the appropriate control strategy in order to deal with the modeled uncertainty is not discussed.

Three different methods to deal with the uncertainty in the power systems are discussed in [12]. The first method simply considers some plants in the standby operation mode in order to provide excess electricity if it is needed. The second method uses stochastic optimization in order to set an appropriate power production profile for each plant. However, this method requires a priori knowledge about the uncertainties in the system. The last method is robust optimization which guarantees the stability of the system in presence of all the possible disturbances. The advantage of the third method compared to the second method is that accurate information about the uncertainties is not necessary.

The main contributions of the current paper are as follows. Firstly, we develop a hybrid model that describes the operation of the system, and takes into account both the uncertainty in the demand profile and the hybrid nature of

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the system dynamics, including the way in which the latter impacts the operational cost. Secondly, this model is used to design a controller for the virtual power plant based on fuel cell cars. In particular, a min-max control scheme is developed for a controller that minimizes the operational cost in the presence of uncertainty in the prediction of the demand inside the microgrid. Finally, the effectiveness of the proposed control strategy is illustrated in a representative case study.

This paper is organized as follows. In Section II, the problem statement is introduced. Section III is dedicated to the system model and the definition of the operational cost. The min-max controller design is proposed in Section IV. In section V, a numerical example illustrates the operation of the system. Finally, section VI concludes the paper.

II. PROBLEM STATEMENT

We consider a parking lot that contains several fuel cell cars. It is assumed that the parking lot uses the fuel cell of the cars in order to produce electricity. If this kind of parking lot is connected to the electricity grid, it can be employed as a power plant. For example, if there are 100 cars inside the parking lot and the power capacity of electricity production in each fuel cell car is 30 kW, then the total capacity of the parking lot is 3 MW. That is why this scenario is called Car as Power Plant (CaPP).

This kind of parking lot is assumed to be in a microgrid that is connected to the power network. In this scenario, we assume that the power exchange between the microgrid and the power network is limited and also that the demand of electricity in the microgrid is not controllable. However, it is assumed that an estimate of the demand profile for the next few hours can be predicted, e.g. based on historical data in combination with smart grid devices. Subsequently, we consider the difference between the actual and predicted electricity demand as an uncertainty in the system.

The CaPP is employed in order to guarantee the power balance in the microgrid. Therefore, a control system is necessary in the parking lot in order to use all the fuel cell and batteries of the cars in such a way that the power production meets the expectations and also the operational cost of the microgrid is minimized.

Figure 1 depicts an overview of the CaPP. The hydrogen and electricity exchange between different devices are shown with solid and dashed lines, respectively. The inputs of controller, i.e. measurements, consist of the amount of available hydrogen and the state of charge of the batteries. The outputs of controller, i.e. actuation commands, consist of both binary and continuous signals in order to determine the on/off mode of each fuel cell and the amount of power production for each fuel cell. Moreover, to manage the function of the batteries, binary outputs of the controller determine the charge/discharge mode of each battery. In addition, continuous controller outputs determine the amount of input or output power to/from the battery. In this paper, we aim to develop such a centralized controller that minimizes
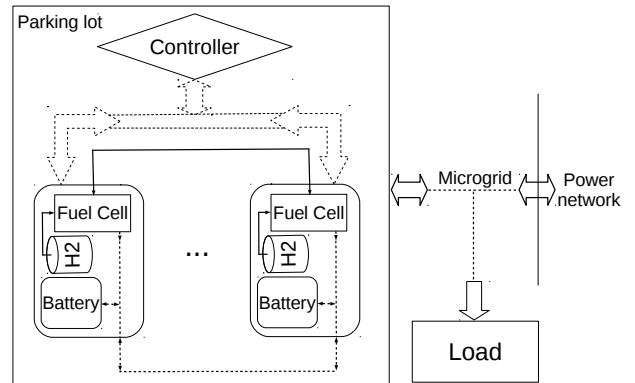

Fig. 1.   The scenario of the car as power plant.

operational cost while aiming to guarantee the power balance in the microgrid.

III. MODELING

A. Fuel cell model

To control a fuel cell device with a model-based algorithm, we need a mathematical model that describes the behavior of the device. In this section, the functionality of the device is explained, and next a hybrid piecewise affine (PWA) model for the fuel cell system is developed.

A fuel cell is a device that extracts the chemical energy of the hydrogen oxidation reaction in the form of electricity. Usually, the required hydrogen is extracted from a storage tank and it will be conducted to one side of the fuel cell. Air is pumped to the other side of the fuel cell and it will be used as the source of oxygen. Reaction of the hydrogen with the oxygen occurs inside the fuel cell and the output is heat and electricity. In this research, the fuel cell will be controlled at a high level and, hence, the detailed internal function of the fuel cell is not considered. For a discussion of the internal function and low-level control of fuel cells, we refer the interested reader to [13].

One of the important aspects in high-level control of the fuel cell is the hydrogen consumption related to the net power generation. Due to the limited life-time of fuel cells, the degradation of the fuel cell is considered in the scope of high-level control. Moreover, maximum power generation is an important physical constraint in every fuel cell system.

The model should be able to describe the financial costs and benefits of the fuel cell operation. Two important factors in determining the operational cost of a fuel cell are the price of the fuel and the degradation of the device. In order to determine the fuel cost, the relationship between the hydrogen consumption and the net power production is considered. Because some part of the power production of the fuel cell is used to run an air compressor, the net power production of the fuel cell should be considered. Based on [14], the hydrogen consumption of the fuel cell per time unit has an affine relation with the net fuel cell power.

A typical fuel cell consumes hydrogen with a constant rate in the standby operation mode, i.e. when the output power

is equal to zero. We define a constant parameter, β_{f} $[g/s]$, to denote this rate. The rate of hydrogen consumption due to the net power generation is equal to another constant parameter, α_{f} $[g/kJ]$. Therefore, if h and p represent the rate of the hydrogen consumption and net power production respectively, the following relation holds:

$$h(t) = \int_{\tau=t_0}^{t} (\alpha_{\mathrm{f}} p(\tau) + \beta_{\mathrm{f}})\, d\tau, \tag{1}$$

where $[t_0, t]$ is the time span that the fuel cell is turned on.

The reserved amount of hydrogen in the storage tank is considered as the state of the fuel cell system model, denoted by x_{f}. There are two inputs for the model of a fuel cell system. The first input, u_{f}, is a continuous variable that determines the amount of net power production of the system and the second input, s_{f}, is a binary variable that indicates the off ($s_{\mathrm{f}} = 0$) or on ($s_{\mathrm{f}} = 1$) operation mode of the fuel cell. Based on the definition of input variables, we have:

$$\text{if } u_{\mathrm{f}}(k) > 0 \text{ then } s_{\mathrm{f}}(k) = 1. \tag{2}$$

Using (1) as the basic operation function of the fuel cell, the following piecewise affine function describes the system behavior in discrete time:

$$x_{\mathrm{f}}(k+1) = \begin{cases} x_{\mathrm{f}}(k) & \text{if } s_{\mathrm{f}}(k) = 0 \\ x_{\mathrm{f}}(k) - (\alpha_{\mathrm{f}} u_{\mathrm{f}}(k) + \beta_{\mathrm{f}}) T_{\mathrm{s}} & \text{if } s_{\mathrm{f}}(k) = 1, \end{cases} \tag{3}$$

where T_s is the sampling interval of the discrete-time system. If the minimum and maximum power production of the fuel cell are equal to 0 and $\bar{P}_{\mathrm{f}}$, respectively, then the constraint

$$0 \le u_{\mathrm{f}}(k) \le \bar{P}_{\mathrm{f}}, \qquad \text{for all } k, \tag{4}$$

should be considered when controlling the system.

B. Battery model

The battery of the fuel cell car can store energy at one time and deliver it at another time. Therefore, two operational modes, namely charging and discharging, are considered for the battery. In a simplified model for the battery [8], a continuous input, u_{b}, determines the input or output power of the battery system. In the charging mode of the battery, the input power, u_{b}, is determined with a negative real number. Conversely, a positive value for u_{b} indicates the discharging power. The system state, x_{b}, represents the energy stored in the battery. Therefore, the simplified discrete-time model of the battery can be formulated as:

$$x_{\mathrm{b}}(k+1) = \begin{cases} x_{\mathrm{b}}(k) - \frac{T_{\mathrm{s}}}{\eta_{\mathrm{d}}} u_{\mathrm{b}}(k) & \text{if } s_{\mathrm{b}}(k) = 0 \\ x_{\mathrm{b}}(k) - T_{\mathrm{s}} \eta_{\mathrm{c}} u_{\mathrm{b}}(k) & \text{if } s_{\mathrm{b}}(k) = 1, \end{cases} \tag{5}$$

where s_{b} is an auxiliary binary variable that determines the discharge ($s_{\mathrm{b}} = 0$) or charge ($s_{\mathrm{b}} = 1$) mode of the battery. The discharging and charging efficiencies are determined by η_{c} and η_{d}, respectively.

Due to the physical limits in the maximum power of charging or discharging a battery, the following constraints are in effect:

$$\underline{P}_{\mathrm{b}} \le u_{\mathrm{b}}(k) \le \bar{P}_{\mathrm{b}}, \qquad \text{for all } k. \tag{6}$$

C. System model

The central controller of the parking lot operates several fuel cell cars inside the parking. Therefore, N_{veh} fuel cells and batteries should be included in the system model. Using the models (3) and (5), the model of the total system can be described as follows:

$$\mathbf{x}(k+1) = \mathbf{x}(k) + T_{\mathrm{s}} \begin{bmatrix} -\mathbf{s}_{\mathrm{f}}(k)\left(\boldsymbol{\alpha}_{\mathrm{f}} \mathbf{u}_{\mathrm{f}}(k) + \boldsymbol{\beta}_{\mathrm{f}}\right) \\ \mathbf{s}_{\mathrm{b}}(k)\left(\tilde{\eta}_{\mathrm{d}} - \tilde{\eta}_{\mathrm{c}}\right)\mathbf{u}_{\mathrm{b}}(k) - \tilde{\eta}_{\mathrm{d}}\mathbf{u}_{\mathrm{b}}(k) \end{bmatrix} \tag{7}$$

with:

$$\mathbf{x}(k) \triangleq \begin{bmatrix} x_{\mathrm{f},1}(k) & \dots & x_{\mathrm{f},N_{\mathrm{veh}}}(k) & x_{\mathrm{b},1}(k) & \dots & x_{\mathrm{b},N_{\mathrm{veh}}}(k) \end{bmatrix}^T$$

$$\mathbf{s}_{\mathrm{f}}(k) \triangleq \mathrm{diag}\{s_{\mathrm{f},1}(k),\, s_{\mathrm{f},2}(k),\, \dots,\, s_{\mathrm{f},N_{\mathrm{veh}}}(k)\}$$

$$\boldsymbol{\alpha}_{\mathrm{f}} \triangleq \mathrm{diag}\{\alpha_{\mathrm{f},1},\, \alpha_{\mathrm{f},2},\, \dots, \alpha_{\mathrm{f},N_{\mathrm{veh}}}\}$$

$$\mathbf{u}_{\mathrm{f}}(k) \triangleq \begin{bmatrix} u_{\mathrm{f},1}(k) & u_{\mathrm{f},2}(k) & \dots & u_{\mathrm{f},N_{\mathrm{veh}}}(k) \end{bmatrix}^T$$

$$\tilde{\eta}_{\mathrm{d}} \triangleq \mathrm{diag}\{1/\eta_{\mathrm{d},1},\, /\eta_{\mathrm{d},2},\, \dots,\, 1/\eta_{\mathrm{d},N_{\mathrm{veh}}}\}$$

$$\tilde{\eta}_{\mathrm{c}} \triangleq \mathrm{diag}\{\eta_{\mathrm{c},1},\, \eta_{\mathrm{c},2},\, \dots,\, \eta_{\mathrm{c},N_{\mathrm{veh}}}\},$$

and $x_{\mathrm{f},i}$ and $x_{\mathrm{b},i}$ denote the states of ith vehicle and a similar notational convention is employed for the other variables. The vectors and matrices $\mathbf{s}_{\mathrm{b}}$, $\boldsymbol{\beta}_{\mathrm{f}}$, and $\mathbf{u}_{\mathrm{b}}$ are defined similarly.

The model in (7) can be written as a Mixed Logical Dynamical (MLD) model [15] by defining auxiliary variables $\mathbf{z}_i(k) = \begin{bmatrix} z_{\mathrm{f},i}(k) & z_{\mathrm{b},i}(k) \end{bmatrix}^T$, for $i = 1, \dots, N_{\mathrm{veh}}$, where:

$$z_{\mathrm{f},i}(k) = s_{\mathrm{f},i}(k) u_{\mathrm{f},i}(k) \tag{8}$$

$$z_{\mathrm{b},i}(k) = s_{\mathrm{b},i}(k) u_{\mathrm{b},i}(k). \tag{9}$$

Following the procedure of [15] and defining $\mathbf{x}_i(k) = \begin{bmatrix} x_{\mathrm{f},i}(k) & x_{\mathrm{b},i}(k) \end{bmatrix}^T$ and $\mathbf{u}_i(k) = \begin{bmatrix} u_{\mathrm{f},i}(k) & u_{\mathrm{b},i}(k) & s_{\mathrm{f},i}(k) \end{bmatrix}^T$, the MLD model of the system can be written in the form:

$$\mathbf{x}(k+1) = \mathbf{x}(k) + B_1 \mathbf{u}(k) + B_3 \mathbf{z}(k), \tag{10}$$

where the vector of auxiliary variables is defined as follows:

$$\mathbf{z}(k) = \begin{bmatrix} \mathbf{z}_1(k)^T & \dots & \mathbf{z}_{N_{\mathrm{veh}}}(k)^T \end{bmatrix}^T.$$

The vector of inputs, $\mathbf{u}(k)$, and states, $\mathbf{x}(k)$, are defined similar to $\mathbf{z}(k)$. If we define

$$b_{1,i} \triangleq \begin{bmatrix} 0 & 0 & -T_{\mathrm{s}}\beta_{\mathrm{f},i} \\ 0 & -T_{\mathrm{s}}/\eta_{\mathrm{d},i} & 0 \end{bmatrix} \tag{11}$$

$$b_{3,i} \triangleq \begin{bmatrix} -T_{\mathrm{s}}\alpha_{\mathrm{f},i} & 0 \\ 0 & T_{\mathrm{s}}(\frac{1}{\eta_{\mathrm{d},i}} - \eta_{\mathrm{c},i}) \end{bmatrix}, \tag{12}$$

then $B_1 = \mathrm{diag}\{b_{1,1}, \dots, b_{1,N_{\mathrm{veh}}}\}$ and $B_3 = \mathrm{diag}\{b_{3,1}, \dots, b_{3,N_{\mathrm{veh}}}\}$ in (10).

D. Operational cost

The cost of the system operation consists of several factors. The price of consumed hydrogen in the parking lot is one factor of the operational cost. It is assumed that the price of hydrogen per kilogram is constant and equal to C_{H_2}. In addition, the price of extracting some part of stored energy in the batteries should be considered as a part of the operational

cost. It is assumed that the stored energy in the battery has a predefined price per kWh, which is given by $C_{\mathrm{e,batt}}$.

Degradation of the fuel cells and batteries is another source of the operational cost in the parking lot. There are several factors that determine the lifetime of a fuel cell. However, it is assumed that the degradation cost of a fuel cell is only associated with turning the fuel cell on or off. Due to the high influence of changing the operation mode in the lifetime of a fuel cell [16], this assumption is realistic. It is also assumed that changing the operation mode of the battery causes degradation and, hence, is accounted in the operational cost.

The amount of power that is injected to the microgrid should be equal to the amount of power that is consumed. Therefore, there is a power balance constraint in the system, which means that the excess or shortage of energy should be traded with the power network, outside of the microgrid. This means that the operational cost also includes the cost of importing or the benefit of exporting electricity. As a result, the operational cost of the microgrid can be written as:

$$
\begin{aligned}
J(k) = \sum_{i=1}^{N_{\mathrm{veh}}} \Bigg(& \sum_{j=0}^{N_{\mathrm{p}}-1} \Big(W_{\mathrm{f},i}|\Delta s_{\mathrm{f},i}(k+j)| + W_{\mathrm{b},i}|\Delta s_{\mathrm{b},i}(k+j)| \\
& - C_{\mathrm{e}}(k+j)(u_{\mathrm{f},i}(k+j) + u_{\mathrm{b},i}(k+j)) \Big) \\
& - C_{\mathrm{H_2}} \left(x_{\mathrm{f},i}(k+N_{\mathrm{p}}) - x_{\mathrm{f},i}(k) \right) \\
& - C_{\mathrm{e,batt}} \left(x_{\mathrm{b},i}(k+N_{\mathrm{p}}) - x_{\mathrm{b},i}(k) \right) \Bigg) \\
& + \sum_{j=0}^{N_{\mathrm{p}}-1} C_{\mathrm{e}}(k+j)e_{\mathrm{in}}(k+j), \qquad (13)
\end{aligned}
$$

where the parameter N_{veh} is the total number of the fuel cell cars in the parking lot. The operator Δ indicates the difference between two consecutive values of its operand. The conversion factors $W_{\mathrm{f},i}$ are defined for all $i \in \{1, \ldots, N_{\mathrm{veh}}\}$ in such a way that the term $W_{\mathrm{f},i}|\Delta s_{\mathrm{f},i}(k+j)|$ in (13) indicates the degradation cost of the fuel cells due to on/off mode switching. A similar reasoning holds for $W_{\mathrm{b},i}|\Delta s_{\mathrm{b},i}(k+j)|$.

The value of $C_{\mathrm{e}}(k)$ indicates the price of electricity at time step k. The term $e_{\mathrm{in}}(k)$ indicates the amount of electricity that is imported to the system at time step k and hence, the last term in the cost function (13) determines the overall cost of importing electricity to the microgrid.

IV. CONTROL STRATEGY

To minimize the cost of system operation defined in (13), the controller should determine, firstly, a suitable production profile for all the fuel cells, and, secondly, a charge or discharge profile plan for all the batteries in the system.

In the parking lot, outputs of the controller contain the net power generation, $u_{\mathrm{f},i}$, the on or off operation mode of each fuel cell, $s_{\mathrm{f},i}$ and the power exchange with each battery, $u_{\mathrm{b},i}$. The operation mode of batteries, i.e. $\delta_{\mathrm{b},i}$, are not considered as a controller output, because they are determined based on the value of $u_{\mathrm{b},i}$.

Considering the presence of auxiliary variables (8) and (9), minimization of the operational cost is nonlinear and hence,

hard to solve. However, the problem can be solved more efficiently by formulating it as a Mixed Integer Linear Programming (MILP) problem. Therefore, the input constraints in (4) and (6) are used and the procedure of [15] is followed in order to formulate the optimization problem as an MILP. As a result, the auxiliary variable $z_{\mathrm{f},i}(k)$ can be treated as an optimization parameter with the following inequalities:

$$
\begin{aligned}
z_{\mathrm{f},i}(k) &\leq \bar{P}_{\mathrm{f},i}s_{\mathrm{f},i}(k) \\
z_{\mathrm{f},i}(k) &\geq 0 \\
z_{\mathrm{f},i}(k) &\leq u_{\mathrm{f},i}(k) \\
z_{\mathrm{f},i}(k) &\geq u_{\mathrm{f},i}(k) - \bar{P}_{\mathrm{f},i}(1 - s_{\mathrm{f},i}(k)),
\end{aligned}
$$

for all $i \in \{1, \ldots, N_{\mathrm{veh}}\}$. A similar argument also holds for the auxiliary variable $z_{\mathrm{b},i}(k)$. In addition, the rule in (2) is equivalent to:

$$
u_{\mathrm{f},i}(k) \leq \bar{P}_{\mathrm{f},i}s_{\mathrm{f},i}(k),
$$

for all k and $i \in \{1, \ldots, N_{\mathrm{veh}}\}$.

In order to obtain a linear cost function, auxiliary variables $\theta_i(k) = \begin{bmatrix} \theta_{\mathrm{f},i}(k) & \theta_{\mathrm{b},i}(k) \end{bmatrix}^T$ ($i \in \{1, \ldots, N_{\mathrm{veh}}\}$) are introduced which have the following constraints for all time steps k and vehicles $i \in \{1, \ldots, N_{\mathrm{veh}}\}$:

$$
\theta_{\mathrm{f},i}(k) \geq |\Delta s_{\mathrm{f},i}(k)| \qquad (14)
$$
$$
\theta_{\mathrm{b},i}(k) \geq |\Delta s_{\mathrm{b},i}(k)|. \qquad (15)
$$

Moreover, the user (i.e. car owner) preference on the minimum level of fuel, $\underline{x}_{\mathrm{f},i}$, and minimum and maximum state of charge of the battery, $\underline{x}_{\mathrm{b},i}$ and $\bar{x}_{\mathrm{b},i}$, will result in another set of constraints. In addition, the amount of power that can be imported from or exported to the grid, e_{in}, is limited. Therefore, the designed controller should minimize the operational cost of the microgrid system subject to:

$$
\begin{aligned}
&\underline{x}_{\mathrm{f},i} \leq x_{\mathrm{f},i}(k) \\
&\underline{x}_{\mathrm{b},i} \leq x_{\mathrm{b},i}(k) \leq \bar{x}_{\mathrm{b},i} \\
&\underline{e}_{\mathrm{in}} \leq e_{\mathrm{in}}(k) \leq \bar{e}_{\mathrm{in}} \\
&\sum_{i=1}^{N_{\mathrm{veh}}} \Big(u_{\mathrm{b},i}(k) + u_{\mathrm{f},i}(k) \Big) + e_{\mathrm{in}}(k) = P_{\mathrm{d}}(k) + \omega(k), \quad (16)
\end{aligned}
$$

for all k and $i \in \{1, \ldots, N_{\mathrm{veh}}\}$, where $P_{\mathrm{d}}(k)$ is the predicted demand in the microgrid. The unknown variable $\omega(k)$ represents the uncertainty in the prediction of power demand in the microgrid at time step k. Hence, $P_{\mathrm{d}}(k) + \omega(k)$ is the actual demand in the microgrid and the constraint in (16) is related to the power balance situation. All the constraints in the system at time step k can be easily expressed in matrix form as:

$$
E_1 \mathbf{u}(k) + E_4 \mathbf{x}(k) + E_{51}(k) + E_{52}\omega(k) \geq E_3. \qquad (17)
$$

The matrices E_1, E_3, E_4, and E_{52} are constant at all times. The matrix $E_{51}(k)$ is a function of k; however, by using the predicted values of demand, it can be determined in the prediction horizon. The variable $\omega(k)$ indicates the deviation between the predicted and actual demand in the

microgrid and, hence, its value is unknown. Assumption 1 below expresses a bound on this uncertainty.

Assumption 1: There exists a finite bound for the deviation of the predicted demand from the actual value, $\omega(k)$, at each time step k. Therefore, it is possible to determine $\bar{\omega}$ and $\underline{\omega}$ such that for all k, $\underline{\omega} \leq \omega(k) \leq \bar{\omega}$.

We define a vector, denoted by $\tilde{V}$, that contains all the parameters that should be determined by solving the optimization problem. This vector contains the sequence of all the future inputs, $\tilde{\mathbf{u}}(k)$ and auxiliary variables, $\tilde{\mathbf{z}}(k)$, $\tilde{\theta}(k)$. The variables with a tilde indicate the stacked value of that variable in the prediction horizon.

Using the definition of vector $\tilde{V}(k)$ and defining constant matrices G_0, G_1, G_2, and G_3, the inequalities in (17) can be written for all time steps $k \in \{0, \ldots, N_\mathrm{p} - 1\}$ as follows:

$$G_0\tilde{V}(k) \leq G_1 + G_2\mathbf{x}(k) + G_3\tilde{\omega}(k), \qquad (18)$$

where $\tilde{\omega}(k) = \begin{bmatrix} \omega(k) & \omega(k+1) & \ldots & \omega(k+N_\mathrm{p}) \end{bmatrix}^T$ is the vector of unknown future deviations of the demand from its predicted values and N_p is the prediction horizon.

Using the power balance equality constraint in (16), the cost function in (13) can be written as follows:

$$J(k) = W_0\tilde{V}(k) + W_\mathrm{d}\tilde{\omega}(k).$$

The aim of the controller is to achieve a guaranteed benefit for the microgrid in case of any possible deviation $\tilde{w}$ from the prediction of the demand. Therefore, the system cost should be minimized with respect to the variable $\tilde{V}$ for the maximum possible value of deviation of demand from the predicted values. In other words, the aim is to solve the following optimization problem:

$$\min_{\tilde{V}(k)} \max_{\tilde{\omega}(k)} \{W_0\tilde{V}(k) + W_\mathrm{d}\tilde{\omega}(k)\} \qquad (19)$$

with respect to the constraints in (18).

Given the fact that the constraint (18) should hold for any realization of the unknown parameter $\tilde{\omega}(k)$, the optimization of (19) is hard to solve. However, with the assumption of bounded deviation of actual demand from the predicted values (Assumption 1) and using Lemma 1 below, the optimization problem can be formulated as an MILP problem.

Lemma 1: Defining

$$\tilde{\omega}^1 = \begin{bmatrix} \bar{\omega} & \ldots & \bar{\omega} \end{bmatrix}^T_{N_\mathrm{p} \times 1}$$
$$\tilde{\omega}^2 = \begin{bmatrix} \underline{\omega} & \ldots & \underline{\omega} \end{bmatrix}^T_{N_\mathrm{p} \times 1},$$

the inequality (18) holds for all possible disturbances ω satisfying Assumption 1 if the following two inequalities hold:

$$G_0\tilde{V}(k) \leq G_1 + G_2\mathbf{x}(k) + G_3\tilde{\omega}^1 \qquad (20)$$
$$G_0\tilde{V}(k) \leq G_1 + G_2\mathbf{x}(k) + G_3\tilde{\omega}^2. \qquad (21)$$

Proof: The existence of a maximum and minimum value for variable ω shows that it lies in a polytope. Therefore,

$$\forall \omega \; \exists \lambda_1, \lambda_2 \in [0,1] \; : \; \lambda_1\underline{\omega} + \lambda_2\bar{\omega} = \omega \text{ and } \lambda_1 + \lambda_2 = 1. \qquad (22)$$

Now assume an arbitrary realization of $\tilde{\omega}(k)$ as follows:

$$\tilde{\omega}(k) = \begin{bmatrix} \omega(k) & \omega(k+1) & \ldots & \omega(k+N_\mathrm{p}-1) \end{bmatrix}^T.$$

The inequality constraint (18) consists of several inequalities belonging to each time step in the prediction horizon. Considering the structure of G_3, it can be shown that (18) consists of the following inequalities, for $j \in \{0, \ldots, N_\mathrm{p} - 1\}$:

$$G_{0,k+j}\tilde{V}(k) \leq G_{1,k+j} + G_{2,k+j}\mathbf{x}(k) + g_{3,k+j}\omega(k+j) \qquad (23)$$

where $G_{0,k+j}$, $G_{1,k+j}$, $G_{2,k+j}$, and $g_{3,k+j}$ are the $j+1$th row of G_0, G_1, G_2, and G_3, respectively.

From (20) and (21), we have that

$$G_{0,k+j}\tilde{V}(k) \leq G_{1,k+j} + G_{2,k+j}\mathbf{x}(k) + g_{3,k+j}\underline{\omega}(k+j) \qquad (24)$$
$$G_{0,k+j}\tilde{V}(k) \leq G_{1,k+j} + G_{2,k+j}\mathbf{x}(k) + g_{3,k+j}\bar{\omega}(k+j), \qquad (25)$$

Property (22) shows that for any realization of $\omega(k+j)$, there exists a pair (λ_1, λ_2) such that $\lambda_1\underline{\omega} + \lambda_2\bar{\omega} = \omega(k+j)$ and $\lambda_1 + \lambda_2 = 1$. By multiplying these factors to (24) and (25), it can be easily seen that (23) holds. This reasoning can be done for all $j \in \{0, \ldots, N_\mathrm{p} - 1\}$ and, hence, (18) holds. $\square$

Lemma 1 shows that if the optimization problem in (19) is solved subject to the two constraints (20) and (21) that are stated in Lemma 1, the inequalities will be hold for all possible disturbances satisfying Assumption 1. Therefore, by using a similar procedure as in the proof of Lemma 1, the optimization problem in (19) can be written as follows:

$$\min_{\tilde{V}(k)} \left\{\max\left\{W_0\tilde{V}(k) + W_\mathrm{d}\tilde{\omega}^1, W_0\tilde{V}(k) + W_\mathrm{d}\tilde{\omega}^2\right\}\right\} \quad (26)$$

subject to

$$G_0\tilde{V}(k) \leq G_1 + G_2\mathbf{x}(k) + G_3\tilde{\omega}^1$$
$$G_0\tilde{V}(k) \leq G_1 + G_2\mathbf{x}(k) + G_3\tilde{\omega}^2,$$

which can be solved by a variety of MILP solvers, such as GLPK, CPLEX, or Gurobi.

V. NUMERICAL EXAMPLE

The algorithm developed in Section IV is now implemented in a scenario where two fuel cell cars in the microgrid have to guarantee the power balance. A fuel cell with maximum power generation, $\bar{P}_\mathrm{f} = 30$kW and a battery with 24 kWh capacity, i.e. $\bar{x}_{\mathrm{b},1} = \bar{x}_{\mathrm{b},2} = 24$, exist in each of the two cars. We assume that the maximum power exchange with the batteries is limited to 2 kW, i.e. $|\underline{P}_{\mathrm{b},i}| = \bar{P}_{\mathrm{b},i} = 2$ for $i \in \{1, 2\}$. In addition, the charging and discharging efficiency is considered to be equal with values 0.9 and 0.8 for each battery, respectively. It is assumed that the maximum power that can be imported to the microgrid $\bar{e}_\mathrm{in}$ is 66 kW. The uncertainty in the power demand is in the interval [-3,2] kW and a min-max model predictive controller based on the optimization problem (26) is designed. The sampling interval, T_s, of the control system is 15 minutes and the simulation shows the system behavior in 24 hours.
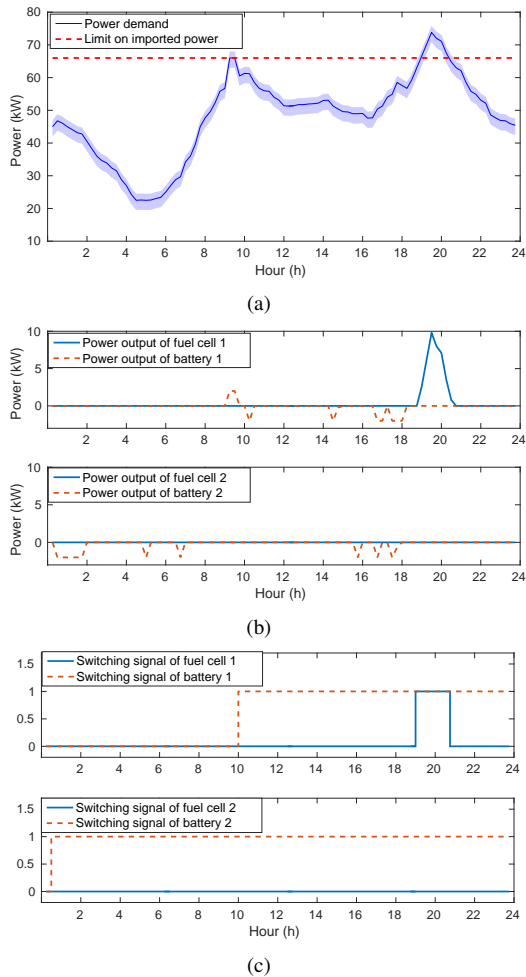
Fig. 2. Operation of the control system in presence of two fuel cell cars. (a) Demand profile in the microgrid and the maximum power capacity of imported power. The solid line is the predicted demand profile and the shaded area represents the possible realization of the demand. (b) Energy production by fuel cells and batteries. (c) Operation mode of fuel cells and batteries.

Figure 2 illustrates the simulation results. In Figure 2(a), the predicted demand profile is depicted. Considering the uncertainty in the system, in two time intervals the actual demand may exceed the maximum power capacity that can be imported to the microgrid. Therefore, the fuel cell car has to produce electricity in these time intervals. Figure 2(b) depicts the production profiles of the fuel cell and battery. During the high demand period in the morning, one of the batteries compensates the shortage of electricity in the microgrid and as a result, the fuel cells remain out of operation. However, the high demand in the afternoon can only be compensated by a fuel cell, as the results also show that in the afternoon one of the fuel cells is turned on in order to guarantee the power balance of the system. Figure 2(c) indicates the changes in the operation mode of fuel cell and battery. The switching signals show that during the simulation, one of the fuel cells is not used in the task of power balance in the microgrid.

VI. CONCLUSIONS

We have considered a parking lot for fuel cell cars as a virtual power plant that is employed to produce electricity in a microgrid. A high-level, hybrid system model has been derived from the energy point of view and in order to achieve the power balance situation in addition to minimizing the operational cost of the virtual power plant, a min-max controller has been designed. The operation of the control system is simulated for a case study. Future work will focus on distributed and scenario-based robust control of the presented system.

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